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ON THE EXTENSION OF THE PRODUCT MODEL IN POLSAR PROCESSING FOR UNSUPERVISED CLASSIFICATION USING INFORMATION GEOMETRY OF COVARIANCE MATRICES

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ABSTRACT

We discuss in the paper the use of the Riemannian mean given by the differential geometric tools. This geometric mean is used in this paper for computing the centers of class in the polarimetric H/α unsupervised classification process. We can show that the centers of class will remain more stable during the iteration process, leading to a different interpretation of the H/α/A classification. This technique can be applied both on classical SCM and on Fixed Point covariance matrices. Used jointly with the Fixed Point CM estimate, this technique can give nice results when dealing with high resolution and highly textured polarimetric SAR images classification.

***Index Terms*—** SAR, Polarimetry, Classification, Estimation, Differential Geometry.

1. INTRODUCTION

The recently launched POLSAR systems are now capable of producing high quality polarimetric SAR images of the Earth surface under meter resolution. The additional polarimetric information allows the discrimination of different scattering mechanisms. In [1] was introduced the entropy-alpha-anisotropy (H/α/A) classification based on the eigenvalues of the polarimetric (or coherency) Covariance Matrix (CM). This CM is usually estimated, under homogeneous and Gaussian assumptions, with the well known Sample Covariance Matrix (SCM) which is Wishart distributed. Based on this decomposition, unsupervised classification of SAR images can be performed by an iterative algorithm based on a complex Wishart density function. It uses the H/α decomposition results to get an initial segmentation into clusters, then the K-means clustering is implemented by considering the polarimetric CM as the feature vectors. This technique needs however to derive by a Euclidian mean operation the averaged CM of each class and to compute by Wishart distance the mininal distance between each pixel CM and with all the class centers.

The decrease of the resolution cell leads however to more complicated effects like spatial heterogeneity or non Gaussianity. Hence, some areas (grass, trees, ...) usually considered as random backscattering mechanisms can become punctual deterministic backscattering mechanisms. The usual techniques of classification, detection, speckle filtering, used for decametric resolution have to be adapted to these new challenging problems. Recent studies have shown that the spatial heterogeneity of the observed scene allows for non-Gaussian clutter modelling. One commonly used fully polarimetric non-Gaussian clutter model is the compound Gaussian model: the polarimetric clutter information m-vector $\mathbf{k}$ is modeled as a SIRV (Spherically Invariant Random Vector), i.e. the product between the square root of a scalar random variable τ (called the texture) and an independent, zero mean, complex circular Gaussian random vector $\mathbf{z}$ (called the polarimetric speckle) and characterized by an unknown zero-mean and circular CM $\mathbf{M}$:

$$\mathbf{k} = \sqrt{\tau}\,\mathbf{z}, \tag{1}$$

In this model, τ can represent the spatial variation of the intensity of the wave vector $\mathbf{k}$ from pixel to pixel. All the polarimetric information (phase relationships within the wave vector) is then contained only in the normalized CM $\mathbf{M}$ ($\mathrm{Tr}(\mathbf{M}) = m$ where $\mathrm{Tr}(.)$ is the trace operator). Conditionally to a given pixel (equivalently to a given τ), the wave vector is then fully Gaussian distributed.

The aim of this proposed paper is twofold. Firstly, we propose in this paper to briefly recall original results obtained recently in [2] for the joint Maximum Likelihood estimation of the texture and the polarimetric CM. These results, based on the above Fixed Point CM estimate, allowed to derive a new distance for SIRV CM and to propose a new technique of speckle filtering in heterogeneous environment. Secondly, we introduce a metric-based mean for the space of positive-definite Hermitian CM. An emerging theory [3, 4, 5] allows to take into account the fact that Euclidian representation cannot describe the space of positive-definite Hermitian CM.

2. ESTIMATION OF THE COHERENCY COVARIANCE MATRIX

2.1. Gaussian homogeneous case

In homogeneous and Gaussian clutter assumption, the texture τ is assumed to be constant and the same for all pixels. In that case, the statistic of the secondary data is Gaussian and the CM can be estimated by Maximum Likelihood (ML) theory with a set of N secondary data $\mathbf{k}_n$, $n \in [1, N]$ as:

$$\widehat{\mathbf{M}}_{SCM} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{k}_n \, \mathbf{k}_n^H \, . \tag{2}$$

This unbiased CM estimate is Hermitian, Wishart-distributed with N degrees of freedom. Another well known POLSAR parameter maximizing the mean-to-variance ratio is the Polarimetric Whitening Filter (PWF) as $\text{PWF}_{\text{SCM}} = \mathbf{k}_n^H \, \widehat{\mathbf{M}}_{SCM}^{-1} \, \mathbf{k}_n$. For supervised and unsupervised POLSAR data clustering in the Gaussian case, a LR distance D_W between a given pixel i characterized by its SCM $\widehat{M}^i$ and a class center characterized by the SCM $\widehat{M}_\omega$ has been derived [6], [7] :

$$D_W \left(\widehat{\mathbf{M}}_i, \widehat{\mathbf{M}}_\omega \right) = \ln \frac{|\widehat{\mathbf{M}}_\omega|}{|\widehat{\mathbf{M}}_i|} + \text{Tr} \left(\widehat{\mathbf{M}}_\omega^{-1} \, \widehat{\mathbf{M}}_i \right) \, . \tag{3}$$

When dealing with heterogeneous and/or non-Gaussian clutter, the SCM is no longer robust. Under SIRV assumption, the SCM takes the form:

$$\widehat{\mathbf{M}}_{SCM} = \frac{1}{N} \sum_{n=1}^{N} \tau_n \, \mathbf{z}_n \, \mathbf{z}_n^H \, . \tag{4}$$

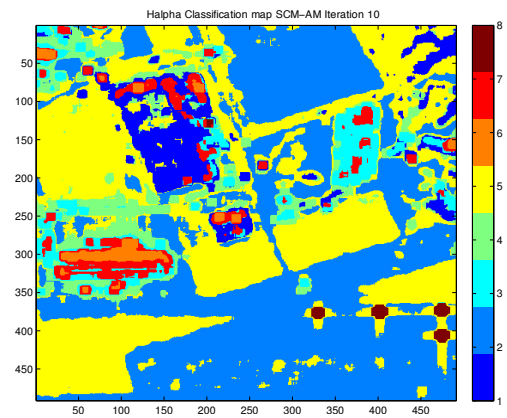
The presence of all textures τ_n of the N secondary wave vectors k_n makes this estimate strongly polluted.

Figure 1 presents the dataset used for the discussion (polarimetric SAR images of ONERA SETHI system in Bretigny in France) and the conventional H/α classification using SCM. We can see clearly that the conventional classification is strongly connected to the intensity variation of the RGB Pauli representation. It can be explained by the fact that the SCM (and so, the Wishart distance, the centers of class) is dependent on the intensity (or texture) present in the reference cells. Figure 2 shows, after 10 iterations, the locations of all the pixels in the H-α plan (for height classes). The distribution of pixels belonging to a given class seems very unordered.

2.2. Non-Gaussian and Heterogeneous Case

In the SIRV model, the CM is generally an unknown parameter which can be estimated from Maximum Likelihood (ML) Theory. In [8], Gini et al. derived the ML estimate $\widehat{\mathbf{M}}_{FP}$ of the covariance matrix $\mathbf{M}$ for deterministic texture, which is

(a) ONERA SETHI polarimetric SAR Image in RGB Pauli basis



(b) Conventional H/α classification after 10 iterations

Fig. 1. Comparison between Pauli representation and conventional unsupervised H/α classification after 10 iterations.

the solution of the following equation:

$$\widehat{\mathbf{M}}_{FP} = f(\mathbf{M}) = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_i \, \mathbf{k}_i^H}{\mathbf{k}_i^H \, \widehat{\mathbf{M}}_{FP}^{-1} \, \mathbf{k}_i} \, . \tag{5}$$

This approach has been used in [9] by Conte et al. to derive a recursive algorithm for estimating the solution matrix $\mathbf{M}_{FP}$ called the Fixed Point Covariance matrix. It has been shown in [8] and [9] that the estimation scheme from (5), developed under the deterministic texture case, yields also an Approximate ML (AML) estimator under stochastic texture hypothesis. This study has been completed by the work of Pascal et al. [10], which recently established the existence and the uniqueness of the Fixed Point estimate, as well as the convergence of the recursive algorithm whatever the initialization. This matrix estimate is shown to be robust, unbiased, consistent and asymptotically distributed as Gaussian PDF [11]. One could suppose legitimately that it has the same behavior as a Wishart distributed matrix with $m\,N/(m+1)$ degrees of freedom. The generalized ML estimator for the τ_n texture for

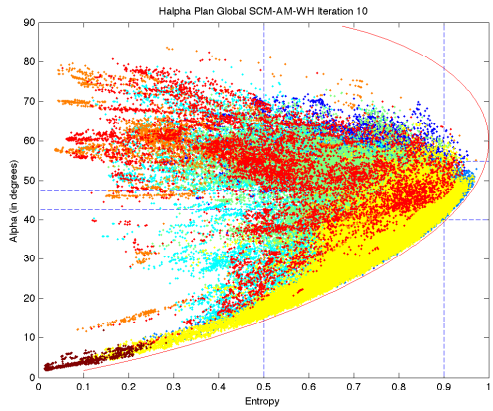


Fig. 2. Location of the pixels in the H/α plan after 10 iterations in the Gaussian case.

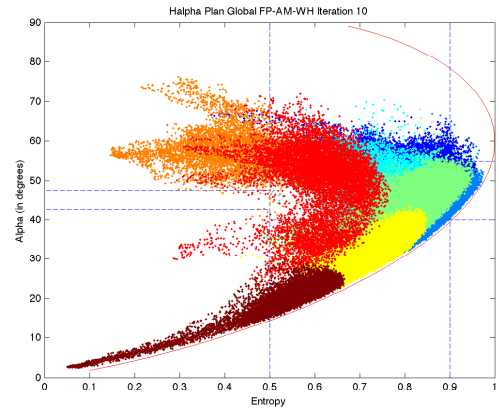


Fig. 3. Location of the pixel in the H/α plan after 10 iterations in the SIRV case for the classical mean.

the primary data $\mathbf{k}_n$ is given by:

$$\widehat{\tau}_n = \frac{\mathbf{k}_n^H \, \widehat{\mathbf{M}}_{FP}^{-1} \, \mathbf{k}_n}{m} \, , \tag{6}$$

which is proportional to the SIRV Polarimetric Whitening Filter $\text{PWF}_{\text{SIRV}} = \mathbf{k}_n^H \, \widehat{\mathbf{M}}_{FP}^{-1} \, \mathbf{k}_n$. The role of the matrix $\widehat{\mathbf{M}}_{FP}$ is here to whiten the possible correlation existing within the polarimetric channels without being polluted by the power of the secondary data used to estimate this matrix. A new LR SIRV distance D_S between two FP CM $\widehat{\mathbf{M}}_i$ and $\widehat{\mathbf{M}}_\omega$ has also been derived in [2]:

$$D_S\left(\widehat{\mathbf{M}}_i, \widehat{\mathbf{M}}_\omega\right) = \ln \frac{|\widehat{\mathbf{M}}_\omega|}{|\widehat{\mathbf{M}}_i|} + \frac{m}{N} \sum_{n=1}^{N} \frac{\mathbf{k}_n^H \, \widehat{\mathbf{M}}_\omega^{-1} \, \mathbf{k}_n}{\mathbf{k}_n^H \, \widehat{\mathbf{M}}_i^{-1} \, \mathbf{k}_n} \, . \tag{7}$$

This equation depends on the secondary data $(\mathbf{k}_n)_{n=1,N}$ but can nicely be simplified as:

$$D_S\left(\widehat{\mathbf{M}}_i, \widehat{\mathbf{M}}_\omega\right) = D_W\left(\widehat{\mathbf{M}}_i, \widehat{\mathbf{M}}_\omega\right) \, . \tag{8}$$

This result shows that, again, the FP CM could be considered as a Wishart CM. Figure 3 shows, after 10 iterations, the locations of all the pixels in the H-α plane (for eight classes). The distribution of pixels belonging to a given class seems much better ordered.

3. RIEMANNIAN DISTANCE

Rigorously, the averaged CM $\mathbf{M}_{\omega_l}$ (SCM or Fixed Point) of a H/α/A cluster l cannot be computed with the Euclidean metric, i.e. usual arithmetic mean as $\mathbf{M}_{\omega_l} = 1/K \sum_{k=1}^{K} \mathbf{M}_k^l$ where $\mathbf{M}_k^l$, $k \in [1, K]$ are the K CM of all pixels belonging to the class ω_l in the H/α plan. It is well known that after a few iterations of the unsupervised classification, all the class centers move significantly within the H/α plane, leading to a more difficult physical interpretation of the final classification. The mean associated with the Riemannian metric corresponds to the geometric mean:

$$\mathbf{M}_{\omega_l} = \underset{\mathbf{M}_\omega \in \mathcal{P}(m)}{\arg\min} \sum_{k=1}^{K} \left|\left| \log\left(\mathbf{M}_k^{l\,-1} \, \mathbf{M}_\omega\right) \right|\right|_F^2 \, , \tag{9}$$

where $||.||_F$ stands for the Frobenius norm and $\mathcal{P}(m)$ is the set of the Hermitian definite-positive CM of size m. The solution $\mathbf{M}_{\omega_l}$ can easily be found using a simple gradient algorithm given by the following iterative procedure:

$$\mathbf{M}_k = \mathbf{M}_{k-1}^{H/2} \, \exp\left(\epsilon \sum_{i=1}^{K} \log\left(\mathbf{M}_{k-1}^{-H/2} \, \mathbf{M}_i^l \, \mathbf{M}_{k-1}^{-1/2}\right)\right) \mathbf{M}_{k-1}^{1/2} \, ,$$

where $k \in \mathbb{N}^*$, where ϵ controls the speed of the gradient descent and where $\mathbf{M}_0 = \mathbf{M}_1^l \left(\mathbf{M}_1^{l\,-1} \, \mathbf{M}_2^l\right)^{1/2}$ is, for example, the geometrical mean of the first two matrices.

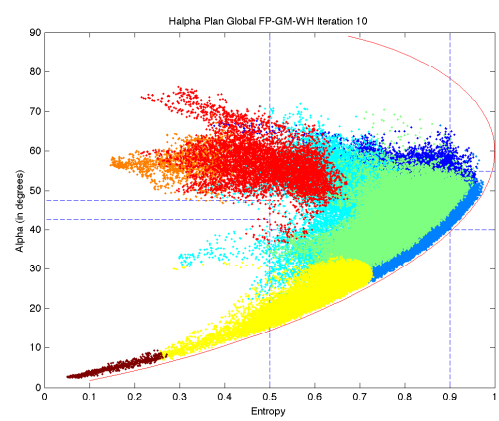
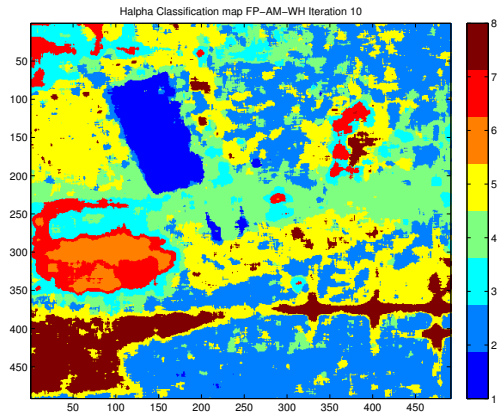


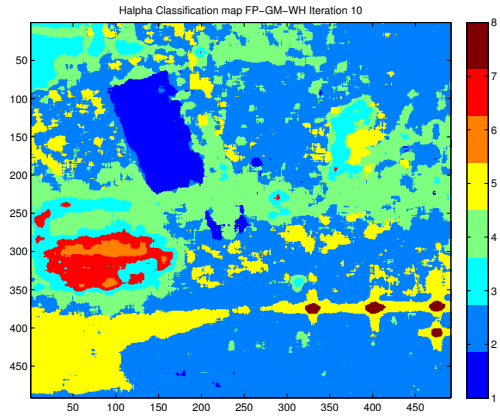
Fig. 4. Location of the pixels in the H/α plan after 10 iterations in the SIRV case for the geometrical mean.

Figure 4 shows, after 10 iterations, the locations of all the pixels in the H-α plan (for eight classes). Figure 5 repre-

sents a comparison between the unsupervised H/α classification results obtained with classical Arithmetical mean and the Riemannian mean (after 10 iterations) with the Fixed Point Covariance matrices.



(a) Fixed Point and Arithmetical Mean



(b) Fixed Point and Geometrical Mean

Fig. 5. Comparison of unsupervised H/α classifications after 10 iterations for arithmetical and geometrical mean

4. CONCLUSION

After recalling some recent techniques useful for modeling the heterogeneity and/or the non-Gaussian behavior of the polarimetric SAR images, this paper[1] has presented the use of a new technique, based on the differential geometry, for computing the barycenter of K Hermitian CM. This mean is used in the H/α unsupervised process and allows, jointly with the Fixed Point Covariance Matrix estimate, to propose a new classification scheme. The latter seems promising (much more stable location in the H/α plan) but needs however to be validated with ground truth.

[1]The authors would like to thank the DGA for supporting this research.

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