

## Appendix S1: Activation or deactivation of submodels: the Bayesian switch

Let  $A$  and  $B$  be probabilistic variables that share the same domain  $\mathcal{D}$ . Let  $\lambda$  be a binary variable, taking 0 or 1 values. We show here how  $\lambda$  can be used as a ‘‘probabilistic switch’’ between variables  $A$  and  $B$ .

We define the joint probability distribution as follows:

$$P(A \ B \ \lambda) = P(A)P(B)P(\lambda \mid A \ B) .$$

The terms  $P(A)$  and  $P(B)$  can be arbitrarily defined, and  $P(\lambda \mid A \ B)$  is defined by:

$$P([\lambda = 1] \mid [A = a] [B = b]) = \begin{cases} 1 & \text{if } a = b \\ 0 & \text{otherwise} \end{cases}$$

Firstly, consider  $P([A = a])$ . It can be trivially shown that it is simply  $P(A)$  as defined in the model. By assumption, it is thus independent of  $P(B)$ .

Secondly, consider  $P([A = a] \mid [\lambda = 1])$ :

$$\begin{aligned} & P([A = a] \mid [\lambda = 1]) \\ &= \frac{P([A = a] [\lambda = 1])}{P([\lambda = 1])} \\ &\propto \sum_B P([A = a])P(B)P([\lambda = 1] \mid [A = a] \ B) . \end{aligned}$$

In the above summation, there is a single value of  $B$  for which  $P([\lambda = 1] \mid [A = a] \ B)$  is not zero: when  $B = a$ . Therefore the summation collapses:

$$\begin{aligned} & P([A = a] \mid [\lambda = 1]) \\ &\propto P([A = a])P([B = a])P([\lambda = 1] \mid [A = a] [B = a]) \\ &\propto P([A = a])P([B = a]) . \end{aligned}$$

Therefore,  $P([A = a] \mid [\lambda = 1])$  is not independent of  $P(B)$ , contrary to  $P([A = a])$ . When  $\lambda$  is specified in the right-hand side of the computed term,  $B$  is forced to take the value taken by  $A$ .

In this manner,  $\lambda$  can be used as a switch. When we want the submodels about  $A$  and  $B$  to be disconnected,  $\lambda$  is not specified; from the point of view of  $B$ ,  $A$  is deactivated. On the other hand, if we want submodel  $A$  to influence submodel  $B$ , we specify  $\lambda = 1$  in the questions we compute, and both submodels are activated.