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► **To cite this version:**

Flavio Mercati, Diego Mazón, Giovanni Amelino-Camelia, José Manuel Carmona, José Luis Cortés, et al.. Probing the quantum-gravity realm with slow atoms. *Classical and Quantum Gravity*, 2010, 27 (21), pp.215003. 10.1088/0264-9381/27/21/215003 . hal-00638667

HAL Id: hal-00638667

<https://hal.science/hal-00638667>

Submitted on 7 Nov 2011

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Probing the quantum-gravity realm with slow atoms

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Abstract. For the study of Planck-scale modifications of the energy-momentum dispersion relation, which had been previously focused on the implications for ultrarelativistic particles, we consider the possible role of experiments involving nonrelativistic particles, and particularly atoms. We extend a recent result establishing that measurements of “atom-recoil frequency” can provide insight that is valuable for some theoretical models. And from a broader perspective we analyze the complementarity of the nonrelativistic and the ultrarelativistic regimes in this research area.

1. Introduction

Over the last decade there has been growing interest in the possibility to investigate experimentally some candidate effects of quantum gravity. The development of this “quantum-gravity phenomenology” [1] of course focuses on rare contexts in which the minute effects induced by the ultra-high “Planck scale” $M_P (\equiv \sqrt{\hbar c/G} \simeq 1.2 \cdot 10^{28} \text{ eV})$ are not completely negligible. Several contexts of this sort have been found particularly in the study of quantum-gravity/quantum-spacetime effects for the propagation of ultrarelativistic particles (see, *e.g.*, Refs. [2, 3, 4, 5, 6, 7, 8, 9, 10]), and often specifically for cases in which the ultrarelativistic on-shell condition[§], $E \simeq p + m^2/(2p)$, is modified by Planck-scale effects.

In the recent Ref. [11] some of us observed that experiments involving cold (slow, nonrelativistic) atoms, and particularly measurements of the atom-recoil frequency, can provide valuable insight on certain types of modifications of the dispersion relation which had been previously considered in quantum-gravity literature. We here extend the scopes of the analysis briefly reported in Ref. [11], also adopting a style of presentation that allows to comment in more detail the derivation of the result. Concerning the conceptual perspective that guides this recent research proposal, we here expose some previously unnoticed aspects of complementarity between the nonrelativistic and the ultrarelativistic regimes in the study of Planck-scale modifications of the dispersion relation. And we offer several observations on how the insight gained from studies of slow atoms might translate into limits of different strength depending on some details of the overall framework within which the modifications of the dispersion relation are introduced. We also report a preliminary exploration of the relativistic issues involved in these studies, which have been already well appreciated in the ultrarelativistic regime but appear to provide novel challenges when the focus is instead on the nonrelativistic regime.

2. Complementarity of nonrelativistic and ultrarelativistic regimes

Results in support of the possibility of modifications of the energy/momentum (“dispersion”) relation have been reported in studies of several approaches to the quantum-gravity problem, and perhaps most notably in analyses inspired by Loop Quantum Gravity [6, 12], and in studies that assumed a “noncommutativity” of spacetime coordinates [13, 14, 15]. The analyses of these quantum-gravity approaches that provide encouragement for the presence of corrections to the dispersion relation have become increasingly robust over the last decade [12, 16, 13, 14, 15], but in the majority of cases they are still unable to establish robustly the functional dependence of the correction on momentum. This has led to the proposal that perhaps on this occasion experiments might take the lead by establishing some experimental facts (at

[§] We adopt units in which the speed-of-light scale c is set to 1 (whereas we shall explicitate the role of the Planck constant \hbar).

least amounting to constraints on the form of the dispersion relation) that may provide guidance for the ongoing investigations on the theory side. From this perspective the fact that presently-available results on the theory side are insufficient to provide narrowly defined phenomenological models is not viewed as sufficient reason for being discouraged: the alternative is giving up on any experimental guidance in the search for quantum gravity, and instead even the constraints produced by a phenomenology of rather broad scopes can be of some value on the theory side, hopefully in turn allowing theorists to provide sharper indications to the phenomenologists.

In light of these considerations the majority of phenomenological studies of Planck-scale corrections to the dispersion relation have assumed a rather general *ansatz*,

$$E^2 = p^2 + m^2 + \Delta_{QG}(p, m, M_P) , \quad (1)$$

denoting with E the energy of the particle and with Δ_{QG} a model-dependent function of the Planck mass M_P and of the spatial momentum p and of the mass m of the particle.

Different models do give (more or less detailed) guidance on the form of Δ_{QG} , and we shall consider this below, but even at a model-independent level a few characteristics can be assumed with reasonable robustness^{||}. As most authors in the field, we shall here focus our analysis on cases in which the mass m still is the rest-energy and the dispersion relation regains its ordinary special-relativistic form in the limit where the Planck scale is removed ($M_P \rightarrow \infty$):

$$\Delta_{QG}(p, m, M_P) \xrightarrow{p \rightarrow 0} 0 , \quad \Delta_{QG}(p, m, M_P) \xrightarrow{M_P \rightarrow \infty} 0 . \quad (2)$$

And, since the relevant phenomenology clearly can at best hope to gain insight on the leading terms of a small- M_P^{-1} expansion, it is natural to focus on a power-series expansion,

$$E^2 = p^2 + m^2 + \frac{1}{M_P} \Delta_{QG}^{(1)}(p, m) + \frac{1}{M_P^2} \Delta_{QG}^{(2)}(p, m) + \dots , \quad (3)$$

where the terms in the power series are subject to the condition $\Delta_{QG}^{(1)}(p, m) \Big|_{p=0} = 0 = \Delta_{QG}^{(2)}(p, m) \Big|_{p=0}$.

This past decade of vigorous investigations of these modifications of the dispersion relation focused primarily (but not exclusively) on terms linear in M_P^{-1} and reached its most noteworthy results in analyses of observational astrophysics data, which of course concern the ultrarelativistic ($p \gg m$) regime of particle kinematics [2, 3, 4, 17, 18, 7]. For these applications the function $\Delta_{QG}^{(1)}(p, m)$ can of course be usefully parametrized in such a way that the relation between energy and spatial momentum takes the following form:

$$E \simeq p + \frac{m^2}{2p} + \frac{1}{2M_P} (\eta_1 p^2 + \eta_2 m p + \eta_3 m^2) , \quad (4)$$

^{||} We should stress however that, while the perspective schematized in our Eqs. (2)-(3) is by far the most studied in the relevant quantum-gravity-inspired literature, in principle more general possibilities may well deserve investigation. For example, one might contemplate non-integer powers of M_P to appear, and this would not be too surprising, especially in light of the rather common expectation that the correct description of quantum gravity might require sizable nonlocality.

where, considering the large value of M_P , we only included correction terms that are linear in $1/M_P$, and, considering that this formula concerns the ultrarelativistic regime of $p \gg m$, the labels on the parameters η_1, η_2, η_3 reflect the fact that in that regime p^2/M_P is the leading correction, mp/M_P is next-to-leading, and so on.

Evidence that at least some of these η_1, η_2, η_3 parameters have nonzero values is indeed found in studies inspired by the Loop-Quantum-Gravity approach and by the approach based on spacetime noncommutativity, and most importantly some of these studies [6, 12, 13, 14, 15] provide encouragement for the presence of the strongest imaginable ultrarelativistic correction, the leading-order term $\eta_1 p^2/(2M_P)$.

Unfortunately, as usual in quantum-gravity research, even the most optimistic estimates represent a gigantic challenge from the perspective of phenomenology. This is because, if the Planck scale is indeed roughly the characteristic scale of quantum-gravity effects then correspondingly parameters such as η_1, η_2, η_3 should take (positive or negative) values that are within no more than 1 or 2 orders of magnitude of 1. And this in turn implies that, for example, all effects induced by Eq. (4) could only affect the running of our present particle-physics colliders at the level [1] of at best 1 part in 10^{14} . In recent years certain semi-heuristic renormalization-group arguments (see, *e.g.*, Refs. [1, 19] and references therein), have encouraged the intuition that the quantum-gravity scale might be plausibly even 3 orders of magnitude smaller than the Planck scale (so that it could coincide [19] with the “grand unification scale” which appears to play a role in particle physics). But even assuming for η_1, η_2, η_3 values plausibly as “high” as 10^3 is not enough help at traditional high-energy particle-collider experiments.

It was therefore rather exciting for many quantum-gravity researchers when it started to emerge that some observations in astrophysics could be sensitive to manifestations of the parameter η_1 all the way down to $|\eta_1| \sim 1$ and even below [2, 3, 4, 17, 18, 7], thereby providing for that parameter the ability to explore the full range of values that could be motivated from a quantum-gravity perspective. These studies are presently being conducted at the Fermi Space Telescope [20, 21, 22, 23, 24], and other astrophysics observatories.

In the recent Ref. [11] some of us observed that it would be very valuable to combine to these astrophysics studies of the ultrarelativistic regime of the dispersion relation also a complementary phenomenology program of investigation of the nonrelativistic regime of $p \ll m$ (which of course is not accessible to massless particles). When $p \ll m$ the 3 largest contributions to $\Delta_{QG}^{(1)}(p, m)$ have behavior $\P m^2 p, mp^2$ and p^3 , allowing to cast the relation between energy and spatial momentum in the following form:

$$E \simeq m + \frac{p^2}{2m} + \frac{1}{2M_P} \left(\xi_1 mp + \xi_2 p^2 + \xi_3 \frac{p^3}{m} \right), \quad (5)$$

\P Note that a contribution of form m^3 (*i.e.* momentum-independent) to $\Delta_{QG}^{(1)}(p, m)$ cannot be included in the nonrelativistic regime because of the requirement $\Delta_{QG}^{(1)}(p = 0, m) = 0$. A contribution to $\Delta_{QG}^{(1)}(p, m)$ of form m^3 is instead admissible in the ultrarelativistic regime (since in that regime the requirement $\Delta_{QG}^{(1)}(p = 0, m) = 0$ of course is not relevant), but we ignored it since m^3 is too small with respect to p^3, mp^2 and $m^2 p$ in the nonrelativistic regime.

where, again, ξ_1, ξ_2, ξ_3 are dimensionless parameters.

Evidence that at least some of these dimensionless parameters ξ_1, ξ_2, ξ_3 should be non-zero has been found for example in the much-studied framework introduced in Refs. [6, 25], which was inspired by Loop Quantum Gravity, and produces a term linear in p in the nonrelativistic limit (the effect here parametrized by ξ_1). And for the purposes of this Section, which we are devoting to the complementarity of the nonrelativistic and ultrarelativistic regimes of the dispersion relation, it is particularly insightful to consider two of the most studied scenarios that have emerged in the literature on noncommutative-geometry-inspired deformations of Poincaré symmetries. These are the scenarios proposed in Refs. [26, 27] and in Ref. [28], which respectively produce the following proposals for the exact form of the dispersion relation:

$$\left(\frac{2M_P}{\eta}\right)^2 \sinh^2\left(\frac{\eta E}{2M_P}\right) = \left(\frac{2M_P}{\eta}\right)^2 \sinh^2\left(\frac{\eta m}{2M_P}\right) + e^{-\eta\frac{E}{M_P}} p^2, \quad (6)$$

and

$$\frac{m^2}{(1 - \eta\frac{m}{M_P})^2} = \frac{E^2 - p^2}{(1 - \eta\frac{E}{M_P})^2}, \quad (7)$$

Both of these proposals have the same description in the nonrelativistic regime

$$E \simeq m + \frac{p^2}{2m} - \eta\frac{p^2}{2M_P}, \quad (8)$$

i.e. the type of correction term in the nonrelativistic regime that we are here parameterizing with ξ_2 . But these proposals have significantly different behavior in the ultrarelativistic regime. From Eq. (6) in the ultrarelativistic regime one finds

$$E \simeq p + \frac{m^2}{2p} - \eta\frac{p^2}{2M_P}, \quad (9)$$

whereas from Eq. (7) in the ultrarelativistic regime one finds

$$E \simeq p + \frac{m^2}{2p} - \eta\frac{m^2}{M_P}. \quad (10)$$

Therefore the example of these two much studied deformed-symmetry proposals is such that by focusing exclusively on the nonrelativistic regime one could not (not at the leading order at least) distinguish between them, but one could discriminate between the two proposals using data on the ultrarelativistic regime. The opposite is of course also possible: different candidate dispersion relations with the same ultrarelativistic limit, but with different leading-order form in the nonrelativistic regime. And in general it would be clearly very valuable to constrain the form of the dispersion relation both using experimental information on the leading nonrelativistic behavior and experimental information on the leading ultrarelativistic behavior.

3. Probing the nonrelativistic regime with cold atoms

Our main objective here is to show that cold-atom experiments can be valuable for the study of Planck-scale effects. We illustrate this point mainly by considering the possibility, already preliminarily characterized in Ref. [11], to use cold-atom studies for the derivation of meaningful bounds on the parameters ξ_1 and ξ_2 , *i.e.* the leading and next-to-leading terms in (5) for the nonrelativistic limit:

$$E \simeq m + \frac{p^2}{2m} + \frac{1}{2M_P} (\xi_1 mp + \xi_2 p^2) . \quad (11)$$

In this Section we work exclusively from a laboratory-frame perspective, as done in Ref. [11], but, as for most relativistic studies, it is valuable to also perform the analysis in one or more frames that are boosted with respect to the laboratory frame, and we shall discuss this in Sec. 5.

The measurement strategy proposed in Ref. [11] is applicable to measurements of the “recoil frequency” of atoms with experimental setups involving one or more “two-photon Raman transitions” [29, 30, 31]. Let us initially set aside the possibility of Planck-scale effects, and discuss the recoil of an atom in a two-photon Raman transition from the perspective adopted in Ref. [31], which provides a convenient starting point for the Planck-scale generalization we shall discuss later. One can impart momentum to an atom through a process involving absorption of a photon of frequency ν and (stimulated [29, 30, 31]) emission, in the opposite direction, of a photon of frequency ν' . The frequency ν is computed taking into account a resonance frequency ν_* of the atom and the momentum the atom acquires, recoiling upon absorption of the photon: $\nu \simeq \nu_* + (h\nu_* + p)^2/(2m) - p^2/(2m)$, where m is the mass of the atom (*e.g.* $m_{Cs} \simeq 124$ GeV for Caesium), and p its initial momentum. The emission of the photon of frequency ν' must be such to de-excite⁺ the atom and impart to it additional momentum: $\nu' + (2h\nu_* + p)^2/(2m) \simeq \nu_* + (h\nu_* + p)^2/(2m)$. Through this analysis one establishes that by measuring $\Delta\nu \equiv \nu - \nu'$, in cases (not uncommon) where ν_* and p can be accurately determined, one actually measures h/m for the atoms:

$$\frac{\Delta\nu}{2\nu_*(\nu_* + p/h)} = \frac{h}{m} . \quad (12)$$

This result has been confirmed experimentally with remarkable accuracy. A powerful way to illustrate this success is provided by comparing the results for atom-recoil measurements of $\Delta\nu/[\nu_*(\nu_* + p/h)]$ and for measurements [32] of α^2 , the square of the fine structure constant. α^2 can be expressed in terms of the mass m of any given particle [31] through the Rydberg constant, R_∞ , and the mass of the electron, m_e , in

⁺ We only give a schematic and simplified account of the process, which suffices for the scopes of our analysis. A more careful description requires taking into account that, rather than a single ground state, the relevant two-photon Raman transition involve hyperfine-splitted ground states [29, 30, 31]. And that, rather than tuning the two lasers exactly on some energy differences between levels, some detuning is needed [29, 30, 31].

the following way [31]: $\alpha^2 = 2R_\infty \frac{m}{m_e} \frac{h}{m}$. Therefore according to Eq. (12) one should have

$$\frac{\Delta\nu}{2\nu_*(\nu_* + p/h)} = \frac{\alpha^2}{2R_\infty} \frac{m_e}{m_u} \frac{m_u}{m}, \quad (13)$$

where m_u is the atomic mass unit and m is the mass of the atoms used in measuring $\Delta\nu/[\nu_*(\nu_* + p/h)]$. The outcomes of atom-recoil measurements, such as the ones with Caesium reported in Ref. [31], are consistent with Eq. (13) with the accuracy of a few parts in 10^9 .

The fact that Eq. (12) has been verified to such a high degree of accuracy proves to be very valuable for our purposes as we find that modifications of the dispersion relation require a modification of Eq. (12). Our derivation can be summarized briefly by observing that the logical steps described above for the derivation of Eq. (12) establish the following relationship

$$h\Delta\nu \simeq E(p + h\nu + h\nu') - E(p) \simeq E(2h\nu_* + p) - E(p), \quad (14)$$

and therefore Planck-scale modifications of the dispersion relation, parametrized in Eq. (5), would affect $\Delta\nu$ through the modification of $E(2h\nu_* + p) - E(p)$, which compares the energy of the atom when it carries momentum p and when it carries momentum $p + 2h\nu_*$.

Since our main objective here is to expose sensitivity to a meaningful range of values of the parameter ξ_1 , let us focus on the Planck-scale corrections with coefficient ξ_1 . In this case the relation (12) is replaced by

$$\Delta\nu \simeq \frac{2\nu_*(h\nu_* + p)}{m} + \xi_1 \frac{m}{M_P} \nu_*, \quad (15)$$

and in turn in place of Eq. (13) one has

$$\frac{\Delta\nu}{2\nu_*(\nu_* + p/h)} \left[1 - \xi_1 \left(\frac{m}{2M_P} \right) \left(\frac{m}{h\nu_* + p} \right) \right] = \frac{\alpha^2}{2R_\infty} \frac{m_e}{m_u} \frac{m_u}{m}. \quad (16)$$

We have arranged the left-hand side of this equation placing emphasis on the fact that our quantum-gravity correction is as usual penalized by the inevitable Planck-scale suppression (the ultrasmall factor m/M_P), but in this specific context it also receives a sizable boost by the large hierarchy of energy scales $m/(h\nu_* + p)$, which in typical experiments of the type here of interest can be [29, 30, 31] of order $\sim 10^9$.

Our result (16) for the case of modification of the dispersion relation by the term with coefficient ξ_1 can be straightforwardly generalized to the case of a modified dispersion relation of the form

$$E \simeq m + \frac{p^2}{2m} + \frac{\xi_\beta}{2} \frac{m^{2-\beta}}{M_P} p^\beta \quad (17)$$

which reproduces our terms with parameters ξ_1 and ξ_2 respectively when $\beta = 1$ and $\beta = 2$ (but in principle could be examined even for non-integer values of β).

One then finds

$$\frac{\Delta\nu}{2\nu_*(\nu_* + p/h)} \left[1 - \xi_\beta \left(\frac{m^{2-\beta} [(p + 2h\nu_*)^\beta - p^\beta]}{4M_P h\nu_*} \right) \left(\frac{m}{h\nu_* + p} \right) \right] = \frac{\alpha^2}{2R_\infty} \frac{m_e}{m_u} \frac{m_u}{m}, \quad (18)$$

which indeed reproduces (16) for $\beta = 1$ and gives [11]

$$\frac{\Delta\nu}{2\nu_*(\nu_* + p/h)} \left[1 - \xi_2 \frac{m}{M_P} \right] = \frac{\alpha^2}{2R_\infty} \frac{m_e}{m_u} \frac{m_u}{m} \quad (19)$$

for $\beta = 2$.

We have so far assumed that the only Planck-scale corrections to the analysis come from parameters such as ξ_1 and ξ_2 , characteristic of the nonrelativistic regime, for particles of nonzero mass. In the experimental setups we are considering all particles are indeed nonrelativistic with the exception of course of the photons involved. Clearly massless particles are inevitably ultrarelativistic and actually (at leading order in $1/M_P$) there is a single possible modification of the dispersion relation for massless particles, the one with coefficient η_1 and quadratic dependence on momentum (see Eq. (4)). Of course, in a given quantum-gravity scenario one might have, say, $\xi_1 \neq 0$ and $\eta_1 = 0$, in which case the derivations we gave above would immediately apply. It is natural to also contemplate the possibility of cases in which both ξ_1 and η_1 are roughly of order 1. We find however that for the analysis of atom-recoil studies the effects produced by an η_1 of order 1 are completely negligible with respect to the effects produced by a ξ_1 of order 1. This is essentially due to the fact that photons enter the derivation of the recoil frequency through momentum transfers that never have a chance to pick up the ‘‘amplification’’ coming from the only large energy scale in the problem which is the atom mass. The amplification of the effects of ξ_1 which was underlined in the comments we offered just after Eq. (16) is not found for the effects of η_1 . Indeed by redoing all the steps of our derivation allowing for a nonzero η_1 one ends up replacing Eq. (14) with

$$h\Delta\nu \simeq E(2h\nu_* + p - \eta_1 h^2 \nu_*^2 / M_P) - E(p). \quad (20)$$

And from this one arrives at a rather intelligible characterization of the different roles of ξ_1 and η_1 in atom-recoil analysis:

$$\frac{\Delta\nu}{2\nu_*(\nu_* + p/h)} = \frac{h}{m} + \xi_1 \frac{hm}{2M_P(p + h\nu_*)} - \eta_1 \frac{hm}{2M_P(p + h\nu_*)} \left(\frac{h\nu_* p + 2h^2 \nu_*^2}{m^2} \right), \quad (21)$$

which shows that the effects of η_1 are suppressed with respect to the ones of ξ_1 by a factor of order $(h\nu_*/m)^2$ or $h\nu_* p/m^2$ (notice the two powers of the mass in the denominator, and that the mass of the atoms in the setup here of interest is much larger than both p and ν_*).

The balance of strengths changes a bit, but not enough, in the case of scenarios with $\xi_1 = 0$ but both ξ_2 and η_1 of order 1. In such cases one should compare the effects

of ξ_2 (which we established to be smaller than the ones of ξ_1) to the effects of η_1 . What one finds is summarized by the formula

$$\frac{\Delta\nu}{2\nu_*(\nu_*+p/h)} = \frac{h}{m} + \xi_2 \frac{h}{M_P} - \eta_1 \frac{h}{M_P} \frac{h\nu_*}{2m} \left(1 + \frac{h\nu_*}{p+h\nu_*}\right), \quad (22)$$

which shows that the effects of η_1 are significantly suppressed with respect to the ones of ξ_2 (here it is only one power of the mass in the denominator, but that is plenty of suppression, considering the large hierarchy between mass of the atoms and spatial momenta available in atom-recoil studies).

4. Limits on different models

From a phenomenological perspective the most noteworthy observation one can ground on the results reported in the previous Section is that the accuracies achievable in cold-atom studies allow us to probe values of ξ_1 that are not distant from $|\xi_1| \sim 1$. This is rather meaningful since, as stressed in the previous Section, the quantum-gravity intuition for parameters such as ξ_1 is that they should be (in models where a nonzero value for them is allowed) within a few orders of magnitude of 1. Besides discussing this point, in this Section we also consider the case of the term with ξ_2 parameter and we comment on the relevance of these analyses from the perspective of a class of phenomenological proposals which is broader than the one here discussed in Section II. The closing remarks of this Section are devoted to observations that may be relevant for attempts to further improve the relevant experimental limits.

4.1. Limits on ξ_1 and ξ_2

The fact that our analysis provides sensitivity to values of ξ_1 of order 1 is easily verified by examining our result for the case of the ξ_1 parameter, which we rewrite here for convenience

$$\frac{\Delta\nu}{2\nu_*(\nu_*+p/h)} \left[1 - \xi_1 \left(\frac{m}{2M_P}\right) \left(\frac{m}{h\nu_*+p}\right)\right] = \frac{\alpha^2}{2R_\infty} \frac{m_e}{m_u} \frac{m_u}{m}, \quad (23)$$

and taking into account some known experimental accuracies. Let us focus in particular on the Caesium-atom recoil measurements reported in Ref. [31], which were ideally structured for our purposes. Let us first notice that R_∞ , m_e/m_u and m_u/m_{Cs} are all known experimentally with accuracies of better than 1 part in 10^9 . When this is exploited in combination with the value of α^{-1} recently determined from electron-anomaly measurements [32], which is $\alpha^{-1} = 137.035999084(51)$, the results of Ref. [31, 33] then allow us to use (23) to determine that $\xi_1 = -1.8 \pm 2.1$. This amounts to the bound $-6.0 < \xi_1 < 2.4$, established at the 95% confidence level, and shows that indeed the cold-atom experiments we here considered can probe the form of the dispersion relation (at least in one of the directions of interest) with sensitivity that is meaningful from a Planck-scale perspective.

As mentioned in Section 2 among the models that could be here of interest there are some where, by construction, $\xi_1 = 0$ but $\xi_2 \neq 0$. In such cases it is then of interest to establish bounds on ξ_2 derived assuming $\xi_1 = 0$, for which one can easily adapt the derivation discussed above. These are therefore cases in which our result (19) is relevant, and one easily then finds that the atom-recoil results for Caesium atoms reported in Refs. [31, 33] can be used to establish that $-3.8 \cdot 10^9 < \xi_2 < 1.5 \cdot 10^9$. This bound is still some 6 orders of magnitude above even the most optimistic quantum-gravity estimates. But it is a bound that still carries some significance from the broader perspective of tests of Lorentz symmetry [11].

We should stress that, since we relied on the results of Ref. [32], our noteworthy bounds on ξ_1 and ξ_2 could in principle be affected by the hypothesis of Planck-scale effects that happened to be relevant for the determination of α from electron-anomaly measurements. One could consider the possibility of a conspiracy between the “Planck-scale-kinematics effects” that appear on the left-hand side of (23) and the “Planck-scale gravity-interaction effects” that could be relevant for the determination of α from electron-anomaly measurements. At the present stage of understanding of the quantum-gravity problem such a conspiracy appears implausible, since in the relevant models (see, *e.g.*, Refs. [6, 12]) anomalous behavior of gravitational interactions are only expected to start at order M_P^{-2} . And we should also stress that electron-anomaly measurements are not the only way to determine accurately α (although they presently provide the most precise determination): one could for example use our strategy of analysis to obtain a bound weakened by not more than one order of magnitude without relying on electron-anomaly measurements, but rather comparing the results of atom-recoil experiments with different types of atoms (*e.g.* Caesium and Rubidium).

4.2. Relevance for other quantum-gravity-inspired scenarios

Up to this point we have assumed “universal” effects, *i.e.* modifications of the dispersion relation that have the same form for all particles, independently of spin and compositeness, and with dependence on the mass of the particles rigidly inspired by the quantum-gravity arguments suggesting correction terms of the form $m^j p^k / M_P^l$ (*i.e.* with a characteristic dependence on momentum and with a momentum-independent coefficient written as a ratio of some power of the mass of the particle versus some power of the Planck-scale).

While this universality is indeed assumed in the majority of studies of the fate of Poincaré symmetry at the Planck scale, alternatives have been considered by some authors [34] and there are good reasons to at least be open to the possibility of nonuniversality. One reason of concern toward universality originates from the fact that clearly modifications of the dispersion relation at the Planck scale are a small effect for microscopic particles (always with energies much below the Planck scale in our experiments), but would be a huge (and unobserved) effect for macroscopic bodies, such as planets and, say, soccer balls. Even the literature that assumes universality

is well aware of this issue, and in fact the opening remarks of papers on this subject always specify a restriction to microscopic particles. With our present (so limited) understanding of the quantum-gravity realm we can indeed contemplate for example the possibility that such effects be confined to motions which admit description in terms of coherent quantum systems (by which we simply mean that the focus is on the type of particles whose quantum properties could also be studied in the relevant class of phenomena, unlike the motions of planets and soccer balls). This is clearly (at least at present) a plausible scenario that many authors are studying and for which atoms provide an extraordinary opportunity of investigation of the nonrelativistic regime. Let us compare for example our study to the popular studies of the ultrarelativistic regime with photons. The best limits on the ultrarelativistic side are obtained [23] through observations of photons with energies of a few tens of GeV's. The limit we here obtained in the nonrelativistic regime involves very small speeds ($\ll c$) but for particles, the atoms, with (rest) energies in the ~ 100 GeV range.

While it is therefore rather clear that atoms are excellent probes of scenarios with universality for “quantum-mechanically microscopic particles”, their effectiveness can be sharply reduced in models with some forms of nonuniversality. In particular, one could consider the compositeness of particles as a possible source of nonuniversality [35]. And this would imply that in the study of processes involving, say, protons and pions one should adopt a “parton picture” with the number of partons acting in the direction of averaging out the effects: if quantum-spacetime effects affect primarily the partons then a particle composed of 3 partons could feel the net result of 3 such fundamental features, with a possible suppression (*e.g.* by a factor of $\sqrt{3}$) of the effect for the particle with respect to the fundamental effect for partons. These ideas have not gained much attention, probably also because things might change only at the level of factors of order 1 if one was for example to devise ways to keep track of the different number of partons for nucleons and for pions. But in the case of atoms, that we are now bringing to the forefront of quantum-gravity phenomenology, clearly these concerns cannot be taken lightly: for the description of an atom one might have to consider hundreds of partons (or at least ~ 100 nucleons). We therefore expect that our strategy to place limits on ξ_1 and ξ_2 will be less effective (limits more distant from the Planck scale) in scenarios based on one or another form of “parton model” for the implications of spacetime quantization on quantum-mechanical particles. We do not dwell much on this here at the quantitative level since the literature does not offer us definite models of this sort that we could compare to data.

Even assuming that the effect is essentially universal one could consider alternatives to the most common assumption that quantum-gravity corrections have the form $m^j p^k / M_P^l$. In particular, some authors (see, *e.g.*, Refs. [36, 37, 38]) have argued that the density of energy (or mass) of a given particle (be it elementary or composite) should govern the magnitude of the effect, rather than simply the mass of the particle. This is another possibility which is also under investigation [36, 37, 38] as a mechanism for effectively confining the new effects to elementary particles. In the simplest scenarios

this proposal might amount to replacing terms such as our $\xi_1 mp/(2M_P)$ with terms of the general form $\tilde{\xi}_1 \rho^{1/4} p/(2M_P)$, but of course the implications of such pictures depend crucially on exactly which density ρ one adopts. For different choices of ρ the limits derived from atom-recoil experiments can be more or less stringent than those derived in studies of lighter particles, such as electrons.

Another framework which can be used to illustrate the different weight that cold-atom studies can carry in different scenarios for the deformation of the dispersion relation is the one already studied in Refs. [39, 40], parameterized by a single scale λ such that $E^2 = m^2 + p^2 + 2\lambda p$. Limits on this form of the dispersion relation have been obtained for neutrinos in Ref. [39], and for electrons, in Ref. [40]. Taking into account that from $E^2 = m^2 + p^2 + 2\lambda p$ it follows that in the nonrelativistic limit $E = m + p^2/(2m) + \lambda p/m$, one easily finds that the parametrization we introduced in Eq. (5) and the parametrization of Refs. [39, 40] are related by $\xi_1 m/M_P \equiv 2\lambda/m$. And in light of this one can quickly estimate that the study of atom-recoil measurements can provide access to $|\lambda| \sim 10^{-6}$ eV. This shows that the cold-atom-based strategy is suitable also for studies of the λ -parameter picture of Refs. [39, 40]. But, while, as some of us already stressed in Ref. [11], these atom-based studies on λ are more powerful (by roughly 6 orders of magnitude) than bounds previously obtained on λ using neutrino data [39], we should here notice that the best present bound on λ is the electron-based bound derived in Ref. [40], which is at the level $|\lambda| \leq 10^{-7}$ eV. We stress that there is no contradiction between the remarks we offered above on the unique opportunities that cold-atom studies provide for setting bounds on the parameter ξ_1 , and the fact that instead for the λ parameter electron studies are competitive with (and actually still slightly more powerful than) atom-based studies: this difference between the strategies for bounding the ξ_1 parameter and the λ parameter is easily understood in light of the relation $\xi_1 m/M_P \leftrightarrow 2\lambda/m$ and of the large difference of masses between electrons and (Caesium or Rubidium) atoms.

Finally, in closing this Subsection on alternative models, let us mention the possibility of intrinsically non-universal modifications of the dispersion relation, *i.e.* phenomenological scenarios in which the modifications of the dispersion relation are assumed to be different for different particles without introducing any specific prescription linking these differences to the mass, the spin or other specific properties of the particles. For example, in Ref. [34], and references therein, the authors introduce a free parameter for each different type of particle. In such cases studies of Caesium and, say, Rubidium atoms could be used to set constraints on parameters that are specialized to those types of atoms. In essence, according to this (certainly legitimate) perspective, we might learn that for Caesium and Rubidium ξ_1 is small but without assuming any implications for the values of ξ_1 for other particles. And another noteworthy example is the one of Ref. [41], and references therein, where it is argued, within a specific scenario for quantum gravity, that the effects of modification of the dispersion relation should be confined to a single type of particle, the photon (in which case of course atoms cannot possibly be of any help).

4.3. Strategies for improving the limits

As a contribution toward the development of experimental setups which in some cases may be optimized for our proposal it is important for us to stress that, while essentially here we structured our analysis in a way that might appear to invite interpretation as “quantum-gravity corrections to h/m measurements”, not all improvements in the sensitivity of measurements of h/m will translate into improved bounds on the parameters we here considered.

First we should notice that our result for the ξ_1 -dependent correction to $\Delta\nu/[2\nu_*(\nu_*+p/h)]$ would not appear as a constant shift of h/m , identically applicable to all experimental setups. This is primarily due to the fact that, as shown in Eq. (23), our quantum-gravity correction factor has the form $1 - \xi_1 m^2/[2M_P(h\nu_* + p)]$, and therefore at the very least should be viewed as a momentum-dependent shift of h/m . Different h/m measurements, even when relying on the same atoms (same m), are predicted to find different levels of inconsistency with the uncorrected relationship between h/m and α^2 . This is particularly important because the remarkable accuracy of some measurements of h/m relies crucially [31, 42] on imparting to the atoms high values of momentum, but from our perspective one should notice that the magnitude of the ξ_1 -governed effect decreases with the magnitude of momentum. This is after all one of the reasons why the bound on ξ_1 which we discussed here relied on the determinations of h/m reported in Ref. [31, 33]: a more accurate determination of h/m was actually obtained in the cold-atom (Rubidium) studies reported in Refs. [43, 44], but those more accurate determinations of h/m relied on much higher values of momentum, thereby producing a bound on ξ_1 which is not competitive [11] with the one obtainable using the h/m determination of Ref. [31, 33]. The challenge we propose is therefore the one of reaching higher accuracies in the measurement of h/m without increasing significantly the momentum imparted to the atoms.

Interestingly these concerns do not apply to our result for the ξ_2 parameter. In fact, our result for the ξ_2 -dependent correction to $\Delta\nu/[2\nu_*(\nu_*+p/h)]$ would actually appear as a constant shift of h/m , a mismatch between h/m results and α^2 results of identical magnitude in all experimental setups using the same atoms (same m). This is due to the fact that, as shown in Eq. (19), our quantum-gravity correction factor has the form $[1 - \xi_2 m/M_P]$, and therefore can indeed be viewed as a (mass-dependent but) momentum-independent shift of h/m .

Besides these issues connected with the role played by the momentum of the atoms in our analysis, there are clearly other issues that should be taken into consideration by colleagues possibly contemplating measurements of h/m that could improve the limits on our parameters. One of these clearly deserves mention here, and concerns the setup of h/m measurements as differential measurements. In this respect it is rather significant that our derivation of dependence of the measured $\Delta\nu$ on the Planck-scale effects shows that the sign of the correction term depends on the “histories” (beam-splitting/beam-recombination histories) of the atoms whose interference is eventually

measured. Even from this perspective our result is therefore not to be viewed simply as “a shift in h/m ”: often in the relevant cold-atom experiments one achieves a very accurate determination of h/m by comparing (in the sense of a differential measurement) two different values of $\Delta\nu$ obtained by interference of different pairs of beams produced in the beam-splitting/beam-recombination sequence of a given experimental setup. We should therefore warn our readers that for some differential measurements the effect measured would be twice as large as the one we here computed (same effect but with opposite sign on the two sides of the differential measurement), but on the other hand it is not hard to arrange* for a differential measurement that is insensitive to the quantum-gravity effects (if the “histories” are such that the correction carries the same sign on the two sides of the differential measurement).

5. Atom velocity, energy-momentum conservation and other relativistic issues

We have so far focused on schemes which assume that the only new relevant quantum-gravity-induced law amounts to a modification of the energy-momentum dispersion relation. The main results here derived in Section 3 relied on a strategy of analysis that only requires a specification in the “laboratory frame” of the form of the dispersion relation (which is used to establish, for example, the energy gained by an atom when its spatial momentum is increased) and the law of energy-momentum conservation (which is used to establish, for example, the spatial momentum imparted to an atom upon absorption of a photon of known wavelength). Even within that scheme of analysis one clearly should consider also the possibility of modifications of the law of energy-momentum conservation, especially in light of the fact that certain quantum-gravity scenarios establish (see below) a direct link between modifications of the dispersion relation and some corresponding modifications of the law of energy-momentum conservation.

Moreover, the laboratory-frame perspective is of course too narrow for the investigation of the relativistic issues that clearly must be involved in scenarios that introduce modifications of the dispersion relation. Also from this perspective the quantum-gravity literature offers significant motivation for a careful investigation, since modifications of the laws of transformation between reference frames have been very actively studied (see below). And, as we shall here stress, connected to this issue of boost transformations between reference frames one also finds intriguing challenges for what concerns the description of the velocity of particles.

In this Section we offer an exploratory discussion of these issues. Even in the quantum-gravity literature on ultrarelativistic modifications of the dispersion relation the study of these issues has proven very challenging, and many unsolved puzzles remain. So we shall not even attempt here to address fully these issues in the novel domain of the

* The careful reader will for example notice that Ref. [45] provides an example of setup in which our Planck-scale effects would cancel out.

nonrelativistic limit, which we are here advocating. But we hope that the observations we report here may provide a valuable starting point for more detailed future studies.

Among the “exploratory aspects” of our discussion we should in particular stress that we assume here, as done in most of the related quantum-gravity-inspired literature, that concepts such as energy, spatial momentum and velocity can still be discussed in standard way, so that the novelty of the pictures resides in new laws linking symbols that admit a conventional/traditional physical interpretation. Of course, alternative possibilities also deserve investigation: a given quantum-gravity/quantum-spacetime picture might well (when fully understood) provide motivation not only for novel forms of, say, the dispersion relation but also impose upon us a novel description of the entities, such as the energy E , that appear in the dispersion relation. But we shall already highlight several challenges for the more conservative scenario (with traditional “interpretation of symbols”), and therefore we postpone to future works the investigation of alternative interpretations.

5.1. Velocity and boosted-frame analysis

As a partial remedy to the laboratory-frame limitation of the strategy of analysis discussed in Section 3, we take as our next task the one of obtaining the same result using a scheme of derivation involving boosting and the Doppler effect. The role played by transformation laws between different observer-frames motivates part of our interest for this calculation, since investigations of the fate of Poincaré symmetry in models with Planck-scale modifications of the dispersion relation must in general address the issue of whether the symmetries are “deformed”, in the sense of the “Doubly Special Relativity” scenario [26, 27], or simply “broken”. When the symmetry transformations are correspondingly “deformed” the dispersion relation will be exactly the same for all observers [26, 27]. In the symmetry-breaking alternative scenario the laws of boosting are unmodified and as a result one typically finds that the chosen form of the dispersion relation only holds for one class of observers (at the very least one must expect [46] observer dependence of the parameters that characterize the modification of the dispersion relation). And another aspect of interest for such analyses originates from the fact that the description of the Doppler effect requires a corresponding description of the velocity of the atoms, and therefore requires a specification of the law that fixes the dependence of speeds on momentum/energy at the Planck scale: this too is a debated issue, with many authors favoring $v(p) = \partial E/\partial p$, but some support in the literature also for some alternatives, the most popular of which is $v = p/E$.

As stressed in the opening remarks of this Section, we are just aiming for a first exploratory characterization of these issues and their possible relevance for our atom-recoil studies. Consistently with these scopes we assume that the Doppler effect (boosting) is undeformed and that the dispersion relation is an invariant law. This of course is only one (and a particularly peculiar) example of combination of the possible formulations of the main issues here at stake, but it suffices for exposing the potentially

strong implications that the choice of these formulations can have for the analysis.

Let us start by reanalyzing the recoil of atoms in terms of a Doppler effect, neglecting initially the possible Planck-scale effects (which we shall reintroduce later in this Section). When an atom absorbs a photon whose frequency is ν in the laboratory frame, in the rest frame of the atom the photon has frequency $\tilde{\nu} = \nu(1 - v)$, where v is the speed of the atom in the lab frame (and for definiteness we are considering the case of photon velocity parallel to the atom velocity). Then in the rest frame, if the absorption of the photon takes the atom to an energy level $h\nu_*$, energy conservation takes the form

$$\tilde{\nu} \simeq \nu_* + \frac{h\nu_*^2}{2m}, \quad (24)$$

which of course can also be equivalently rewritten in terms of the lab-frame frequency of the photon

$$\nu \simeq \nu_*(1 + v) + \frac{h\nu_*^2}{2m}, \quad (25)$$

also neglecting a contribution of order $v h\nu_*^2/m$ which is indeed negligible in the nonrelativistic ($v \ll 1$) regime.

This photon absorption also takes the atom from velocity v to velocity v' ,

$$v' \simeq v + h\nu_*/m, \quad (26)$$

in the laboratory frame (where we also observed that the gain of momentum of the atom is approximately $h\nu_*$).

For the stage of (stimulated) emission of a second photon, whose frequency in the lab frame we denote with ν' , the atom would then be moving at this speed v' , and in the rest frame of the atom the frequency of this emitted photon is $\tilde{\nu}' = \nu'(1 + v')$ (also taking into account that if, in the lab frame, the absorbed photon moved in parallel with the atom, the emitted photon must then move in anti-parallel direction). In the case of photon emission, conservation of energy in the rest frame has a different sign with respect to Eq. (24), *i.e.*

$$\tilde{\nu}' \simeq \nu_* - \frac{h\nu_*^2}{2m}, \quad (27)$$

which again one may prefer to re-express in terms of the lab-frame frequency of the photon

$$\nu' \simeq \nu_*(1 - v') - \frac{h\nu_*^2}{2m}. \quad (28)$$

So the lab-frame frequency difference between the two photons is

$$\Delta\nu = \nu_*(v + v') + \frac{h\nu_*^2}{m} \simeq 2v\nu_* + \frac{2h\nu_*^2}{m}, \quad (29)$$

and this (as easily seen upon noticing that in the nonrelativistic limit $v = p/m$) of course perfectly agrees with the corresponding result (12), which we had obtained relying exclusively on lab-frame kinematics.

It is easy to verify that redoing this Doppler-effect-based derivation in presence of our Planck-scale corrections to the dispersion relation (but setting aside, at least for now, possible Planck-scale dependence of the Doppler effect) one ends up replacing (29) with

$$\Delta\nu = \nu_* [v(p) + v(p + h\nu_*)] + \frac{h\nu_*^2}{m} + \xi_1 \frac{m}{M_P} \nu_* . \quad (30)$$

This is the formula that should reproduce our main result (15). Indeed this is the point where one might encounter the necessity of Planck-scale modifications of the boost/Doppler-effect laws and/or of Planck-scale modifications of the law that fixes the dependence of speeds on momentum/energy. Concerning speeds, if one assumes (as done by most authors [2, 4, 6, 9, 12]) $v = \partial E/\partial p$ then in our context (nonrelativistic regime, with ξ_1 parameter) one finds $v(p) = p/m + \xi_1 m/M_P$. If instead, as argued by other authors [47, 48, 49], consistency of the Planck-scale laws requires that $v = p/E$ should be enforced then in our nonrelativistic context one of course has $v(p) = p/m$.

We find that the desirable agreement between (30) and (15) is found upon assuming $v(p) = p/m$, which indeed allows one to rewrite (30) as

$$\Delta\nu = \frac{2\nu_*(p + h\nu_*)}{m} + \xi_1 \frac{m}{M_P} \nu_* . \quad (31)$$

If instead one insists on the alternative $v(p) = \partial E/\partial p = p/m + \xi_1 m/M_P$, then (30) takes the form

$$\Delta\nu = \frac{2\nu_*(p + h\nu_*)}{m} + 2\xi_1 \frac{m}{M_P} \nu_* , \quad (32)$$

which is sizably different from (15).

Our observation that the law $v = p/m$ is automatically consistent with a plausible symmetry-deformation perspective is intriguing, but might well be just a quantitative accident. We thought it might still be worth reporting just as a way to illustrate the complexity of the issues that come into play if our cold-atom studies are examined within a symmetry-deformation scenario, issues that we postpone to future studies. The Doppler effect in models with deformed Poincaré symmetries had not been previously studied, and there are several alternative “schools” on how to derive from the energy-momentum dispersion relation a law giving the speed as a function of energy. In the specific case of the correction term we here parametrized with ξ_1 it would seem that $v = p/m$ is a natural choice, at least in as much as the choice $v(p) = \partial E/\partial p$ appears to be rather pathological/paradoxical since it leads to $v(p) = p/m + \xi_1 m$, *i.e.* a law that assigns nonzero speed to the particle even when the spatial momentum vanishes.

5.2. Testing energy-momentum conservation

Up to this point our analysis has focused on tests of the Lorentz sector of Poincaré symmetry. But of course there is also interest in testing the translation sector, and indeed there has been a corresponding effort, particularly over the last decade. The aspect of the translation sector on which these studies have primarily focused is the

law of energy-momentum conservation in particle-physics processes, and particularly noteworthy are some results [50, 51] which exposed “Planck-scale sensitivity” for the analysis of certain classes of “ultraviolet” (high-energy) modifications of the law of energy-momentum conservation. Even for these studies one can contemplate the alternative between breaking and deforming Poincaré symmetry, and from this perspective it is rather noteworthy that the scenarios in which one deforms Poincaré symmetry require [26, 35] a consistency \sharp between the scheme of modification of the dispersion relation and the scheme of modification of the law of energy-momentum conservation. Instead of course if one is willing to break Poincaré symmetry one can consider independently (or in combination) both modifications of the dispersion relation and modifications of the law of energy-momentum conservation.

In this Section we want to point out that our cold-atom-based strategy also provides opportunities for studies of the form of the law of energy-momentum conservation in the nonrelativistic regime. The observations on cold-atom experiments that some of us reported in Ref. [11] already inspired the recent analysis of Ref. [52], which provides preliminary encouragement for the idea of using cold-atom experiments for the study of the form of the law of energy-momentum conservation in the nonrelativistic regime. The scopes of the analysis reported in Ref. [52] were rather limited, since it focused on one specific model, which in particular codifies no modifications of the dispersion relation: the only modification allowed in Ref. [52] appeared in the law of energy-momentum conservation and appeared only at subleading order (in the sense here introduced in Sections 2-3) in the nonrelativistic limit.

While maintaining the perspective of a first exploratory investigation of these issues, we shall here contemplate a more general scenario, with modifications of both energy-momentum conservation and dispersion relation, and with correction terms strong enough to appear even at the leading order in the nonrelativistic regime. Besides aiming for greater generality, our interest in this direction is also motivated by the desire of setting up future analysis which might consider in detail the interplay between modifications of the dispersion relation and modifications of energy-momentum conservation, particularly from the perspective of identifying scenarios with deformation (rather than breakdown) of Poincaré symmetries, for which, as mentioned, this interplay is in many instances required [26, 35]. While we shall not here attempt to formulate a suitable deformed-symmetry scenario, the observations we here report are likely to be relevant for the possible future search of such a formulation.

In light of the exploratory nature of our investigation of this point we shall be satisfied illustrating the possible relevance of the interplay between dispersion relation and energy-momentum conservation for the specific case of modified laws of conservation

\sharp These consistency requirements for a deformation of Poincaré symmetry are very restrictive but may not suffice to fully specify the form of the law of energy-momentum conservation by insisting on compatibility with a chosen form of the dispersion relation [26, 35].

of spatial momentum (ordinary conservation of energy):

$$\begin{aligned} \vec{p}_1 + \vec{p}_2 - \frac{\rho_1}{4M_P} \left(\frac{E_1^2}{E_2} \vec{p}_1 + \frac{E_2^2}{E_1} \vec{p}_2 \right) - \frac{\rho_2}{2M_P} (E_1 \vec{p}_2 + E_2 \vec{p}_1) = \\ = \vec{p}_3 + \vec{p}_4 - \frac{\rho_1}{4M_P} \left(\frac{E_3^2}{E_4} \vec{p}_3 + \frac{E_4^2}{E_3} \vec{p}_4 \right) - \frac{\rho_2}{2M_P} (E_3 \vec{p}_4 + E_4 \vec{p}_3). \end{aligned} \quad (33)$$

We are focusing on the case of two incoming and two outgoing particles (relevant for processes in which a photon is absorbed and one is emitted by an atom), and we characterized the modification in terms of parameters ρ_1 and ρ_2 . As announced, we shall keep track of these parameters ρ_1 and ρ_2 together with the parameters ξ_1 and ξ_2 that parametrized the modifications of the dispersion relation in the nonrelativistic limit^{††}.

For a two-photon Raman transition our modified law of conservation of spatial momentum has significant implications along the common direction of the laser beams used to excite/de-excite the atoms:

$$\begin{aligned} h|\vec{k}| + |\vec{p}| - \frac{\rho_1}{4M_P} \left(\frac{h^2\nu^2}{m} h|\vec{k}| + \frac{m^2}{h\nu} |\vec{p}| \right) - \frac{\rho_2}{2M_P} (h\nu|\vec{p}| + Eh|\vec{k}|) = \\ - h|\vec{k}'| + |\vec{p}'| - \frac{\rho_1}{4M_P} \left(-\frac{h^2\nu'^2}{m} h|\vec{k}'| + \frac{m^2}{h\nu'} |\vec{p}'| \right) - \frac{\rho_2}{2M_P} (h\nu'|\vec{p}'| - E'h|\vec{k}'|), \end{aligned} \quad (34)$$

In Section 3 we used ordinary momentum conservation, $h|\vec{k}| + |\vec{p}| = -h|\vec{k}'| + |\vec{p}'|$, but if instead one adopts (34) the following result is then straightforwardly obtained:

$$\frac{\Delta\nu}{2\nu_*(\nu_* + p/h)} \simeq \frac{h}{m} + \frac{1}{M_P} [m(\xi_1 - \rho_1) + (2\xi_2 - \rho_2)p + 2(\xi_2 - \rho_2)h\nu_*] \frac{h\nu_*}{2\nu_*(h\nu_* + p)}. \quad (35)$$

While this is, as stressed, only an exploratory investigation of the role that could be played by modifications of energy-momentum conservation (in particular there is clearly a strong influence of the specific *ansatz* we adopted for the modified law of conservation of energy-momentum) it is still noteworthy that the parameter ρ_1 enters the final result at the same order as the parameter ξ_1 and similarly the parameter ρ_2 enters the final result at the same order as the parameter ξ_2 . In particular, this implies that even at the type of leading order we here mainly focused on (the order where ξ_1 appears) the possibility of modifications of the law of energy-momentum conservation may well be relevant, with nonnegligible effects even in cases where $\xi_1 = 0$ but $\rho_1 \neq 0$.

6. Closing remarks

We have here used the noteworthy example of atom-recoil measurements to explore whether it is possible to setup a phenomenology for the nonrelativistic limit of the

^{††}For simplicity we here simply assume that the photon dispersion relation (bound to be in the ultrarelativistic regime) is undeformed. As stressed at the end of Sec. 3, one could introduce for photons the parameter η_1 , characteristic of the ultrarelativistic regime, but in the analysis of atom-recoil studies η_1 turns out to have effects much smaller than the ones of ξ_1 and ξ_2 (as already discussed at the end of Sec. 3) and also of ρ_1 and ρ_2 .

energy-momentum dispersion relation that adopts the same spirit of a popular research program focusing instead on the corresponding ultrarelativistic regime. It appears that this is indeed possible and that on the one hand there is a strong complementarity of insight to be gained by combining studies of the nonrelativistic regime and of the ultrarelativistic regime, and on the other hand the nature of the conceptual issues that must be handled (particularly the relativistic issues associated with the possibility of breaking or deforming Poincaré symmetry) are closely analogous. We therefore argue that by adding the nonrelativistic limit to the relevant phenomenology agenda we could improve our ability to constrain certain scenarios, and we could also gain a powerful tool from the conceptual side, exploiting the possibility to view the same conceptual challenges within regimes that are otherwise very different.

For what concerns the phenomenology we here proposed it is noteworthy that, particularly considering the values of ξ_1 being probed, any improvement in sensitivity that will be achieved could also be viewed as a (slim but valuable) chance for a striking discovery. We therefore feel that our analysis should motivate experimentalists to tailor some of their plans in this direction (also using the remarks we offered in Subsection 4.3) and should motivate theorists toward a vigorous effort aimed at overcoming the technical difficulties on the quantum-gravity-theory side that presently obstruct the derivation of more detailed quantitative predictions.

Acknowledgments

We gratefully acknowledge valuable conversations with H. Müller and L. Urrutia. G. A.-C. is supported in part by grant RFP2-08-02 from The Foundational Questions Institute (fqxi.org). C. L. is supported in part by the German Research Foundation and the Centre for Quantum Engineering and Space-Time Research QUEST. D. M., J. M. C., J. L. C. and J. I. are supported by CICYT (grants FPA2006-02315 and FPA2009-09638) and DGIID-DGA (grant 2009-E24/2). J. I. acknowledges a FPU grant and D. M. a FPI grant from MICINN.

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