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Arches and contact forces in a granular pile

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Assemblies of granular particles mechanically stable under their own weight contain arches. These are structural units identified as mutually stable grains. It is generally assumed that these arches shield the weight above them and should bear most of the stress in the system. We test such hypothesis by studying the stress born by in-arch and out-of-arch grains. We show that, in average, particles in arches withstand larger stresses. However, out-of-arch grains with large stresses and in-arch grains with low stresses exist and the two populations cannot be distinguished by simply looking at the stress distributions.

The study of mechanically stable granular beds has become a major point of interest in granular physics. Studies range from analysis of force network properties [1–4] to structural characterization [5–7] to thermodynamic and statistical descriptions [8–12]. A recurrent question in the subject is related to the existence or not of a relation between force chains and arches [13, 14].

Arches are multiparticle structures where all grains are mutually stable [13, 15–17], i.e. fixing the positions of all other particles in the assembly the removal of any particle in the arch leads to the collapse of the other particles in it. For an arch to be formed, it is necessary (although not sufficient) that two or more falling particles be in contact at the time they reach equilibrium in order to create mutually stabilizing structures [18]. Like arches in architecture, granular arches are assumed to sustain the weight of grains above.

Highly stressed grains in static deposits are generally found to form linear structures: the so-called force chains. A correlation as been pointed out [13, 15, 19] between the distribution of horizontal span of arches in a granular pile and the distribution of normal forces at the grain contacts. Therefore, it is assumed that a strong correlation has to be present between arches and highly stressed grains in a granular deposit. In this paper we assess this general belief in the frame of a simulation of granular packings prepared by tapping.

We used a velocity Verlet algorithm to integrate the Newton equations for 512 monosized disks in a rectangular box. The disk–disk and disk–wall contact interaction comprises a linear spring–dashpot in the normal direction and a tangential friction force that implements the Coulomb criterion to switch between dynamic and static friction. Units are reduced with the diameter of the disks, d, the disk mass, m, and the acceleration of gravity, g. Details on the force equations and the interaction parameters can be found elsewhere [18, 20]. Tapping is simulated by moving the confining box in the vertical direction following a half sine wave trajectory of frequency $\nu = \pi/2(g/d)^{1/2}$. The intensity of the excitation is controlled through the amplitude, A, of the sinusoidal trajectory; and it is characterized by the parameter $\Gamma = A(2\pi\nu)^2/g$. A new tap is applied after the system has come to mechanical equilibrium defined via the stability of each particle-particle contact [18]. After many taps, the system reaches a steady state where the properties of the static configurations generated have well defined mean values and fluctuations. Averages were taken over 500 taps (configurations) in the steady state corresponding to each value of Γ.

Details on the algorithms used to identify arches can be found in previous works [15, 16, 18]. Briefly, we need first to identify the supporting grains of each particle in

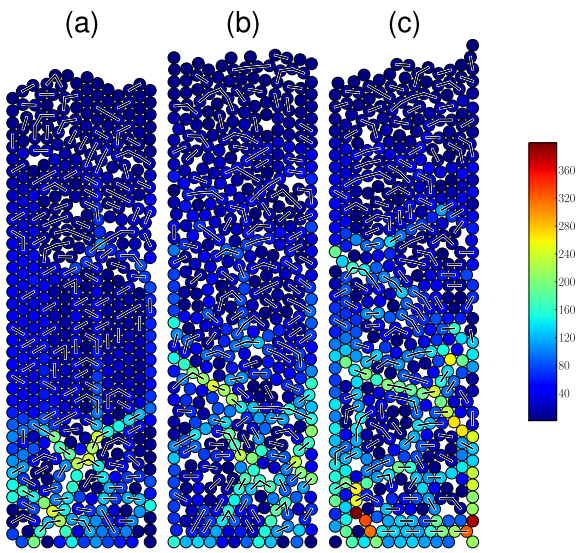

FIG. 1: (Color online). Sample images of the simulated granular columns for different Γ: (a) $\Gamma = 2.2$, (b) $\Gamma = 4.93$ and (c) $\Gamma = 15.4$. The color code indicates the trace, $\mathrm{Tr}(\sigma)$, of the stress tensor for each particle. The joining segments indicate the arches detected in the system.

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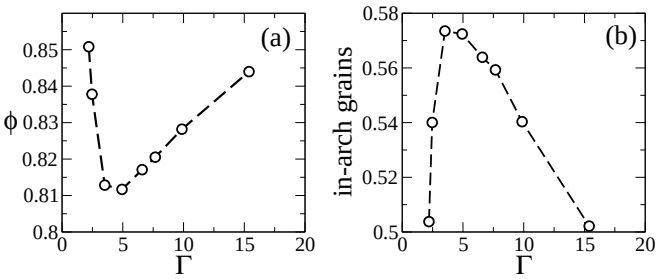

FIG. 2: (a) The mean packing fraction, ϕ, as a function of tapping intensity Γ. (b) Fraction of in-arch grains as a function of Γ.

the packing. In 2D, there are two disks that support any given grain. Two grains in contact with a given particle are able to provide support if the segment defined by the contact points lies below the center of mass of the particle. Some of these supporting contacts may be provided by the walls of the container. Then, we find all mutually stable particles. Two grains A and B are mutually stable if A supports B and B supports A. Arches are defined as sets of particles connected through these mutually stabilizing contacts (MSC). The fact that the supporting particles of each grain have to be known implies that contacts, and the chronological order in which they form, have to be clearly defined in the model. In the case of MD simulations, an algorithm that tracks the contacts that support each particle has been designed [18]. We use this scheme in the present work.

We measure the stress tensor σ for grain i as:

$$\sigma_i^{\alpha\beta} = \frac{1}{\pi(d/2)^2} \sum_{j=1}^{N_c} f_{ij}^{\alpha} r_{ij}^{\beta}, \qquad (1)$$

where, f_{ij} is the force exerted by grain j on grain i and r_{ij} is the branch vector that goes from the center of grain i to the contact point with grain j. The sum runs over the N_c particles in contact with particle i.

Figure 1 shows some examples of the distribution of pressures and arches inside a granular pile. As it is to be expected, particles are subjected to higher pressures, in average, in the lower part of the pile as compared with the upper layers. The mean pressure grows continuously with the tapping intensity Γ [12, 22]. This contrasts with the appearance of a minimum packing fraction, ϕ, at intermediate Γ [see Fig. 2(a)] as previously observed in various models [20, 21] and experiments [12, 22]. This minimum in ϕ is related with the existence of a maximum in the number of grains involved in arches [see Fig. 2(b)] [20].

It is worth mentioning that the system does not display force chains that span the system from top to bottom as is commonly seen in many experiments and simulations. This is due to the fact that the system is in mechanical equilibrium under its own weight; no external compression is applied to the sample in any direction.

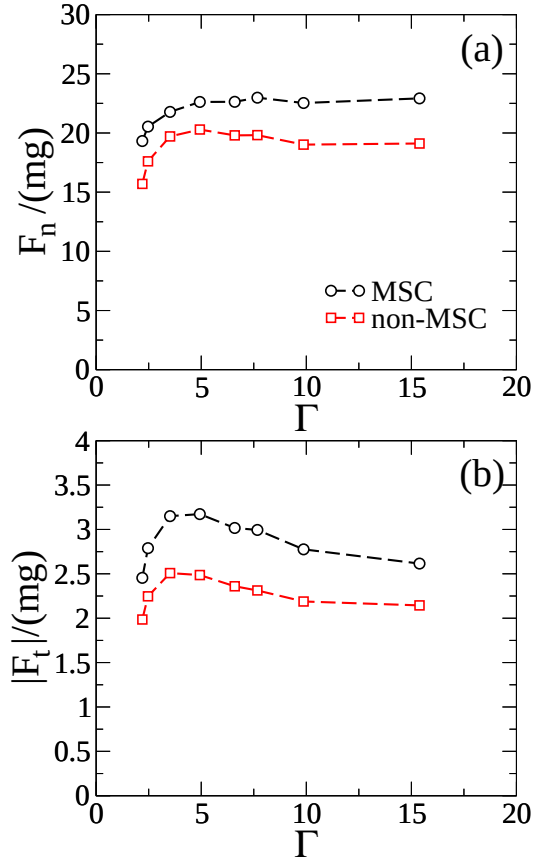

FIG. 3: (Color online). The mean value of the contact force for mutually stabilizing contacts (black circles) and non-mutually stabilizing contacts (red-squares). (a) Normal contact forces, and (b) tangential contact forces.

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It can be observed from Fig. 1 that, at any depth into the pile, grains can present high and low stress irrespective of whether they belong to an arch or not. For a more quantitative analysis we plot in Fig. 3 the mean value of the contact forces (normal and tangential to the contact). MSC and non-MSC have been separated in the analysis. The mean normal contact force increases with Γ in agreement with the increasing mean pressure reported elsewhere [12, 22]. It is clear that MSC (i.e., contacts within arches) have, in average, larger normal and tangential forces. However, Fig. 4 shows that the distribution of contact forces for MSC and non-MSC are rather similar. The exponential tail for contact forces adobe the mean contact force characteristic of granular packs subjected to external compression [2, 3, 23] or stable under their own weight [24] is also present in our systems. The behavior for forces below the mean resembles the weak divergence found for packings without external compression when bulk contacts (as opposed to contacts made between the grains and the container) are considered [23]. The different mean values observed in Fig. 3 for MSC and non-MSC are only due to a somewhat lower preponderance of large forces for the non-MSC and not

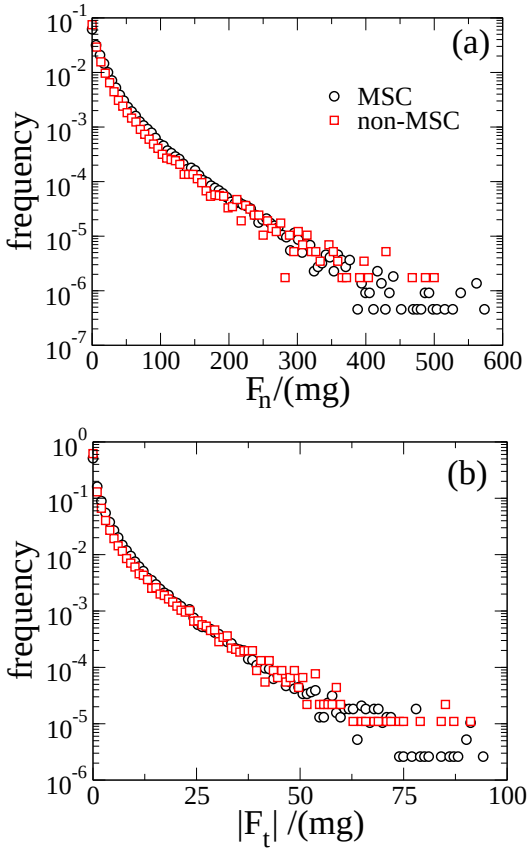

FIG. 4: (Color online). The PDF of contact forces for MSC (black circles) and non-MSC (red-squares) for $\Gamma = 2.2$. (a) Normal contact forces, and (b) tangential contact forces.

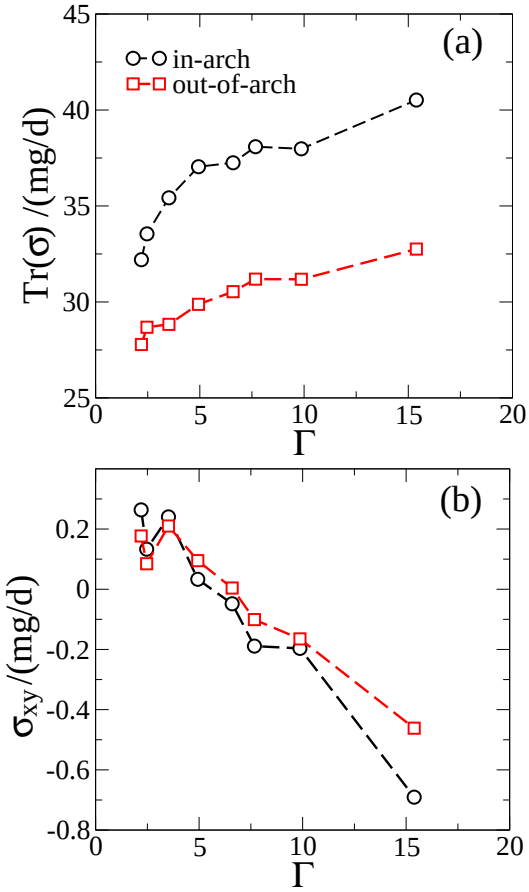

FIG. 5: (Color online). Stress tensor for in-arch (black circles) and out-of-arch (red squares) grains. (a) The trace $\mathrm{Tr}(\sigma)$ of the stress, and (b) the off diagonal component σ_{xy} of the stress.

to a clear separation of the two population of contacts. Deciding if a given contact is or not mutually stabilizing based on the strength of the forces involved is in fact impossible.

In Fig. 5, we show the results of a similar analysis but now the stress tensor on each particle, as defined in Eq. (1), is considered. The stress tensor accounts for both MSC and non-MSC on each grain. We separate in-arch grains from out-of-arch grains in the analysis. As observed in previous studies [12, 22], $\mathrm{Tr}(\sigma)$ increases monotonically with Γ, which is in correspondence with the increasing normal contact forces [see Fig. 3(a)]. In-arch grains support, in average, pressures about 30% higher than off-arch grains. In contrast, the shear stress, σ_{xy}, seems to be rather similar for both types of grains. These results indicate that in-arch grains are essentially subjected to larger compressive stresses. However, the stress tensor distributions do not show any peculiar difference between in-arch and off-arch grains. Figure 6 displays the characteristic exponential decays for both compressive and shear stresses distributions. While important differences in mean stress between in-arch and out-of-arch grains are observed in the compressive terms [see Fig. 5(a)], the stress distributions are hardly distinguishable for large $\mathrm{Tr}(\sigma)$. In contrast, the shear component, σ_{xy}, for the in-arch grains present longer tails in comparison with out-of–arch grains even though their means are similar. Once again, the differences found between in-arch and out-of-arch grains are not the cause of a bimodal character of the stress distribution and particles with high and low stresses are likely to be found forming arches.

It is important to comment on some effects that might cause the lack of a clear separation of the two grain classes (i.e., in-arch or out-of-arch grains) in the contact force distribution. On the one hand, arches can always exist in the structure that sustain little weight since they are shielded by other arches above. This leads to the preponderance of small forces in the distributions for MSC. On the other hand, force chains can also develop without the need of arches. A deposit carefully built by sequential deposition of grains contains no arches in the structure, yet it will present force chains and the ubiquitous exponential PDF of contact forces. An interesting conclusion drawn in Ref. [17] —were the term bridge is used in the same sense we use arch— is directly connected with our results: *"...bridges can be neither load-bearing nor load-induced... Bridges thus characterize the load-bearing*

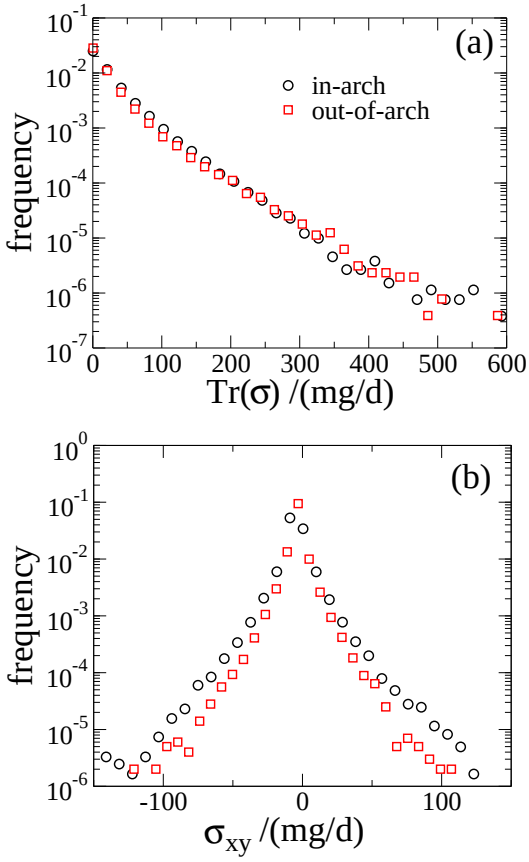

FIG. 6: (Color online). The PDF of the stress tensor for in-arch (black circles) and out-of-arch (red-squares) for $\Gamma = 2.2$. (a) The trace, $\mathrm{Tr}(\sigma)$, of the stress tensor, and (b) the shear component, σ_{xy} of the stress.

ability, not necessarily the actual degree of load-bearing, of a packing." Such conclusions came from the fact that arch size distributions proved to be independent of gravity. Here, we went one step further and showed that in a single packing the weight is not born by the arches much more than by the rest of the grains.

In summary, we have shown that MSC, which define arches, present higher normal and tangential components of the contact forces as compared with non-MSC contacts. Grains belonging to arches are generally subjected to larger pressures but similar shear. However, the PDF of contact forces and particle stresses present similar exponential decays for in-arch and out-of-arch grains. A clear separation of the two populations of grains cannot be made. Future research in this respect should look into the effect that applied external loads can make to the force distributions for in-arch and out-of-arch grains.

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