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Wronskian representation of solutions of NLS equation, and seventh order rogue wave.

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Abstract

In this paper, we use the representation of the solutions of the focusing nonlinear Schrödinger equation we have constructed recently, in terms of wronskians; when we perform a special passage to the limit, we get quasi-rational solutions expressed as a ratio of two wronskian determinants.

We have already construct Peregrine breathers of orders $N = 4, 5, 6$ in preceding works; we give here the Peregrine breather of order seven.

1 Introduction

From fundamental work of Zakharov and Shabat in 1968, a lot of research has been carried out on the nonlinear Schrödinger equation (NLS). The case of periodic and almost periodic algebro-geometric solutions to the focusing NLS equation were first constructed in 1976 by Its and Kotlyarov [13]. The first quasi-rational solutions of NLS equation were constructed in 1983 by Peregrine; they are nowadays called worldwide Peregrine breathers. In 1986 Eleonski, Akhmediev and Kulagin obtained the two-phase almost periodic solution to the NLS equation and obtained the first higher order analogue of the Peregrine breather[3]. Other families of higher order were constructed in a series of articles by Akhmediev et al. [1, 2] using Darboux transformations. In this paper, we use a result [9] giving a new representation of the solutions

of the NLS equation in terms of a ratio of two wronskians determinants of even order $2N$ composed of elementary functions; the related solutions of NLS are called of order N . When we perform the passage to the limit when some parameter tends to 0, we got families of multi-rogue wave solutions of the focusing NLS equation depending on a certain number of parameters. It allows to recognize the famous Peregrine breather [16] and also higher order Peregrine's breathers constructed by Akhmediev [1, 4].

Recently, an other representation of the solutions of the focusing NLS equation, as a ratio of two determinants has been given in [11] using generalized Darboux transform.

A new approach has been done in [15] which gives a determinant representation of solutions of the focusing NLS equation, obtained from Hirota bilinear method, derived by reduction of the Gram determinant representation for Davey-Stewartson system.

Here, we construct the Peregrine breather of order $N = 7$ which shows the efficiency of this method.

2 Expression of solutions of NLS equation in terms of Wronskian determinant and quasi-rational limit

2.1 Solutions of NLS equation in terms of Wronskian determinant

We briefly recall results obtained in [9, 10]. We consider the focusing NLS equation

$$iv_t + v_{xx} + 2|v|^2v = 0. \quad (1)$$

From [10], the solution of the NLS equation can be written in the form

$$v(x, t) = \frac{\det(I + A_3(x, t))}{\det(I + A_1(x, t))} \exp(2it - i\varphi). \quad (2)$$

In (2), the matrix $A_r = (a_{\nu\mu})_{1 \leq \nu, \mu \leq 2N}$ is defined by

$$a_{\nu\mu} = (-1)^{\epsilon_\nu} \prod_{\lambda \neq \mu} \left| \frac{\gamma_\lambda + \gamma_\nu}{\gamma_\lambda - \gamma_\mu} \right| \exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu}). \quad (3)$$

The terms ϵ_ν are defined by :

$$\begin{aligned}\epsilon_\nu &= 0, & 1 \leq \nu \leq N \\ \epsilon_\nu &= 1, & N + 1 \leq \nu \leq 2N.\end{aligned}\tag{4}$$

We consider the following functions

$$\begin{aligned}\phi_\nu^r(y) &= \sin(\kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y), & 1 \leq \nu \leq N, \\ \phi_\nu^r(y) &= \cos(\kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y), & N + 1 \leq \nu \leq 2N.\end{aligned}\tag{5}$$

We use the following notations :

$$\Theta_\nu = \kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y, \quad 1 \leq \nu \leq 2N.$$

$W_r(y) = W(\phi_1, \dots, \phi_{2N})$ is the wronskian

$$W_r(y) = \det[(\partial_y^{\mu-1} \phi_\nu)_{\nu, \mu \in [1, \dots, 2N]}].\tag{6}$$

We consider the matrix $D_r = (d_{\nu\mu})_{\nu, \mu \in [1, \dots, 2N]}$ defined by

$$\begin{aligned}d_{\nu\mu} &= (-1)^{\epsilon_\nu} \prod_{\lambda \neq \mu} \left| \frac{\gamma_\lambda + \gamma_\nu}{\gamma_\lambda - \gamma_\mu} \right| \exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu}), \\ &1 \leq \nu \leq 2N, \quad 1 \leq \mu \leq 2N,\end{aligned}$$

with

$$x_{r,\nu} = (r-1) \ln \frac{\gamma_\nu - i}{\gamma_\nu + i}.$$

Then we get the following link between Fredholm and Wronskian determinants [10]

Theorem 2.1

$$\det(I + D_r) = k_r(0) \times W_r(\phi_1, \dots, \phi_{2N})(0),\tag{7}$$

where

$$k_r(y) = \frac{2^{2N} \exp(i \sum_{\nu=1}^{2N} \Theta_\nu)}{\prod_{\nu=2}^{2N} \prod_{\mu=1}^{\nu-1} (\gamma_\nu - \gamma_\mu)}.$$

It can be deduced the following result :

Theorem 2.2 *The function v defined by*

$$v(x, t) = \frac{W_3(0)}{W_1(0)} \exp(2it - i\varphi).\tag{8}$$

is solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0.$$

2.2 Quasi-rational solutions of NLS equation

In the following, we take the limit when the parameters $\lambda_j \rightarrow 1$ for $1 \leq j \leq N$ and $\lambda_j \rightarrow -1$ for $N + 1 \leq j \leq 2N$.

For simplicity, we denote d_j the term $\frac{c_j}{\sqrt{2}}$.

We consider the parameter λ_j written in the form

$$\lambda_j = 1 - 2\epsilon^2 d_j^2, \quad 1 \leq j \leq N. \quad (9)$$

When ϵ goes to 0, we realize limited expansions at order p , for $1 \leq j \leq N$, of the terms

$$\begin{aligned} \kappa_j &= 4d_j\epsilon(1 - \epsilon^2 d_j^2)^{1/2}, \quad \delta_j = 4d_j\epsilon(1 - 2\epsilon^2 d_j^2)(1 - \epsilon^2 d_j^2)^{1/2}, \\ \gamma_j &= d_j\epsilon(1 - \epsilon^2 d_j^2)^{-1/2}, \quad x_{r,j} = (r - 1) \ln \frac{1 + i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}}{1 - i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}}, \\ \kappa_{N+j} &= 4d_j\epsilon(1 - \epsilon^2 d_j^2)^{1/2}, \quad \delta_{N+j} = -4d_j\epsilon(1 - 2\epsilon^2 d_j^2)(1 - \epsilon^2 d_j^2)^{1/2}, \\ \gamma_{N+j} &= 1/(d_j\epsilon)(1 - \epsilon^2 d_j^2)^{1/2}, \quad x_{r,N+j} = (r - 1) \ln \frac{1 - i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}}{1 + i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}}. \end{aligned}$$

We have the central result given in [10] :

Theorem 2.3 *With the parameters λ_j defined by (9), the function v defined by*

$$v(x, t) = \exp(2it - i\varphi) \lim_{\epsilon \rightarrow 0} \frac{W_3(0)}{W_1(0)}, \quad (10)$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0,$$

depending on $3N$ parameters $d_j, \tilde{a}_j, \tilde{b}_j, 1 \leq j \leq N$.

3 Quasi-rational solutions of order N

To get solutions of NLS equation written in the context of fiber optics

$$iu_x + \frac{1}{2}u_{tt} + u|u|^2 = 0, \quad (11)$$

from these of (1), we can make the following changes of variables

$$\begin{aligned} t &\rightarrow X/2 \\ x &\rightarrow T. \end{aligned} \quad (12)$$

3.1 Case N=7

In the case of order $N = 7$, we make an expansion at order 13. Taking the limit when $\epsilon \rightarrow 0$, the solution of NLS equation (11) takes the form

$$v(x, t) = \frac{n(x, t)}{d(x, t)} \exp(2it - i\varphi).$$

Because of the length of the complete analytical expression, we only give it in the appendix.

We give here the expression of the solution $v(x, t) = \frac{n(x, t)}{d(x, t)}$ in the case of all the parameters equal to 0 :

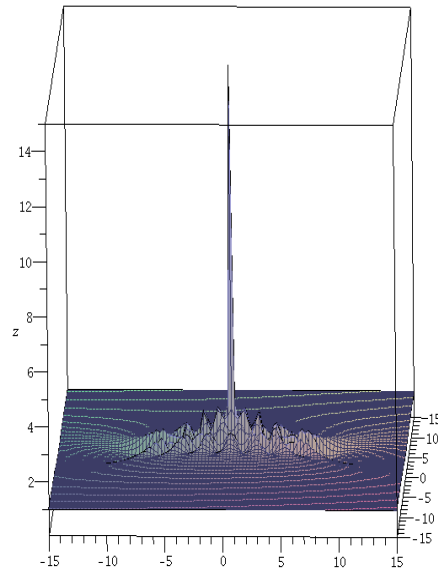


Figure 1: Solution to the NLS equation, N=7.

4 Conclusion

The method described in the present paper provides a powerful tool to get explicitly solutions of the NLS equation.

As my knowledge, it is the first time that the Peregrine breather of order seven is presented.

It confirms the conjecture about the shape of the breather in the (x, t) coordinates, the maximum of amplitude equal to $2N + 1 = 15$ and the degree of polynomials in x and t here equal to 56. This new formulation gives an infinite set of non singular solution of NLS equation. It opens a large way to future researches in this domain.

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Appendix

The solution of NLS equation takes the form

$$v_N(x, t) = \frac{n(x, t)}{d(x, t)} \exp(2it) = \left(1 - \alpha_N \frac{G_N(2x, 4t) + iH_N(2x, 4t)}{Q_N(2x, 4t)}\right) e^{2it}$$

with

$$G_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{g}_k(T) X^k$$

$$H_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{h}_k(T) X^k$$

$$Q_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{q}_k(T) X^k$$

$$\begin{aligned}
& \alpha_7 = 4, \quad \mathbf{g}_{56} = 0, \quad \mathbf{g}_{55} = 0, \quad \mathbf{g}_{54} = 28, \quad \mathbf{g}_{53} = 0, \quad \mathbf{g}_{52} = 2268T^2 + 2268, \quad \mathbf{g}_{51} = 0, \quad \mathbf{g}_{50} = 49140T^4 \\
& -125496T^2 + 272916, \quad \mathbf{g}_{49} = 0, \quad \mathbf{g}_{48} = 573300T^6 - 8158500T^4 - 4630500T^2 + 33736500, \quad \mathbf{g}_{47} = 0, \\
& \mathbf{g}_{46} = 4422600T^8 - 145000800T^6 + 144471600T^4 - 288943200T^2 + 3645923400, \quad \mathbf{g}_{45} = 0, \\
& \mathbf{g}_{44} = 24864840T^{10} - 1405819800T^8 + 8654642640T^6 + 1328821200T^4 - 27569467800T^2 + 285603683400, \\
& \mathbf{g}_{43} = 0, \quad \mathbf{g}_{42} = 107747640T^{12} - 9018923760T^{10} + 129131097480T^8 - 92144939040T^6 \\
& -160080089400T^4 - 4068407574000T^2 + 9697930011000, \quad \mathbf{g}_{41} = 0, \quad \mathbf{g}_{40} = 372972600T^{14} \\
& -41946985080T^{12} + 1042152554520T^{10} - 3985886824920T^8 - 2424305683800T^6 - 38041300420200T^4 \\
& -532602960535800T^2 - 462639709917000, \quad \mathbf{g}_{39} = 0, \quad \mathbf{g}_{38} = 1056755700T^{16} - 148997772000T^{14} \\
& +5529226071600T^{12} - 48020254682400T^{10} + 9567791263800T^8 - 277615181844000T^6 \\
& -3997430875746000T^4 - 27691049938860000T^2 - 11433617241007500, \quad \mathbf{g}_{37} = 0, \quad \mathbf{g}_{36} = 2493390900T^{18} \\
& -417526886700T^{16} + 21097721106000T^{14} - 312305697673200T^{12} + 750925867368600T^{10} \\
& -1296117088497000T^8 - 19549525119894000T^6 + 178015776175314000T^4 + 2547484073859532500T^2 \\
& -257622602070487500, \quad \mathbf{g}_{35} = 0, \quad \mathbf{g}_{34} = 4960535580T^{20} - 942501760200T^{18} \\
& +60895186381260T^{16} - 1320509170965600T^{14} + 8073640409095800T^{12} - 10433330397428400T^{10} \\
& -94059388591245000T^8 + 4212714864528324000T^6 + 81525901261652347500T^4 \\
& +83880191483144595000T^2 + 4532925200381077500, \quad \mathbf{g}_{33} = 0, \quad \mathbf{g}_{32} = 8396404380T^{22} \\
& -1736314646220T^{20} + 137179010388420T^{18} - 3955742649118260T^{16} + 44742788370264600T^{14} \\
& -87890985649467000T^{12} - 581266358785035000T^{10} + 34489428486040143000T^8 \\
& +916422481982789743500T^6 - 267128299891396597500T^4 + 3533027762243314912500T^2 \\
& +1052175273585966337500, \quad \mathbf{g}_{31} = 0, \quad \mathbf{g}_{30} = 12168702000T^{24} - 2628289863360T^{22} \\
& +246156751098720T^{20} - 8802207660818880T^{18} + 156743234946421200T^{16} \\
& -400285925208892800T^{14} - 4082398971145387200T^{12} + 182299012392364560000T^{10} \\
& +6993601895564462178000T^8 - 29136271554263384280000T^6 + 22705339934145925980000T^4 \\
& +133644440887635613320000T^2 + 69720100670714536350000, \quad \mathbf{g}_{29} = 0, \quad \mathbf{g}_{28} = 15164074800T^{26} \\
& -3271508730000T^{24} + 356805698608800T^{22} - 14963164581103200T^{20} + 373275824011952400T^{18} \\
& -1646661408941934000T^{16} - 23044246961534568000T^{14} + 647137266547990680000T^{12} \\
& +44440332791868485010000T^{10} - 368524266068389959990000T^8 + 515157395379005679300000T^6 \\
& +964818990199979907300000T^4 + 3432008280166633455750000T^2 + 2522450003846320374750000, \\
& \mathbf{g}_{27} = 0, \quad \mathbf{g}_{26} = 16287339600T^{28} - 3325987072800T^{26} + 422344463612400T^{24} \\
& -19765833506798400T^{22} + 609616438648232400T^{20} - 8426887778118348000T^{18} \\
& -198426421456868682000T^{16} - 2190017982667295088000T^{14} + 260606376243375811110000T^{12} \\
& -2667540176320654889100000T^{10} + 8847209138357814736050000T^8 + 13096014915145148494200000T^6 \\
& +6581743865224894916550000T^4 + 11025412640863669471500000T^2 + 47293802784572214197250000, \\
& \mathbf{g}_{25} = 0, \quad \mathbf{g}_{24} = 15089190480T^{30} - 2709127486800T^{28} + 412978164031440T^{26} - 20496272265378000T^{24} \\
& +625906405138669200T^{22} - 35718951161305011600T^{20} - 1333506673014425910000T^{18} \\
& -34418796637312024938000T^{16} + 1244339447399806908150000T^{14} - 11990961186295394893590000T^{12} \\
& +65945519948476262964870000T^{10} + 160150242555541152164250000T^8 \\
& +91138495041978507273150000T^6 - 1554908002287761890518750000T^4 \\
& -2392952896647698174039250000T^2 + 889739092856294586155250000, \quad \mathbf{g}_{23} = 0, \\
& \mathbf{g}_{22} = 12047014980T^{32} - 1685346505920T^{30} + 339670088966880T^{28} - 16767007875455040T^{26} \\
& +195015256305433200T^{24} - 103115102184824184000T^{22} - 4963164090380892396000T^{20} \\
& -73350016919927243496000T^{18} + 5521291279776971601903000T^{16} - 39856907136517355724840000T^{14} \\
& +376003222446071990870100000T^{12} + 718852738860427750979400000T^{10} \\
& +3176243015946106500944550000T^8 - 17990585286630587798567400000T^6 \\
& -67776208060401761741320500000T^4 - 27136987658593437499875000000T^2
\end{aligned}$$

$$\begin{aligned}
& +12087940146214297919616562500, \quad \mathbf{g}_{21} = 0, \quad \mathbf{g}_{20} = 8267559300 T^{34} - 689768832060 T^{32} \\
& +242818935559200 T^{30} - 10771451892321120 T^{28} - 609328959012219600 T^{26} \\
& -203253573603293413200 T^{24} - 10395034180133435632800 T^{22} + 165116685980326092012000 T^{20} \\
& +17170916828136913280439000 T^{18} - 102607297371198744509385000 T^{16} \\
& +1635023327366588192773740000 T^{14} + 2860768497736673150887260000 T^{12} \\
& +45606671281786995394764750000 T^{10} - 80947367097903820900836450000 T^8 \\
& -700286840100765740788465500000 T^6 + 130801873396469808053020500000 T^4 \\
& -447283391478568632911861437500 T^2 + 56768583214608413567481562500, \quad \mathbf{g}_{19} = 0, \\
& \mathbf{g}_{18} = 4855550700 T^{36} - 40277853000 T^{34} + 158719690968300 T^{32} - 5194244162923200 T^{30} \\
& -1360528622797251600 T^{28} - 281596511270242476000 T^{26} - 11850683160765907458000 T^{24} \\
& +920175424144902165240000 T^{22} + 25468628785050976429905000 T^{20} \\
& -310836193534541858413230000 T^{18} + 5807378097843796606757625000 T^{16} \\
& +15634234802719441673364600000 T^{14} + 278887408065007528609572750000 T^{12} \\
& +52000683646906354993132500000 T^{10} - 4439081082994819230111392250000 T^8 \\
& +8438816045170493665968285000000 T^6 - 3001122035235356030511325312500 T^4 \\
& -7391743593040300996805428125000 T^2 - 1054152239431812041879170312500, \\
& \mathbf{g}_{17} = 0, \quad \mathbf{g}_{16} = 2424321900 T^{38} + 211776711300 T^{36} + 99477568505700 T^{34} \\
& -1389209187712500 T^{32} - 1621264449868232400 T^{30} - 280260753940566630000 T^{28} \\
& -4787780690058717582000 T^{26} + 1656692481162132035622000 T^{24} + 5482826579394141962985000 T^{22} \\
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& -74996679144078506620333965937500 T^4 - 61056959125851227464611623437500 T^2 \\
& -5245972489497170017957682812500, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 1019458440 T^{40} + 209361952800 T^{38} \\
& +59799859202160 T^{36} + 597930018285600 T^{34} - 1327830040347010200 T^{32} - 203680458354584534400 T^{30} \\
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& -954575061914762846454243847500000 T^6 - 411166271799374891645351756250000 T^4 \\
& +783333878093152389727736512500000 T^2 - 20793848949925378145840896875000, \quad \mathbf{g}_{13} = 0, \\
& \mathbf{g}_{12} = 356396040 T^{42} + 123877270440 T^{40} + 32610284395920 T^{38} + 1216306932881040 T^{36} \\
& -784563891537195000 T^{34} - 110761814779398783000 T^{32} + 7937796416339529432000 T^{30} \\
& +978775172774012225304000 T^{28} - 21292516324448140972662000 T^{26} + 161923476389289550585650000 T^{24} \\
& +34432591861323164507298300000 T^{22} + 897588666218543018306687100000 T^{20} \\
& +10399172873750652367819292850000 T^{18} + 12734590641823469076245037450000 T^{16} \\
& -1056247157487187710224365185000000 T^{14} + 1832056249038285845965084695000000 T^{12} \\
& +2959286708382169575258073850625000 T^{10} - 344028559792148330017460416875000 T^8 \\
& -12140076235349263409707132383750000 T^6 + 21659795132230009469150342156250000 T^4 \\
& +3602055625460117946415936265625000 T^2 + 61049167368118769138877703125000, \quad \mathbf{g}_{11} = 0, \\
& \mathbf{g}_{10} = 101719800 T^{44} + 52472463120 T^{42} + 15005352063240 T^{40} + 1021506129594720 T^{38}
\end{aligned}$$

$$\begin{aligned}
& -331006205910303000 T^{36} - 47731541013103978800 T^{34} + 3909120362191961947800 T^{32} \\
& + 302275195306483257936000 T^{30} - 3397149051079935164706000 T^{28} \\
& + 1063471996099453676509620000 T^{26} + 35282865975833690856131490000 T^{24} \\
& + 1009461102866985900033662760000 T^{22} + 28108708355943404795842686450000 T^{20} \\
& - 85747360020983326179491877900000 T^{18} - 1491463246568322076250046095250000 T^{16} \\
& + 2775278138548494265001581266000000 T^{14} - 18540128543730526297289281181625000 T^{12} \\
& - 21944626454220073070250932898750000 T^{10} - 80973913998934031957478576354375000 T^8 \\
& + 148114784421877559463874334137500000 T^6 + 31126078957047574894186796128125000 T^4 \\
& + 20673599609280586675821470606250000 T^2 + 1398479083269650104689513421875000, \\
& \mathbf{g}_9 = 0, \quad \mathbf{g}_8 = 23095800 T^{46} + 16444701000 T^{44} + 5475036936600 T^{42} \\
& + 566976092898600 T^{40} - 93441539643190200 T^{38} - 17935367491764261000 T^{36} \\
& - 34358185653025311000 T^{34} - 84340417236266770425000 T^{32} - 6247372784431620892050000 T^{30} \\
& + 79423291493938895975010000 T^{28} - 6517484194549893449935050000 T^{26} \\
& + 1157932673902046452477117050000 T^{24} + 48145337637619603341498788250000 T^{22} \\
& - 238274280945074139753039302250000 T^{20} + 1614124491632111637086543885250000 T^{18} \\
& + 30463748856261460407343234308750000 T^{16} - 118755251721826815926043684095625000 T^{14} \\
& - 48773592689963392959811287159375000 T^{12} - 1738438081260949813862744404903125000 T^{10} \\
& + 1572448964546520061265933822278125000 T^8 - 673017920052474095253612817734375000 T^6 \\
& + 326795486450765136002224260609375000 T^4 + 79407567379987217482876217578125000 T^2 \\
& + 2882552147106173292709630546875000, \quad \mathbf{g}_7 = 0, \quad \mathbf{g}_6 = 4013100 T^{48} \\
& + 3764023200 T^{46} + 1497943465200 T^{44} + 217159761218400 T^{42} - 14221439411394600 T^{40} \\
& - 6062992711379604000 T^{38} - 868091861569403394000 T^{36} - 137559109216434396876000 T^{34} \\
& - 9260888323767068417467500 T^{32} + 83092589684282324500200000 T^{30} \\
& + 45951189509544653085374700000 T^{28} + 2043751713304237133419114200000 T^{26} \\
& + 39823148838347256006248674050000 T^{24} - 107645574930283654095040209000000 T^{22} \\
& + 5704220460549886178354194243500000 T^{20} + 47477860076491575514786707705000000 T^{18} \\
& + 96760004701426703802202262537812500 T^{16} - 2746609454519613461575760731267500000 T^{14} \\
& - 5055092442696255371030072885681250000 T^{12} + 7685340668503114406318287145887500000 T^{10} \\
& - 2527654302854142287937009789515625000 T^8 + 2549540272922798483165127227437500000 T^6 \\
& + 1681003830938891487377650144593750000 T^4 - 119038912584481624295814690937500000 T^2 \\
& + 4248891856349740702685512617187500, \quad \mathbf{g}_5 = 0, \quad \mathbf{g}_4 = 501228 T^{50} + 599186700 T^{48} \\
& + 286948221840 T^{46} + 55465811326800 T^{44} + 225191978134200 T^{42} - 1592867890825734600 T^{40} \\
& - 378975454715995830000 T^{38} - 49698971523700677486000 T^{36} - 2416070416982018904607500 T^{34} \\
& + 163449545090537650746532500 T^{32} + 22630419780133976885691540000 T^{30} \\
& + 658066531416844960184890500000 T^{28} + 17958744056622134874813046650000 T^{26} \\
& + 265938368682687944669400224250000 T^{24} + 3373546912165127652278618794500000 T^{22} \\
& + 49992632917159774293562606996500000 T^{20} + 243622840935340303792373213507812500 T^{18} \\
& - 3472965533440290793642368859414687500 T^{16} - 3909930410917269828231259536168750000 T^{14} \\
& - 2373854538016335651007933345893750000 T^{12} + 22505528586240563125522950136321875000 T^{10} \\
& + 1725433101762243379165143781171875000 T^8 + 20602166714282000488422754253531250000 T^6 \\
& - 256192442301384365332296834843750000 T^4 - 79260192530178496317997648945312500 T^2 \\
& - 3304693666049798324310954257812500, \quad \mathbf{g}_3 = 0, \quad \mathbf{g}_2 = 40068 T^{52} \\
& + 59556168 T^{50} + 34248844980 T^{48} + 8538182019360 T^{46} + 489064062475800 T^{44} \\
& - 249702181686342000 T^{42} - 69203448982691793000 T^{40} - 5213133014477381532000 T^{38} \\
& + 438714917019434320303500 T^{36} + 84931805893311124163955000 T^{34}
\end{aligned}$$

$$\begin{aligned}
& +4218956433098176838806237500 T^{32} + 31107276944713383532410600000 T^{30} \\
& +3271649949938812288254997050000 T^{28} + 201849235241947964535548861100000 T^{26} \\
& +4953109227394118516246184914250000 T^{24} - 25630541428745966760074602575000000 T^{22} \\
& -412192793582829745372981826294062500 T^{20} - 4700092011184358352745951843198125000 T^{18} \\
& +12509179325796087908708757937602187500 T^{16} + 17173693466353903144949117196862500000 T^{14} \\
& -36421201667691057461711715138515625000 T^{12} + 130348386619893149350129124043656250000 T^{10} \\
& +135527319688597478057052486959484375000 T^8 + 12269848488887082318515323846312500000 T^6 \\
& +4223241138846592268073003781757812500 T^4 - 374846681549077124214699668671875000 T^2 \\
& -1101564555349932774770318085937500, \quad \mathbf{g}_1 = 0, \quad \mathbf{g}_0 = 1540 T^{54} + 2791404 T^{52} \\
& +1916196156 T^{50} + 600986059380 T^{48} + 60893832396600 T^{46} - 14981298604738200 T^{44} \\
& -5448980120702995800 T^{42} - 677699093948317749000 T^{40} - 28109651584658009404500 T^{38} \\
& +2321568723921179741662500 T^{36} + 483153674937785992894792500 T^{34} \\
& +34432196292857178176426257500 T^{32} + 1329648591046758598027462050000 T^{30} \\
& +48030465435235520461569818550000 T^{28} + 871144519174189671772071537750000 T^{26} \\
& +862287173564925372614929829250000 T^{24} + 37733399259532601896905168972937500 T^{22} \\
& -97131347340613907133312877564687500 T^{20} - 6464853953097864723190272724680937500 T^{18} \\
& -13312771923417438005165293807457812500 T^{16} - 21734401086635138140243391915653125000 T^{14} \\
& +162602072233593709389730376910215625000 T^{12} + 57203947835898442945082247296390625000 T^{10} \\
& +95479747942182277597570867016671875000 T^8 + 4823751187877355620719222898320312500 T^6 \\
& -1973321762271862907337361717851562500 T^4 - 20929726551648722720636043632812500 T^2 \\
& +157366365049990396395759726562500,
\end{aligned}$$

$$\begin{aligned}
\mathbf{h}_{56} &= 0, \quad \mathbf{h}_{55} = 0, \quad \mathbf{h}_{54} = 28T, \quad \mathbf{h}_{53} = 0, \quad \mathbf{h}_{52} = 756 T^3 - 2268 T, \quad \mathbf{h}_{51} = 0, \quad \mathbf{h}_{50} = 9828 T^5 \\
& -181944 T^3 - 147420 T, \quad \mathbf{h}_{49} = 0, \quad \mathbf{h}_{48} = 81900 T^7 - 3660300 T^5 + 4630500 T^3 - 15214500 T, \quad \mathbf{h}_{47} = 0, \\
\mathbf{h}_{46} &= 491400 T^9 - 39160800 T^7 + 310428720 T^5 + 79380000 T^3 - 1631259000 T, \quad \mathbf{h}_{45} = 0, \\
\mathbf{h}_{44} &= 2260440 T^{11} - 275020200 T^9 + 5106674160 T^7 - 3889302480 T^5 + 392931000 T^3 - 148377889800 T, \\
\mathbf{h}_{43} &= 0, \quad \mathbf{h}_{42} = 8288280 T^{13} - 1398806640 T^{11} + 45284861160 T^9 - 207951559200 T^7 - 42200789400 T^5 \\
& +253204736400 T^3 - 8291708548200 T, \quad \mathbf{h}_{41} = 0, \quad \mathbf{h}_{40} = 24864840 T^{15} - 5449225320 T^{13} \\
& +264073498920 T^{11} - 2825524908360 T^9 + 1514969046360 T^7 + 6573185526600 T^5 + 137993987993400 T^3 \\
& +149047765713000 T, \quad \mathbf{h}_{39} = 0, \quad \mathbf{h}_{38} = 62162100 T^{17} - 16844335200 T^{15} + 1110003879600 T^{13} \\
& -20635793085600 T^{11} + 60341881030200 T^9 + 76117021596000 T^7 + 2171428651854000 T^5 \\
& +25299618434244000 T^3 + 62466943452952500 T, \quad \mathbf{h}_{37} = 0, \quad \mathbf{h}_{36} = 131231100 T^{19} \\
& -42306887700 T^{17} + 3541157207280 T^{15} - 98466907770000 T^{13} + 661042062865800 T^{11} \\
& -16499618011800 T^9 + 18885110233374000 T^7 + 314318361244734000 T^5 \\
& +1773658200690337500 T^3 + 2282181897675487500 T, \quad \mathbf{h}_{35} = 0, \quad \mathbf{h}_{34} = 236215980 T^{21} \\
& -87763321800 T^{19} + 8848450735740 T^{17} - 335891899617120 T^{15} + 3894124207743000 T^{13} \\
& -10086562817377200 T^{11} + 107633107470735000 T^9 + 2631781328889636000 T^7 \\
& +12651337028921629500 T^5 - 44712998532434925000 T^3 + 94738725025613707500 T, \quad \mathbf{h}_{33} = 0, \\
\mathbf{h}_{32} &= 365061060 T^{23} - 152080693380 T^{21} + 17658627372540 T^{19} - 861904089654300 T^{17} \\
& +14783141715388200 T^{15} - 91110040017935400 T^{13} + 397997325082942200 T^{11} \\
& +17756117603405181000 T^9 + 69992510515501612500 T^7 - 1798102986888203572500 T^5 \\
& -663927578365162072500 T^3 + 3221445733967484292500 T, \quad \mathbf{h}_{31} = 0, \quad \mathbf{h}_{30} = 486748080 T^{25} \\
& -221807355840 T^{23} + 28457171376480 T^{21} - 1718595944936640 T^{19} + 39120073399970640 T^{17} \\
& -415203708424982400 T^{15} + 817743548090145600 T^{13} + 98690112473697360000 T^{11} \\
& +525169842047690994000 T^9
\end{aligned}$$

$$\begin{aligned}
& -20729338406078319864000 T^7 + 10946551849110426780000 T^5 - 30430914937630182360000 T^3 \\
& + 57758643716593431150000 T, \quad \mathbf{h}_{29} = 0, \quad \mathbf{h}_{28} = 561632400 T^{27} - 273514978800 T^{25} \\
& + 37167025240800 T^{23} - 2726075908432800 T^{21} + 74609232551607600 T^{19} - 1241876967913890000 T^{17} \\
& + 2806282455935112000 T^{15} + 490091992910111592000 T^{13} + 4335307814553317910000 T^{11} \\
& - 150136948798788777210000 T^9 + 517137578894482137900000 T^7 - 221596077714765761700000 T^5 \\
& - 2219284940307326995950000 T^3 - 1317730766813851524750000 T, \quad \mathbf{h}_{27} = 0, \\
\mathbf{h}_{26} & = 561632400 T^{29} - 285683680800 T^{27} + 39160805609520 T^{25} - 3512223888264000 T^{23} \\
& + 102167332243376400 T^{21} - 2763398175005714400 T^{19} + 16607639793911166000 T^{17} \\
& + 1283509837177544592000 T^{15} + 33361595631780673950000 T^{13} - 845829484238863818540000 T^{11} \\
& + 5929023781997105418090000 T^9 - 2765051286836672017800000 T^7 - 27148916935817462743650000 T^5 \\
& - 115902479613006870052500000 T^3 - 104983925561520215883750000 T, \quad \mathbf{h}_{25} = 0, \\
\mathbf{h}_{24} & = 486748080 T^{31} - 252547369200 T^{29} + 32714359619760 T^{27} - 3765812506026480 T^{25} \\
& + 93108670306470000 T^{23} - 4900225415888602800 T^{21} + 73064176305750630000 T^{19} \\
& + 3317316129490661298000 T^{17} + 240092239695587184906000 T^{15} - 3820939255995521710050000 T^{13} \\
& + 36755694097751666247210000 T^{11} - 28748698362619854869250000 T^9 \\
& - 227283456202004553501150000 T^7 - 1415778815362819915057050000 T^5 \\
& - 2649841070338215122118750000 T^3 - 1615660724286293639204250000 T, \quad \mathbf{h}_{23} = 0, \\
\mathbf{h}_{22} & = 365061060 T^{33} - 188259180480 T^{31} + 20712348488160 T^{29} \\
& - 3475991858875200 T^{27} + 34258831092486000 T^{25} - 7053263233444209600 T^{23} \\
& + 254155505018359744800 T^{21} + 21963909822726159288000 T^{19} \\
& + 1097079878815670724975000 T^{17} - 13987261578728541022920000 T^{15} \\
& + 156374316828871749628020000 T^{13} - 233287710053247185295960000 T^{11} \\
& - 497446516059346978774650000 T^9 - 13650573914640909339093000000 T^7 \\
& - 22310053614024779723620500000 T^5 + 25842933923369284130409000000 T^3 \\
& - 25005235940076871386845437500 T, \quad \mathbf{h}_{21} = 0, \quad \mathbf{h}_{20} = 236215980 T^{35} - 117484137540 T^{33} \\
& + 8643874266720 T^{31} - 2885517466745760 T^{29} - 53721876349920240 T^{27} - 8070710526481402800 T^{25} \\
& + 703400025108222842400 T^{23} + 87738460958393897652000 T^{21} + 2207780690580728420157000 T^{19} \\
& - 47159498268521646014007000 T^{17} + 496567256795595289083780000 T^{15} \\
& - 978609893331396737190780000 T^{13} + 1437856090599426449801850000 T^{11} \\
& - 92732171219122521020284350000 T^9 - 240924172353273324602586900000 T^7 \\
& + 882173042531830165664047500000 T^5 + 1507151362901343475803361875000 T^3 \\
& - 245470559631182354558151562500 T, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = 131231100 T^{37} - 60628768200 T^{35} \\
& + 713083902300 T^{33} - 2235366203716800 T^{31} - 128502034840040400 T^{29} - 7113116608897068000 T^{27} \\
& + 1435474058223587382000 T^{25} + 179236025682574973112000 T^{23} \\
& + 584958278967735397725000 T^{21} - 114564525812965190232270000 T^{19} \\
& + 1365628046065395347760525000 T^{17} - 4320053654794030396643400000 T^{15} \\
& + 24178922285644163410723350000 T^{13} - 526170087460361614389619500000 T^{11} \\
& - 2187241005250348610938625250000 T^9 + 8215068646799644855790565000000 T^7 \\
& - 3234615439021971072725219062500 T^5 + 4466772809861963778344041875000 T^3 \\
& + 628766109312195978709523437500 T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 62162100 T^{39} - 25347791700 T^{37} \\
& - 2330970600420 T^{35} - 1619478410665500 T^{33} - 157961463444615600 T^{31} - 4857821543087528400 T^{29} \\
& + 2057113535504220270000 T^{27} + 201257821271491146162000 T^{25} - 4914048257261533415025000 T^{23} \\
& - 104146699011073208745255000 T^{21} + 3611762205877271065424805000 T^{19} \\
& - 6525946067600925299859525000 T^{17} - 90372116556342562197806550000 T^{15} \\
& - 1707727156947430436644076250000 T^{13} - 19914514153947633893074652250000 T^{11}
\end{aligned}$$

$$\begin{aligned}
&+60441504322604592785420072250000T^9 - 121990662905227310507903277187500T^7 \\
&+32024799656441660037917134687500T^5 + 133347159699666487023785273437500T^3 \\
&+50897855165920147555334226562500T, \quad \mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 24864840T^{41} - 8275528800T^{39} \\
&-2290606108560T^{37} - 1056111041351520T^{35} - 139995360430695000T^{33} - 3012968529441763200T^{31} \\
&+2016715929359309640000T^{29} + 122776919545138700688000T^{27} - 8724617561136462399414000T^{25} \\
&-8776215034264551673800000T^{23} + 5320414469496529196653860000T^{21} \\
&+34291992550412388290499000000T^{19} - 742197812663450900200994550000T^{17} \\
&-9969205728097481189275551600000T^{15} - 161880090490472685603186915000000T^{13} \\
&+248980392840144022927183314000000T^{11} - 818299264516747040989821234375000T^9 \\
&-476713419004836751657597177500000T^7 + 1758336091397966713046828163750000T^5 \\
&+1769350768557853190505858562500000T^3 + 280160068877261722604358028125000T \quad \mathbf{h}_{13} = 0, \\
\mathbf{h}_{12} = 8288280T^{43} - 1950296040T^{41} - 1303260734160T^{39} - 587448080326800T^{37} \\
&-96181467152740200T^{35} - 2333545862710847400T^{33} + 1303880857145840635200T^{31} \\
&+34549368376057348968000T^{29} - 6423142555199586204450000T^{27} + 84991830908599974348990000T^{25} \\
&+5196658815883842091604820000T^{23} + 71216267628306383393150340000T^{21} \\
&-537186993036056736281959050000T^{19} - 49680822174712697320494392250000T^{17} \\
&-510089192998572154824243387000000T^{15} + 648961140374231120423300313000000T^{13} \\
&-4595842673483238620153978392125000T^{11} - 2306917135543113677700059068125000T^9 \\
&+10465408425092291217199521618750000T^7 + 24940685779394420437367058843750000T^5 \\
&-1891607794892583614798379553125000T^3 + 1383666073198822939633454221875000T \\
\mathbf{h}_{11} = 0, \quad \mathbf{h}_{10} = 2260440T^{45} - 256299120T^{43} - 520433325720T^{41} - 265398457595040T^{39} \\
&-51963689173971960T^{37} - 2078330803534062000T^{35} + 504414194258306316600T^{33} \\
&+2354920000955531856000T^{31} - 1514616736300246258218000T^{29} + 131371987844966806057428000T^{27} \\
&+2424830356405951553429370000T^{25} + 74039904708285822523537320000T^{23} \\
&+996048278667469926140492250000T^{21} - 109841718054872507713652342700000T^{19} \\
&-527139316065717040229677183050000T^{17} + 948629826710126902238243346000000T^{15} \\
&-15738010488860491791083359360125000T^{13} - 21195405167566928689123351368750000T^{11} \\
&+14136303156631427798978019193125000T^9 + 216127929068602452779971891297500000T^7 \\
&-47561677461591404329676492409375000T^5 + 4018232176163239787982535556250000T^3 \\
&+2017953028228342458755361234375000T, \quad \mathbf{h}_9 = 0, \quad \mathbf{h}_8 = 491400T^{47} + 17098200T^{45} \\
&-150724589400T^{43} - 93307202742600T^{41} - 21786354176617800T^{39} - 1497011705550207000T^{37} \\
&+74558739462560199000T^{35} + 2282444312171889789000T^{33} + 1284486001836552906930000T^{31} \\
&+158764283968215852809190000T^{29} + 5961808009517232701540250000T^{27} \\
&+300170542939126568966807550000T^{25} + 3037790768826275206178984550000T^{23} \\
&-178058394981408498570197934750000T^{21} + 706918769910782227397813964750000T^{19} \\
&+3211142985153408253476494216250000T^{17} - 82873544921836068155438221914375000T^{15} \\
&+43774520932599987667701130441875000T^{13} - 150787474955869364748865826971875000T^{11} \\
&+2021475165637011479161476994659375000T^9 - 818085617975150180472289613765625000T^7 \\
&+557849276405116114837268303578125000T^5 - 77987440013694817376566123453125000T^3 \\
&-23844751123289021015776545234375000T, \quad \mathbf{h}_7 = 0, \quad \mathbf{h}_6 = 81900T^{49} + 17287200T^{47} \\
&-31138868880T^{45} - 24379010964000T^{43} - 6845248148092200T^{41} - 741093475985200800T^{39} \\
&-25555815521743122000T^{37} + 3158961025226410404000T^{35} + 1425135751337418094012500T^{33} \\
&+175549516977252515133960000T^{31} + 9472551469732244487395340000T^{29} \\
&+141872271768241531599713400000T^{27} - 2611527288780357895294497150000T^{25} \\
&-159876628457546422579162005000000T^{23} + 904821081291606171001080271500000T^{21}
\end{aligned}$$

$$\begin{aligned}
& +2911105425590491035120995781000000T^{19} - 57856971350556380128450774137187500T^{17} \\
& -597240785276937569608838354857500000T^{15} + 2056519193919467475565331182968750000T^{13} \\
& +12154683904194670162802727887137500000T^{11} + 13863900825388320276306901257843750000T^9 \\
& +1098571541450992569418057287187500000T^7 + 607947066875348083972181687343750000T^5 \\
& -947695220634386609405575242187500000T^3 - 35757135169692262292147626757812500T, \\
\mathbf{h}_5 = 0, \quad \mathbf{h}_4 = & 9828 T^{51} + 4101300 T^{49} - 4340604240 T^{47} - 4436265289680 T^{45} \\
& -1514282595903000 T^{43} - 234322521203647800 T^{41} - 15633540600684930000 T^{39} \\
& +1586816675525592126000 T^{37} + 603412183386569812909500 T^{35} \\
& +70495161343635553258387500 T^{33} + 3381977148695834843343420000 T^{31} \\
& +2314480249177743285393500000 T^{29} - 508001687496347812118539650000 T^{27} \\
& -45317179030459158057822637050000T^{25} - 639392657721512904650453512500000T^{23} \\
& -633453674783199881723813200500000T^{21} - 87308156097322179307308497039062500T^{19} \\
& -1355979332634292253868900567460312500T^{17} + 4411540235306765090271379701813750000T^{15} \\
& +10057603832362589442041629620843750000T^{13} + 26911295530554930144489745621340625000T^{11} \\
& +17125918398489101416653911676328125000T^9 + 18954282091963852165062854667093750000T^7 \\
& -6660318081890442429357638396906250000T^5 - 552093664050382974021790374023437500T^3 \\
& -34463233945947896810671380117187500T, \quad \mathbf{h}_3 = 0, \quad \mathbf{h}_2 = 756 T^{53} + 495432 T^{51} \\
& -362210940 T^{49} - 500636959200 T^{47} - 209842694598600 T^{45} - 42645497546559600 T^{43} \\
& -3498152086126744200 T^{41} + 377281024347463236000 T^{39} + 109988509720439880337500 T^{37} \\
& +7421060572419282448275000 T^{35} + 35933479335640711244227500 T^{33} \\
& -11661105491106988983093720000 T^{31} + 439741080879143818859780850000 T^{29} \\
& +19783711746209042465457739500000T^{27} - 5375663257081727637430152750000T^{25} \\
& -22493569190218996211616545271000000T^{23} - 186281502287073108403281941459812500T^{21} \\
& -1059084822334974004802067028318125000T^{19} + 6329153949445054314997820083110937500T^{17} \\
& -14918769495839465654360619394237500000T^{15} - 29373548513181426947900813984803125000T^{13} \\
& +152565680355947795987851167072956250000T^{11} + 54541012524300447784197649387171875000T^9 \\
& -62401428931462991825188978932187500000T^7 + 8198315520009349681029894474726562500T^5 \\
& -834776111135182389414040096171875000T^3 + 29742242994448184918798588320312500T, \\
\mathbf{h}_1 = 0, \quad \mathbf{h}_0 = & 28 T^{55} + 25956 T^{53} - 13362300 T^{51} - 26340465060 T^{49} - 13674573352440 T^{47} \\
& -3395567206168200 T^{45} - 366196973040498600 T^{43} + 12483830514741201000 T^{41} \\
& +8840223660767963656500 T^{39} + 1211740695196554052183500 T^{37} \\
& +95626598418360513349807500 T^{35} + 4134578042051998735362052500 T^{33} \\
& +79851486142884785720633550000 T^{31} + 664734262949553830334181650000T^{29} \\
& -97834567689265552651609176150000T^{27} - 3703009772457379493336677700250000T^{25} \\
& -14475894457788465791146209089437500T^{23} - 226810862867256551420326103376562500T^{21} \\
& -1684904275255975197128945251574062500T^{19} + 9448363637590584401034224408586562500T^{17} \\
& +28548596595041297866371213386578125000T^{15} + 146225151676068938646301685253571875000T^{13} \\
& +31326045499715628103678345844109375000T^{11} + 25092831591187433843822670758765625000T^9 \\
& -44918394960897223683724633522007812500T^7 - 7823888482100405864538916831992187500T^5 \\
& -57123990513146513891660780742187500T^3 + 5507822776749663873851590429687500)T,
\end{aligned}$$

$$\begin{aligned}
\mathbf{q}_{56} = 1, \quad \mathbf{q}_{55} = 0, \quad \mathbf{q}_{54} = & 28 T^2 + 28, \quad \mathbf{q}_{53} = 0, \quad \mathbf{q}_{52} = 378 T^4 - 2268 T^2 + 3402, \quad \mathbf{q}_{51} = 0, \\
\mathbf{q}_{50} = & 3276 T^6 - 62748 T^4 + 21924 T^2 + 483084, \quad \mathbf{q}_{49} = 0, \quad \mathbf{q}_{48} = 20475 T^8 - 749700 T^6
\end{aligned}$$

$$\begin{aligned}
&+2315250 T^4 + 5953500 T^2 + 66976875, \quad \mathbf{q}_{47} = 0, \quad \mathbf{q}_{46} = 98280 T^{10} - 5556600 T^8 + 48157200 T^6 \\
&+65091600 T^4 + 959704200 T^2 + 8132481000, \quad \mathbf{q}_{45} = 0, \quad \mathbf{q}_{44} = 376740 T^{12} - 29037960 T^{10} \\
&+479058300 T^8 - 442940400 T^6 + 9798270300 T^4 + 117272043000 T^2 + 765518890500, \quad \mathbf{q}_{43} = 0, \\
\mathbf{q}_{42} &= 1184040 T^{14} - 114123240 T^{12} + 2988021960 T^{10} - 13647803400 T^8 + 81493889400 T^6 \\
&+1166454966600 T^4 + 9193406607000 T^2 + 45889232697000, \quad \mathbf{q}_{41} = 0, \quad \mathbf{q}_{40} = 3108105 T^{16} \\
&-350020440 T^{14} + 13152128220 T^{12} - 117779757480 T^{10} + 606076420950 T^8 + 10517426904600 T^6 \\
&+74383606489500 T^4 + 157761403569000 T^2 + 1272788332979625, \quad \mathbf{q}_{39} = 0, \quad \mathbf{q}_{38} = 6906900 T^{18} \\
&-855924300 T^{16} + 43638210000 T^{14} - 571636018800 T^{12} + 4633520938200 T^{10} + 91699486263000 T^8 \\
&+670471524954000 T^6 - 2950507091070000 T^4 - 33200878716967500 T^2 + 74380367755732500, \\
\mathbf{q}_{37} &= 0, \quad \mathbf{q}_{36} = 13123110 T^{20} - 1685814900 T^{18} + 114021611550 T^{16} - 1773231490800 T^{14} \\
&+32095272062700 T^{12} + 488569383779400 T^{10} + 7285991633959500 T^8 - 37353863706390000 T^6 \\
&-712242268565231250 T^4 + 766523807513887500 T^2 + 4232772093141033750, \quad \mathbf{q}_{35} = 0, \\
\mathbf{q}_{34} &= 21474180 T^{22} - 2671057620 T^{20} + 242800796700 T^{18} - 3452383449900 T^{16} \\
&+173542141218600 T^{14} + 2616311024019000 T^{12} + 72996496011747000 T^{10} - 112701870219015000 T^8 \\
&-7619235080076427500 T^6 + 26397875963474977500 T^4 + 56266542474799387500 T^2 \\
&+207628508681964562500, \quad \mathbf{q}_{33} = 0, \quad \mathbf{q}_{32} = 30421755 T^{24} - 3341712780 T^{22} \\
&+434948276070 T^{20} - 2762553786300 T^{18} + 707628327869925 T^{16} + 16452498066820200 T^{14} \\
&+420040677679276500 T^{12} + 1905343386976953000 T^{10} - 55819031246158516875 T^8 \\
&+383292609490411582500 T^6 + 758856538412299923750 T^4 + 2764219001238915412500 T^2 \\
&+7818616326630954796875, \quad \mathbf{q}_{31} = 0, \quad \mathbf{q}_{30} = 37442160 T^{26} - 3107699280 T^{24} \\
&+676957508640 T^{22} + 7303633777440 T^{20} + 2213013639790800 T^{18} + 80237629739005200 T^{16} \\
&+2078146188047160000 T^{14} + 28199965799057976000 T^{12} - 257869432151320374000 T^{10} \\
&+3386896499825497890000 T^8 + 5008882358117664180000 T^6 + 35493664601967887700000 T^4 \\
&+79785466324073952750000 T^2 + 191031684895268043750000, \quad \mathbf{q}_{29} = 0, \quad \mathbf{q}_{28} = 40116600 T^{28} \\
&-1684897200 T^{26} + 943969761000 T^{24} + 36029988237600 T^{22} + 5459806605468600 T^{20} \\
&+266337649042542000 T^{18} + 8144921696066001000 T^{16} + 188833010171624280000 T^{14} \\
&-749375912196779175000 T^{12} + 26593887688740291150000 T^{10} + 13857652471353057675000 T^8 \\
&+490019806344263094900000 T^6 + 847091638120081055625000 T^4 + 76231684688082635250000 T^2 \\
&+2590948189562408935875000, \quad \mathbf{q}_{27} = 0, \quad \mathbf{q}_{26} = 37442160 T^{30} + 561632400 T^{28} \\
&+1204554985200 T^{26} + 88218653418000 T^{24} + 10930955933444400 T^{22} + 618649682178805200 T^{20} \\
&+19905633824246478000 T^{18} + 212421481759657170000 T^{16} - 7909388178373163790000 T^{14} \\
&+190838377838882403630000 T^{12} + 246648947765545606050000 T^{10} + 5106259955755665733950000 T^8 \\
&+14509298027190311210250000 T^6 - 16822775828173706102250000 T^4 - 57377741897262022755750000 T^2 \\
&+64893616962420115311750000, \quad \mathbf{q}_{25} = 0, \quad \mathbf{q}_{24} = 30421755 T^{32} + 2733277680 T^{30} \\
&+1411247917800 T^{28} + 156504460165200 T^{26} + 18220186626232500 T^{24} + 1063917237344598000 T^{22} \\
&+24341468738209623000 T^{20} - 1458407669898168990000 T^{18} - 38195651665360037598750 T^{16} \\
&+704274365043021222450000 T^{14} + 5599053528058930314375000 T^{12} \\
&+37978367503214552163750000 T^{10} + 257388209612512632466312500 T^8 \\
&-76827096194526390462750000 T^6 - 1281634404886971940179375000 T^4 \\
&+831668698074883802883750000 T^2 + 1475186709752105729050546875, \quad \mathbf{q}_{23} = 0, \\
\mathbf{q}_{22} &= 21474180 T^{34} + 3959508420 T^{32} + 1496491506720 T^{30} + 220136214564000 T^{28} \\
&+25819817141790000 T^{26} + 1464130513297998000 T^{24} + 431038469000412000 T^{22} \\
&-5567239710825100308000 T^{20} - 51972702340355822025000 T^{18} + 2111596128156974433975000 T^{16} \\
&+26110454746607837505900000 T^{14} + 370837660144520166097500000 T^{12}
\end{aligned}$$

$$\begin{aligned}
& +2211046006855432481484750000 T^{10} + 3768821426548444795278750000 T^8 \\
& -15057540757380336212497500000 T^6 + 24166290139115638826812500000 T^4 \\
& +22727364203519577125586562500 T^2 + 26036255163840613991361562500, \\
& \mathbf{q}_{21} = 0, \quad \mathbf{q}_{20} = 13123110 T^{36} + 3942989820 T^{34} + 1403651839590 T^{32} \\
& +254097412763040 T^{30} + 31521548873014200 T^{28} + 1794671024233626000 T^{26} \\
& -35581106620452393000 T^{24} - 8152518727359282276000 T^{22} + 80631055883822660716500 T^{20} \\
& +5932728449553959198145000 T^{18} + 82578636925153162005712500 T^{16} \\
& +1543335783106772812657500000 T^{14} + 14290988763259977767373375000 T^{12} \\
& +80369144237433402713612250000 T^{10} - 65257978237151882814757125000 T^8 \\
& +298324647301009641815002500000 T^6 + 423803283423874977134856093750 T^4 \\
& +357328418589665267175368437500 T^2 + 289950943789653252140878593750, \quad \mathbf{q}_{19} = 0, \\
& \mathbf{q}_{18} = 6906900 T^{38} + 3038504700 T^{36} + 1138241065500 T^{34} + 243867643658100 T^{32} \\
& +33248280773307600 T^{30} + 2138061788277030000 T^{28} - 18711013316805810000 T^{26} \\
& -3565489869883051110000 T^{24} + 375784427183868517815000 T^{22} + 8589634936729811329365000 T^{20} \\
& +177884937944352775261125000 T^{18} + 5022188934743990092458375000 T^{16} \\
& +34767700807474654377608250000 T^{14} + 346392882720924307150632750000 T^{12} \\
& +7298403068043804798058477500000 T^{10} + 3349869712218751272882581250000 T^8 \\
& +5016164083568572942014820312500 T^6 + 6682149789566477649221235937500 T^4 \\
& +1308993528629685934471373437500 T^2 + 1438708282930546005298232812500, \quad \mathbf{q}_{17} = 0, \\
& \mathbf{q}_{16} = 3108105 T^{40} + 1879208100 T^{38} + 782694539550 T^{36} + 194946808587300 T^{34} \\
& +30068684997553125 T^{32} + 2434041863757781200 T^{30} + 79798925261816145000 T^{28} \\
& +8035953559797385530000 T^{26} + 754836005796200279201250 T^{24} \\
& +17125179786494377633035000 T^{22} + 613061169627775399022992500 T^{20} \\
& +13213975547514313000303875000 T^{18} + 82582615880709112432795781250 T^{16} \\
& -277980267954943276814841750000 T^{14} - 3948507840549888704785071375000 T^{12} \\
& +55686750696026949631741859250000 T^{10} + 42615372067309740929873220703125 T^8 \\
& +151368250313086725334355001562500 T^6 + 17141985927211128923407436718750 T^4 \\
& -42562069342332573564516923437500 T^2 + 14036118651669035253714022265625, \\
& \mathbf{q}_{15} = 0, \quad \mathbf{q}_{14} = 1184040 T^{42} + 942951240 T^{40} + 448820341200 T^{38} + 129061854306000 T^{36} \\
& +22950971378735400 T^{34} + 2420746857044901000 T^{32} + 189431470627899192000 T^{30} \\
& +19740165844212003960000 T^{28} + 1288418099780576436210000 T^{26} + 39634356458318874582330000 T^{24} \\
& +932948607189954436181100000 T^{22} + 24964784950300898037835500000 T^{20} \\
& +336813899817214828406648250000 T^{18} - 4277060110872501238036500750000 T^{16} \\
& -15793226076541858476785253000000 T^{14} + 182147996494954471930263315000000 T^{12} \\
& +957228897383337168450769033125000 T^{10} + 1659229465068162746724356015625000 T^8 \\
& +1250994378672604495344045881250000 T^6 - 1046218980520023853006332618750000 T^4 \\
& +233488960703366614557013828125000 T^2 + 114491186085844621649987015625000, \\
& \mathbf{q}_{13} = 0, \quad \mathbf{q}_{12} = 376740 T^{44} + 383173560 T^{42} + 211091556060 T^{40} + 69936531310800 T^{38} \\
& +14478829033214700 T^{36} + 1951093276439491800 T^{34} + 219781394892621292500 T^{32} \\
& +23753718553640461848000 T^{30} + 1737564922252746032985000 T^{28} + 85455658543552552907190000 T^{26} \\
& +3155694614350839848822535000 T^{24} + 83228531888006342213676300000 T^{22} \\
& +980904596645249105852397375000 T^{20} - 8069972179322250266601782250000 T^{18} \\
& +5852053736347304386211936625000 T^{16} + 470130559276041642828328479000000 T^{14}
\end{aligned}$$

$$\begin{aligned}
&+3826968310962656491781882545312500T^{12} + 14400168925110609944407683954375000T^{10} \\
&+35422981894412214489355285504687500T^8 - 8045800083267738229862592918750000T^6 \\
&+5267694033280469186100991223437500T^4 + 2241399641805817999957998421875000T^2 \\
&+580522174177198819681404351562500, \mathbf{q}_{11} = 0, \mathbf{q}_{10} = 98280T^{46} + 124671960T^{44} \\
&+79862456520T^{42} + 30469610860920T^{40} + 7363086516739800T^{38} + 1221618341096029800T^{36} \\
&+165380901580623267000T^{34} + 18523557951212005461000T^{32} + 1521418822317545771466000T^{30} \\
&+94208790464887079928870000T^{28} + 4142878394428875557173410000T^{26} \\
&+108603899031880110735226350000T^{24} + 1586101557851603861258940750000T^{22} \\
&+5626299013637437585429229250000T^{20} + 95942669081847030488951655750000T^{18} \\
&+1600756201260760979683508510250000T^{16} + 12940726819204628290681342633125000T^{14} \\
&+41771730297204236476700239561875000T^{12} + 118010611986482678310004663130625000T^{10} \\
&+154559704011983463621551783034375000T^8 + 53946862592799320363224126996875000T^6 \\
&+60136820917315236553097341828125000T^4 + 8188712841236219843772580734375000T^2 \\
&+824966108420796215602744640625000, \mathbf{q}_9 = 0, \mathbf{q}_8 = 20475T^{48} + 31789800T^{46} + 23681605500T^{44} \\
&+10399502698200T^{42} + 2927281902340950T^{40} + 577627006715451000T^{38} + 87402057712417255500T^{36} \\
&+9919859107719402765000T^{34} + 800327687122633415563125T^{32} + 42769550356395608294010000T^{30} \\
&+1015850709135001823928075000T^{28} - 9859626751919274812824650000T^{26} \\
&-12994083461579363825869687500T^{24} + 20016288702561441180144003750000T^{22} \\
&+679783469479385743524604426875000T^{20} + 2547659051955188113304398286250000T^{18} \\
&+23936520687149123724088500660703125T^{16} + 186593162117858937503104921003125000T^{14} \\
&-236735342801561911802080068295312500T^{12} - 413881814536318993632578613928125000T^{10} \\
&+2319699480672946592561913286558593750T^8 + 645133155249795867525146260359375000T^6 \\
&+272571033660634953096852335273437500T^4 - 34555655461929637202205397734375000T^2 \\
&+2590924796001627597801615498046875, \mathbf{q}_7 = 0, \mathbf{q}_6 = 3276T^{50} + 6129900T^{48} \\
&+5297821200T^{46} + 2674898024400T^{44} + 872848514676600T^{42} + 199686264011249400T^{40} \\
&+33197856289357914000T^{38} + 3837403641408383970000T^{36} + 276908778929881432372500T^{34} \\
&+9159559733209296842392500T^{32} - 131290592583932779023180000T^{30} \\
&-15032038313822098605428700000T^{28} + 271522692937305895882970250000T^{26} \\
&+23727100949644226008739114250000T^{24} + 768309999321268014688880302500000T^{22} \\
&+12902152525905099940011381352500000T^{20} + 120504248624518289819676115540312500T^{18} \\
&+645622592643611459396432157557812500T^{16} - 1119578306397850896579839825718750000T^{14} \\
&+10319330688733274219793888356250000T^{12} + 6342730065378280526865194003596875000T^{10} \\
&+12132603370280467942856600784046875000T^8 + 9468629274147888824201932240781250000T^6 \\
&-615966923109006853803313738593750000T^4 + 60481139634212975681436988242187500T^2 \\
&+5507822776749663873851590429687500, \mathbf{q}_5 = 0, \mathbf{q}_4 = 378T^{52} + 841428T^{50} \\
&+840568050T^{48} + 487096318800T^{46} + 183138162788700T^{44} + 47831592399129000T^{42} \\
&+8850859090368025500T^{40} + 1129359739079859210000T^{38} + 93052542964654276368750T^{36} \\
&+4403146806459838814107500T^{34} + 89202381643571262806643750T^{32} \\
&-1179658500799710888841500000T^{30} + 23432583245724520630079625000T^{28} \\
&+10578094500969856231695150750000T^{26} + 499339457548290824562364892625000T^{24} \\
&+10735921664568725480441788702500000T^{22} + 75790371288166572312787304136093750T^{20} \\
&+608524156666257484535247119442187500T^{18} + 2677685608955016886758750786536718750T^{16} \\
&+672486304831536963675354803606250000T^{14}
\end{aligned}$$

$$\begin{aligned}
&+9131740007447079465862895562023437500T^{12} + 45475192297272688169637000223265625000T^{10} \\
&+21369215838930001455459090380273437500T^8 + 13508958241351375588197597967031250000T^6 \\
&+348382904493170405887479407988281250T^4 + 146822818591641039837243824882812500T^2 \\
&+1652346833024899162155477128906250, \quad \mathbf{q}_3 = 0, \quad \mathbf{q}_2 = 28T^{54} + 73332T^{52} + 84306852T^{50} \\
&+55935513900T^{48} + 24073903614600T^{46} + 7132291668797400T^{44} + 1498506622541775000T^{42} \\
&+230871155311572237000T^{40} + 27771970845932873752500T^{38} + 2783359442956321295797500T^{36} \\
&+231023465050074485008027500T^{34} + 14304208964922571107875962500T^{32} \\
&+609392496844662576050832750000T^{30} + 17180550932698920283485680250000T^{28} \\
&+351839533333824978599094572250000T^{26} + 2915178549212598578022656292750000)T^{24} \\
&-32498957280378769107354302813437500T^{22} - 102838157202293944963833561572812500T^{20} \\
&+1615150792954957399277819919173437500T^{18} + 28742506839178493428111859797795312500T^{16} \\
&+72905835863993641253168075755228125000T^{14} + 158368618939599733643864923772109375000T^{12} \\
&-65146061199633257026804758778265625000T^{10} - 17057919476262103450025081489296875000T^8 \\
&+37949266119990300735095714833242187500T^6 + 5943570241573087281471449112539062500T^4 \\
&-76007954319145361459151947929687500T^2 + 1101564555349932774770318085937500, \quad \mathbf{q}_1 = 0, \\
&\mathbf{q}_0 = T^{56} + 3052T^{54} + 4019274T^{52} + 3044365884T^{50} + 1490175636075T^{48} + 501200329379400T^{46} \\
&+120293856369130500T^{44} + 21109132070495469000T^{42} + 2743830229192474739625T^{40} \\
&+264032656532001259402500T^{38} + 18705499355839478697033750T^{36} \\
&+967356830845193024425612500T^{34} + 36317543301591844669914796875T^{32} \\
&+1182862195373087251907172750000T^{30} + 43265271525507252011412727875000T^{28} \\
&+1146101963874736797501493362750000T^{26} + 18763596889105886868604052170546875T^{24} \\
&+156011886701245799140278339790312500T^{22} + 1621973055294513702812375078578593750)T^{20} \\
&+10334106828233210967813542074964062500T^{18} + 11130688397493816047672073880222265625T^{16} \\
&+53932911302618520117796345322953125000T^{14} + 171043092322485209840747207763351562500T^{12} \\
&+99160652131611586296198617527453125000T^{10} + 90541065978105828712959516510498046875T^8 \\
&+4768253316469725674256984968085937500T^6 + 1723869845940119797317349924628906250T^4 \\
&+40757888547947512666501769179687500T^2 + 39341591262497599098939931640625.
\end{aligned}$$