

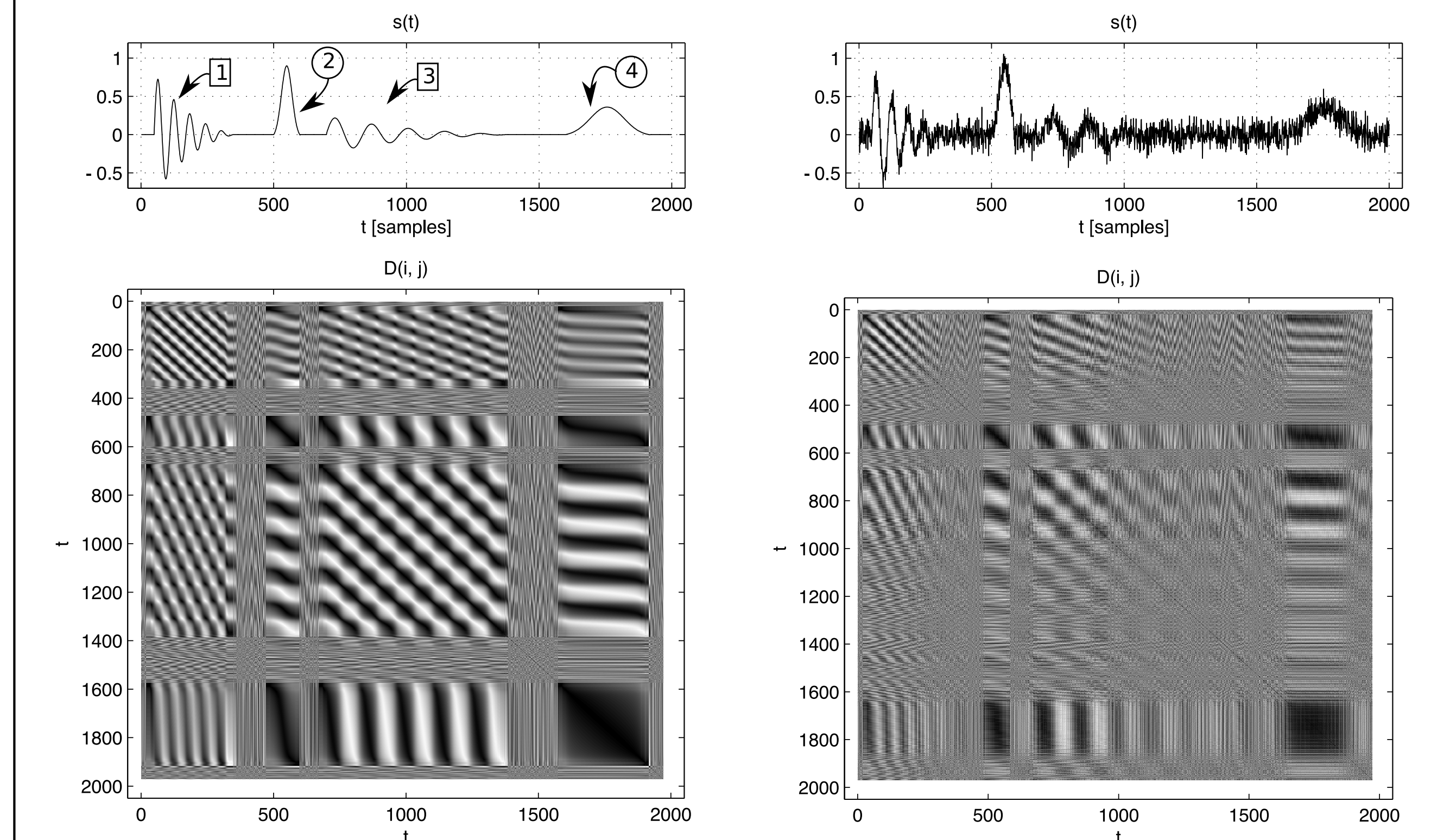
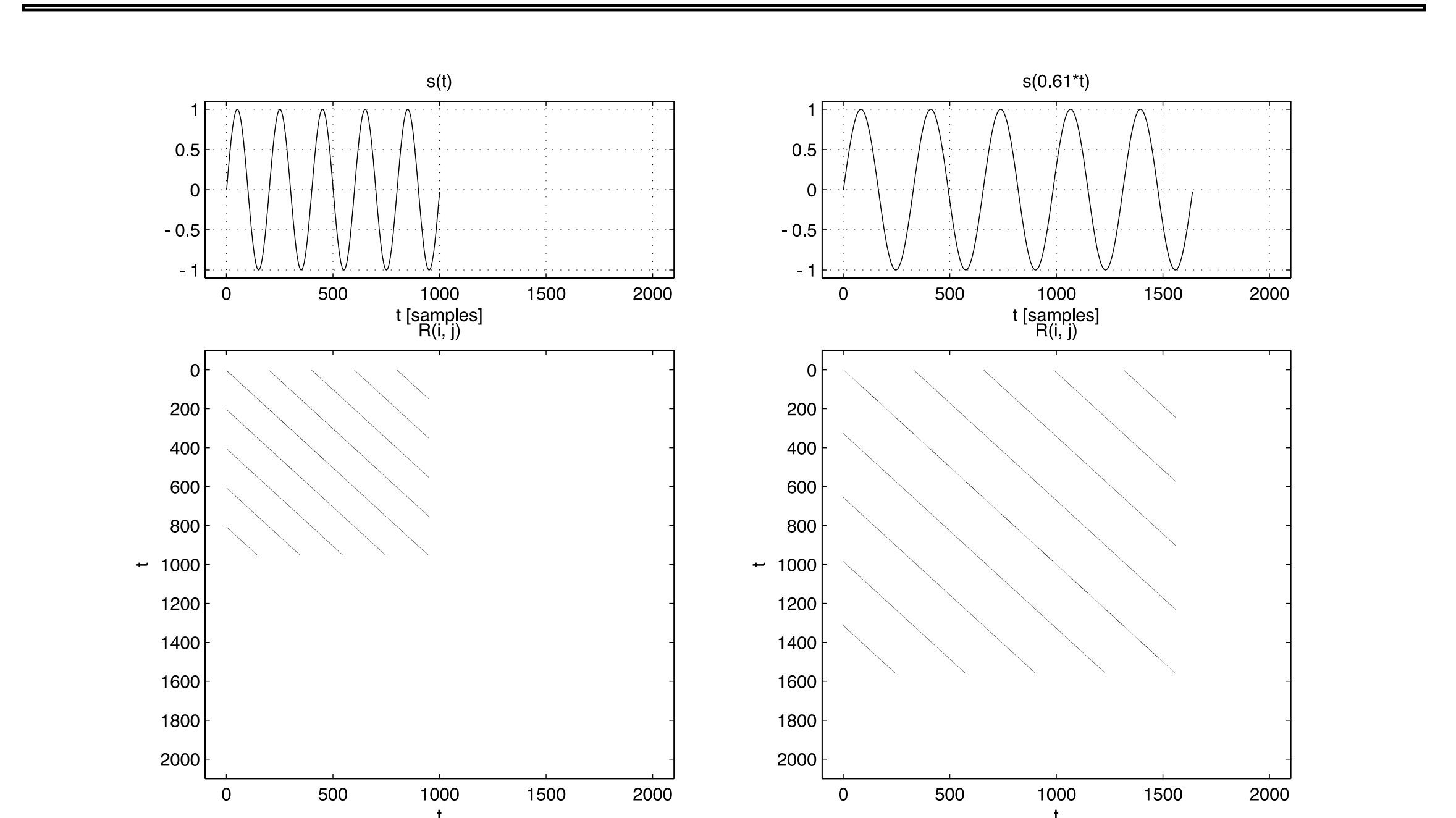
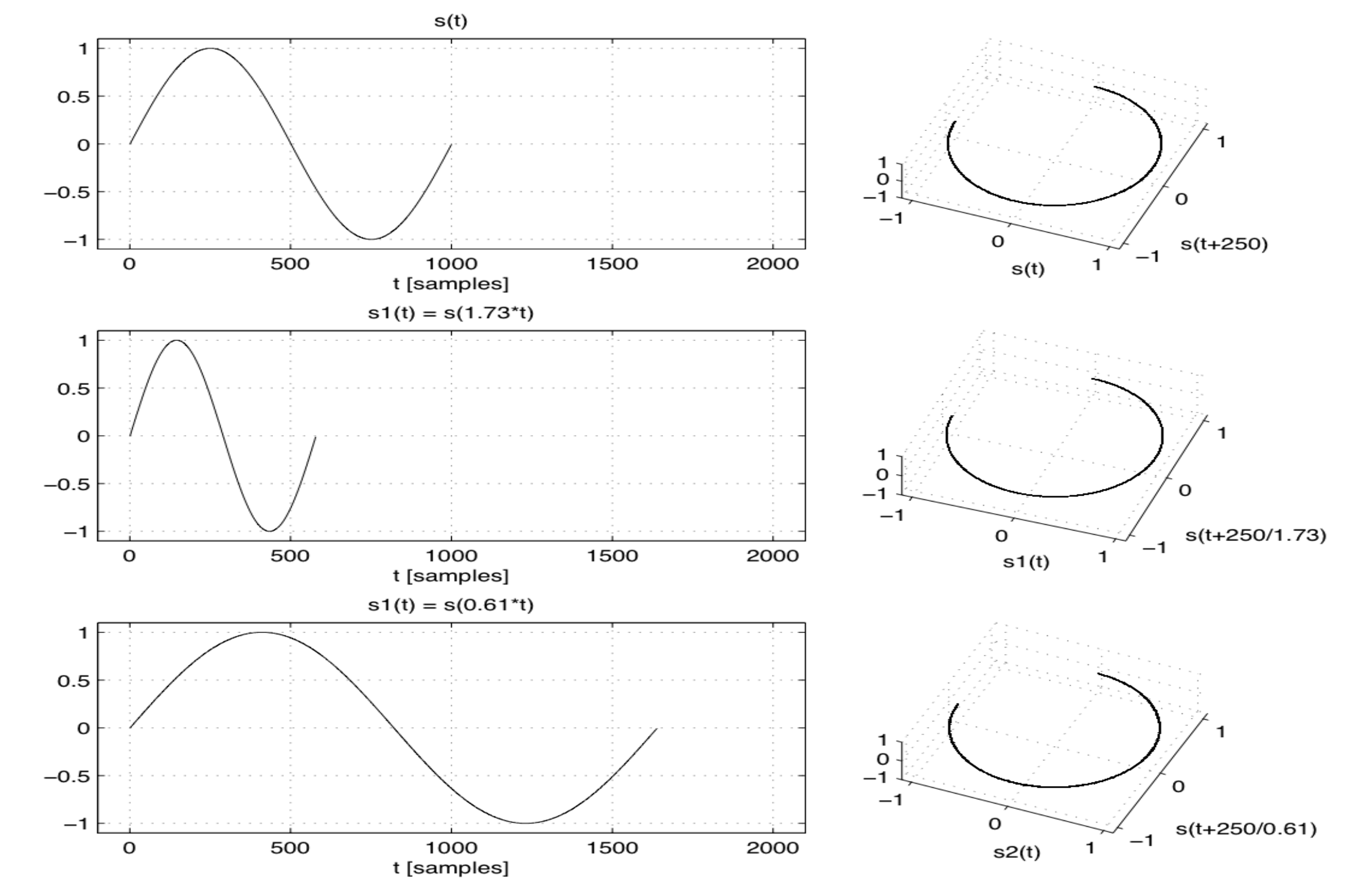
Context and Purpose

- (1987): **Recurrence Plot** = "a new graphical tool for measuring the time constancy of dynamical systems" [Eckmann et al.]
- dynamical systems \leftrightarrow nonlinear time series analysis \leftrightarrow **Signal Processing**
- **Frequency** \leftrightarrow **Recurrence**
- What are the effects of scaling the **amplitude** and the **time** axis?

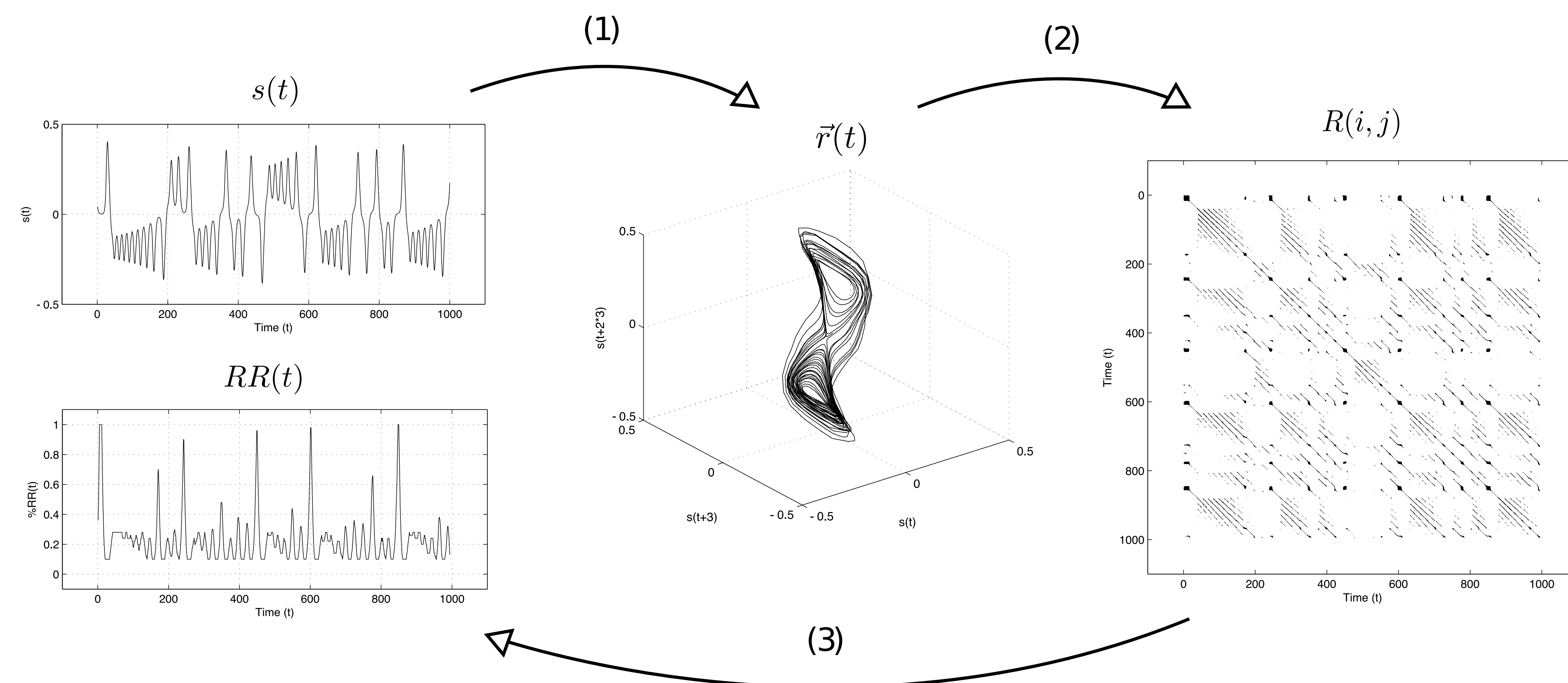
Conclusions

- **RPA**: a noteworthy alternative to time-frequency and time-scale analysis
- a new (angular) metric, that leads to an amplitude-invariant RP
- **amplitude scaling**: $\varepsilon' = \alpha \cdot \varepsilon \leftrightarrow$ same RP (scaled trajectory)
- **time scaling**: $\tau' = \tau/\beta \leftrightarrow$ stretched RP (same trajectory, different speed)

Results



Method



Mathematical Details

Recurrence Plot Analysis

$$s(t) \rightsquigarrow \vec{r}(t) = \sum_{k=1}^m s(t + (k-1)\tau) \cdot \vec{e}_k \rightsquigarrow R(i, j) = \Theta(\varepsilon(i) - D(\vec{r}(i), \vec{r}(j))) \rightsquigarrow Q$$

Amplitude Scaling

$$s'(t) = \alpha \cdot s(t) \Rightarrow \vec{r}'(t) = \sum_{k=1}^m s'(t + (k-1)\tau) \cdot \vec{e}_k = \sum_{k=1}^m \alpha \cdot s(t + (k-1)\tau) \cdot \vec{e}_k = \alpha \cdot \vec{r}(t)$$

$$D(\vec{r}(i), \vec{r}(j)) = \|\vec{r}(i) - \vec{r}(j)\| \Rightarrow \bullet \text{ Angular distance: } D(\vec{r}(i), \vec{r}(j)) = \arccos\left(\frac{\vec{r}(i) \cdot \vec{r}(j)}{\|\vec{r}(i)\| \cdot \|\vec{r}(j)\|}\right)$$

$$\Rightarrow D(\alpha \cdot \vec{r}(i), \alpha \cdot \vec{r}(j)) = \alpha \cdot D(\vec{r}(i), \vec{r}(j))$$

Time Scaling

$$s'(t) = s(\beta \cdot t) \Rightarrow \vec{r}'(t) = \sum_{k=1}^m s'(t + (k-1)\tau') \cdot \vec{e}_k =$$

$$= \sum_{k=1}^m s(\beta(t + (k-1)\tau')) \cdot \vec{e}_k = \sum_{k=1}^m s(\beta t + (k-1)\beta\tau') \cdot \vec{e}_k$$

