Kolmogorov cascade and equipartition of kinetic energy in numerical simulation of Superfluid turbulence

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Abstract. The turbulence of a superfluid is investigated by direct numerical simulations at finite temperature and high Reynolds numbers using the continuous model. The superfluid component is described by the Euler equation while the normal fluid component is described by the Navier-Stokes equation, both being coupled by mutual friction. In the high temperature limit, the Kolmogorov cascade is recovered, as expected from previous numerical and experimental studies. As the temperature decreases, the Kolmogorov cascade remains present at large scales while, at small scales, the system evolves towards a statistical equipartition of kinetic energy among spectral modes.

1. Introduction

What is the velocity spectrum of a turbulent superfluid such as ^{4}He ?

On the one hand, one can argue that superfluids should undergo the same inertial instability as any other fluids. Therefore, a Kolmogorov $k^{-5/3}$ spectrum is expected as a signature of a cascade of kinetic energy from low to large wave-numbers. Two independent experimental studies actually report such a scaling Maurer & Tabeling (1998); Salort *et al.* (2010).

On the other hand, one can argue that the energy distribution across spectral modes should tend towards equipartition since there is no viscous dissipation: kinetic energy piles up and spreads equally through excited modes. A k^2 energy spectrum is expected. However, in real systems (at finite temperature) there is always some sort of energy dissipation, for instance due to "mutual friction" which result from interaction between vortices and the residual phonons/rotons bath in ^4He. It is therefore unclear which picture is relevant to *real* superfluids. As we argue in a recent contribution [Salort *et al.* (2011)], the answer is eventually composite: a cascade operates at low wavenumbers while equipartition organizes at large wavenumbers.

2. Approach & Results

We simulated the continuous equations for superfluid at finite temperature by DNS at resolution up to 512^3. These equations, obtained by coarse-graining the superfluid dynamics, smooth out the atomic-core vortex singularities. Nevertheless, a numerical truncation at finite scale is introduced to account for the quantization of vortex singularities. This procedure is validated by comparisons with experimental data obtained in ^{4}He in similar conditions. In particular, to

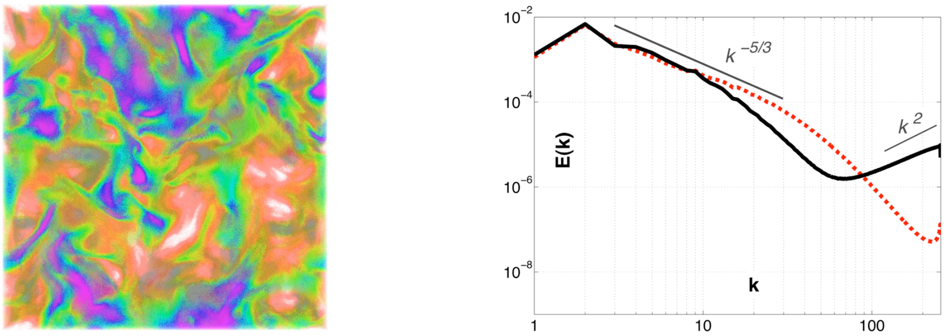

Figure 1. Left : Superfluid kinetic energy field exhibits large-scale coherency. Right : Superfluid velocity power spectrum at high temperature (dash line, $T = 2.1565\,K$) and low temperature (solid line, $T = 1.15\,K$) .

the best of our knowledge, our simulations are the first to account for the absolute value of the density of vortex lines at a given Reynolds number, without any adjustable parameter [Salort *et al.* (2011)]. Contrary to previous simulations at high Reynolds numbers [Roche *et al.* (2009); Tchoufag & Sagaut (2010)], no additional *artificial* dissipation is added to the HVBK model.

In the low temperature limit ($T = 1.15\ K$) for which the dissipation is still possible but difficult, we found evidence that the system evolves to a stationary state that exhibits a Kolmogorov scaling $k^{-5/3}$ at large scales and a nearly k^2 scaling at small scales [Salort *et al.* (2011)]. As argued in this reference, the strong increase of the superfluid spectrum near the quantum cut-off can be seen as a response of the system to favour dissipation through mutual friction (between the superfluid and nomal components) at small scales and reach a stationary state.

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References

Maurer, J. & Tabeling, P. 1998 Local investigation of superfluid turbulence. *Europhys. Lett.* **43** (1), 29–34.

Roche, P.-E., Barenghi, C. F. & Lévêque, E. 2009 Quantum turbulence at finite temperature: the two-fluids cascade. *EPL* **87**, 54006.

Salort, J., Baudet, C., Castaing, B., Chabaud, B., Daviaud, F., Didelot, T., Diribarne, P., Dubrulle, B., Gagne, Y., Gauthier, F., Girard, A., Hébral, B., Rousset, B., Thibault, P. & Roche, P.-E. 2010 Turbulent velocity spectra in superfluid flows. *Phys. Fluids* **22**, 125102.

Salort, J., Roche, P.-E. & Lévêque, E. 2011 Mesoscale equipartition of kinetic energy in quantum turbulence. *Europhys. Lett.* **94**, 24001.

Tchoufag, J. & Sagaut, P. 2010 Eddy damped quasinormal Markovian simulations of superfluid turbulence in helium II. *Phys. Fluids* **22**, 125103.