

On the Recurrence Plot Analysis Method Behaviour under Scaling Transform



Florin-Marian Birleanu ^(a,d), Cornel Ioana ^(a), Cedric Gervaise ^(b), Jocelyn Chanussot ^(a), Alexandru Serbanescu ^(c), and Gheorghe Serban ^(d)



^(a) GIPSA-Lab / DIS, Grenoble Institute of Technology, Grenoble, France

^(b) ENSIETA, Brest, France

^(c) Military Technical Academy, Bucharest, Romania

^(d) Department of Electronics and Computers, University of Pitesti, Romania

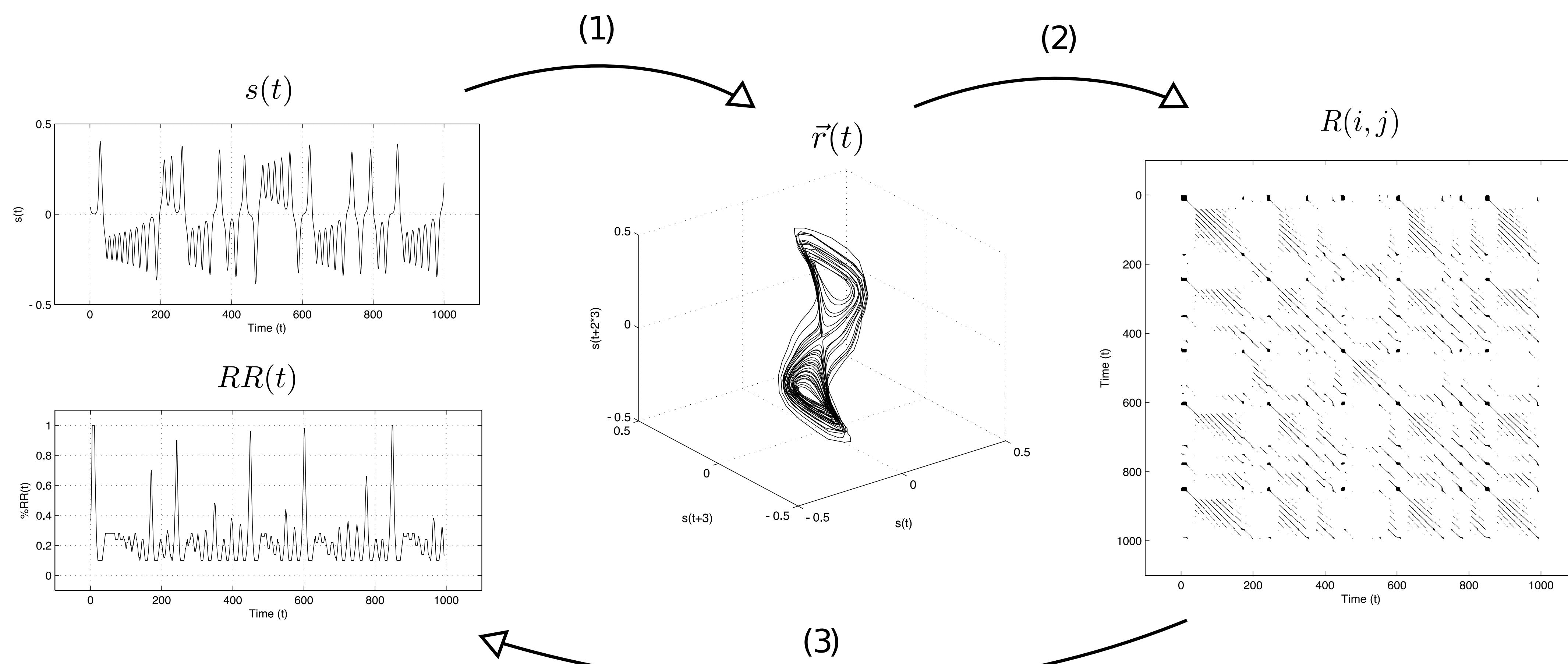
Context and Purpose

- (1987): Recurrence Plot = "a new graphical tool for measuring the time constancy of dynamical systems" [Eckmann et al.]
- dynamical systems \Rightarrow nonlinear time series analysis \Rightarrow Signal Processing
- Frequency \leftrightarrow Recurrence
- What are the effects of scaling the amplitude and the time axis?

Conclusions

- RPA: a noteworthy alternative to time-frequency and time-scale analysis
- a new (angular) metric, that leads to an amplitude-invariant RP
- amplitude scaling: $\varepsilon' = \alpha \cdot \varepsilon \Rightarrow$ same RP (scaled trajectory)
- time scaling: $\tau' = \tau/\beta \Rightarrow$ stretched RP (same trajectory, different speed)

Method



Mathematical Details

Recurrence Plot Analysis

$$s(t) \rightsquigarrow \vec{r}(t) = \sum_{k=1}^m s(t + (k-1)\tau) \cdot \vec{e}_k \rightsquigarrow R(i, j) = \Theta(\varepsilon(i) - D(\vec{r}(i), \vec{r}(j))) \rightsquigarrow Q$$

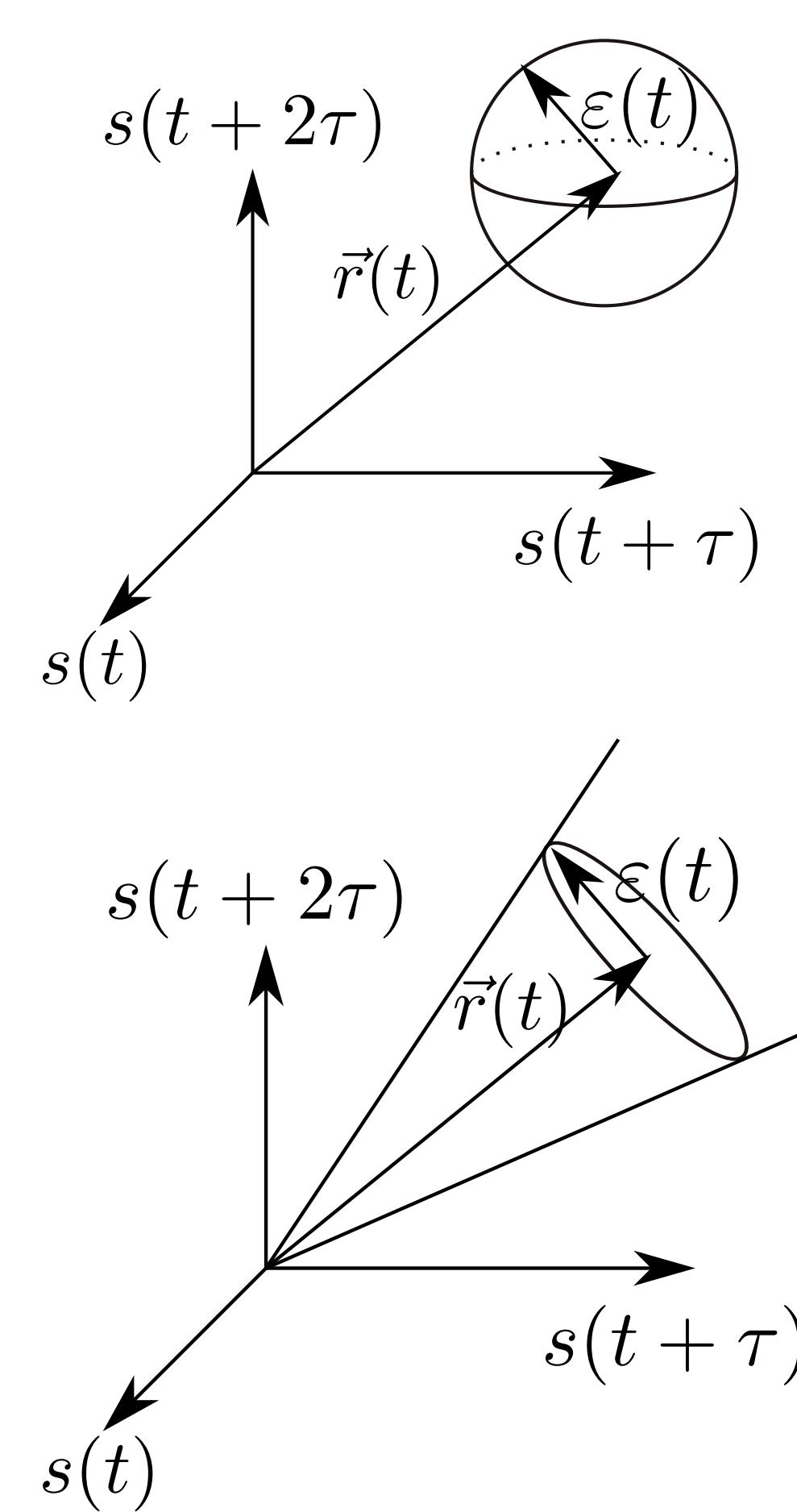
Amplitude Scaling

$$\begin{aligned} s'(t) &= \alpha \cdot s(t) \Rightarrow \vec{r}'(t) = \sum_{k=1}^m s'(t + (k-1)\tau) \cdot \vec{e}_k = \sum_{k=1}^m \alpha \cdot s(t + (k-1)\tau) \cdot \vec{e}_k = \alpha \cdot \vec{r}(t) \\ D(\vec{r}(i), \vec{r}(j)) &= \|\vec{r}(i) - \vec{r}(j)\| \Rightarrow D(\vec{r}(i), \vec{r}(j)) = \|\vec{r}(i) - \vec{r}(j)\| \\ \Rightarrow D(\alpha \cdot \vec{r}(i), \alpha \cdot \vec{r}(j)) &= \alpha \cdot D(\vec{r}(i), \vec{r}(j)) \end{aligned}$$

Angular distance: $D(\vec{r}(i), \vec{r}(j)) = \arccos\left(\frac{\vec{r}(i) \cdot \vec{r}(j)}{\|\vec{r}(i)\| \cdot \|\vec{r}(j)\|}\right)$

Time Scaling

$$\begin{aligned} s'(t) &= s(\beta \cdot t) \Rightarrow \vec{r}'(t) = \sum_{k=1}^m s'(\beta t + (k-1)\tau') \cdot \vec{e}_k = \\ &= \sum_{k=1}^m s(\beta t + (k-1)\tau') \cdot \vec{e}_k = \sum_{k=1}^m s(\beta t + (k-1)\beta\tau') \cdot \vec{e}_k \end{aligned}$$



Results

