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A Cooperative Local Search-Based Algorithm for the Multiple-Scenario Max-Min Knapsack Problem

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Abstract

The purpose of this article is to present a novel method to approximately solve the Multiple-Scenario Max-Min Knapsack Problem (MSM²KP). This problem models many real world situations, e.g. when for many scenarios noted $\pi \in \mathcal{P} = \{1, \dots, P\}$, the aim is to identify the one offering a better alternative in term of maximizing the worst possible outcome.

Herein is presented a cooperative approach based on two local search algorithms : (i) a limited-area local search applied in the elite neighborhood and which accepts the first solution with some deterioration threshold of the current solution, (ii) a wide range local search is applied to perform a sequence of paths exchange to improve the current solution.

Results have been analyzed by means state-of-the art methods and via problem instances obtained by a generator code taken from the literature. The tests were executed in completely comparable scenarios to those of the literature. The results are promising and the efficiency of the proposed approach is also shown.

Key words: Combinatorial optimization; Knapsack; Max-min optimization; Robust optimization; Heuristics; Cooperative
2008 MSC: 2008, 90C10, 90C35, 90C59, 90C90

1. Introduction

The Multiple-Scenario Max-Min Knapsack Problem (MSM²KP) is a variant of the well known single 0-1 Knapsack Problem (KP) (e.g. Martello and Toth (1990), Martello et al. (2000)). MSM²KP is a max-min binary knapsack problem with multiple scenarios. The aim of this max-min optimization

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9 problem is to maximize the minimum of a family of linear objective functions
10 subject to a single functional constraint. Several applications of MSM²KP
11 (linear and non-linear types) match with the real world situations where the
12 aim is to find an equilibrium for many criterias or an efficient solution. In this
13 case the solution is said to be Pareto-optimal (see e.g. Branke et al. (2008)).
14 However, there are several multiobjective problems where the Pareto-optimal
15 solution building strategy is not effective enough to reach an efficient solu-
16 tion since the equity or fairness among uniform individual outcomes is an
17 important issue.

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19
20 This type of model is also applied when we expect to forecast a situation
21 depending on many scenarios or simply when the objective is formulated as
22 a maximum or a minimum of a class of criterias. As an application of this
23 model, we can cite resources allocation (e.g. see Tang (1988)), matroid the-
24 ory (e.g. see Averbach et al. (1995)), economics (e.g. see Yu (1996)), game
25 theory (e.g. see Nash and Sofer (1996)) or multiobjective optimization (e.g.
26 see Bitran (1977) and Ecker and Shoemaker (1981)). Recently, Taniguchi et
27 al. (2008) have studied MSM²KP and proposed an algorithm to optimally
28 solve the problem. The method was able to solve instance problems with a
29 number of variables up to 1000 items and 30 scenarios.

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32 Our approach investigated this problem under the framework of the max-
33 min optimization, where we maximize the minimum of all the objectives.
34 In some literature it is referred to as the robust optimization (e.g. see Yu
35 (1996)). While, in general, such a multiobjective optimization problem is
36 considered as a multi-criteria decision making (e.g. see Steuer (1986)). We
37 recall that the concept of robust optimization finds its origins in engineering,
38 specifically, control theory.

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41 Let \mathcal{P} be the set of scenarios describing all possible payoffs. We assume
42 that each payoff might occur with some positive and unknown probability.
43 Then maximizing the minimal payoff $Z^\pi(X)$, $\forall \pi \in \mathcal{P}$ and X is the binary so-
44 lution vector related to the scenario π can be seen as “reasonable-worstcase
45 optimization” and where the decision-maker protects revenue against the
46 worst-case values. Hence, the max-min optimization approach is an alterna-
47 tive method to stochastic programming which can be applied for any problem
48 whose parameter values are unknown, variable, and their distributions are
49 uncertain. The max-min optimization consists of computing the cost of ro-
50 bustness of the robust counterpart to the maximum outcome optimization
51 problem. Due to the scenario uncertainty, a max-min optimization model is
52 likely to be more useful to a risk-averse decision maker.

In spite of its comparatively long history (e.g. Gilmore and Gomory (1966)), many variants of the knapsack problem are still being studied intensively. In this paper, we are concerned with the Multiple-Scenario Max-Min Knapsack Problem; namely MSM²KP. This problem generalizes the classical knapsack problem with respect to: (i) each item $j \in J$ is associated with P values $(v_j^\pi)_{(j=1, \dots, n)}^{(\pi=1, \dots, P)}$, for which each one corresponds to a different scenario, and a single weight w_j , $j = 1, \dots, n$, (ii) instead of trying to compute a single solution with a maximum total profit, we compute a set of feasible solutions of different values and covering all the possible scenarios $\pi = 1, \dots, P$. Then the worst scenario is identified in order to perform a maximization. Given a knapsack of a fixed capacity C , the MSM²KP can formally be stated as follows:

$$(MSM^2KP) \begin{cases} \max_X & Z(X) = \min_{1 \leq \pi \leq P} \left\{ \sum_{j \in J} v_j^\pi x_j \right\} \\ \text{Subject to:} & \sum_{j \in J} w_j x_j \leq C \\ & x_j \in \{0, 1\}, \text{ for } j = 1, \dots, n \end{cases}$$

$X = (x_1, \dots, x_j, \dots, x_{n=|J|})$, the variable x_j is either equal to 0, implying item j is not picked, or equal to 1 implying item j is picked, knowing the worst scenario π^* .

We may assume without loss of generality that w_j , v_j^π (for $j = 1, \dots, n$ and $\pi = 1, \dots, P$) and C are positive integers, and $\sum_{j \in J} w_j > C$.

In the following, the items $j \in J$, for each scenario $\pi = 1, \dots, P$ are sorted regarding a specific order based on the efficiency of the items and which we detail in section 3. This order-based efficiency determines priority to some items to select in the solution. Notice that if two efficiencies (for the same scenario π) have the same value, then we consider, in first, the efficiency corresponding to the greater profit. The so-defined problem is NP-hard since for $P = 1$, it simply represents the single Knapsack Problem (KP) which is NP-hard (see Martello and Toth (1990)).

The remaining of the paper is organized as follows. First (Section 2), we briefly review some related works on the classical knapsack problem and some of its variants, also on the max-min (or min-max) combinatorial problems dealing with sequential exact and approximate algorithms. Second (Section 3), we detail the algorithm which ensures the feasibility to the obtained solution. This procedure consists to construct, in a greedy way, a feasible

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9 sub-solution to complete by adding items realizing the minimum objective
10 value without exceeding the remaining capacity. This algorithm identifies
11 some critical elements j regarding the current scenario π^* for the solution
12 such that $\sum_{k=1}^{j-1} w_k \leq C < \sum_{k=1}^j w_k$ which allow to diversify the search in
13 the neighborhood. Then the approach will apply some complex local search
14 strategies to attempt some improvements.
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16 The purpose of this heuristic is to find a starting feasible solution to
17 MSM²KP in order to evaluate the influence of the initial solution's quality
18 in the overall performance of the cooperative search. In Section 4, we detail
19 the cooperative approach. It consists to improve the current solution by en-
20 hancing the local search around several types of feasible neighborhood search
21 space. The neighborhoods selection is decided regarding a certain criterion
22 and the main local search is run regarding the type of the selected neigh-
23 borhood. So that it can be seen as a hierarchical depth search around the
24 solution neighborhood. In Section 5, we present several series of computa-
25 tional results on problem instances obtained by a generator code taken from
26 the literature. We show the efficiency of the proposed approach regarding
27 the obtained solutions. Finally, in the conclusion (Section 6), we summarize
28 the main results of the paper and give a conclusion.
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34 2. Literature survey

35 The Knapsack Problem (KP) is an NP-hard combinatorial problem (for
36 more details on complexity theory the reader may refer to Garey and Johnson
37 (1979)) and has been widely studied in the literature. The way of seeking a
38 solution depends on the particular framework of the application (leading to
39 the particular knapsack problem) and the available computational resources.
40 Hence, a good review of the single knapsack problem and its associated exact
41 and heuristic algorithms is available in Martello and Toth (1990). For the
42 (un)bounded single constraint KP, we can find in the literature a large variety
43 of solution methods (see Martello et al. (1999); Balas and Zemel (1980);
44 Fayard and Plateau (1982) and Pisinger (1997)). The problem has been
45 solved optimally and approximately by dynamic programming, search tree
46 procedures or other hybrid approaches.
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48 Several approaches have also been developed in other previous works that
49 adressed the general case, i.e., when the number of constraints is extended to
50 more than one or the aim is to optimize a class of functions (multiobjective
51 KP). A first variant of the KP is called Multidimensional (or Multiconstraint)
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9 Knapsack Problem (MDKP) (see Chu and Beasley (1998)). The Multiple-
10 Choice Knapsack Problem (MCKP) (see Pisinger (1995)) is another vari-
11 ant where the picking criterion of items is more restrictive. Another more
12 harder variant is the Multiple-choice Multidimensional Knapsack Problem
13 (MMKP) (see Hifi et al. (2006) and Sbihi (2007)). A recent detailed review
14 for the MMKP is given in Sbihi (2007). If we deal with only two knapsack
15 constraints, then the problem is referred to as the Bidimensional Knapsack
16 Problem (BKP) (see Freville and Plateau (1997)). Another type of combina-
17 torial optimization problem is the max-min (min-max) allocation problem.
18 This problem has widely been studied in the literature (see Brown (1991);
19 Luss (1992); Pang and Yu (1989) and Tang (1988)). We also can find in the
20 literature another max-min optimization problem of knapsack type which
21 has been approximately and optimally solved via tabu search based-heuristic,
22 branch and bound and binary search methods (see Hifi et al. (2002) and
23 Yamada et al. (1998)).

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28 To the best of our knowledge, there exists only three papers dealing di-
29 rectly or indirectly with the Multiple-Scenario Max-Min Knapsack Problem
30 (MSM²KP) (see Yu (1996); Iida (1999) and Taniguchi et al. (2008)). The
31 first paper (Yu (1996)) proposed to optimally solve MSM²KP by a branch
32 and bound algorithm. The approach was able to solve instance problems
33 up to 60 items. In the second paper (Iida (1999)), the author developed a
34 method to obtain new lower and upper bounds for the max-min 0-1 knapsack
35 problem thanks to a derivative procedure-based lagrangean relaxation. The
36 approach used also a branch and bound procedure developed in Yu (1996)
37 as well as the obtained lower and upper bounds. The author showed that
38 these bounds were good enough to solve the problem in a acceptable comput-
39 ing time. However, the method failed to increase the total number of items
40 more than 60. Finally, in Taniguchi et al. (2008), the paper developed
41 an approach in two steps to optimally solve MSM²KP. First, the authors
42 proposed a surrogate relaxation heuristic to reduce the problem, then they
43 compute some upper and lower bounds. This technique allowed to reduce the
44 problem and permits to optimally solve the remaining problem by a branch
45 and bound procedure. In this paper, we analyze and present an alternative
46 optimization method. It can be seen as a first step to extend to more than a
47 simple method which iteratively attempts to select a good heuristic amongst
48 many.
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3. A greedy algorithm for the MSM²KP

Prior to design our main approach, we propose an algorithm called herein GH in order to build a starting feasible solution. This initial solution is obtained iteratively and the aim is to perform better improvement throughout the process by a very sophisticated method.

Let $X = (x_j)_{j=1,\dots,n}$ be a solution vector, where $x_j = 1$ if item j is selected in the solution, and $x_j = 0$ otherwise. The aim is to maximize the total profit regarding the minimal π -objective value $Z^\pi(X)$ obtained over all the P scenarios.

In this heuristic, items are initially sorted according to a specific decreasing order. For each item $j = 1, \dots, n$, we compute first its efficiency regarding the scenario π as $e_j^\pi = v_j^\pi/w_j$. We say that $(j \prec k)$ iff $e_j^\pi \geq e_k^\pi$. Considering this order, we apply a reordering of the total items as a preprocessing step.

Then, GH builds iteratively a partial feasible solution. Indeed, in the main steps, for each selected item j , the procedure attempts to complete the solution. The procedure locates a minimal scenario π^* and computes $Z(\bar{X}) = \max_x \sum_{j \in J} v_j^{\pi^*} x_j$. GH terminates once a feasible solution $Z(\bar{X})$ is obtained. We recall that the obtained solution could be of bad quality. Herein, we describe the main steps of the greedy algorithm GH :

Algorithm GH (Greedy Heuristic)

Input: An MSM²KP instance I

Ouput: A feasible solution \bar{X} for MSM²KP

Initialization:

1. FOR $j = 1, \dots, n$ and $\pi = 1, \dots, P$ DO
 - Sort items in the decreasing order of the efficiency e_j^π
2. $\pi^* = 1$; X^0 : initial; // π^* : scenario with the minimum (sub)solution
3. Set $\bar{X} \leftarrow X^0$; $Z(\bar{X}) \leftarrow Z^{\pi^*}(X^0)$; // \bar{X} : feasible starting point e.g. 0

Main steps:

4. REPEAT
 5. IF $w_j \leq (C - \sum_{k \leq j-1} w_k)$ THEN //the solution is feasible for π^*

$$Z^{\pi^*}(\bar{X}) \leftarrow Z^{\pi^*}(\bar{X}) + v_j^{\pi^*};$$

$$j \leftarrow j + 1;$$
 Let $\pi^* = \arg \min_{1 \leq \pi \leq P} \{Z^\pi(\bar{X})\}$;
6. UNTIL $j > |J|$
7. EXIT with best feasible solution \bar{X} of best value $Z(\bar{X})$;

4. A cooperative local search

In this section, we present the main principle of the cooperative local search-based approach namely CLS. The main algorithm contains several steps. Tabu search starts by a feasible solution obtained thanks to GH. All the visited solutions are feasible. The exploration of the solutions space is executed with some add/drop swaps. The elite solutions list is generated by improving the objective value where the minimal scenario has already been identified. The core of the approach is to build neighborhoods and perform several local searches in order to reach a best near-optimal solution.

Our cooperative strategy takes place when a search procedure gives away information about its best solutions and another procedure adapts its search trajectory based on this information. The cooperative local search framework is composed of a decision parameter and of two local search heuristics (notice that we can extend the method to more than 2 local search heuristics). The decision parameter is in charge of the selection and the execution of the appropriate local search heuristic at each decision point of the search process. The greedy strategy selects the best, not necessarily an improving, scenario. The tabu based strategy, incorporates a tabu list in the selection mechanism that forbids the selection of the non-improving solution for a certain tabu tenure. The decision parameter allows to accept the selected local search heuristic which accepts only to improve the current solution or either to improve the current solution or to lead to the smallest deterioration of the current solution if no improvement can be achieved.

The aim of using the cooperative heuristic is to raise the level of generality so as to be able to apply the same solution method to several criteria problems. Perhaps at the expense of reduced but still acceptable solution quality when compared to a tailor-made approach. Hence, we define two types of local search that address MSM²KP neighborhood search : (i) a generalized search outlined herein by GS and (ii) a limited search represented herein by RS.

The GS algorithm is a very large local search and applied if the list L has not included enough moves regarding a certain threshold S . Otherwise the RS algorithm is called in order to perform a limited-area local search in the elite neighborhood. RS accepts the first solution which slightly deteriorates the current solution $X^{current}$. The limited-area local search is supposed to perform in a very rich elite neighborhood.

To ensure the feasibility of the solution, we apply a feasible state phase regarding the best recorded infeasible solution. It corresponds to set to ‘zero’ the solution components x_j ($j = 1, \dots, n$) which are equal to ‘one’ and have the lowest efficiency $e_j^{\pi^*}$.

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9 **Algorithm FSH** (Feasible State Heuristic)

11 **Initialization:**
 12 1. Set $\bar{X} \leftarrow \underline{X} // \underline{X}$ is the recorded best infeasible solution
 13 **Main steps:**
 14 2. REPEAT
 15 3. Select $j_{min} = \operatorname{argmin}_{1 \leq j \leq n} \{e_j^{\pi^*} \mid \bar{x}_j = 1\}$
 16 4. $\bar{x}_{j_{min}} \leftarrow 0$;
 17 5. UNTIL $\sum_{1 \leq j \leq n} w_j \bar{x}_j \leq C$;
 18 6. END

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23 The tabu approach uses a tabu list L that includes all the tabu moves. The
 24 tabu status for these moves is amended regarding a period τ such that $\alpha\sqrt{n} \leq$
 25 $\tau \leq 2\alpha\sqrt{n}$, ($\alpha \in [0.5; 1.5]$ and n is the problem size). The tabu status of a move
 26 is removed if it belongs to the list L and it exceeds τ iterations, or if it matches
 27 with the aspiration criteria.
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30 *4.1. Intensification and diversification*

31 It is achieved via GH which is called once an improvement of the current
 32 solution is realized. We set L an elite moves list and $\pi^* = \operatorname{argmin}_{1 \leq \pi \leq P} \left\{ \sum_{j \in J} v_j^\pi x_j \right\}$
 33 is the index corresponding to the minimal scenario corresponding to the current
 34 solution $X^{current}$.
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39 *4.1.1. The very large local search*

40 The very large local search or generalized search (algorithm GS) starts with a
 41 feasible solution and performs a sequence of path exchange to improve the solution.
 42 The list L contains the moves that improves the objective value $Z(\cdot) = Z^{\pi^*}(X)$
 43 and $j \in L \Leftrightarrow (v_j^{\pi^*} \geq 0$ and $x_j = 1)$.
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9 **Algorithm GS** (Generalized Search)

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11 **Initialization:**
12 1. $L = \{j_1, j_2, \dots, j_r\}; j^* = j_1;$
13 2. FOR $\pi = 1$ to P DO
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15 3. $Value^\pi = \sum_{j \neq j^*} v_j^\pi x_j + v_{j^*}^\pi (x_{j^*} \pm \epsilon_{j^*}); \epsilon_j = \begin{cases} +1 & \text{if } \bar{x}_j = 0 \\ -1 & \text{if } \bar{x}_j = 1 \end{cases};$
16
17 4. $\mu = \min_{1 \leq \pi \leq P} \{Value^\pi\};$
18
19 **Main steps:**
20 5. FOR $j = j_2$ to j_r DO
21 6. $\pi \leftarrow 1; m = \sum_{k \neq j} v_k^\pi x_k + v_j^\pi (x_j \pm \epsilon_j);$
22 7. WHILE ($Value^\pi \geq \mu$ and $\pi \leq P$) DO //abandon candidate j once
23 it leads to a smaller value of μ
24
25 8. $\pi \leftarrow \pi + 1;$
26 9. IF ($Value^\pi \leq m$) THEN
27 10. $m \leftarrow Value^\pi;$
28 //save the minimum of all $Value^\pi, \pi = 1, \dots, P$
29
30 11. END_WHILE
31 12. IF $\pi = P$ THEN $j^* \leftarrow j; \mu \leftarrow m;$
32 13. END

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34 Furthermore, we consider a set L_{swap} of couples of items (j_1, j_2) that are
35 candidates for a swap such that the exchange operation allows to improve the
36 current value of the solution.

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$$(j_1, j_2) \in L_{\text{swap}} \Leftrightarrow \left(\begin{cases} v_{j_1}^{\pi^*} \geq 0; x_{j_1} = 1 \\ \text{and} \\ x_{j_2} = 0 \end{cases} \text{ or } \begin{cases} v_{j_2}^{\pi^*} \geq 0; x_{j_2} = 1 \\ \text{and} \\ x_{j_1} = 0 \end{cases} \right)$$

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40 This procedure performs systematically a search of all the solutions contained
41 in the list L .
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44 *4.1.2. The restricted guided local search*

45 Contrary to GS algorithm, RS algorithm is a targeting limited-area local
46 search. It allows to make savings in term of locating the best solution or to
47 accept degrading solution with some threshold. This is achieved via a filtering
48 process thanks to a decreasing threshold acceptance function $\Phi(\theta)$, where θ is a
49 parameter to set up. The threshold acceptance function $\Phi(\cdot)$ can be seen as a
50 penalization-based concept. We detail in what follows the RS algorithm :
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9 **Algorithm RS** (Restricted Search)

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11 **Initialization:**
12 1. $L = \{j_1, j_2, \dots, j_\ell\}$; $j^* = j_1$, and $\ell > S$;
13 // L is a solution subspace with ℓ 1-components
14 // S is a parameter controlling the search complexity
15 2. $Value^\pi = \sum_{j \neq j^*} v_j^\pi x_j^{current} + v_{j^*}^\pi (x_{j^*}^{current} \pm \epsilon_{j^*})$, $\pi = 1, \dots, P$;
16 3. $\mu = \min_{1 \leq \pi \leq P} \{Value^\pi\}$;
17
18 **Main steps:**
19 5. WHILE (not Stopping Condition and $k \leq S$) DO
20 6. $j_k \leftarrow \text{random}(j, L)$; $L \leftarrow L \setminus \{j\}$; // random selection of item j
21 7. $Value^\pi = \sum_{j \neq j_k} v_j^\pi x_j^{current} + v_{j_k}^\pi (x_{j_k}^{current} \pm \epsilon_{j_k})$, $\pi = 1, \dots, P$;
22 8. $\mu = \min_{1 \leq \pi \leq P} Value^\pi$;
23 9. IF $\mu \geq Z(X^{current}) + \Phi(\theta) \times Z(X^{current})$ THEN // $Z(\cdot)$ best value
24 10. $j^* \leftarrow j_k$;
25 11. ELSE $k \leftarrow k + 1$;
26 12. END_WHILE
27 13. END
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33 The algorithm starts by setting the current solution to a selected one belonging
34 to the region L with $|L| = \ell$. Then it applies a restricted swaps sequence in order
35 to remain in the same region to better explore it. Another feature is to guide the
36 search to unexplored or not enough explored regions thanks to a changing region
37 movement and some swaps to explore the new region.

38 The idea is to accept less and less degraded solutions progressively of the search
39 process. It uses memory to guide the search to promising regions of the solution
40 space. This is performed by increasing the current cost function with a penalty
41 term that penalizes bad features of previously visited solutions. However, $\Phi(\cdot)$ can
42 trap the local search around some local optima.

43 This reduction in the search spaces enables a fast execution of the global algo-
44 rithm. Unfortunately, it also imposes serious limitations on the ability to provide
45 good quality intermediate solutions.

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49 *4.1.3. An improved 2-phase tabu search for MSM²KP*

50 The concept of the approach is to help guide the search towards the opti-
51 mization of the different individual objectives. It starts with a feasible solution
52 obtained thanks to the GH algorithm. For the local search, we apply both large
53 and limited local search (algorithm GS and algorithm RS). It starts with an initial
54 complete solution and then makes a request for informations to perform the right
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local search using the assigned heuristic either for a certain number of iterations, or until no further improvement is possible. The process performs search through the same solution space, starting from the same current solution. If the obtained solution is not feasible, then the feasible state phase (algorithm FSH) is applied to convert the solution in order to build a new solution regarding the best solution found in the list L . This operation permits also to release the solution trapped around a local optima.

Algorithm CLS (Cooperative Local Search)

Input: An MSM²KP instance I

Ouput: A best near-optimal solution X^* for MSM²KP

Initialization:

1. $\bar{X} = (\bar{x}_1, \dots, \bar{x}_j, \dots, \bar{x}_n) := \text{GH}()$; // a feasible initial solution
2. $X^{\text{current}} \leftarrow X^* \leftarrow \bar{X}$;
3. $Iter \leftarrow 0$; $L \leftarrow \emptyset$;
4. $Z(X^{\text{current}}) \leftarrow Value^{\text{current}} \leftarrow \min_{1 \leq \pi \leq P} \left\{ \sum_{j \in J} v_j^\pi x_j^{\text{current}} \right\}$;

Main steps:

5. $\text{FSH}() \leftarrow \text{FALSE}$; // the solution is feasible
6. $\pi^* = \arg \min_{1 \leq \pi \leq P} \sum_{j \in J} v_j^\pi x_j^{\text{current}}$; // if many, select the first obtained one
7. $L \leftarrow \text{construct}()$; // generate the list of moves elite
8. $t \leftarrow 0$; // internal counter for the local search
9. **IF** $|L| \leq S$ **THEN**
 $\text{GS}(x^{\text{current}}, j^*)$;
 $x_{j^*}^{\text{current}} \leftarrow x_{j^*}^{\text{current}} \pm \epsilon_{j^*}$;
 $t \leftarrow t + 1$;
10. **IF** $|L| > S$ **THEN**
 $\text{RS}(x^{\text{current}}, j^*)$;
 $x_{j^*}^{\text{current}} \leftarrow x_{j^*}^{\text{current}} \pm \epsilon_{j^*}$;
 $t \leftarrow t + 1$;
11. $Value^{\text{current}} \leftarrow \min_{\pi} \left\{ \sum_{j \neq j^*} (v_j^\pi x_j^{\text{current}}) + v_{j^*}^\pi x_{j^*}^{\text{current}} \right\}$;
12. **IF** $(Value^{\text{current}} \geq Z(X^{\text{current}}))$ **and** $\text{FSH}()$ **THEN**
 $Z(X^{\text{current}}) \leftarrow Value^{\text{current}}$;
 $X^* \leftarrow X^{\text{current}}$;
13. **UPDATE** $L, Iter$;
14. **IF** $Iter \geq Max_Iter$ **THEN STOP**;
15. **EXIT** with solution X^* of value $Z(X^*)$;

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9 We detail the method as a chosen heuristic to guide the search towards the
10 desired regions of the trade-off area. It takes into account the localization of
11 the current solution in the objective space and the ability of each neighbourhood
12 exploration heuristic to achieve improvements on each of the individual objectives.
13 The local search process allows to run the selected heuristic to the current solution
14 for a certain number of iterations, or until no further improvement is possible.
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16 The strategy attempts to *restrict* the likelihood of generating solutions in
17 *crowded* regions of the trade-off surface and *enhance* the likelihood of generating
18 solutions in *under-populated* regions. The algorithm using this strategy, attempts
19 to perform a more “intelligent” path finding by applying the neighbourhood search
20 heuristic that is more likely to “guide” the solution in the desired direction.
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23 5. Computational results

24 We have run initial tests to analyze the computational performances of both
25 greedy heuristic GH and the developed approach CLS. The proposed algorithm is
26 coded in C++ and run on Sony VAIO Centrino Duo Core laptop (with 1.66 Ghz
27 and 1 GB/Go of Ram). Our computational study was conducted on 132 problem
28 instances of various sizes and densities. These test problem instances (detailed in
29 Table 0) are standard and their optimal solution values are not known.
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32 First of all, we notice that we did not succeed to have access to the same
33 problem instances tested by Taniguchi et al. (2008). We only had access to the
34 problems generator used by these authors who provided us with the code. We may
35 also notice that since we used a different computer with a different processor than
36 the one used by Taniguchi et al. (2008), the generator code generated problem
37 instances with different values but the design technique remains the same.
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40 The problem instances are generated as follows: each weight w_j and the ordinal
41 profit v_j^0 ($j = 1, \dots, n$) are randomly, uniformly and independently generated in
42 the integer interval $[1, 100]$. For a given scenario π , the value v_j^π of each item
43 j is uniform and random integer in the interval $[(1 - \sigma)v_j^0, (1 + \sigma)v_j^0]$, where
44 $\sigma \in \{0.3; 0.6; 0.9\}$ is a parameter to set up the correlation level between different
45 scenarios. For instance, more σ is close to 0, more the scenarios are strongly
46 correlated and consequently the profits are also strongly correlated. The knapsack
47 capacity is set to $C = \frac{1}{D} \sum_{1 \leq j \leq n} w_j$. D is constant to set.
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50 These instances are divided into three different groups depending on the σ
51 value. The first group ($\sigma = 0.9$) represents the “uncorrelated” or “likelihood un-
52 correlated” instances, the second one ($\sigma = 0.6$) contains the “weakly correlated”
53 instances and the last group ($\sigma = 0.3$) represents the “strongly correlated” in-
54 stances. In addition to this, we set D the availability parameter that indicates
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whether the capacity C is tight enough or not. Practically, we designed our computational test protocol by setting three types of sets S_1 , S_2 and S_3 .

Class	Inst.	n	P	Inst.	n	P
S_1	I01x.D	1000	40	I02x.D	1500	40
	I03x.D	2000	40	I04x.D	2500	40
	I05x.D	3000	40	I06x.D	3500	20
	I07x.D	4000	20	I08x.D	4500	20
	I09x.D	5000	20	I10x.D	6000	20
	I11x.D	10000	20			
S_2	I12x.D	2500	50	I13x.D	3000	50
	I14x.D	3500	50	I15x.D	5000	30
	I16x.D	8000	20			
S_3	I17x.D	500	60	I18x.D	1000	60
	I19x.D	1500	60	I20x.D	2000	60
	I21x.D	200	80	I22x.D	200	100

Table 0: Test problem instances details: x=u, w, s; D=2, 4

Here **u** ($\sigma = 0.9$) denotes uncorrelated problem instances, **w** ($\sigma = 0.6$) denotes weakly correlated problem instances and **s** ($\sigma = 0.3$) denotes strongly correlated problem instances.

On one hand, S_1 includes problem instances with relatively a big number of items n and a small number of scenarios P , S_2 contains problem instances with a relative medium n and P and on the other hand, S_3 contains problem instances with a relative small n and a big P .

5.1. The results summary

The purpose of this section is twofold: (i) to evaluate the performance of the GH and CLS and (ii) to determine the best trade-off between the running time and the used parameters for the cooperative CLS algorithm: the maximum number of iterations, the length of the list L and the penalty $\Phi(\cdot)$ function. Each parameter of CLS algorithm has been calibrated in order to obtain the best possible performances for each problem. The length of the tabu list L varies dynamically. Indeed, if P is the number of the different scenarios, then the parameter S is automatically and randomly taken in the interval $[\lfloor \sqrt{P} \rfloor + 25, \lfloor \sqrt{P} \rfloor + 35]$. Other parameters settings were necessary. We summarize them in the table 1.

Iter	α	τ	θ	$\Phi(\theta)$
4500 (<i>uncor</i>)	0.9 (<i>uncor</i>)	random($[\alpha\sqrt{n}, 2\alpha\sqrt{n}]$)	random[1,10]	$\frac{\theta}{(Iter)^{3/2}}$
2500 (<i>w_cor</i>)	1.1 (<i>w_cor</i>)			
1500 (<i>s_cor</i>)	1.3 (<i>s_cor</i>)			

Table 1: Parameters settings

In a preliminary experiment, we have solved the problem instances by considering several parameters of the algorithm. The details of the test results appear in Tables 2-10.

To analyze the method's performance, we first compute the upper bound UB (we used Dantzig (1957)'s upper bound) to determine : (i) the gap $\gamma := (UB - Z(X^*))$ between UB and the best obtained solution value $Z(X^*)$ and (ii) the average percentage deviation $\rho := 100 \cdot (\frac{UB - Z(X^*)}{Z(X^*)})$, since we don't know the optimal solution for the treated problem instances. We explain in the Upper bound computation Appendix how to compute UB.

In our numerical tests, we solved the same problem instances sets for each fixed parameter D , α , τ , $Iter$ and $\Phi(\cdot)$.

We may notice that we have improved these results by increasing the size of the problems regarding the number of the items as well as the number of the scenarios. CLS is able to process a very large number of variables within a few amount of cpu consuming time. In some cases, the algorithm is either able to lead to the optimal solution. We remark that for several problems of different sizes and types and belonging to S_1 and S_2 , the obtained solution $Z(X^*)$ is equal to the upper bound UB. In this case the algorithm has reached the optimal solution since it is the biggest feasible solution. The percentage of these obtained solutions depends on the set we have considered.

Furthermore, the algorithm is able to produce high quality near-optimal solutions for these sets ($n = 10000$ and $P = 100$ scenarios) within a relatively small cpu consuming time. While in Taniguchi et al. (2008), the approach was able to solve the problem with a number of items n less than 1000 and a number of scenarios P less than 30 and in less than a few seconds. Indeed, these solutions were obtained within 1% of relative errors. Our method has solved approximately larger problems and faster regarding the size of the treated problems. Comparing these obtained results to those of Taniguchi et al. (2008) and based on the same problems design, we can remark that our approach is able to lead to solutions of good quality for the big size problems (in term of number of items and scenarios).

On average, the first set S_1 (for uncorrelated, weakly and strongly correlated problems) gives the highest amount of reached optimal solutions by CLS and particularly for the uncorrelated set with 13 solutions out of 22 problems. The average running time is less than one minute and does not exceed 22 *sec*. The average relative percentage error is less than 1% and is located in the intervals [0,0.08%] for the uncorrelated, [0,0.221%] the weakly correlated and [0,0.073%] the strongly correlated problems. Furthermore, the algorithm is able to produce high quality near-optimal solutions for these sets ($n = 10000$ and $P = 40$ scenarios) within a relatively small cpu consuming time.

Inst.	GH	UB	$Z(X^*)$	γ	$\rho\%$	$CPU(s)$
I01u.2	40725	40740.00	40740	0	0	2.54
I01u.4	31328	31427.17	31403	24.17	0.076	27.2
I02u.2	61173	61218.00	61218	0	0	2.15
I02u.4	44147	44226.93	44207	19.93	0.045	2.78
I03u.2	82818	82830.00	82830	0	0	2.53
I03u.4	58872	61156.00	61156	0	0	3.31
I04u.2	104025	104044.34	104043	1.34	0.001	3.51
I04u.4	73877	73951.44	73911	40.44	0.054	3.78
I05u.2	123280	123297.72	123297	0.72	0	2.95
I05u.4	90817	90946.45	90939	7.45	0.080	3.72
I06u.2	146094	146118.00	146118	0	0	3.28
I06u.4	103424	103472.00	103451	21	0.020	3.81
I07u.2	163523	163529.00	163529	0	0	3.75
I07u.4	119007	119114.32	119088	26.32	0.022	3.86
I08u.2	187034	187040.00	187040	0	0	3.62
I08u.4	135952	136004.00	136004	0	0	3.77
I09u.2	208114	208136.00	208136	0	0	4.12
I09u.4	148808	148830.00	148830	0	0	4.19
I10u.2	256397	256416.23	256415	1.23	0	4.91
I10u.4	191406	191491.00	191491	0	0	5.15
I11u.2	428807	428849.00	428849	0	0	5.68
I11u.4	318754	318813.00	318813	0	0	7.73

Table 2: Uncorrelated set S_1 results

The second set S_2 of computational experiments (uncorrelated, weakly and strongly correlated) gives an overview of the algorithm behaviour regarding both the obtained upper bound and solution. The algorithm reached 8 optimal solutions out of 10 problems. For this set, the results present the same quality regarding the running cpu time which is less than 24 *sec* and the worst percentage deviation $\rho = 0.082\%$. The average relative percentage error is located in the intervals [0,0.033%] for the uncorrelated, [0,0.0083%] the weakly correlated

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Inst.	GH	UB	$Z(X^*)$	γ	$\rho\%$	$CPU(s)$
I01w.2	41095	41172.64	41133	39.64	0.096	3.21
I01w.4	28343	28446.76	28384	62.76	0.221	3.39
I02w.2	60256	60328.80	60291	37.8	0.062	3.54
I02w.4	44609	44654.00	44654	0	0	3.67
I03w.2	79872	79936.00	79896	40	0.050	3.81
I03w.4	58854	58864.16	58863	1.16	0.001	4.25
I04w.2	100545	100677.94	100566	111.94	0.111	4.43
I04w.4	71988	72108.92	72093	15.92	0.022	4.72
I05w.2	120991	121031.44	121030	1.44	0.001	5.37
I05w.4	86756	86837.35	86821	16.35	0.018	5.56
I06w.2	140795	14837.00	14837	0	0	5.84
I06w.4	99827	99861.00	99861	0	0	6.17
I07w.2	162739	162756.56	162755	1.56	0	6.28
I07w.4	113325	113367.00	113367	0	0	6.63
I08w.2	180524	180538.70	180538	0.7	0	6.72
I08w.4	130718	130801.93	130744	57.93	0.044	7.14
I09w.2	202081	202139.00	202099	40	0.019	7.23
I09w.4	143549	143806.18	143699	107.18	0.074	7.31
I10w.2	245519	245549.56	245547	2.56	0.001	7.42
I10w.4	178630	178658.00	178655	3.32	0.001	7.66
I11w.2	416328	416378.12	416371	7.12	0.001	8.23
I11w.4	303786	303944.56	303938	6.65	0.002	8.45

Table 3: Weakly correlated set S_1 results

and $[0, 0.082\%]$ the strongly correlated problems. Furthermore, this set seems to be more easy than the first set S_1 regarding the global results. Problems with medium size seems to be relatively easy to tackle.

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Inst.	GH	UB	$Z(X^*)$	γ	$\rho\%$	$CPU(s)$
I01s.2	41651	41681.89	41679	2.89	0.006	7.91
I01s.4	46909	46947.00	46947	0	0	8.24
I02s.2	63029	63083.28	63037	46.28	0.073	9.36
I02s.4	45331	45425.60	45396	29.6	0.065	9.76
I03s.2	82378	82426.34	82417	9.344	0.011	9.97
I03s.4	63661	63666.65	63664	2.65	0.004	10.32
I04s.2	102602	105720.75	105715	5.75	0.005	10.73
I04s.4	76435	76502.83	76481	21.83	0.028	11.36
I05s.2	126306	126368.68	126349	19.68	0.015	13.55
I05s.4	93579	93594.97	93589	5.97	0.006	13.99
I06s.2	151932	151952.00	151952	0	0	14.36
I06s.4	110497	110511.00	110511	0	0	14.82
I07s.2	168199	168211.00	168211	0	0	15.94
I07s.4	128169	128273.46	128223	50.46	0.039	16.21
I08s.2	191228	191242.20	191235	7.2	0.003	17.34
I08s.4	143825	143917.26	143885	32.26	0.022	17.78
I09s.2	191228	211728.00	211728	0	0	18.33
I09s.4	158404	158530.73	158477	53.73	0.033	18.54
I10s.2	247302	247873.84	247855	18.84	0.007	19.35
I10s.4	172299	172381.09	172379	2.09	0.001	19.66
I11s.2	405655	405709.71	405709	0.79	0	21.45
I11s.4	290357	290417.00	290417	0	0	21.93

Table 4: Strongly correlated set S_1 results

Inst.	GH	UB	$Z(X^*)$	γ	$\rho\%$	$CPU(s)$
I12u.2	102608	102608.00	102608	0	0	8.93
I12u.4	72767	72784.49	72782	2.49	0.003	9.12
I13u.2	122302	122315.16	122310	3.16	0.002	9.73
I13u.4	87527	88549.00	88549	0	0	11.17
I14u.2	143316	143328.00	143328	0	0	13.96
I14u.4	101067	101103.70	101070	33.7	0.033	14.23
I15u.2	217652	217652.00	217652	0	0	18.73
I15u.4	158841	158921.00	158876	45	0.028	19.34
I16u.2	343368	343399.00	343399	0	0	7.23
I16u.4	253303	253316.00	253316	0	0	8.56

Table 5: Uncorrelated set S_2 results

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Inst.	GH	UB	$Z(X^*)$	γ	$\rho\%$	$CPU(s)$
I12w.2	106592	106624.23	106623	1.23	0.001	9.35
I12w.4	79249	79286.63	79284	2.63	0.003	9.63
I13w.2	127545	127566.00	127566	0	0	10.72
I13w.4	95303	95401.82	95322	79.82	0.083	11.13
I14w.2	147832	147848.00	147848	0	0	12.58
I14w.4	108703	108786.82	108757	29.82	0.027	12.84
I15w.2	208762	208817.54	208775	42.54	0.020	13.57
I15w.4	151046	151060.00	151060	0	0	14.36
I16w.2	329312	329336.00	329336	0	0	9.92
I16w.4	242503	242574.45	242570	4.45	0.001	11.06

Table 6: Weakly correlated set S_2 results

Inst.	GH	UB	$Z(X^*)$	γ	$\rho\%$	$CPU(s)$
I12s.2	103173	103180.00	103180	0	0	11.24
I12s.4	75012	75099.24	75037	62.24	0.082	11.78
I13s.2	123190	123243.25	123218	25.25	0.020	12.43
I13s.4	88446	88545.20	88502	43.2	0.004	12.84
I14s.2	142592	142661.67	142631	30.67	0.002	13.41
I14s.4	103385	103386.00	103386	0	0	13.75
I15s.2	204113	204669.42	204662	7.42	0.003	15.62
I15s.4	145587	145717.32	145709	8.32	0.005	16.85
I16s.2	325303	325375.00	325375	0	0	20.14
I16s.4	231209	231300.81	231295	5.81	0.002	23.62

Table 7: Strongly correlated set S_2 results

The third set S_3 (uncorrelated, weakly and strongly correlated problems) shows that the obtained solutions are at most 5.306% far from the upper bound. No optimal solution was reached by the algorithm. Also all the obtained solutions are computed in a big cpu consuming time comparing to the first two sets. In addition, the reached solutions are obtained with a bigger gap than those obtained in the second set S_2 . The average relative percentage error is located in the intervals [0.648%, 4.490%] for the uncorrelated, [0.817%, 4.039%] the weakly correlated and [0.062, 5.306%] the strongly correlated problems. We remark that this set is a significantly hard set of instances (uncorrelated, weakly and strongly correlated) and the obtained solutions present less good quality than those of S_1 and S_2 .

Inst.	GH	UB	$Z(X^*)$	γ	$\rho\%$	$CPU(s)$
I17u.2	20317	21574.00	21347	227	1.063	3.92
I17u.4	15651	16538.00	16236	302	1.860	4.15
I18u.2	40837	42144.42	41873	271.42	0.648	4.53
I18u.4	30668	33476.80	32789	687.8	2.097	4.72
I19u.2	62433	66216.33	65129	1087.33	1.669	5.56
I19u.4	47511	49674.00	48973	701	1.431	5.13
I20u.2	85301	89940.66	88126	1814.66	2.059	5.42
I20u.4	60937	65922.50	65028	894.5	1.375	5.59
I21u.2	7904	8282.54	8125	157.54	1.938	6.46
I21u.4	5749	6078.92	5893	185.92	3.154	6.87
I22u.2	5736	5982.63	5843	139.63	2.389	7.15
I22u.4	3740	4103.34	3927	176.34	4.490	9.35

Table 8: Uncorrelated set S_3 results

Consequently, it is relatively easy to solve problem instances with a relatively big n and a small P or medium size of both n and P . While n is of small size and P of big size, the algorithm reaches less good quality solutions and what ever if they are uncorrelated, weakly or strongly correlated problems. In average, the algorithm is able to compute these solutions in an acceptable cpu consuming time. S_2 contains problem instances with n up to 8000 and $P = 50$ and contrary to the set S_3 , the obtained solutions are of better quality regarding the computed percentage deviation error ρ . We recall that S_3 contains problem instances with n up to 2000 and $P = 100$.

The interaction between scenarios becomes more complex and the algorithm needs more processing to compute the solution. The correlation between items is also a parameter which impacts the quality of the solution.

Also, it appears that if the total of iterations is a small one, it is better to not allow the intensification and diversification. We can explain this by the fact that the recorded informations are not representative enough of the solutions space.

Inst.	GH	UB	$Z(X^*)$	γ	$\rho\%$	$CPU(s)$
I17w.2	20589	21721.72	21223	498.72	2.349	5.61
I17w.4	14924	16534.00	15892	642	4.039	5.96
I18w.2	40675	43573.22	42527	1046.22	2.460	6.25
I18w.4	29051	32251.93	31175	1076.93	3.454	6.74
I19w.2	61013	64841.00	63794	1047	1.641	7.22
I19w.4	44024	47085.70	6154	931.7	2.020	7.53
I20w.2	82352	86474.00	85105	1369	1.610	7.96
I20w.4	58812	63230.50	61772	1458.5	2.361	8.84
I21w.2	7623	7796.23	7733	63.23	0.817	9.16
I21w.4	4645	4835.35	4735	100.35	2.119	10.27
I22w.2	5552	5776.49	5663	113.49	2.004	11.65
I22w.4	4020	4223.52	4140	83.52	2.017	12.56

Table 9: Weakly correlated set S_3 results

Inst.	GH	UB	$Z(X^*)$	γ	$\rho\%$	$CPU(s)$
I17s.2	20207	21544.00	21256	488	2.317	7.91
I17s.4	13966	14848.73	14217	631.73	4.443	8.24
I18s.2	40932	43941.00	42345	1596	3.769	8.63
I18s.4	28979	31025.59	30453	572.59	1.880	8.97
I19s.2	61799	63727.22	62236	1491.22	2.396	9.23
I19s.4	43197	46090.00	44987	1103	2.451	9.56
I20s.2	79741	85187.50	84254	933.5	1.107	9.92
I20s.4	58115	63514.00	62478	1036	1.658	10.46
I21s.2	7702	8150.74	8012	138.74	1.731	10.93
I21s.4	5189	5381.37	5378	3.37	0.062	11.56
I22s.2	5123	6152.00	5842	310	5.306	12.53
I22s.4	2912	3630.42	3625	5	0.137	13.58

Table 10: Strongly correlated set S_3 results

6. Conclusion

In this article, we have investigated the max-min binary knapsack with multiple scenarios (MSM²KP). Finding optimal solutions for the Multiple-Scenario Max-Min Knapsack Problem (MSM²KP) becomes a challenging and important issue due to its complex structure and possibly large size problems to tackle. An approximate algorithm might be an appropriate way to seek near-optimal solutions in an acceptable consuming cpu. To do so, we have developed a cooperative approximate algorithm. The approach is mainly based upon tabu search and a combination of two cooperative procedures; a generalized and a restricted local

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9 search. The principle of the method is to identify the scenario π^* realizing the
10 minimum total profit as a current solution and to tailor a neighborhood search
11 to improve the obtained solution. We have used a spreading search strategy and
12 designed a heuristic feasibility in order to improve the performance of the algo-
13 rithm. Computational results showed that the two cooperative procedures applied
14 together are able to generate high-quality solutions for the Multiple-Scenario Max-
15 Min Knapsack Problem, within a reasonable computing time.

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17 Our approach investigated MSM²KP under the framework of the max-min
18 optimization, where we maximize the minimum of all the objectives. As a result,
19 we were able to approximately solve the problem with n up to 10000 variables
20 and P up to 100 scenarios and in less than one minute. The obtained solutions
21 were usually within 0.221% for S_1 , 0.083% for S_2 and 5.306% for S_3 of relative
22 percentage deviation errors.
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24 In the local search, each neighborhood solution is evaluated at worst in $\mathcal{O}(nP^2)$
25 time and the neighborhood search is pruned heuristically by the neighbor list. Dur-
26 ing the search, infeasible solutions are allowed to be visited while the amount of
27 violation is penalized. The computational results on a representative benchmark
28 instances indicate that the proposed algorithm is efficient enough to tackle prob-
29 lem instances of a certain size (in term of number of items and scenarios). The
30 algorithm was able to reach 30 solutions which are equal to the computed upper
31 bound UB among a total of 132 problem instances.
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39
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Appendix: An upper bound computation

Herein we show how to compute UB . First, the MSM²KP is reduced to a single knapsack problem by considering the item j of profit $p_j(\lambda)$ as a convex combination of all the P alternative profits : $p_j(\lambda) := \sum_{\pi=1}^P \lambda^\pi v_j^\pi$; $\sum_{\pi} \lambda^\pi = 1$. Then the MSM²KP relaxation is formulated as :

$$(LP) \begin{cases} \text{Maximize} & Z(X) = \sum_{j \in J} p_j(\lambda) x_j \\ \text{Subject to} & \sum_{j \in J} w_j x_j \leq C \\ & 0 \leq x_j \leq 1, \text{ for } j = 1, \dots, n \end{cases}$$

To determine the parameters (λ^π) , we consider at once an auxiliary problem Aux as a single scenario MSM²KP :

$$(Aux) \begin{cases} \text{Maximize} & Z(X) = \sum_{j \in J} v_j^\pi x_j \\ \text{Subject to} & \sum_{j \in J} w_j x_j \leq C \\ & x_j \in \{0, 1\}, \text{ for } j = 1, \dots, n \end{cases}$$

Aux denotes a single knapsack problem KP. Its LP relaxation is given by Aux_{LP} :

$$(Aux_{LP}) \begin{cases} \text{Maximize} & Z(X) = \sum_{j \in J} v_j^\pi x_j \\ \text{Subject to} & \sum_{j \in J} w_j x_j \leq C \\ & 0 \leq x_j \leq 1, \text{ for } j = 1, \dots, n \end{cases}$$

It exists a critical item $\ell = \min\{k : \sum_{j=1}^k w_j > C\}$ such that $\sum_{j=1}^{\ell-1} w_j \leq C < \sum_{j=1}^{\ell} w_j$.

Then the Aux_{LP} optimal solution is given by:

$$x_j := \begin{cases} 1 & \text{if } j \leq \ell - 1 \\ \frac{c - \sum_{k=1}^{\ell-1} w_k}{w_\ell} & \text{if } j = \ell \\ 0 & \text{if } j \geq \ell + 1 \end{cases}$$

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10 And of objective value $Z_{AuxLP}(X^*) = \sum_{j=1}^{\ell-1} v_j^\pi + \frac{c - \sum_{j=1}^{\ell-1} w_j}{w_\ell} v_\ell^\pi$. It represents an
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12 upper bound for Aux.

13 The Dantzig Dantzig (1957) upper bound for Aux is then given by :

$$14 \quad UB^\pi := \sum_{j=1}^{\ell-1} v_j^\pi + \left[\frac{c - \sum_{j=1}^{\ell-1} w_j}{w_\ell} v_\ell^\pi \right].$$

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19 Then, we consider $\lambda^\pi := \frac{UB^\pi}{\sum_\pi UB^\pi}, \forall \pi = 1, \dots, P$. Knowing these parameters,
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21 it is then easy to compute the combined profits $p_j(\lambda) := \sum_{\pi=1}^P \left(\frac{UB^\pi}{\sum_\pi UB^\pi} \right) v_j^\pi$. We
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23 solve LP and the Dantzig upper bound UB for MSM²KP is simply the obtained
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25 objective value of LP.
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