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► To cite this version:

Yassine Bouteraa, Jawhar Ghommam, Nabil Derbel, Gérard Poisson. Adaptive synchronization control of multi-robot teams: Cooperative and coordinated schemes. Mediterranean Conference on Control & Automation (MED), 2010, Jun 2010, Marrakech, Morocco. pp.586 - 591, 10.1109/MED.2010.5547733 . hal-00647537

HAL Id: hal-00647537

<https://hal.science/hal-00647537v1>

Submitted on 27 Jan 2025

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Adaptive synchronization control of multi-agent robotic systems

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Abstract In this article, an internal and an external synchronization have been investigated, in the presence of uncertain parameters. The proposed controller is designed to synchronize the movement of robot group following the same desired trajectory. To this effect, using a consensus algorithm, we provide an adaptive control under two different schemes. In the mutual synchronization topology, robot network requires only local neighbor-to-neighbor information exchange between robots. The interaction topology of agents network is represented using an undirected graph. However the objective of the external synchronization coordinated scheme is to design interconnections and feedback controllers for slaves, such that their positions and velocities synchronize to those of the leader robot. It's assumed that robots in cooperation or in coordination have the same joints number and equivalent joint work spaces. The proposed adaptive law based on combination of lyapunov direct method and concepts from graph theory has been developed to guarantee asymptotic convergence to zero of both position and synchronization errors.

Keywords: Cooperative control, decentralized trajectory tracking control, synchronization control, adaptive control, graph theory.

1. Introduction

Currently, the great need for efficient production and high quality have resulted in the presence of complex and integrated systems such as multi-robot systems. In many industrial applications that can be performed by a single robot, either because limitations of the robot or the complexity of the task, the use of robot network as a controlled-synchronized system, have proved to be a good alternative.

1.1 Previews Work

In the literature there are several types of synchronization [8]. The natural synchronization occurs in the case of disconnected systems that behaves in a synchronized way. In case of connected systems where synchronization is ensured by proper interconnections in the system and without any outside intervention, we talk about the self-synchronized. The connected system undergoing external action such as a control law or artificial interconnections is called controlled-synchronized. For the controlled-synchronized system, depending on the formulation of the controlled synchronization, we define two synchronization kinds: mutual synchronization, when all agents occur on equal term, cooperative systems case, and external synchronization, when one object is more powerful than the others and its motion can be considered as independent of the motion of the other objects, master-slave systems case [1]. Research on synchronization has been launched in several areas including lagrangian systems [14, 6–1]; tele-operated systems [11]; chaotic systems [4, 15]; wireless sensors networks [5, 10]; space- craft formation flight [13]; neuroscience [7] etc. Under a cooperative scheme, an approach control that solves the problem of position synchronization of two (or more) robot systems when only position measurements are available is presented in [1]. Using contraction analysis as nonlinear stability tool, Soon-Jo chung presented a simple synchronization framework that can be directly applied to cooperative control of multi-agent systems and oscillation synchronization in robotic manipulation and teleoperation [14]. Adaptive control is an effective strategy used to address the synchronization problem [2]. A new control approach to position synchronization of multiple motion axes is developed, by incorporating cross-coupling technology into adaptive control architecture in [6].

1.2 Main Contribution

In this study, we present a control approach to position synchronization of multiple robots while following a common desired trajectory, in presence of parametric uncertainties. In [2], authors developed trajectory-tracking control ensuring mutual synchronization. The synchronization error is defined to be the positional errors sum multiplied by a communication term inspired of the adjacency matrix of the network undirected graph. Most earlier control strategies and consensus algorithm for multi-agent networks study simple dynamic models and cannot be used for highly nonlinear systems (e.g. manipulator robots) [9], [12], [16]. In contrast, the present work deals with highly non linear systems. Most previous work on consensus and coordination of multi-agent systems using graph theory and laplacian [17], [18], [19], [20], [21] have presented a synchronization to the weighted average of initial conditions but they do not consider multi-agent systems where there is a desired path to follow. In contrast, the developed approach achieves not only global and exponential synchronization of the configuration variables, but also global exponential convergence to the desired trajectory. The present work extends consensus algorithms to second-order dynamics to be applied in highly nonlinear systems. It is showed in theory that the proposed control law guarantee asymptotic convergence to zero of both position and synchronization errors. Simulations done on a network of robot manipulators demonstrates the effectiveness of the proposed method.

1.3 Organization

This paper is organized as follows: Section II describes some background and preliminaries containing modeling of robots based on the Lagrangian formulation and basic concepts in graph theory. In Section III a control strategy for the synchronization of n cooperative Lagrangian systems is proposed. In section IV, we consider the case of external synchronization under a coordination scheme: master-slave. The proof of exponential synchronization is more involved and treated separately in Sections III and IV. In section V, we present a case study where we apply the proposed control strategy to synchronize 3 robot manipulators. Conclusion and further research work are given in section VI.

2. Background and preliminaries

2.1 Dynamics of Robot Manipulator in the Joint Space

The dynamic equation of a general rigid link manipulator having n degrees of freedom in a free space and in absence of any external disturbance can be written as:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i \quad (1)$$

where i , ($1 \leq i \leq p$) denotes the i -th robot index in the network, and p is the total number of the individual elements. In addition, $q_i \in \mathfrak{R}^n$ denotes the vector of generalized displacements of the i th robot coordinates, $\tau_i \in \mathfrak{R}^n$ denotes the vector of generalized control input torques in robot coordinates; $M_i(q_i) \in \mathfrak{R}^{n \times n}$ inertia matrix which is symmetric uniformly bounded and positive definite,

$C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathfrak{R}^n$ is a vector function containing coriolis and centrifugal forces, $g_i(q_i) \in \mathfrak{R}^n$ is a vector function consisting of gravitational forces. We have some fundamental properties of motion equations

(i) The inertia matrix $M_i(q_i)$ is symmetric, positive definite, and uniformly bounded.

$$M_i(q_i) = M_i^T(q_i) > 0. \quad (2)$$

(ii) Using a proper definition: $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is a skew symmetric matrix, satisfying

$$X^T(\dot{M}_i(q_i) - 2C_i(q, \dot{q}))X = 0 \quad (3)$$

where X^T is the transpose of a vector $X \in \mathfrak{R}^n$.

(iii) The Euler-Lagrange equation (1) is linear with respect to the structural parameters θ , hence,

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = Y(q_i, \dot{q}_i, \ddot{q}_i)\theta_i \quad (4)$$

where, $Y \in \mathfrak{R}^{n \times a}$ is the regressor matrix composed of known functions of q , \dot{q} and \ddot{q} , $\theta_i \in \mathfrak{R}^a$ is the vector of structural parameters of the manipulator and a is the number of unknown parameters.

2.2 Graph Theory Notions and Communication Topology

We start by introducing some concepts and notations in graph theory. Let $G = (V, E)$ is a digraph with N nodes, the set of nodes $V = 1, 2, \dots, n$

and edges $E \subseteq V \times V$. Each node is labeled by $v_i \in V$ and each edge is denoted by $e_{ij} = (v_i, v_j)$. The adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ of a weighted digraph is defined as:

$$\begin{cases} a_{ij} > 0 & \text{if } (v_i, v_j) \in E \\ a_{ij} = 0 & \text{elsewhere} \end{cases} .$$

The neighbors of agent v_i are denoted by $N = \{v_j \in V / (v_i, v_j) \in E\}$. Agent i communicates with agent j if j is a neighbor of i or if $a_{ij} \neq 0$. Note that an edge e_{ij} in a directed graph means that robot j can reach information from robot i , but not necessarily vice versa. In contrast, in an undirected graph, pairs of node are unordered and an edge e_{ij} implies that robots i and j can get information from one another. The adjacency matrix of an undirected graph is defines same as that of the directed graph except that $a_{ij} = a_{ji}$. The degree matrix of the digraph $G = (V, E)$ is a diagonal matrix defined as :

$$d_{ij} = \begin{cases} deg_{out}(v_i) & \text{if } (i = j) \in E \\ 0 & \text{otherwise} \end{cases} .$$

where $deg_{out}(v_i) = \sum_{j=1}^n a_{ij}$.

The graph laplacian of G is defined as: $L = D - A$, where $D = [d_{ii}]$ the degree matrix of G . In the undirected graph case, L is symmetric positive semidefinite. In the present topology, the edge represents bidirectional communication links. This consists on a group of n manipulators interchanging information that can be viewed as an undirected graph (Fig. 1.)

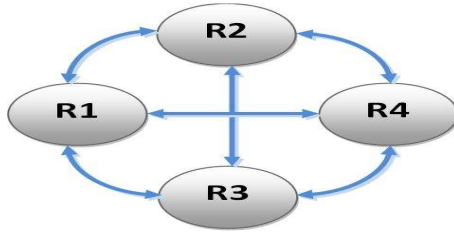


Fig. 1. Network manipulators group interchanging information via bidirectional communication.

3. Adaptive Mutual Synchronization: cooperative scheme

This section addresses the problem of mutual synchronization (internal synchronization). We consider a robot network formed by n robot manipulators tracking a common desired trajectory (q_d, \dot{q}_d) (see fig. 2). It is assumed that mass, coriolis and gravity matrix, are badly known. Internal synchronization control problems can be presented to conceive interconnections and controllers τ_i for all agents in the system, such that angular positions and velocities $q_i, \dot{q}_i \in \mathbb{R}^n$ of the i -th robot in the system are synchronized with respect to the common desired trajectory q_d, \dot{q}_d and the other angular robots positions and velocities $q_j, \dot{q}_j \in \mathbb{R}^n$, $j = 1, \dots, p; j \neq i$.

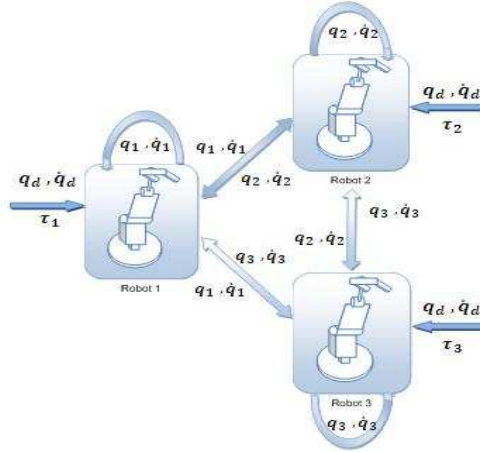


Fig. 2. Multi-robot system under mutual synchronization scheme.

3.1 Cooperative feedback control law

The major constraint to design the synchronization controller is that the uncertainty parameters. This problem is solved by using adaptive approach. To this end, we define: the position tracking error of the i -th robot manipulator as:

$$e_{1i}(t) = q_i(t) - q_d(t) + \int_{t_0}^t \Lambda_i(q_i(\lambda) - q_d(\lambda))d\lambda \quad (5)$$

where Λ_i is a diagonal positive definite matrix. Information on the vector e_{1i} will give insight on the convergence of the joints position to the desired trajectory. It is necessary to know the performance of the controller, especially how the trajectory of each robot manipulator converges with respect to each other. There is various ways to choose the synchronization error. For example in [1], authors include the error information of all systems involved in the synchronization. Our approach will make use of the graph theory to propose a feasible and efficient synchronization error, which consists on a measure of the synchronization for robot manipulator as defined as follows:

$$e_{2i}(t) = \sum_{j \neq i} K_{ij}(q_i - q_j) \quad (6)$$

where, k_{ij} is a positive definite matrix containing element from the laplacian matrix associated with a given graph that describes the communication topology.

Our objective is to design individual tracking controller for the n manipulators such that they coordinate their motions and track synchronously a desired trajectory. To this end, we combine these two errors to define the global errors as:

$$e_i = e_{1i} + \int_{t_0}^t e_{2i}(\lambda) d\lambda \quad (7)$$

A similar relation to (4) holds when the estimates of the parameters $\hat{\theta}$ are used to replace the exact parameter θ .

$$\hat{M}_i(q_i)\ddot{q}_i + \hat{C}_i(q_i, \dot{q}_i)\dot{q}_i + \hat{g}_i(q_i) = Y_i(q_i, \dot{q}_i, \ddot{q}_i)\hat{\theta}_i \quad (8)$$

Where, $\theta_i \in R^a$ is the vector of structural parameters of the manipulator and a is the number of unknown parameters. Since the value of the dynamic parameter θ_i is hard to be known exactly in practice, one defines $\hat{\theta}_i(t)$ as the estimate of θ_i . \hat{M}_i , \hat{C}_i and \hat{g}_i are estimates, respectively, of M_i , C_i and g_i . $Y_i(q_i, \dot{q}_i, \ddot{q}_i)$ denotes a regression matrix. Let the controller τ_i for the i-th robot given by:

$$\begin{aligned} \Gamma_i \hat{C}_i(q_i, \dot{q}_i) + \hat{g}_i(q_i) + \\ \hat{M}_i(q_i)[\ddot{q}_d - K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d)] + \\ \hat{M}_i(q_i)[\sum_{j \neq i} K_{ij}(e_{2i} - e_{2j}) + \Lambda e_{2i}] \end{aligned} \quad (9)$$

The control law (9) can be seen as a computed torque control with two additional terms owing to the presence of the particular choice of

the tracking error and an expression inspired from consensus algorithm ensuring an exponential synchronization. Notice that the controller law (9) contains PD controller terms. However, these terms are premultiplied by the inertia matrix $M_i(q_i)$. Therefore it is clear that this is not a linear controller as the PD control law, since the position and velocity gains are not constant but they depend explicitly on the position error. The estimated parameter $\hat{\theta}_i$ is subject to the adaptation law:

$$\dot{\hat{\theta}}_i = \Gamma_i^{-1}(\hat{M}_i^{-1}Y_i)^T e_i \quad (10)$$

where Γ_i is a diagonal positive-definite control gain.

3.2 Stability analysis

Theorem: *If $K_{di} > \sum_{j \neq i} K_{ij}$, the proposed adaptive controllers (8) guarantees asymptotic convergence to zero of position errors and synchronization errors of the robot network.*

Proof: Substituting (9) into the dynamic model (1) leads to the following closed-loop dynamics

$$\begin{aligned} & M_i(q_i)\ddot{q}_i - \hat{M}_i(q_i)\ddot{q}_d + C_i(q_i, \dot{q}_i)\dot{q}_i - \hat{C}_i(q_i, \dot{q}_i)\dot{q}_i \\ & + g_i(q_i) - \hat{g}_i(q_i) = \hat{M}_i(q_i)[-K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d) \\ & \quad + \sum_{j \neq i} K_{ij}(e_{2i} - e_{2j}) + \Lambda e_{2i}] \end{aligned} \quad (11)$$

Defining

$$\tilde{M}_i(q_i) = M_i(q_i) - \hat{M}_i(q_i) \quad (12)$$

$$\tilde{C}_i(q_i, \dot{q}_i) = C_i(q_i, \dot{q}_i) - \hat{C}_i(q_i, \dot{q}_i) \quad (13)$$

$$\tilde{g}_i(q_i) = g_i(q_i) - \hat{g}_i(q_i) \quad (14)$$

$$\tilde{\theta}_i = \theta_i - \hat{\theta}_i \quad (15)$$

and subtracting $\hat{M}_i(q_i)\ddot{q}_i$ in both sides yields

$$\begin{aligned} & (M_i(q_i) - \hat{M}_i(q_i))\ddot{q}_i + (C_i(q_i, \dot{q}_i) - \hat{C}_i(q_i, \dot{q}_i))\dot{q}_i \\ & + (g_i(q_i) - \hat{g}_i(q_i)) + \hat{M}_i(q_i)(\ddot{q}_i - \ddot{q}_d) = \\ & \hat{M}_i(q_i)[-K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d) \\ & \quad + \sum_{j \neq i} K_{ij}(e_{2i} - e_{2j}) + \Lambda e_{2i}] \end{aligned} \quad (16)$$

As a result

$$\begin{aligned}
 & Y_i(q_i, \dot{q}_i, \ddot{q}_i) \tilde{\theta}_i + \hat{M}_i(q_i) (\ddot{q}_i - \ddot{q}_d) = \\
 & \hat{M}_i(q_i) [-K_{pi} e_i - K_{di} \dot{e}_i - \Lambda_i (\dot{q}_i - \dot{q}_d) \\
 & + \sum_{j \neq i} K_{ij} (e_{2i} - e_{2j}) + \Lambda e_{2i}] \quad (17)
 \end{aligned}$$

Adding $\hat{M}_i(q_i) \dot{e}_{2i}$ both sides and using the expression of the synchronization error, yields:

$$\begin{aligned}
 & Y_i(q_i, \dot{q}_i, \ddot{q}_i) \tilde{\theta}_i + \hat{M}_i(q_i) [(\ddot{q}_i - \ddot{q}_d) + \Lambda_i (\dot{q}_i - \dot{q}_d)] \\
 & + \hat{M}_i(q_i) \sum_{j \neq i} [K_{ij} (\dot{q}_i - \dot{q}_d) - (\dot{q}_j - \dot{q}_d)] = \\
 & \hat{M}_i(q_i) [-K_{pi} e_i - K_{di} \dot{e}_i + \sum_{j \neq i} K_{ij} (e_{2i} - e_{2j})] \\
 & + \hat{M}_i(q_i) \sum_{j \neq i} \Lambda_i K_{ij} [(q_i - q_d) - (q_j - q_d)] \\
 & + \hat{M}_i(q_i) \sum_{j \neq i} [K_{ij} (\dot{q}_i - \dot{q}_d) - (\dot{q}_j - \dot{q}_d)] \quad (18)
 \end{aligned}$$

Further calculation, will result in

$$\begin{aligned}
 & \hat{M}_i(q_i) \ddot{e}_i + Y_i(q_i, \dot{q}_i, \ddot{q}_i) \tilde{\theta}_i = \hat{M}_i(q_i) [-K_{pi} e_i - K_{di} \dot{e}_i + \\
 & \sum_{j \neq i} K_{ij} (\dot{e}_i - \dot{e}_j)] \quad (19)
 \end{aligned}$$

we define a symmetric and positive semi-definite matrix $K_c =$

$$\begin{pmatrix}
 \sum_{1 \neq j} K_{ij} & \cdots & -K_{1j} & \cdots & -K_{1n} \\
 \cdots & \cdots & \cdots & \cdots & \cdots \\
 -K_{i1} & \cdots & \sum_{i \neq j} K_{ij} & \cdots & -K_{in} \\
 \cdots & \cdots & \cdots & \cdots & \cdots \\
 -K_{n1} & \cdots & -K_{nj} & \cdots & \sum_{n \neq j} K_{nj}
 \end{pmatrix}$$

Then, equation 19 can be written for the overall system as

$$\hat{M}[\ddot{e} + K_p e + K_d \dot{e} - K_c \dot{e}] = -Y(q_i, \dot{q}_i, \ddot{q}_i) \tilde{\theta} \quad (20)$$

Multiplying by \hat{M}^{-1} both sides yields

$$\ddot{e} = -K_p e - K_d \dot{e} + K_c \dot{e} - \hat{M}^{-1} Y(q_i, \dot{q}_i, \ddot{q}_i) \tilde{\theta} \quad (21)$$

To prove the stability, the Lyapunov function candidate is presented as:

$$v = 1/2(\dot{e}^T \dot{e} + e^T K_p e + \tilde{\theta}^T \Gamma \tilde{\theta}) \quad (22)$$

The derivative of equation (22) respect to time can be written as:

$$\dot{v} = \ddot{e}^T \dot{e} + e^T K_p \dot{e} + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} \quad (23)$$

Substituting (21) into (23)

$$\dot{v} = -\dot{e}^T (K_d - K_c) \dot{e} + \tilde{\theta}^T [\Gamma \dot{\tilde{\theta}} - (\hat{M}^{-1} Y)^T \dot{e}] \quad (24)$$

According to the adaptation law (10)

$$\dot{v} = -\dot{e}^T (K_d - K_c) \dot{e} < 0. \quad (25)$$

Since $K_{di} > \sum_{j \neq i} K_{ij}$, we have $\dot{v}(t) < 0$, this yields that $v(t) \leq v(0)$, which gives that e, \dot{e} and $\hat{M}^{-1} Y \tilde{\theta}^T$ are bounded.

Differentiating $\dot{v}(t)$ with respect to time yields

$$\ddot{v} = 2(K_p e + (K_d - K_c) \dot{e} + \hat{M}^{-1} Y \tilde{\theta})^T (K_d - K_c) \dot{e} \quad (26)$$

Using Barbalat's lemma:

\ddot{v} is bounded because e, \dot{e} and $\hat{M}^{-1} Y \tilde{\theta}$ are bounded. This implies $\dot{v}(t) \rightarrow 0$ as $t \rightarrow \infty$ and hence $\dot{e} \rightarrow 0$ as $t \rightarrow \infty$.

According to the equation (7) and as $\dot{e} = 0$, we get

$$\dot{q}_i - \dot{q}_d = -\Lambda_i (q_i - q_d) - \sum_{j \neq i} K_{ij} (q_i - q_j) \quad (27)$$

The goal is to ensure convergence of robot angular positions and velocities to a given desired trajectory. To this end, we set $\varepsilon_i = q_i - q_d$, Then equation (27) can be expressed as:

$$\dot{\varepsilon}_i = -\Lambda_i \varepsilon_i - \sum_{j \neq i} K_{ij} (\varepsilon_i - \varepsilon_j) \quad (28)$$

we define $\varepsilon = [\varepsilon_1 \dots \varepsilon_n]^T$ and $\Lambda = [\Lambda_1 \dots \Lambda_n]^T$. Let us define a matrix A as:

$$\begin{pmatrix} -\Lambda_1 - \sum_{j \neq 1} \beta_{1j} & \cdots & \beta_{1j} & \cdots & \beta_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta_{i1} & \cdots & -\Lambda_i - \sum_{j \neq i} \beta_{ij} & \cdots & \beta_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta_{n1} & \cdots & \beta_{nj} & \cdots & -\Lambda_n - \sum_{j \neq n} \beta_{nj} \end{pmatrix}$$

Then equation (28) can be simplified as:

$$\dot{\epsilon} = A \cdot \epsilon \quad (29)$$

A nonnegative piecewise continuous function is defined as:

$$v(t) = \epsilon^T \epsilon \quad (30)$$

The time derivative of (30) can be written as

$$\dot{v} = 2 \sum_{i=1}^n \epsilon_i \dot{\epsilon}_i \quad (31)$$

$$\dot{v} = 2 \sum_{i=1}^n \epsilon_i (-\Lambda_i \epsilon_i - \sum_{j \neq i} \beta_{ij} (\epsilon_i - \epsilon_j)) \quad (32)$$

$$\dot{v} = -2 \sum_{i=1}^n \Lambda_i \epsilon_i^2 - 2 \sum_{i=1}^n \sum_{j \neq i} \beta_{ij} (\epsilon_i - \epsilon_j) \epsilon_i \quad (33)$$

$$\dot{v} = -2 \sum_{i=1}^n \Lambda_i \epsilon_i^2 - 2 \sum_{i=1}^n \sum_{j \neq i} \beta_{ij} \epsilon_i^2 + 2 \sum_{i=1}^n \sum_{j \neq i} \beta_{ij} \epsilon_i \epsilon_j \quad (34)$$

Knowing that

$$\sum_{i=1}^n \sum_{j \neq i} \beta_{ij} \epsilon_i^2 = \sum_{j=1}^n \sum_{i \neq j} \beta_{ji} \epsilon_j^2 \quad (35)$$

Consequently,

$$\dot{v} = -2 \sum_{i=1}^n \Lambda_i \epsilon_i^2 - \sum_{i=1}^n \sum_{j \neq i} \beta_{ij} (\epsilon_i - \epsilon_j)^2 \leq 0. \quad (36)$$

From Lasalle's invariance principle, the origin is globally asymptotically stable. Consequently, $\lim_{t \rightarrow \infty} \epsilon_i(t) \rightarrow 0$ is guaranteed. Finally, since $q_i \rightarrow q_d$ and $\dot{q}_i \rightarrow \dot{q}_d$ as $t \rightarrow \infty$, from (7), we can conclude that $q_i \rightarrow q_j$ for $t \rightarrow \infty$.

4. Adaptive External Synchronization: master-slaves scheme

The synchronization under a master-slave scheme is an external synchronization (see fig. 3). Indeed, since each slave should synchronize its clock to the master which is an external source of time. In this study, we

consider a network topology formed by a group of robots, such that the behavior of one among them is independent of others. This dominant robot is considered as master robot or network leader. The objective for slaves is to follow the trajectory of the master robot and not the desired trajectory, which privileges the implementation task of the given trajectory, i.e., we do not need to implement every time the desired trajectory for each slave robot in network, an implementation for only one times in the master is sufficient each time when the operator seeks to change the desired trajectory. We assume that the physical parameters of the slaves robot are badly known and we consider that the full state of all robots in the multi-composed system is available.

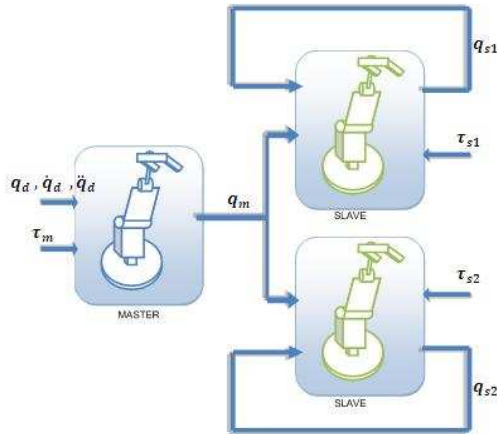


Fig. 3. External synchronization under master-slaves scheme

4.1 Coordinated feedback control law

Design a controller that ensures the master convergence to the desired trajectory, It is absolutely classic. The aim is to conceive interconnections for slaves robots to synchronize their positions and velocities to those of robot master. It is assumed that coordinating identical robots have the same number of joints $q_i \in \mathbb{R}^n$ where $i = m, s_i$ identifies the master (m) and slaves (s_i) robot respectively. The master robot with uncertainty parameters is driven by an input torque τ_m while each slave is driven by an input torque τ_{s_i} . Then we propose the synchronization

controller as follows:

$$\tau_m = \hat{C}_m \dot{q}_m + \hat{g}_m + \hat{M}_m [\ddot{q}_d - K_p e_m - K_d \dot{e}_m - \Lambda(\dot{q}_m - \dot{q}_d)] \quad (37)$$

$$\tau_{s_i} = \hat{C}_{s_i} \dot{q}_{s_i} + \hat{g}_{s_i} + \hat{M}_{s_i} [\ddot{q}_m - K_{p_i} e_{s_i} - K_{d_i} \dot{e}_{s_i} - \Lambda_i(\dot{q}_{s_i} - \dot{q}_m)] \quad (38)$$

where, $K_p \in R^{n \times n}$ and $K_d \in R^{n \times n}$ are two positive matrices defined as a feedback gains for the i -th robot and such as the synchronization tracking errors e_m and e_{s_i} defined by:

$$e_m = q_m - q_d + \int_0^t \Lambda(q_m - q_d) d\Lambda \quad (39)$$

$$e_{s_i} = q_{s_i} - q_m + \int_0^t \Lambda_i(q_{s_i} - q_m) d\Lambda_i \quad (40)$$

4.2 Stability analysis

In this subsection, the goal is to prove the convergence of the slave synchronization-tracking error.

Theorem: For any positive gain matrix K_d , the control law defined by equation (38) applied to the master, the control law defined by equation (39) applied to slaves and the adaptation law described by equation (10) guarantee the asymptotic convergence to zero of synchronization-tracking errors of the robot network defined by equation (1).

Proof: Substituting (39) into (1) yields

$$\begin{aligned} M_{s_i}(q_{s_i}) \ddot{q}_{s_i} + C_{s_i}(q_{s_i}, \dot{q}_{s_i}) \dot{q}_{s_i} + g_{s_i} &= \hat{C}_{s_i}(q_{s_i}, \dot{q}_{s_i}) \dot{q}_{s_i} \\ &+ \hat{g}_{s_i} + \hat{M}_{s_i} [\ddot{q}_m - K_{p_i} e_{s_i} - K_{d_i} \dot{e}_{s_i} - \Lambda_{s_i}(\dot{q}_{s_i} - \dot{q}_m)] \end{aligned} \quad (41)$$

Subtracting $\hat{M}_{s_i} \ddot{q}_{s_i}$ in both sides from equation (42) then it can be written as:

$$\begin{aligned} \hat{M}_{s_i}(q_i) \ddot{e}_{s_i} + Y_{s_i}(q_{s_i}, \dot{q}_{s_i}, \ddot{q}_{s_i}) \tilde{\theta}_{s_i} &= \\ \hat{M}_i(q_{s_i}) [-K_{p_i} e_{s_i} - K_{d_i} \dot{e}_{s_i}] \end{aligned} \quad (42)$$

To prove the stability of the overall synchronized system, let's define

$$e_s^T = [e_{s_1}^T, \dots, e_{s_i}^T, \dots, e_{s_n}^T]^T; Y_s^T = [Y_{s_1}^T, \dots, Y_{s_i}^T, \dots, Y_{s_n}^T]^T$$

$$\theta^{T_s} = [\theta_{s_1}^T, \dots, \theta_{s_i}^T, \dots, \theta_{s_n}^T]^T; \Gamma_s = [\Gamma_{s_1}^T, \dots, \Gamma_{s_i}^T, \dots, \Gamma_{s_n}^T]^T$$

$$K_{d_s} = \text{diag}(k_{d_i}); K_{p_s} = \text{diag}(k_{p_i}); \hat{M}_s = \text{diag}(\hat{M}_{s_i})$$

Using the adaptation law (10), one can easily deduce

$$\dot{v} = -\dot{e}_s^T K_d \dot{e}_s < 0 \quad (43)$$

Using Barbalat's lemma, we show that $\dot{e}_s \rightarrow 0$ for $t \rightarrow \infty$. Hence, we show that q_{s_i} converges to q_m exponentially.

5. Simulations

To demonstrate the proposed approach, simulations were performed on a multi-manipulators motion control system, where the motion of each robot has two degrees of freedom. Simulations are performed on Matlab/Simulink®. Assume that lengths of two links of each robot are $l_1 = 1m$, $l_2 = 1.5m$ with the link masses $m_1 = 2kg$, $m_2 = 1kg$. Define the initial joint coordinations of cooperative robots as following: $(q_{11}, q_{12}) = (-2, -2)$ rad, $(q_{21}, q_{22}) = (3, 3)$ rad, $(q_{31}, q_{32}) = (1.5, 1.5)$ rad. Where (q_{11}, q_{12}) , (q_{21}, q_{22}) and (q_{31}, q_{32}) denote two joint coordinations of each robot. Consider that the common trajectory to be tracked is a periodic form like: $q_d = \sin(t)\sin(2\cos(t))$. In the simulation, the length and the mass of each link of robots are assumed to be unknown.

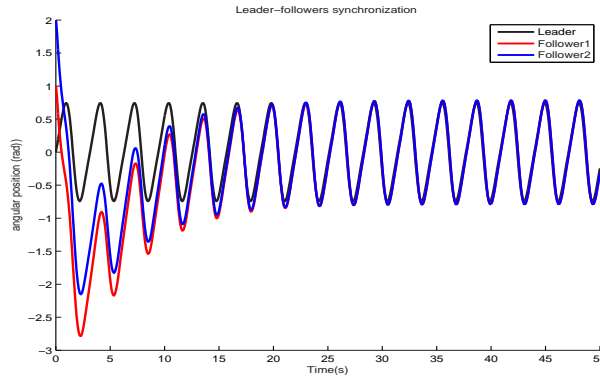


Fig. 4. Coordinated robots synchronization

Figure 3 illustrates the robot trajectory tracking control and it shows in the same time the good performance in position synchronization amongst the three robots, under a master-slaves coordinated scheme. It can be seen from figure 4 the adaptive synchronization accomplishment of three robots, under a cooperative scheme. Figure 5 and figure 6 illustrate the position errors and the synchronization errors in the presence of uncertainty parameters, respectively .

6. CONCLUSIONS

This paper has considered mutual synchronization problems and external synchronization problems for distributed multi-robot systems with uncertainty in the dynamic model. The aim of this work has been to find

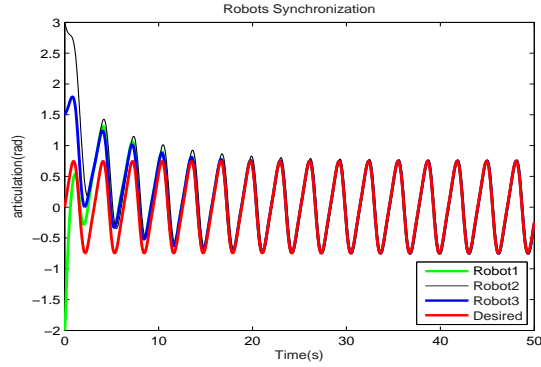


Fig. 5. Cooperative robots Synchronization

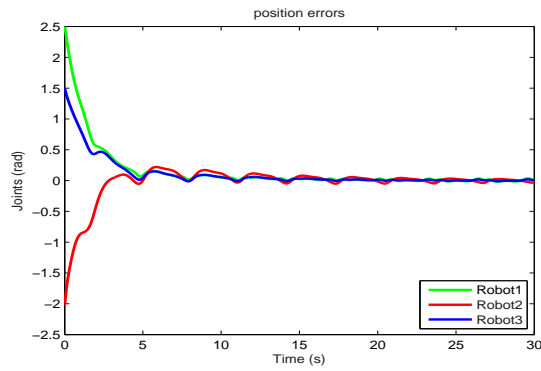


Fig. 6. Position errors

out a decentralized controller, which individually applied to each manipulator, the synchronization is therefore met. It has been shown that the proposed strategy can be coordinate manipulator articulations, without loss position, to track a given time varying trajectory. In mutual synchronization scheme, there is a decentralized control implemented in each robot. Therefore there is not a master who dominates the group. On the other side we should implemented the desired trajectory for each agent. From industrial point of view, it can be seen painful. Moreover, the implementation of a desired trajectory for one robot and others should be

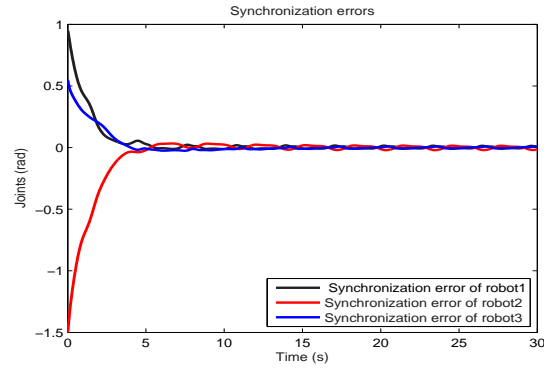


Fig. 7. Synchronization errors

tracking the dominant is easier, faster and efficient. The external synchronization scheme take into account the master-slaves configuration. The main drawback of this scheme occurs in the case of failed master sensors or in the case of perturbed master sensor measurements. In contrast, in the case of mutual synchronization there is change of information flow between agents in the network. These information which refer to an explicit and implicit data for each robot offer a high tolerance on sensors degradation level. Simulation results obtained from a multi-robot motion control system demonstrate the effectiveness of the synchronization approach. In a prospective research work we are interested in extending this methodology considering the time delay transmission and the information transmission losses.

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