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# The ultimate regime of convection over uneven plates

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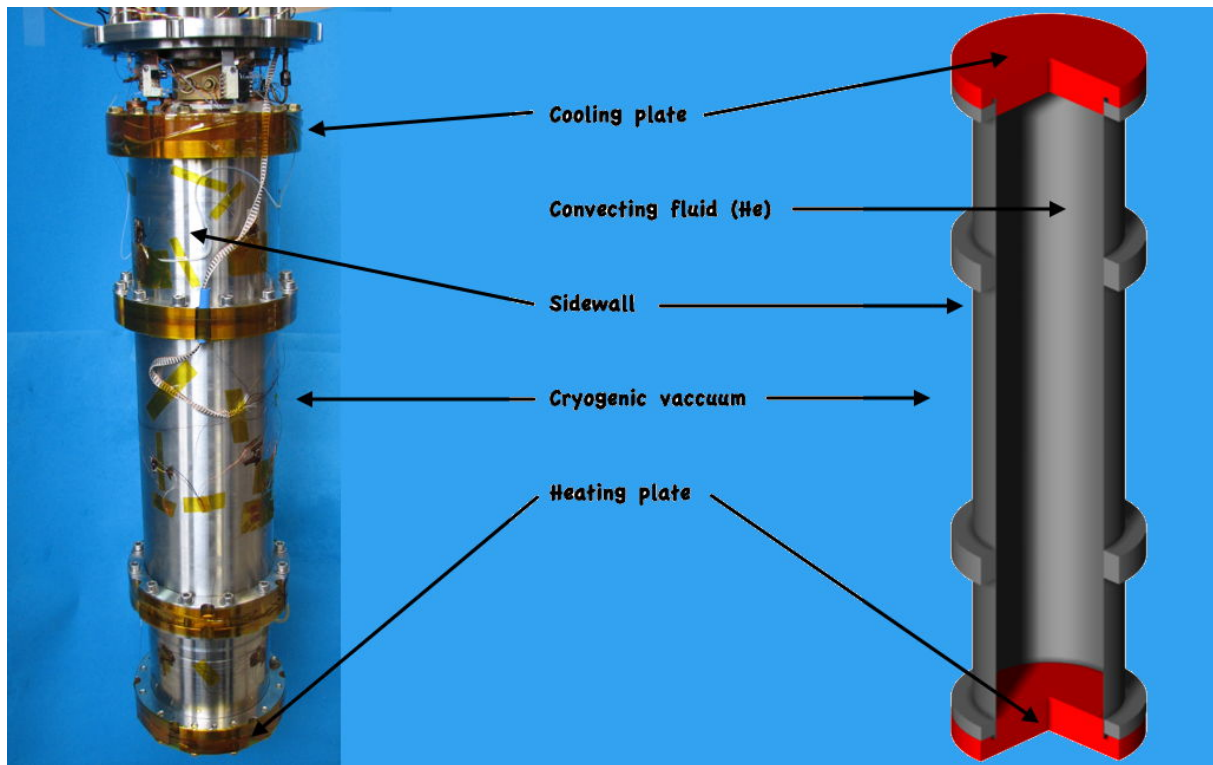
**Abstract.** A new regime of convection, with a unprecedented heat transfer efficiency ( $Nu \sim Ra^{0.38}$ ) has been observed in Grenoble in 1996 and named the *Ultimate Regime*. Following the prediction of Kraichnan in 1962, this regime has been interpreted as the asymptotic regime of convection, expected in the limit of very high thermal forcing ( $Ra \rightarrow \infty$ ). A systematic study of the experimental conditions for the triggering of the *Ultimate Regime* has been conducted over the last decade. It revealed that the transition threshold is dependent on an unknown fixed length scale of the convection cells, in addition to the expected dependence versus the cell height. The cell diameter is a good candidate for this unknown scale and the observed sensitivity to the sidewall conditions tends to support this view. In the present study, we test an alternative candidate length scale associated with flatness defects of the heating and cooling plates. This hypothesis was tested by measuring the heat transfer in an elongated cell (aspect ratio 0.23) before and after introducing a controlled alteration of its surface flatness. Four smooth depressions have been formed on each plate, and their depth is of the order of the thermal boundary thickness at transition. The measurements show that such defect has no significant influence on the transition to the *Ultimate Regime*.

## 1. Introduction

A common model system to investigate thermal convection is the Rayleigh-Bénard cell. Inside a RB-convection cell, flow is driven by temperature difference between the top and bottom plates. Such an experiment is parameterized by a few dimensionless numbers. The Rayleigh number  $Ra = g\alpha\Delta h^3/\nu\kappa$  characterizes the temperature difference between the top and bottom plates  $\Delta = T_{\text{bottom}} - T_{\text{top}}$ . The Prandtl number  $Pr = \nu/\kappa$  specifies the molecular transport properties of the investigated flow. While the aspect ratio  $\Gamma = d/h$  describes the geometrical conditions of the cylindrical RB-cell.  $g$  is the gravity.  $\nu$  and  $\kappa$  are the kinematic viscosity and thermal diffusivity.  $\alpha$  is the thermal expansion coefficient.  $h$  and  $d$  are cell height and cell diameter. For given  $Ra$ ,  $Pr$  and  $\Gamma$ , the system response can be characterized by the Nusselt number  $Nu = \dot{Q}_{\text{convection}}/\dot{Q}_{\text{diffusion}}$ , which is the convective heat transport normalized by the diffusive heat transport that would settle in the absence of convection.

Nearly fifteen years ago, a transition to an enhanced heat transfer, compared to the well established hard turbulence  $Nu \sim Ra^{1/3}$ -scaling, was reported at high  $Ra$  (Chavanne *et al.*, 1997). This observation was interpreted as the asymptotic regime of convection, predicted by Kraichnan (1962) and named *Ultimate Regime*. Over the recent years, intensive experimental efforts were made to understand this *Ultimate Regime* (Roche *et al.* (2010) and reference within). These investigations have shown that for a fixed  $Pr$  the triggering of the *Ultimate Regime*

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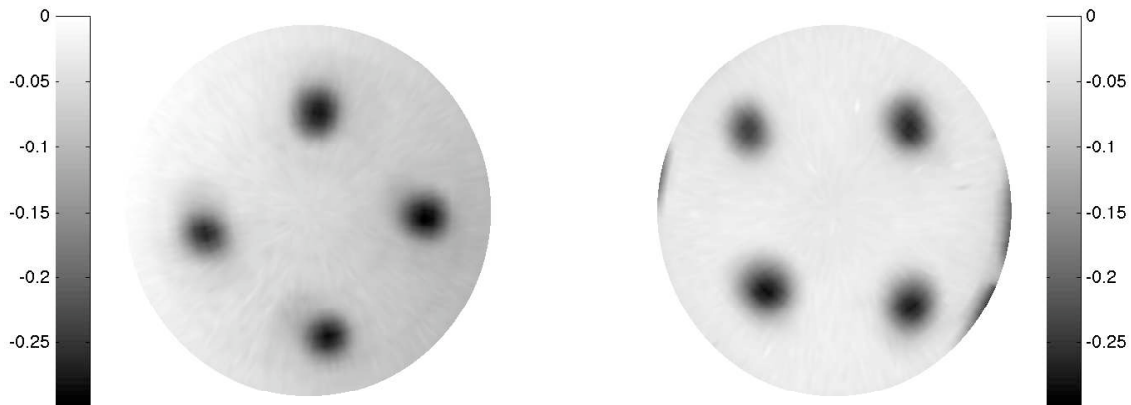
**Figure 1.** Photograph and sketch of the elongated Rayleigh-Bénard cell used in this study.

occurs at different  $Ra$  in cells of different heights but similar diameter. The transition  $Ra$  scales like  $\sim \Gamma^{-3}$  for  $\Gamma$  of order 1, which suggests that a length scale common to all cells controls the transition (Roche *et al.*, 2010). The systematic experimental study which evidenced the existence of this length scale also showed that it cannot be associated with deviations from the Boussinesq approximation, from details on the sidewall nor with a  $Pr$  dependence (Roche *et al.*, 2010). The cryogenic environment and protocol of this previous study allowed to exclude length scales which would be related with residual heat leak (shown to be negligible regarding to cryogenic vacuum isolation of the suspended cell, and low black body radiation around 6 K), plate thermal response static/dynamic cut-off (due to high conductivity and low thermal inertia of cryogenic copper) or cell filling procedure (the cell is operated after being closed by a micro valve located close to the cell and isolated by a thermal siphon). Possible remaining fixed length scales are the cell diameter and length scales related with a residual defect in the flatness of the plates. Such defect on flatness could possibly favour the transition if the thermal boundary layer in the vicinity of the plates is thin enough to feel these defects.

In the present study, we compare the heat transfer in an elongated cylindrical cell ( $\Gamma = 0.23$ , figure 1) before and after alteration of the top and bottom thermal plates. For reference, we point that recent numerical simulations have been done in this elongated geometry (Stevens *et al.*, 2011).

## 2. Experiment description

The diameter of the cells is 10 cm with a height of 43 cm. The plates are made of annealed OFHC copper and are 2.5 cm thick. The conductivity of the plates is 1090 W/mK at 4.2 K and was measured *in situ*. As a first experiment we measured the heat transfer through a cell with very smooth and even plates. First measurements have already been published in (Roche *et al.*,



**Figure 2.** Measurements of the flatness of the top (left) and bottom (left) plates. The color scale is in [mm] and negative values correspond to cavities.

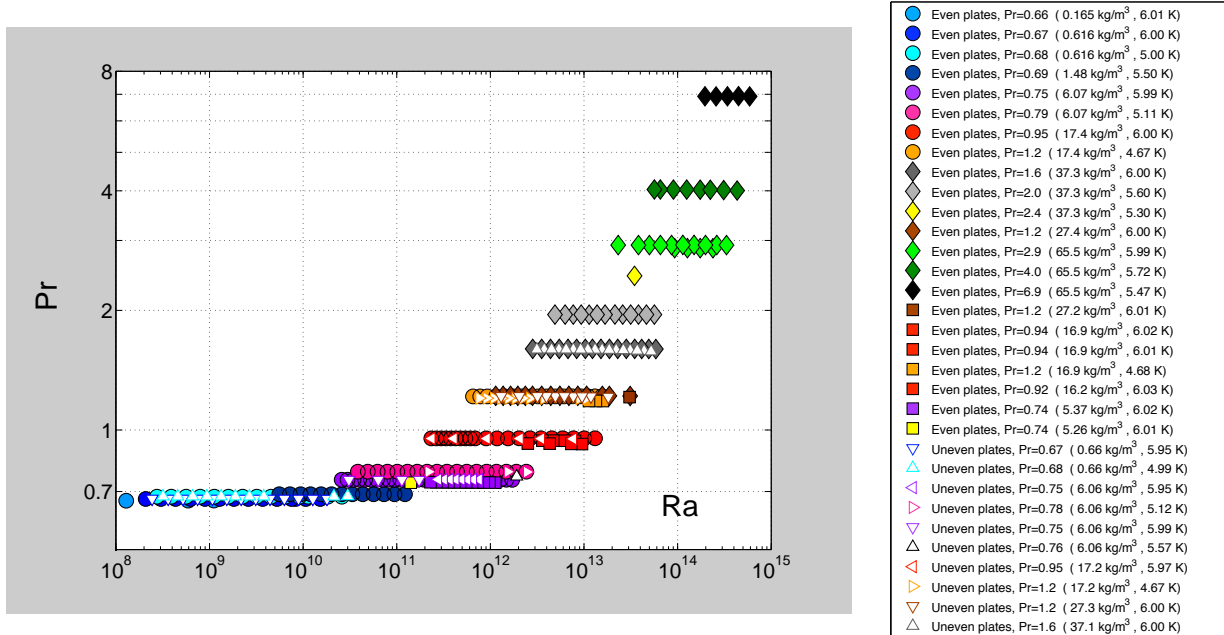
2010) but are extended in the present paper. We then machined these bottom and top plates to alter both, their roughness and flatness. The measured roughness of the smooth plates was between  $ra = 0.15 \mu\text{m}$  and  $ra = 1 \mu\text{m}$  and are planar within  $\pm 4 \mu\text{m}$  except for a  $15 \mu\text{m}$  bump at one point of the perimeter of the bottom plate.

The alteration of the plates consisted in digging four  $250 \mu\text{m}$  deep cavities (figure 2) and sandblasting the surface with glass spheres. The planicity defect has the same characteristic size as the thermal boundary layer thickness  $\lambda_\theta \simeq h/2Nu$  at the high  $Ra$  of interest ( $Ra \sim 2 \cdot 10^{12}$ ). The sandblasting results in an enhanced roughness of the plate surfaces, which is  $ra = (2.95 \pm 0.10) \mu\text{m}$ .

The sidewall is made of seamless stainless steel and has thickness of  $550 \mu\text{m}$ . It has a measured thermal conductance of  $163 \mu\text{W/K}$  at  $4.7 \text{K}$ . The influence of the sidewall conduction was taken into account using the analytical correction described in (Roche *et al.*, 2001) and verified in (Verzicco, 2002). The impact of the sidewall conduction is negligible at very high  $Ra$ . The assembly of the plate-sidewall connection is optimized to prevent “corner” thermal effects, as described in (Gauthier *et al.*, 2007). The measurement protocol is described in (Roche *et al.*, 2010) and its main points are recalled below.

“The top plate is cooled by a helium bath at  $4.2 \text{K}$  through a calibrated thermal resistance (several  $\text{KW}^{-1}$  at  $6 \text{K}$ ). The temperature is regulated by a PID controller. A constant and distributed Joule heating  $P$  is delivered on the bottom plate. The heat leak from the bottom plate to the surroundings has been measured in situ in few experiments ( $\simeq 200 \text{nW}$  at  $4.7 \text{K}$ ) and it is three to four decades smaller than the lowest heating applied on the bottom plate to generate convection. This leak is mainly due to the radiative transfer to the environment at  $4.2 \text{K}$ . This excellent thermal control is one of the advantages of our cryogenic environment over room temperature convection experiments, along with the excellent thermal properties of the Cu, which provide isothermal plates to the highest heat flux (Verzicco, 2004).

The temperature difference  $\Delta$  between the plates is measured with an accuracy down to  $0.1 \text{mK}$ , thanks to specifically designed thermocouples. For comparison, the smallest  $\Delta$  in our experiments are about  $10 \text{mK}$ . The temperature of each plate is measured with various Ge thermistances. Their calibration is checked in situ against the critical temperature  $T_c$  of the fluid with a resolution of  $0.2 \text{mK}$ . To avoid a common misunderstanding, we stress that all the  $Nu(Ra)$  measurements are done far away



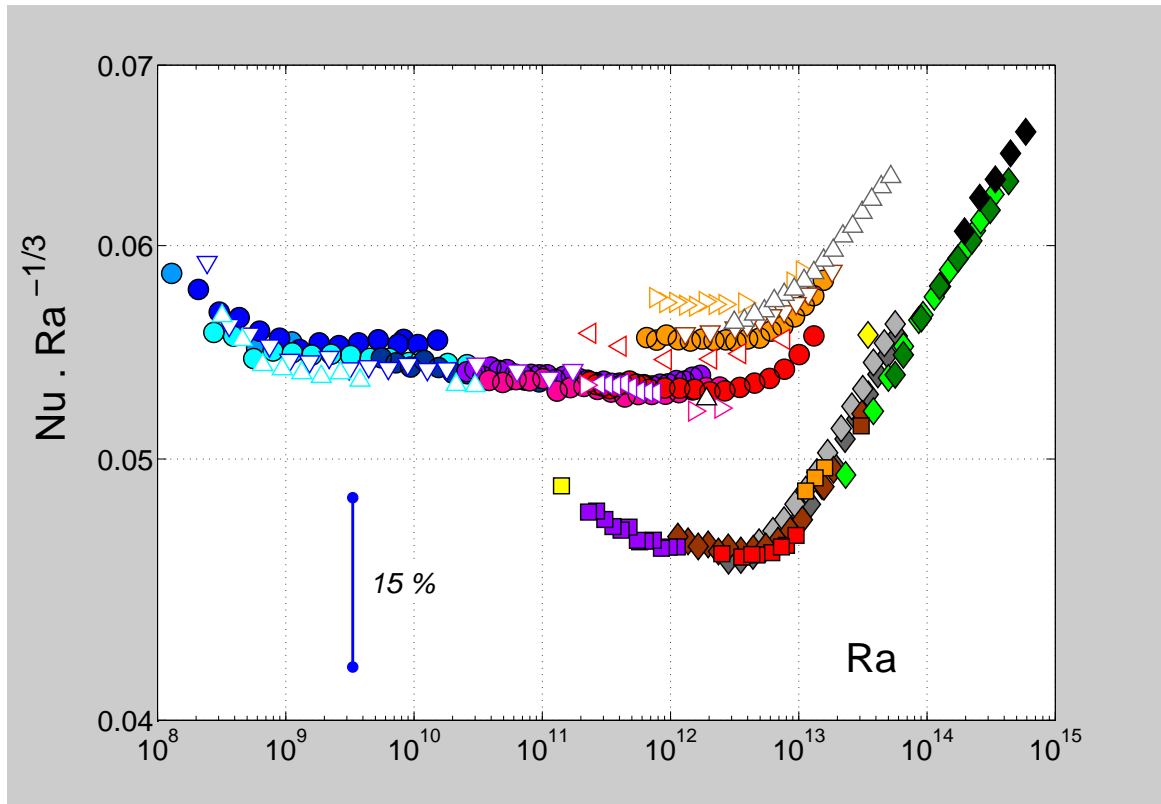
**Figure 3.** Parameter space of the  $Pr$  versus  $Ra$  for an elongated cell with even plates (filled symbols) and the same cell after making the plates uneven sand-blasted plates and four cavities on each (open symbols).

from the critical point, as argued in the appendix [of reference (Roche *et al.*, 2010)]. The critical point is simply used here as a thermodynamical reference to cross-check temperature calibration.” [Roche, P.-E.: On the triggering of the Ultimate Regime of Convection, *New J. Phys.* 12 (2011), p.8]

### 3. Results and Conclusion

The investigated  $Pr$ - $Ra$  parameter space is shown in figure 3 and the heat transfer measurements are plotted in figure 4, using the same symbols. The explanation of the symbols (see figure 3 corresponds to the chronological order of the data acquisition. The measured  $Nu$  and the corresponding  $Ra$  and  $Pr$  are listed in Appendix A for the reference cell and in Appendix B for the altered cell, including the density  $\rho$  in  $\text{kg/m}^3$ , the mean temperature  $T$  in K and the temperature difference between the bottom and the top plates  $\Delta$  in mK.

A bi-valued  $Nu$  is observed with typical 14% difference between the upper and the lower sets of measurement. Such a bi-stability of heat transfer at high  $Ra$ , already reported in a cell with a larger  $\Gamma = 0.5$  (Roche *et al.*, 2002), is interpreted as a result of the bi-stability of the large scale circulation in the flow (Roche *et al.*, 2002; Verzicco & Camussi, 2003). But we have to point out that a direct experimental proof of this interpretation is missing. Regardless of the characteristic of the large scale circulation, both the upper and lower subset of  $Nu$  experience the transition (defined as a significant change of  $Nu(Ra)$  scaling) around  $Ra \simeq 2 \cdot 10^{12}$  leading to a scaling  $Nu \sim Ra^{0.42}$  on the high  $Ra$  side. For each experimental condition, the cell seemed “locked” either on the upper or the lower branch. After the system jumped into the lower branch, it stayed there until the end of the experiment. We note that the lower branch seemed to reconnect smoothly to the upper branch, although further investigations would be need to confirm this point. While an observation of bi-stability could only be seen on the reference cell with smooth and even plates, the transition to the *Ultimate Regime* occurs in both cells, which



**Figure 4.** Compensated heat transfer  $Nu/Ra^{1/3}$  versus  $Ra$  corresponding to the  $Ra - Pr$  parameter space shown in figure 3. Filled symbols correspond to the cell with even plates and open symbols to the same cell with uneven plates. The  $Nu(Ra)$ -scaling above  $Ra \sim 10^{13}$  can be fitted as  $Nu \sim Ra^{0.42}$

is the main result of this paper. We cannot exclude that one of the cell is effected by a residual tilt, which might be cause or prevent bi-stability.

As a main conclusion, planeity defects on the plates of Rayleigh-Bénard cell seem to have little impact of the occurence of a transition to the *Ultimate Regime* of convection, at least when the typical depth of these defects is comparable to the thickness of the thermal boundary layers. Furthermore at a given  $Pr$  the  $Nu \sim Ra^{1/3}$ -regime is evidenced very clearly in this elongated cell, suggesting that the confinement by the sidewall “breaks” the long range correlation which prevents a interaction between the plates. This suggest that small aspect ratio cells are adequate to investigate the  $Nu \sim Ra^{1/3}$  scaling regime.

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## Appendix A. Data - Cigar cell with even plates ( $\Gamma = 0.23$ )

$Ra$	$Nu$	$Pr$	kg/m <sup>3</sup>	$T$ [K]	$\Delta$ [mK]
$1.290 \times 10^8$	$2.960 \times 10^1$	0.66	0.165	6.053	114.9
$5.920 \times 10^8$	$4.616 \times 10^1$	0.66	0.165	6.002	512.5
$1.115 \times 10^9$	$5.730 \times 10^1$	0.66	0.165	5.973	952.1
$8.306 \times 10^9$	$1.122 \times 10^2$	0.67	0.616	5.998	507.2
$5.266 \times 10^9$	$9.632 \times 10^1$	0.67	0.616	6.001	322.2
$3.702 \times 10^9$	$8.541 \times 10^1$	0.67	0.616	6.001	226.8
$2.602 \times 10^9$	$7.577 \times 10^1$	0.67	0.616	6.001	159.7
$1.526 \times 10^{10}$	$1.373 \times 10^2$	0.67	0.616	6.003	933.4
$1.073 \times 10^{10}$	$1.216 \times 10^2$	0.67	0.616	6.002	656.0
$7.527 \times 10^9$	$1.081 \times 10^2$	0.67	0.616	6.001	460.2
$6.246 \times 10^8$	$4.768 \times 10^1$	0.67	0.616	6.001	39.0
$4.340 \times 10^8$	$4.272 \times 10^1$	0.67	0.616	6.001	27.3
$3.026 \times 10^8$	$3.806 \times 10^1$	0.67	0.616	6.001	19.3
$2.085 \times 10^8$	$3.427 \times 10^1$	0.67	0.616	6.001	13.5
$1.823 \times 10^9$	$6.746 \times 10^1$	0.67	0.616	6.001	112.1
$1.284 \times 10^9$	$5.967 \times 10^1$	0.67	0.616	6.001	79.2
$8.949 \times 10^8$	$5.342 \times 10^1$	0.67	0.616	6.001	55.4
$7.946 \times 10^8$	$5.079 \times 10^1$	0.68	0.616	4.998	30.1
$5.629 \times 10^8$	$4.498 \times 10^1$	0.68	0.616	4.999	21.5
$3.914 \times 10^8$	$4.062 \times 10^1$	0.68	0.616	4.999	15.2
$2.749 \times 10^8$	$3.622 \times 10^1$	0.68	0.616	4.999	10.9
$2.597 \times 10^{10}$	$1.605 \times 10^2$	0.68	0.616	4.999	956.5
$1.831 \times 10^{10}$	$1.430 \times 10^2$	0.68	0.616	4.997	674.2
$3.204 \times 10^9$	$8.052 \times 10^1$	0.68	0.616	4.998	118.7
$2.261 \times 10^9$	$7.177 \times 10^1$	0.68	0.616	4.998	84.0
$1.598 \times 10^9$	$6.388 \times 10^1$	0.68	0.616	4.998	59.6
$1.127 \times 10^9$	$5.697 \times 10^1$	0.68	0.616	4.999	42.3
$1.130 \times 10^{10}$	$1.221 \times 10^2$	0.68	0.616	4.997	416.3
$9.126 \times 10^9$	$1.135 \times 10^2$	0.68	0.616	4.997	336.3
$6.431 \times 10^9$	$1.013 \times 10^2$	0.68	0.616	4.997	237.2
$4.542 \times 10^9$	$9.024 \times 10^1$	0.68	0.616	4.997	167.9

$1.555 \times 10^{10}$	$1.349 \times 10^2$	0.69	1.48	5.502	126.1
$1.199 \times 10^{10}$	$1.245 \times 10^2$	0.69	1.48	5.502	97.4
$9.397 \times 10^9$	$1.141 \times 10^2$	0.69	1.48	5.487	76.0
$7.238 \times 10^9$	$1.050 \times 10^2$	0.69	1.48	5.494	58.9
$5.565 \times 10^9$	$9.661 \times 10^1$	0.69	1.48	5.502	45.7
$4.338 \times 10^{10}$	$1.887 \times 10^2$	0.69	1.48	5.503	350.4
$3.362 \times 10^{10}$	$1.732 \times 10^2$	0.69	1.48	5.503	271.8
$2.600 \times 10^{10}$	$1.593 \times 10^2$	0.69	1.48	5.502	210.3
$2.013 \times 10^{10}$	$1.463 \times 10^2$	0.69	1.48	5.502	163.1
$9.471 \times 10^{10}$	$2.434 \times 10^2$	0.69	1.48	5.489	758.9
$7.317 \times 10^{10}$	$2.237 \times 10^2$	0.69	1.48	5.489	586.3
$1.225 \times 10^{11}$	$2.655 \times 10^2$	0.69	1.48	5.488	981.3
$5.660 \times 10^{10}$	$2.054 \times 10^2$	0.69	1.48	5.489	453.7
$1.714 \times 10^{12}$	$6.427 \times 10^2$	0.75	6.07	5.994	892.7
$1.482 \times 10^{12}$	$6.114 \times 10^2$	0.75	6.07	5.992	771.6
$1.284 \times 10^{12}$	$5.808 \times 10^2$	0.75	6.07	5.991	668.1
$1.109 \times 10^{12}$	$5.535 \times 10^2$	0.75	6.07	5.990	577.0
$9.606 \times 10^{11}$	$5.261 \times 10^2$	0.75	6.07	5.990	499.8
$8.308 \times 10^{11}$	$5.011 \times 10^2$	0.75	6.07	5.990	432.4
$2.255 \times 10^{11}$	$3.255 \times 10^2$	0.75	6.07	5.991	118.1
$1.687 \times 10^{11}$	$2.964 \times 10^2$	0.75	6.07	5.991	88.5
$1.267 \times 10^{11}$	$2.688 \times 10^2$	0.75	6.07	5.991	66.7
$9.474 \times 10^{10}$	$2.449 \times 10^2$	0.75	6.07	5.991	50.1
$7.189 \times 10^{11}$	$4.768 \times 10^2$	0.75	6.07	5.990	374.4
$6.215 \times 10^{11}$	$4.547 \times 10^2$	0.75	6.07	5.990	323.7
$5.376 \times 10^{11}$	$4.333 \times 10^2$	0.75	6.07	5.990	280.1
$4.655 \times 10^{11}$	$4.126 \times 10^2$	0.75	6.07	5.990	242.7
$4.022 \times 10^{11}$	$3.938 \times 10^2$	0.75	6.07	5.990	209.8
$3.482 \times 10^{11}$	$3.752 \times 10^2$	0.75	6.07	5.990	181.8
$3.011 \times 10^{11}$	$3.581 \times 10^2$	0.75	6.07	5.990	157.3
$2.609 \times 10^{11}$	$3.410 \times 10^2$	0.75	6.07	5.991	136.4
$7.106 \times 10^{10}$	$2.224 \times 10^2$	0.75	6.07	5.992	37.8
$6.149 \times 10^{10}$	$2.122 \times 10^2$	0.75	6.07	5.991	32.9
$5.310 \times 10^{10}$	$2.029 \times 10^2$	0.75	6.07	5.991	28.5
$4.600 \times 10^{10}$	$1.932 \times 10^2$	0.75	6.07	5.991	24.8
$3.974 \times 10^{10}$	$1.846 \times 10^2$	0.75	6.07	5.991	21.5
$3.461 \times 10^{10}$	$1.749 \times 10^2$	0.75	6.07	5.991	18.9
$2.982 \times 10^{10}$	$1.674 \times 10^2$	0.75	6.07	5.992	16.4
$2.584 \times 10^{10}$	$1.593 \times 10^2$	0.75	6.07	5.992	14.3
$1.951 \times 10^{11}$	$3.106 \times 10^2$	0.75	6.07	5.991	102.3
$1.460 \times 10^{11}$	$2.826 \times 10^2$	0.75	6.07	5.991	76.8
$1.094 \times 10^{11}$	$2.570 \times 10^2$	0.75	6.07	5.991	57.7
$8.201 \times 10^{10}$	$2.335 \times 10^2$	0.75	6.07	5.991	43.5
$3.441 \times 10^{11}$	$3.710 \times 10^2$	0.79	6.07	5.107	111.1
$2.695 \times 10^{11}$	$3.427 \times 10^2$	0.79	6.07	5.107	87.2
$2.110 \times 10^{11}$	$3.167 \times 10^2$	0.79	6.07	5.107	68.5
$1.657 \times 10^{11}$	$2.919 \times 10^2$	0.79	6.07	5.108	54.0
$1.303 \times 10^{11}$	$2.686 \times 10^2$	0.79	6.07	5.108	42.7
$1.164 \times 10^{12}$	$5.566 \times 10^2$	0.79	6.07	5.105	373.6
$9.124 \times 10^{11}$	$5.125 \times 10^2$	0.79	6.07	5.106	293.0
$7.150 \times 10^{11}$	$4.724 \times 10^2$	0.79	6.07	5.106	229.8
$5.605 \times 10^{11}$	$4.356 \times 10^2$	0.79	6.07	5.106	180.4
$4.401 \times 10^{11}$	$4.010 \times 10^2$	0.79	6.07	5.106	141.9
$1.014 \times 10^{11}$	$2.497 \times 10^2$	0.79	6.07	5.108	33.4
$7.970 \times 10^{10}$	$2.300 \times 10^2$	0.79	6.07	5.108	26.5
$6.248 \times 10^{10}$	$2.123 \times 10^2$	0.79	6.07	5.108	20.9
$4.909 \times 10^{10}$	$1.954 \times 10^2$	0.79	6.07	5.108	16.7
$3.845 \times 10^{10}$	$1.805 \times 10^2$	0.79	6.07	5.108	13.2
$1.896 \times 10^{12}$	$6.578 \times 10^2$	0.79	6.07	5.106	607.9
$1.485 \times 10^{12}$	$6.051 \times 10^2$	0.79	6.07	5.106	476.2
$2.424 \times 10^{12}$	$7.142 \times 10^2$	0.79	6.07	5.108	778.0
$2.652 \times 10^{12}$	$7.335 \times 10^2$	0.95	17.4	6.002	109.6
$1.545 \times 10^{12}$	$6.135 \times 10^2$	0.95	17.4	6.003	64.4
$9.014 \times 10^{11}$	$5.132 \times 10^2$	0.95	17.4	6.005	38.0
$1.180 \times 10^{12}$	$5.613 \times 10^2$	0.95	17.4	6.004	49.4
$2.026 \times 10^{12}$	$6.701 \times 10^2$	0.95	17.4	6.003	84.0
$7.741 \times 10^{12}$	$1.068 \times 10^3$	0.95	17.4	6.001	317.7
$4.533 \times 10^{12}$	$8.829 \times 10^2$	0.95	17.4	6.001	186.5
$3.467 \times 10^{12}$	$8.045 \times 10^2$	0.95	17.4	6.002	142.9
$5.930 \times 10^{12}$	$9.695 \times 10^2$	0.95	17.4	6.001	243.6
$6.894 \times 10^{11}$	$4.688 \times 10^2$	0.95	17.4	6.005	29.3
$5.412 \times 10^{11}$	$4.344 \times 10^2$	0.95	17.4	6.006	23.3
$4.262 \times 10^{11}$	$4.011 \times 10^2$	0.95	17.4	6.006	18.5



$3.365 \times 10^{11}$	$3.693 \times 10^2$	0.95	17.4	6.006	14.9
$2.643 \times 10^{11}$	$3.420 \times 10^2$	0.95	17.4	6.006	11.9
$2.341 \times 10^{11}$	$3.292 \times 10^2$	0.95	17.4	6.006	10.7
$2.982 \times 10^{11}$	$3.555 \times 10^2$	0.95	17.4	6.006	13.3
$3.788 \times 10^{11}$	$3.848 \times 10^2$	0.95	17.4	6.006	16.6
$4.794 \times 10^{11}$	$4.183 \times 10^2$	0.95	17.4	6.005	20.7
$6.104 \times 10^{11}$	$4.517 \times 10^2$	0.95	17.4	6.005	26.1
$1.009 \times 10^{13}$	$1.181 \times 10^3$	0.95	17.4	6.001	413.7
$1.312 \times 10^{13}$	$1.310 \times 10^3$	0.95	17.4	6.003	538.2
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$7.800 \times 10^{11}$	$5.095 \times 10^2$	1.21	17.4	4.668	12.1
$6.520 \times 10^{11}$	$4.810 \times 10^2$	1.21	17.4	4.668	10.3
$9.272 \times 10^{12}$	$1.186 \times 10^3$	1.22	17.4	4.668	130.4
$6.093 \times 10^{12}$	$1.019 \times 10^3$	1.21	17.4	4.668	86.1
$3.993 \times 10^{12}$	$8.789 \times 10^2$	1.21	17.4	4.668	56.9
$3.232 \times 10^{12}$	$8.172 \times 10^2$	1.21	17.4	4.668	46.2
$4.941 \times 10^{12}$	$9.444 \times 10^2$	1.21	17.4	4.668	70.1
$7.526 \times 10^{12}$	$1.097 \times 10^3$	1.22	17.4	4.668	106.1
$2.702 \times 10^{12}$	$7.710 \times 10^2$	1.21	17.4	4.669	38.9
$1.764 \times 10^{12}$	$6.693 \times 10^2$	1.21	17.4	4.668	25.8
$1.152 \times 10^{12}$	$5.804 \times 10^2$	1.21	17.4	4.668	17.3
$9.287 \times 10^{11}$	$5.423 \times 10^2$	1.21	17.4	4.668	14.1
$1.427 \times 10^{12}$	$6.226 \times 10^2$	1.21	17.4	4.668	21.1
$2.185 \times 10^{12}$	$7.179 \times 10^2$	1.22	17.4	4.668	31.6
$1.556 \times 10^{13}$	$1.454 \times 10^3$	1.22	17.4	4.666	217.5
$1.102 \times 10^{13}$	$1.268 \times 10^3$	1.22	17.4	4.667	154.7
$1.311 \times 10^{13}$	$1.355 \times 10^3$	1.22	17.4	4.667	183.7
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$3.482 \times 10^{13}$	$1.725 \times 10^3$	1.60	37.3	6.001	119.6
$2.297 \times 10^{13}$	$1.446 \times 10^3$	1.60	37.3	6.001	79.3
$1.520 \times 10^{13}$	$1.210 \times 10^3$	1.60	37.3	6.001	52.9
$1.233 \times 10^{13}$	$1.111 \times 10^3$	1.60	37.3	6.000	43.2
$1.871 \times 10^{13}$	$1.321 \times 10^3$	1.60	37.3	6.000	64.8
$2.832 \times 10^{13}$	$1.577 \times 10^3$	1.60	37.3	6.001	97.5
$4.927 \times 10^{13}$	$2.004 \times 10^3$	1.60	37.3	6.002	168.8
$4.141 \times 10^{13}$	$1.859 \times 10^3$	1.60	37.3	6.001	142.0
$5.866 \times 10^{13}$	$2.158 \times 10^3$	1.60	37.3	6.003	201.0
$1.035 \times 10^{13}$	$1.037 \times 10^3$	1.60	37.3	6.000	36.4
$6.754 \times 10^{12}$	$8.819 \times 10^2$	1.60	37.3	6.000	24.2
$4.392 \times 10^{12}$	$7.538 \times 10^2$	1.60	37.3	6.000	16.2
$3.533 \times 10^{12}$	$6.986 \times 10^2$	1.60	37.3	6.000	13.3
$5.450 \times 10^{12}$	$8.147 \times 10^2$	1.60	37.3	6.000	19.8
$8.365 \times 10^{12}$	$9.555 \times 10^2$	1.60	37.3	6.000	29.7
$2.836 \times 10^{12}$	$6.489 \times 10^2$	1.60	37.3	6.000	10.9
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$5.653 \times 10^{13}$	$2.153 \times 10^3$	1.95	37.3	5.597	109.6
$3.821 \times 10^{13}$	$1.829 \times 10^3$	1.95	37.3	5.597	74.5
$2.596 \times 10^{13}$	$1.549 \times 10^3$	1.95	37.3	5.597	51.1
$2.142 \times 10^{13}$	$1.425 \times 10^3$	1.95	37.3	5.596	42.4
$3.157 \times 10^{13}$	$1.678 \times 10^3$	1.95	37.3	5.597	61.8
$4.645 \times 10^{13}$	$1.984 \times 10^3$	1.95	37.3	5.597	90.3
$1.679 \times 10^{13}$	$1.287 \times 10^3$	1.95	37.3	5.597	33.5
$1.137 \times 10^{13}$	$1.095 \times 10^3$	1.95	37.3	5.598	23.2
$7.684 \times 10^{12}$	$9.364 \times 10^2$	1.95	37.3	5.597	16.1
$6.298 \times 10^{12}$	$8.683 \times 10^2$	1.95	37.3	5.597	13.5
$9.345 \times 10^{12}$	$1.013 \times 10^3$	1.95	37.3	5.597	19.3
$1.383 \times 10^{13}$	$1.187 \times 10^3$	1.95	37.3	5.597	27.9
$4.904 \times 10^{12}$	$7.912 \times 10^2$	1.95	37.3	5.597	10.8
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$3.472 \times 10^{13}$	$1.814 \times 10^3$	2.44	37.3	5.302	38.4
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$1.069 \times 10^{13}$	$1.046 \times 10^3$	1.22	27.4	6.000	112.8
$6.900 \times 10^{12}$	$8.878 \times 10^2$	1.22	27.4	6.000	73.2
$4.425 \times 10^{12}$	$7.604 \times 10^2$	1.22	27.4	5.999	47.4
$3.539 \times 10^{12}$	$7.047 \times 10^2$	1.22	27.4	5.999	38.1
$5.533 \times 10^{12}$	$8.206 \times 10^2$	1.22	27.4	5.999	58.9
$8.589 \times 10^{12}$	$9.634 \times 10^2$	1.22	27.4	5.999	90.9
$3.112 \times 10^{13}$	$1.635 \times 10^3$	1.22	27.4	6.004	327.4
$1.573 \times 10^{13}$	$1.224 \times 10^3$	1.22	27.4	6.001	165.5
$1.866 \times 10^{13}$	$1.314 \times 10^3$	1.22	27.4	6.001	196.2
$2.821 \times 10^{12}$	$6.551 \times 10^2$	1.22	27.4	5.999	30.6
$1.966 \times 10^{12}$	$5.823 \times 10^2$	1.22	27.4	6.000	21.7
$1.370 \times 10^{12}$	$5.177 \times 10^2$	1.22	27.4	6.000	15.5
$1.142 \times 10^{12}$	$4.890 \times 10^2$	1.22	27.4	6.000	13.1
$1.645 \times 10^{12}$	$5.478 \times 10^2$	1.22	27.4	6.000	18.4
$2.363 \times 10^{12}$	$6.154 \times 10^2$	1.22	27.4	6.000	25.9
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$2.388 \times 10^{14}$	$3.770 \times 10^3$	2.87	65.5	5.999	104.9
$1.273 \times 10^{14}$	$2.915 \times 10^3$	2.87	65.5	5.998	56.5
$9.314 \times 10^{13}$	$2.564 \times 10^3$	2.87	65.5	5.998	41.7

$1.742 \times 10^{14}$	$3.314 \times 10^3$	2.87	65.5	5.998	76.8
$8.629 \times 10^{13}$	$2.488 \times 10^3$	2.92	65.5	5.980	37.1
$5.003 \times 10^{13}$	$1.975 \times 10^3$	2.92	65.5	5.980	22.1
$3.823 \times 10^{13}$	$1.755 \times 10^3$	2.92	65.5	5.980	17.3
$6.557 \times 10^{13}$	$2.220 \times 10^3$	2.92	65.5	5.980	28.5
$2.623 \times 10^{14}$	$3.924 \times 10^3$	2.92	65.5	5.980	109.9
$1.501 \times 10^{14}$	$3.123 \times 10^3$	2.92	65.5	5.979	63.4
$1.139 \times 10^{14}$	$2.784 \times 10^3$	2.92	65.5	5.979	48.5
$1.988 \times 10^{14}$	$3.491 \times 10^3$	2.92	65.5	5.980	83.6
$3.328 \times 10^{14}$	$4.344 \times 10^3$	2.92	65.5	5.980	139.3
$2.320 \times 10^{13}$	$1.407 \times 10^3$	2.92	65.5	5.980	11.0
$1.742 \times 10^{14}$	$3.313 \times 10^3$	4.03	65.5	5.722	34.9
$9.031 \times 10^{13}$	$2.529 \times 10^3$	4.03	65.5	5.722	18.8
$6.536 \times 10^{13}$	$2.201 \times 10^3$	4.03	65.5	5.722	14.0
$1.252 \times 10^{14}$	$2.899 \times 10^3$	4.03	65.5	5.722	25.5
$3.111 \times 10^{14}$	$4.190 \times 10^3$	4.02	65.5	5.724	61.4
$2.238 \times 10^{14}$	$3.658 \times 10^3$	4.02	65.5	5.723	44.4
$4.333 \times 10^{14}$	$4.797 \times 10^3$	4.01	65.5	5.725	85.5
$5.645 \times 10^{13}$	$2.061 \times 10^3$	4.03	65.5	5.722	12.3
$5.888 \times 10^{14}$	$5.542 \times 10^3$	6.92	65.5	5.471	34.3
$3.407 \times 10^{14}$	$4.434 \times 10^3$	6.92	65.5	5.471	20.5
$2.581 \times 10^{14}$	$3.979 \times 10^3$	6.91	65.5	5.471	15.9
$4.477 \times 10^{14}$	$4.966 \times 10^3$	6.92	65.5	5.470	26.4
$1.962 \times 10^{14}$	$3.529 \times 10^3$	6.92	65.5	5.471	12.5
$3.071 \times 10^{13}$	$1.611 \times 10^3$	1.21	27.2	6.007	332.6
$8.018 \times 10^{12}$	$9.296 \times 10^2$	0.94	16.9	6.024	363.8
$6.152 \times 10^{12}$	$8.458 \times 10^2$	0.94	16.9	6.018	278.4
$4.706 \times 10^{12}$	$7.720 \times 10^2$	0.94	16.9	6.014	212.7
$3.601 \times 10^{12}$	$7.048 \times 10^2$	0.94	16.9	6.012	162.7
$1.592 \times 10^{13}$	$1.249 \times 10^3$	1.18	16.9	4.683	252.4
$1.341 \times 10^{13}$	$1.169 \times 10^3$	1.18	16.9	4.681	212.3
$1.132 \times 10^{13}$	$1.092 \times 10^3$	1.18	16.9	4.680	179.1
$2.513 \times 10^{12}$	$6.270 \times 10^2$	0.92	16.2	6.012	128.3
$4.304 \times 10^{12}$	$7.496 \times 10^2$	0.92	16.2	6.018	219.8
$9.519 \times 10^{12}$	$9.927 \times 10^2$	0.92	16.2	6.041	491.5
$7.329 \times 10^{12}$	$9.010 \times 10^2$	0.92	16.2	6.031	376.4
$5.748 \times 10^{11}$	$3.873 \times 10^2$	0.74	5.37	5.992	392.3
$1.129 \times 10^{12}$	$4.830 \times 10^2$	0.74	5.37	6.056	793.8
$9.799 \times 10^{11}$	$4.607 \times 10^2$	0.74	5.37	6.046	685.7
$8.505 \times 10^{11}$	$4.390 \times 10^2$	0.74	5.37	6.038	593.0
$7.344 \times 10^{11}$	$4.209 \times 10^2$	0.74	5.37	6.030	510.2
$6.370 \times 10^{11}$	$4.010 \times 10^2$	0.74	5.37	6.025	441.5
$5.519 \times 10^{11}$	$3.826 \times 10^2$	0.74	5.37	6.020	381.8
$4.747 \times 10^{11}$	$3.680 \times 10^2$	0.74	5.37	6.014	327.6
$4.120 \times 10^{11}$	$3.501 \times 10^2$	0.74	5.37	6.011	284.1
$3.563 \times 10^{11}$	$3.346 \times 10^2$	0.74	5.37	6.008	245.4
$3.074 \times 10^{11}$	$3.205 \times 10^2$	0.74	5.37	6.006	211.6
$2.651 \times 10^{11}$	$3.072 \times 10^2$	0.74	5.37	6.003	182.4
$2.298 \times 10^{11}$	$2.927 \times 10^2$	0.74	5.37	6.002	158.1
$1.413 \times 10^{11}$	$2.545 \times 10^2$	0.74	5.26	6.011	102.4

## Appendix B. Data - Cigar cell with uneven plates ( $\Gamma = 0.23$ )

$Ra$	$Nu$	$Pr$	$\rho$ [kg/m <sup>3</sup> ]	$T$ [K]	$\Delta$ [mK]
$7.477 \times 10^8$	$4.999 \times 10^1$	0.67	0.66	5.946	39.5
$5.203 \times 10^8$	$4.475 \times 10^1$	0.67	0.66	5.945	27.8
$3.623 \times 10^8$	$3.996 \times 10^1$	0.67	0.66	5.945	19.6
$2.437 \times 10^8$	$3.695 \times 10^1$	0.67	0.66	5.944	13.4
$6.310 \times 10^9$	$9.984 \times 10^1$	0.67	0.66	5.956	329.2
$4.435 \times 10^9$	$8.890 \times 10^1$	0.67	0.66	5.952	231.2
$3.122 \times 10^9$	$7.895 \times 10^1$	0.67	0.66	5.950	162.8
$2.178 \times 10^9$	$7.070 \times 10^1$	0.67	0.66	5.948	113.8
$1.534 \times 10^9$	$6.266 \times 10^1$	0.67	0.66	5.947	80.3
$1.075 \times 10^9$	$5.574 \times 10^1$	0.67	0.66	5.946	56.5
$1.800 \times 10^{10}$	$1.416 \times 10^2$	0.67	0.66	5.981	948.3
$1.272 \times 10^{10}$	$1.260 \times 10^2$	0.67	0.66	5.970	666.5
$8.940 \times 10^9$	$1.125 \times 10^2$	0.67	0.66	5.961	467.1
$9.395 \times 10^8$	$5.289 \times 10^1$	0.68	0.66	4.980	30.6
$6.611 \times 10^8$	$4.721 \times 10^1$	0.68	0.66	4.980	21.8

4.591 × 10 <sup>8</sup>	4.271 × 10 <sup>1</sup>	0.68	0.66	4.980	15.4
3.186 × 10 <sup>8</sup>	3.859 × 10 <sup>1</sup>	0.68	0.66	4.980	10.9
3.794 × 10 <sup>9</sup>	8.341 × 10 <sup>1</sup>	0.68	0.66	4.983	121.1
2.671 × 10 <sup>9</sup>	7.466 × 10 <sup>1</sup>	0.68	0.66	4.982	85.5
1.892 × 10 <sup>9</sup>	6.635 × 10 <sup>1</sup>	0.68	0.66	4.981	60.8
1.333 × 10 <sup>9</sup>	5.923 × 10 <sup>1</sup>	0.68	0.66	4.981	43.1
2.132 × 10 <sup>10</sup>	1.477 × 10 <sup>2</sup>	0.68	0.66	4.999	683.3
2.996 × 10 <sup>10</sup>	1.653 × 10 <sup>2</sup>	0.68	0.66	5.008	967.3
7.532 × 10 <sup>11</sup>	4.814 × 10 <sup>2</sup>	0.75	6.06	5.959	387.7
6.534 × 10 <sup>11</sup>	4.594 × 10 <sup>2</sup>	0.75	6.06	5.955	335.8
5.666 × 10 <sup>11</sup>	4.384 × 10 <sup>2</sup>	0.75	6.06	5.952	290.9
4.903 × 10 <sup>11</sup>	4.192 × 10 <sup>2</sup>	0.75	6.06	5.949	251.5
4.241 × 10 <sup>11</sup>	4.007 × 10 <sup>2</sup>	0.75	6.06	5.946	217.4
3.683 × 10 <sup>11</sup>	3.815 × 10 <sup>2</sup>	0.75	6.06	5.945	188.7
3.190 × 10 <sup>11</sup>	3.642 × 10 <sup>2</sup>	0.75	6.06	5.943	163.5
2.766 × 10 <sup>11</sup>	3.473 × 10 <sup>2</sup>	0.75	6.06	5.942	141.8
2.464 × 10 <sup>12</sup>	7.053 × 10 <sup>2</sup>	0.78	6.06	5.128	803.9
1.535 × 10 <sup>12</sup>	6.008 × 10 <sup>2</sup>	0.78	6.06	5.112	496.2
2.188 × 10 <sup>11</sup>	3.209 × 10 <sup>2</sup>	0.78	6.06	5.106	71.3
1.737 × 10 <sup>11</sup>	3.010 × 10 <sup>2</sup>	0.75	6.06	5.995	91.7
1.135 × 10 <sup>11</sup>	2.594 × 10 <sup>2</sup>	0.75	6.06	5.995	60.2
6.357 × 10 <sup>10</sup>	2.152 × 10 <sup>2</sup>	0.75	6.06	5.994	34.1
3.080 × 10 <sup>10</sup>	1.700 × 10 <sup>2</sup>	0.75	6.06	5.994	17.0
1.911 × 10 <sup>12</sup>	6.531 × 10 <sup>2</sup>	0.76	6.06	5.573	805.2
7.645 × 10 <sup>12</sup>	1.091 × 10 <sup>3</sup>	0.95	17.2	5.982	322.2
3.476 × 10 <sup>12</sup>	8.288 × 10 <sup>2</sup>	0.95	17.2	5.969	145.9
2.060 × 10 <sup>12</sup>	6.930 × 10 <sup>2</sup>	0.95	17.2	5.968	86.8
9.155 × 10 <sup>11</sup>	5.286 × 10 <sup>2</sup>	0.95	17.2	5.966	39.1
4.061 × 10 <sup>11</sup>	4.077 × 10 <sup>2</sup>	0.95	17.2	5.966	17.9
2.354 × 10 <sup>11</sup>	3.437 × 10 <sup>2</sup>	0.95	17.2	5.966	10.8
8.987 × 10 <sup>12</sup>	1.209 × 10 <sup>3</sup>	1.20	17.2	4.675	133.7
3.772 × 10 <sup>12</sup>	8.893 × 10 <sup>2</sup>	1.20	17.2	4.671	56.5
1.063 × 10 <sup>13</sup>	1.291 × 10 <sup>3</sup>	1.20	17.2	4.678	158.3
2.652 × 10 <sup>12</sup>	7.897 × 10 <sup>2</sup>	1.20	17.2	4.670	40.0
2.222 × 10 <sup>12</sup>	7.445 × 10 <sup>2</sup>	1.20	17.2	4.669	33.7
1.858 × 10 <sup>12</sup>	7.032 × 10 <sup>2</sup>	1.20	17.2	4.669	28.4
1.561 × 10 <sup>12</sup>	6.609 × 10 <sup>2</sup>	1.20	17.2	4.669	24.0
1.307 × 10 <sup>12</sup>	6.234 × 10 <sup>2</sup>	1.20	17.2	4.669	20.3
1.093 × 10 <sup>12</sup>	5.887 × 10 <sup>2</sup>	1.20	17.2	4.668	17.2
9.154 × 10 <sup>11</sup>	5.549 × 10 <sup>2</sup>	1.20	17.2	4.668	14.6
7.639 × 10 <sup>11</sup>	5.246 × 10 <sup>2</sup>	1.20	17.2	4.668	12.4
1.185 × 10 <sup>13</sup>	1.313 × 10 <sup>3</sup>	1.21	27.3	6.002	127.0
7.688 × 10 <sup>12</sup>	1.119 × 10 <sup>3</sup>	1.22	27.3	5.996	82.4
4.951 × 10 <sup>12</sup>	9.596 × 10 <sup>2</sup>	1.22	27.3	5.994	53.4
3.973 × 10 <sup>12</sup>	8.878 × 10 <sup>2</sup>	1.22	27.3	5.993	43.0
6.171 × 10 <sup>12</sup>	1.037 × 10 <sup>3</sup>	1.22	27.3	5.994	66.3
9.557 × 10 <sup>12</sup>	1.212 × 10 <sup>3</sup>	1.22	27.3	5.998	102.3
3.183 × 10 <sup>12</sup>	8.227 × 10 <sup>2</sup>	1.22	27.3	5.992	34.7
2.039 × 10 <sup>12</sup>	7.069 × 10 <sup>2</sup>	1.22	27.3	5.992	22.7
1.305 × 10 <sup>12</sup>	6.081 × 10 <sup>2</sup>	1.22	27.3	5.992	14.9
1.818 × 10 <sup>13</sup>	1.544 × 10 <sup>3</sup>	1.21	27.3	6.010	195.4
4.395 × 10 <sup>13</sup>	2.226 × 10 <sup>3</sup>	1.58	37.1	6.018	156.8
3.125 × 10 <sup>13</sup>	1.944 × 10 <sup>3</sup>	1.59	37.1	6.009	110.6
2.211 × 10 <sup>13</sup>	1.698 × 10 <sup>3</sup>	1.59	37.1	6.003	78.1
1.561 × 10 <sup>13</sup>	1.481 × 10 <sup>3</sup>	1.59	37.1	6.000	55.4
1.312 × 10 <sup>13</sup>	1.383 × 10 <sup>3</sup>	1.59	37.1	5.999	46.7
1.861 × 10 <sup>13</sup>	1.583 × 10 <sup>3</sup>	1.59	37.1	6.001	65.8
2.633 × 10 <sup>13</sup>	1.814 × 10 <sup>3</sup>	1.59	37.1	6.005	93.1
3.709 × 10 <sup>13</sup>	2.081 × 10 <sup>3</sup>	1.58	37.1	6.012	131.6
5.217 × 10 <sup>13</sup>	2.378 × 10 <sup>3</sup>	1.58	37.1	6.024	187.2
1.100 × 10 <sup>13</sup>	1.295 × 10 <sup>3</sup>	1.59	37.1	5.998	39.3
7.703 × 10 <sup>12</sup>	1.136 × 10 <sup>3</sup>	1.59	37.1	5.996	27.9
5.399 × 10 <sup>12</sup>	9.954 × 10 <sup>2</sup>	1.59	37.1	5.996	19.9
3.768 × 10 <sup>12</sup>	8.758 × 10 <sup>2</sup>	1.59	37.1	5.995	14.3
3.141 × 10 <sup>12</sup>	8.229 × 10 <sup>2</sup>	1.59	37.1	5.995	12.1
4.506 × 10 <sup>12</sup>	9.346 × 10 <sup>2</sup>	1.59	37.1	5.996	16.8
6.442 × 10 <sup>12</sup>	1.065 × 10 <sup>3</sup>	1.59	37.1	5.996	23.5
9.216 × 10 <sup>12</sup>	1.212 × 10 <sup>3</sup>	1.59	37.1	5.997	33.1