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Wronskian representation of solutions of NLS equation, and seventh order rogue wave.

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Abstract

In this paper, we use the representation of the solutions of the focusing nonlinear Schrödinger equation we have constructed recently, in terms of wronskians; when we perform a special passage to the limit, we get quasi-rational solutions expressed as a ratio of two wronskian determinants.

We have already construct Peregrine breathers of orders $N = 4, 5, 6$ in preceding works; we give here the Peregrine breather of order seven.

1 Introduction

From fundamental work of Zakharov and Shabat in 1968, a lot of research has been carried out on the nonlinear Schrödinger equation (NLS). The case of periodic and almost periodic algebro-geometric solutions to the focusing NLS equation were first constructed in 1976 by Its and Kotlyarov [13]. The first quasi-rational solutions of NLS equation were constructed in 1983 by Peregrine; they are nowadays called worldwide Peregrine breathers. In 1986 Eleonski, Akhmediev and Kulagin obtained the two-phase almost periodic solution to the NLS equation and obtained the first higher order analogue of the Peregrine breather[3]. Other families of higher order were constructed in a series of articles by Akhmediev et al. [1, 2] using Darboux transformations. In this paper, we use a result [9] giving a new representation of the solutions

of the NLS equation in terms of a ratio of two wronskians determinants of even order $2N$ composed of elementary functions; the related solutions of NLS are called of order N . When we perform the passage to the limit when some parameter tends to 0, we got families of multi-rogue wave solutions of the focusing NLS equation depending on a certain number of parameters. It allows to recognize the famous Peregrine breather [16] and also higher order Peregrine's breathers constructed by Akhmediev [1, 4].

Recently, an other representation of the solutions of the focusing NLS equation, as a ratio of two determinants has been given in [11] using generalized Darboux transform.

A new approach has been done in [15] which gives a determinant representation of solutions of the focusing NLS equation, obtained from Hirota bilinear method, derived by reduction of the Gram determinant representation for Davey-Stewartson system.

Here, we construct the Peregrine breather of order $N = 7$ which shows the efficiency of this method.

2 Expression of solutions of NLS equation in terms of Wronskian determinant and quasi-rational limit

2.1 Solutions of NLS equation in terms of Wronskian determinant

We briefly recall results obtained in [9, 10]. We consider the focusing NLS equation

$$iv_t + v_{xx} + 2|v|^2v = 0. \quad (1)$$

From [10], the solution of the NLS equation can be written in the form

$$v(x, t) = \frac{\det(I + A_3(x, t))}{\det(I + A_1(x, t))} \exp(2it - i\varphi). \quad (2)$$

In (2), the matrix $A_r = (a_{\nu\mu})_{1 \leq \nu, \mu \leq 2N}$ is defined by

$$a_{\nu\mu} = (-1)^{\epsilon_\nu} \prod_{\lambda \neq \mu} \left| \frac{\gamma_\lambda + \gamma_\nu}{\gamma_\lambda - \gamma_\mu} \right| \exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu}). \quad (3)$$

The terms ϵ_ν are defined by :

$$\begin{aligned}\epsilon_\nu &= 0, & 1 \leq \nu \leq N \\ \epsilon_\nu &= 1, & N + 1 \leq \nu \leq 2N.\end{aligned}\tag{4}$$

We consider the following functions

$$\begin{aligned}\phi_\nu^r(y) &= \sin(\kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y), & 1 \leq \nu \leq N, \\ \phi_\nu^r(y) &= \cos(\kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y), & N + 1 \leq \nu \leq 2N.\end{aligned}\tag{5}$$

We use the following notations :

$$\Theta_\nu = \kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y, \quad 1 \leq \nu \leq 2N.$$

$W_r(y) = W(\phi_1, \dots, \phi_{2N})$ is the wronskian

$$W_r(y) = \det[(\partial_y^{\mu-1} \phi_\nu)_{\nu, \mu \in [1, \dots, 2N]}].\tag{6}$$

We consider the matrix $D_r = (d_{\nu\mu})_{\nu, \mu \in [1, \dots, 2N]}$ defined by

$$\begin{aligned}d_{\nu\mu} &= (-1)^{\epsilon_\nu} \prod_{\lambda \neq \mu} \left| \frac{\gamma_\lambda + \gamma_\nu}{\gamma_\lambda - \gamma_\mu} \right| \exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu}), \\ &1 \leq \nu \leq 2N, \quad 1 \leq \mu \leq 2N,\end{aligned}$$

with

$$x_{r,\nu} = (r-1) \ln \frac{\gamma_\nu - i}{\gamma_\nu + i}.$$

Then we get the following link between Fredholm and Wronskian determinants [10]

Theorem 2.1

$$\det(I + D_r) = k_r(0) \times W_r(\phi_1, \dots, \phi_{2N})(0),\tag{7}$$

where

$$k_r(y) = \frac{2^{2N} \exp(i \sum_{\nu=1}^{2N} \Theta_\nu)}{\prod_{\nu=2}^{2N} \prod_{\mu=1}^{\nu-1} (\gamma_\nu - \gamma_\mu)}.$$

It can be deduced the following result :

Theorem 2.2 *The function v defined by*

$$v(x, t) = \frac{W_3(0)}{W_1(0)} \exp(2it - i\varphi).\tag{8}$$

is solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0.$$

2.2 Quasi-rational solutions of NLS equation

In the following, we take the limit when the parameters $\lambda_j \rightarrow 1$ for $1 \leq j \leq N$ and $\lambda_j \rightarrow -1$ for $N + 1 \leq j \leq 2N$.

For simplicity, we denote d_j the term $\frac{c_j}{\sqrt{2}}$.

We consider the parameter λ_j written in the form

$$\lambda_j = 1 - 2\epsilon^2 d_j^2, \quad 1 \leq j \leq N. \quad (9)$$

When ϵ goes to 0, we realize limited expansions at order p , for $1 \leq j \leq N$, of the terms

$$\begin{aligned} \kappa_j &= 4d_j\epsilon(1 - \epsilon^2 d_j^2)^{1/2}, \quad \delta_j = 4d_j\epsilon(1 - 2\epsilon^2 d_j^2)(1 - \epsilon^2 d_j^2)^{1/2}, \\ \gamma_j &= d_j\epsilon(1 - \epsilon^2 d_j^2)^{-1/2}, \quad x_{r,j} = (r - 1) \ln \frac{1 + i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}}{1 - i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}}, \\ \kappa_{N+j} &= 4d_j\epsilon(1 - \epsilon^2 d_j^2)^{1/2}, \quad \delta_{N+j} = -4d_j\epsilon(1 - 2\epsilon^2 d_j^2)(1 - \epsilon^2 d_j^2)^{1/2}, \\ \gamma_{N+j} &= 1/(d_j\epsilon)(1 - \epsilon^2 d_j^2)^{1/2}, \quad x_{r,N+j} = (r - 1) \ln \frac{1 - i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}}{1 + i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}}. \end{aligned}$$

We have the central result given in [10] :

Theorem 2.3 *With the parameters λ_j defined by (9), the function v defined by*

$$v(x, t) = \exp(2it - i\varphi) \lim_{\epsilon \rightarrow 0} \frac{W_3(0)}{W_1(0)}, \quad (10)$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0,$$

depending on $3N$ parameters $d_j, \tilde{a}_j, \tilde{b}_j, 1 \leq j \leq N$.

3 Quasi-rational solutions of order N

To get solutions of NLS equation written in the context of fiber optics

$$iu_x + \frac{1}{2}u_{tt} + u|u|^2 = 0, \quad (11)$$

from these of (1), we can make the following changes of variables

$$\begin{aligned} t &\rightarrow X/2 \\ x &\rightarrow T. \end{aligned} \quad (12)$$

3.1 Case N=7

In the case of order $N = 7$, we make an expansion at order 13. Taking the limit when $\epsilon \rightarrow 0$, the solution of NLS equation (11) takes the form

$$v(x, t) = \frac{n(x, t)}{d(x, t)} \exp(2it - i\varphi).$$

Because of the length of the complete analytical expression, we only give it in the appendix.

We give here the expression of the solution $v(x, t) = \frac{n(x, t)}{d(x, t)}$ in the case of all the parameters equal to 0 :

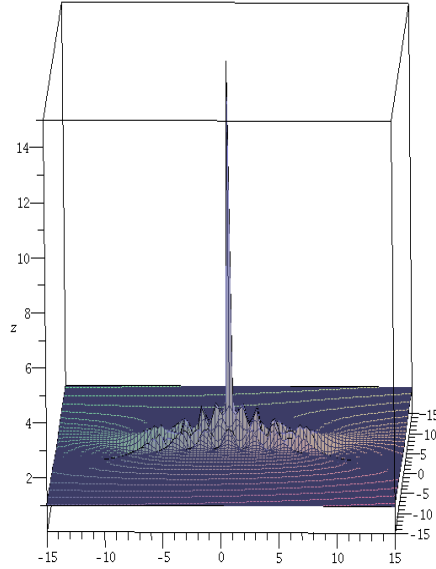


Figure 1: Solution to the NLS equation, N=7.

4 Conclusion

The method described in the present paper provides a powerful tool to get explicitly solutions of the NLS equation.

As my knowledge, it is the first time that the Peregrine breather of order seven is presented.

It confirms the conjecture about the shape of the breather in the (x, t) coordinates, the maximum of amplitude equal to $2N + 1 = 15$ and the degree of polynomials in x and t here equal to 56. This new formulation gives an infinite set of non singular solution of NLS equation. It opens a large way to future researches in this domain.

References

- [1] N. Akhmediev, A. Ankiewicz, J.M. Soto-Crespo, Rogue waves and rational solutions of nonlinear Schrödinger equation, *Physical Review E*, V. **80**, N. 026601, (2009).
- [2] N. Akhmediev, V. Eleonskii, N. Kulagin, Exact first order solutions of the nonlinear Schrödinger equation, *Th. Math. Phys.*, V. **72**, N. 2, 183-196, (1987).
- [3] N. Akhmediev, V. Eleonsky, N. Kulagin, Generation of periodic trains of picosecond pulses in an optical fiber : exact solutions, *Sov. Phys. J.E.T.P.*, V. **62**, 894-899, (1985).
- [4] N. Akhmediev, A. Ankiewicz, P.A. Clarkson, Rogue waves, rational solutions, the patterns of their zeros and integral relations, *J. Phys. A : Math. Theor.*, V. **43**, 122002, 1-9, (2010).
- [5] D. J? Kedziora, A. Ankiewicz, N. Akhmediev, Circular rogue wave clusters, *Phys. Rev. E*, V. **84**, 1-7, (2011).
- [6] E.D. Belokolos, A.i. Bobenko, A.R. its, V.Z. Enolskij and V.B. Matveev, *Algebro-geometric approach to nonlinear integrable equations*, Springer series in nonlinear dynamics, Springer Verlag, 1-360, (1994).
- [7] P. Dubard, P. Gaillard, C. Klein, V.B. Matveev, On multi-rogue waves solutions of the NLS equation and positon solutions of the KdV equation, *Eur. Phys. J. Special Topics*, V. **185**, 247-258, (2010).

- [8] V. Eleonskii, I. Krichever, N. Kulagin, Rational multisoliton solutions to the NLS equation, Soviet Doklady 1986 sect. Math. Phys., V. **287**, 606-610, (1986).
- [9] P. Gaillard, Families of quasi-rational solutions of the NLS equation and multi-rogue waves, J. Phys. A : Meth. Theor., V. **44**, 1-15, 2011
- [10] P. Gaillard, Wronskian representation of solutions of the NLS equation and higher Peregrine breathers, Scientific Advances, V. **13**, N. 2, 71-153, 2012
- [11] B. Guo, L. Ling, Q.P. Liu, Nonlinear Schrodinger equation: Generalized Darboux transformation and rogue wave solutions, Phys. Rev. E, V. **85**, 026607, (2012).
- [12] A.R. Its, A.V. Rybin, M.A. Salle, Exact integration of nonlinear Schrödinger equation, Teore. i Mat. Fiz., V. **74.**, N. 1, 29-45, (1988).
- [13] A.R. Its, V.P. Kotlyarov, Explicit expressions for the solutions of nonlinear Schrödinger equation, Dokl. Akad. Nauk. SSSR, S. A, V. **965.**, N. 11, (1976).
- [14] V.B. Matveev, M.A. Salle, Darboux transformations and solitons, Series in Nonlinear Dynamics, Springer Verlag, Berlin, (1991).
- [15] Y Ohta, J. Yang, General high-order rogue waves and their dynamics in the nonlinear Schrödinger equation, arXiv : 1110.5873 [nlin.SI] 26 Oct. 2011.
- [16] D. Peregrine, Water waves, nonlinear Schrödinger equations and their solutions, J. Austral. Math. Soc. Ser. B, V. **25**, 16-43, (1983).
- [17] V. E. Zakharov, Stability of periodic waves of finite amplitude on a surface of a deep fluid, J. Appl. Tech. Phys, V. **9**, 86-94, (1968)
- [18] V. E. Zakharov, A.B. Shabat Exact theory of two dimensional self focusing and one dimensional self modulation of waves in nonlinear media, Sov. Phys. JETP, V. **34**, 62-69, (1972)

Appendix

The solution of NLS equation takes the form

$$v_N(x, t) = \frac{n(x, t)}{d(x, t)} \exp(2it) = \left(1 - \alpha_N \frac{G_N(2x, 4t) + iH_N(2x, 4t)}{Q_N(2x, 4t)}\right) e^{2it}$$

with

$$G_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{g}_k(T) X^k$$

$$H_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{h}_k(T) X^k$$

$$Q_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{q}_k(T) X^k$$

$$\begin{aligned}
& \alpha_7 = 4, \quad \mathbf{g}_{56} = 0, \quad \mathbf{g}_{55} = 0, \quad \mathbf{g}_{54} = 28, \quad \mathbf{g}_{53} = 0, \quad \mathbf{g}_{52} = 2268T^2 + 2268, \quad \mathbf{g}_{51} = 0, \quad \mathbf{g}_{50} = 49140T^4 \\
& -125496T^2 + 272916, \quad \mathbf{g}_{49} = 0, \quad \mathbf{g}_{48} = 573300T^6 - 8158500T^4 - 4630500T^2 + 33736500, \quad \mathbf{g}_{47} = 0, \\
& \mathbf{g}_{46} = 4422600T^8 - 145000800T^6 + 144471600T^4 - 288943200T^2 + 3645923400, \quad \mathbf{g}_{45} = 0, \\
& \mathbf{g}_{44} = 24864840T^{10} - 1405819800T^8 + 8654642640T^6 + 1328821200T^4 - 27569467800T^2 + 285603683400, \\
& \mathbf{g}_{43} = 0, \quad \mathbf{g}_{42} = 107747640T^{12} - 9018923760T^{10} + 129131097480T^8 - 92144939040T^6 \\
& -160080089400T^4 - 4068407574000T^2 + 9697930011000, \quad \mathbf{g}_{41} = 0, \quad \mathbf{g}_{40} = 372972600T^{14} \\
& -41946985080T^{12} + 1042152554520T^{10} - 3985886824920T^8 - 2424305683800T^6 - 38041300420200T^4 \\
& -532602960535800T^2 - 462639709917000, \quad \mathbf{g}_{39} = 0, \quad \mathbf{g}_{38} = 1056755700T^{16} - 148997772000T^{14} \\
& +5529226071600T^{12} - 48020254682400T^{10} + 9567791263800T^8 - 277615181844000T^6 \\
& -3997430875746000T^4 - 27691049938860000T^2 - 11433617241007500, \quad \mathbf{g}_{37} = 0, \quad \mathbf{g}_{36} = 2493390900T^{18} \\
& -417526886700T^{16} + 21097721106000T^{14} - 312305697673200T^{12} + 750925867368600T^{10} \\
& -1296117088497000T^8 - 19549525119894000T^6 + 178015776175314000T^4 + 2547484073859532500T^2 \\
& -257622602070487500, \quad \mathbf{g}_{35} = 0, \quad \mathbf{g}_{34} = 4960535580T^{20} - 942501760200T^{18} \\
& +60895186381260T^{16} - 1320509170965600T^{14} + 8073640409095800T^{12} - 10433330397428400T^{10} \\
& -94059388591245000T^8 + 4212714864528324000T^6 + 81525901261652347500T^4 \\
& +83880191483144595000T^2 + 4532925200381077500, \quad \mathbf{g}_{33} = 0, \quad \mathbf{g}_{32} = 8396404380T^{22} \\
& -1736314646220T^{20} + 137179010388420T^{18} - 3955742649118260T^{16} + 44742788370264600T^{14} \\
& -87890985649467000T^{12} - 581266358785035000T^{10} + 34489428486040143000T^8 \\
& +916422481982789743500T^6 - 267128299891396597500T^4 + 3533027762243314912500T^2 \\
& +1052175273585966337500, \quad \mathbf{g}_{31} = 0, \quad \mathbf{g}_{30} = 12168702000T^{24} - 2628289863360T^{22} \\
& +246156751098720T^{20} - 8802207660818880T^{18} + 156743234946421200T^{16} \\
& -400285925208892800T^{14} - 4082398971145387200T^{12} + 182299012392364560000T^{10} \\
& +6993601895564462178000T^8 - 29136271554263384280000T^6 + 22705339934145925980000T^4 \\
& +133644440887635613320000T^2 + 69720100670714536350000, \quad \mathbf{g}_{29} = 0, \quad \mathbf{g}_{28} = 15164074800T^{26} \\
& -3271508730000T^{24} + 356805698608800T^{22} - 14963164581103200T^{20} + 373275824011952400T^{18} \\
& -1646661408941934000T^{16} - 23044246961534568000T^{14} + 647137266547990680000T^{12} \\
& +44440332791868485010000T^{10} - 368524266068389959990000T^8 + 515157395379005679300000T^6 \\
& +964818990199979907300000T^4 + 3432008280166633455750000T^2 + 2522450003846320374750000, \\
& \mathbf{g}_{27} = 0, \quad \mathbf{g}_{26} = 16287339600T^{28} - 3325987072800T^{26} + 422344463612400T^{24} \\
& -19765833506798400T^{22} + 609616438648232400T^{20} - 8426887778118348000T^{18} \\
& -198426421456868682000T^{16} - 2190017982667295088000T^{14} + 260606376243375811110000T^{12} \\
& -2667540176320654889100000T^{10} + 8847209138357814736050000T^8 + 13096014915145148494200000T^6 \\
& +6581743865224894916550000T^4 + 11025412640863669471500000T^2 + 47293802784572214197250000, \\
& \mathbf{g}_{25} = 0, \quad \mathbf{g}_{24} = 15089190480T^{30} - 2709127486800T^{28} + 412978164031440T^{26} - 20496272265378000T^{24} \\
& +625906405138669200T^{22} - 35718951161305011600T^{20} - 1333506673014425910000T^{18} \\
& -34418796637312024938000T^{16} + 1244339447399806908150000T^{14} - 11990961186295394893590000T^{12} \\
& +65945519948476262964870000T^{10} + 160150242555541152164250000T^8 \\
& +91138495041978507273150000T^6 - 1554908002287761890518750000T^4 \\
& -2392952896647698174039250000T^2 + 889739092856294586155250000, \quad \mathbf{g}_{23} = 0, \\
& \mathbf{g}_{22} = 12047014980T^{32} - 1685346505920T^{30} + 339670088966880T^{28} - 16767007875455040T^{26} \\
& +195015256305433200T^{24} - 103115102184824184000T^{22} - 4963164090380892396000T^{20} \\
& -73350016919927243496000T^{18} + 5521291279776971601903000T^{16} - 39856907136517355724840000T^{14} \\
& +376003222446071990870100000T^{12} + 718852738860427750979400000T^{10} \\
& +3176243015946106500944550000T^8 - 17990585286630587798567400000T^6 \\
& -67776208060401761741320500000T^4 - 27136987658593437499875000000T^2
\end{aligned}$$

$$\begin{aligned}
& +12087940146214297919616562500, \quad \mathbf{g}_{21} = 0, \quad \mathbf{g}_{20} = 8267559300 T^{34} - 689768832060 T^{32} \\
& +242818935559200 T^{30} - 10771451892321120 T^{28} - 609328959012219600 T^{26} \\
& -203253573603293413200 T^{24} - 10395034180133435632800 T^{22} + 165116685980326092012000 T^{20} \\
& +17170916828136913280439000 T^{18} - 102607297371198744509385000 T^{16} \\
& +1635023327366588192773740000 T^{14} + 2860768497736673150887260000 T^{12} \\
& +45606671281786995394764750000 T^{10} - 80947367097903820900836450000 T^8 \\
& -700286840100765740788465500000 T^6 + 130801873396469808053020500000 T^4 \\
& -447283391478568632911861437500 T^2 + 56768583214608413567481562500, \quad \mathbf{g}_{19} = 0, \\
& \mathbf{g}_{18} = 4855550700 T^{36} - 40277853000 T^{34} + 158719690968300 T^{32} - 5194244162923200 T^{30} \\
& -1360528622797251600 T^{28} - 281596511270242476000 T^{26} - 11850683160765907458000 T^{24} \\
& +920175424144902165240000 T^{22} + 25468628785050976429905000 T^{20} \\
& -310836193534541858413230000 T^{18} + 5807378097843796606757625000 T^{16} \\
& +15634234802719441673364600000 T^{14} + 278887408065007528609572750000 T^{12} \\
& +52000683646906354993132500000 T^{10} - 4439081082994819230111392250000 T^8 \\
& +8438816045170493665968285000000 T^6 - 3001122035235356030511325312500 T^4 \\
& -7391743593040300996805428125000 T^2 - 1054152239431812041879170312500, \\
& \mathbf{g}_{17} = 0, \quad \mathbf{g}_{16} = 2424321900 T^{38} + 211776711300 T^{36} + 99477568505700 T^{34} \\
& -1389209187712500 T^{32} - 1621264449868232400 T^{30} - 280260753940566630000 T^{28} \\
& -4787780690058717582000 T^{26} + 1656692481162132035622000 T^{24} + 5482826579394141962985000 T^{22} \\
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& +12085551247727524652824418750000 T^8 - 53956818495735676912368790312500 T^6 \\
& -74996679144078506620333965937500 T^4 - 61056959125851227464611623437500 T^2 \\
& -5245972489497170017957682812500, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 1019458440 T^{40} + 209361952800 T^{38} \\
& +59799859202160 T^{36} + 597930018285600 T^{34} - 1327830040347010200 T^{32} - 203680458354584534400 T^{30} \\
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& -954575061914762846454243847500000 T^6 - 411166271799374891645351756250000 T^4 \\
& +783333878093152389727736512500000 T^2 - 20793848949925378145840896875000, \quad \mathbf{g}_{13} = 0, \\
& \mathbf{g}_{12} = 356396040 T^{42} + 123877270440 T^{40} + 32610284395920 T^{38} + 1216306932881040 T^{36} \\
& -784563891537195000 T^{34} - 110761814779398783000 T^{32} + 7937796416339529432000 T^{30} \\
& +978775172774012225304000 T^{28} - 21292516324448140972662000 T^{26} + 161923476389289550585650000 T^{24} \\
& +34432591861323164507298300000 T^{22} + 897588666218543018306687100000 T^{20} \\
& +10399172873750652367819292850000 T^{18} + 12734590641823469076245037450000 T^{16} \\
& -1056247157487187710224365185000000 T^{14} + 1832056249038285845965084695000000 T^{12} \\
& +2959286708382169575258073850625000 T^{10} - 344028559792148330017460416875000 T^8 \\
& -12140076235349263409707132383750000 T^6 + 21659795132230009469150342156250000 T^4 \\
& +3602055625460117946415936265625000 T^2 + 61049167368118769138877703125000, \quad \mathbf{g}_{11} = 0, \\
& \mathbf{g}_{10} = 101719800 T^{44} + 52472463120 T^{42} + 15005352063240 T^{40} + 1021506129594720 T^{38}
\end{aligned}$$

$$\begin{aligned}
& -331006205910303000 T^{36} - 47731541013103978800 T^{34} + 3909120362191961947800 T^{32} \\
& + 302275195306483257936000 T^{30} - 3397149051079935164706000 T^{28} \\
& + 1063471996099453676509620000 T^{26} + 35282865975833690856131490000 T^{24} \\
& + 1009461102866985900033662760000 T^{22} + 28108708355943404795842686450000 T^{20} \\
& - 85747360020983326179491877900000 T^{18} - 1491463246568322076250046095250000 T^{16} \\
& + 2775278138548494265001581266000000 T^{14} - 18540128543730526297289281181625000 T^{12} \\
& - 21944626454220073070250932898750000 T^{10} - 80973913998934031957478576354375000 T^8 \\
& + 148114784421877559463874334137500000 T^6 + 31126078957047574894186796128125000 T^4 \\
& + 20673599609280586675821470606250000 T^2 + 1398479083269650104689513421875000, \\
& \mathbf{g}_9 = 0, \quad \mathbf{g}_8 = 23095800 T^{46} + 16444701000 T^{44} + 5475036936600 T^{42} \\
& + 566976092898600 T^{40} - 93441539643190200 T^{38} - 17935367491764261000 T^{36} \\
& - 34358185653025311000 T^{34} - 84340417236266770425000 T^{32} - 6247372784431620892050000 T^{30} \\
& + 79423291493938895975010000 T^{28} - 6517484194549893449935050000 T^{26} \\
& + 1157932673902046452477117050000 T^{24} + 48145337637619603341498788250000 T^{22} \\
& - 238274280945074139753039302250000 T^{20} + 1614124491632111637086543885250000 T^{18} \\
& + 30463748856261460407343234308750000 T^{16} - 118755251721826815926043684095625000 T^{14} \\
& - 48773592689963392959811287159375000 T^{12} - 1738438081260949813862744404903125000 T^{10} \\
& + 1572448964546520061265933822278125000 T^8 - 673017920052474095253612817734375000 T^6 \\
& + 326795486450765136002224260609375000 T^4 + 79407567379987217482876217578125000 T^2 \\
& + 2882552147106173292709630546875000, \quad \mathbf{g}_7 = 0, \quad \mathbf{g}_6 = 4013100 T^{48} \\
& + 3764023200 T^{46} + 1497943465200 T^{44} + 217159761218400 T^{42} - 14221439411394600 T^{40} \\
& - 6062992711379604000 T^{38} - 868091861569403394000 T^{36} - 137559109216434396876000 T^{34} \\
& - 9260888323767068417467500 T^{32} + 83092589684282324500200000 T^{30} \\
& + 45951189509544653085374700000 T^{28} + 2043751713304237133419114200000 T^{26} \\
& + 39823148838347256006248674050000 T^{24} - 107645574930283654095040209000000 T^{22} \\
& + 5704220460549886178354194243500000 T^{20} + 47477860076491575514786707705000000 T^{18} \\
& + 96760004701426703802202262537812500 T^{16} - 2746609454519613461575760731267500000 T^{14} \\
& - 5055092442696255371030072885681250000 T^{12} + 7685340668503114406318287145887500000 T^{10} \\
& - 2527654302854142287937009789515625000 T^8 + 2549540272922798483165127227437500000 T^6 \\
& + 1681003830938891487377650144593750000 T^4 - 119038912584481624295814690937500000 T^2 \\
& + 4248891856349740702685512617187500, \quad \mathbf{g}_5 = 0, \quad \mathbf{g}_4 = 501228 T^{50} + 599186700 T^{48} \\
& + 286948221840 T^{46} + 55465811326800 T^{44} + 225191978134200 T^{42} - 1592867890825734600 T^{40} \\
& - 378975454715995830000 T^{38} - 49698971523700677486000 T^{36} - 2416070416982018904607500 T^{34} \\
& + 163449545090537650746532500 T^{32} + 22630419780133976885691540000 T^{30} \\
& + 658066531416844960184890500000 T^{28} + 17958744056622134874813046650000 T^{26} \\
& + 265938368682687944669400224250000 T^{24} + 3373546912165127652278618794500000 T^{22} \\
& + 49992632917159774293562606996500000 T^{20} + 243622840935340303792373213507812500 T^{18} \\
& - 3472965533440290793642368859414687500 T^{16} - 3909930410917269828231259536168750000 T^{14} \\
& - 2373854538016335651007933345893750000 T^{12} + 22505528586240563125522950136321875000 T^{10} \\
& + 1725433101762243379165143781171875000 T^8 + 20602166714282000488422754253531250000 T^6 \\
& - 256192442301384365332296834843750000 T^4 - 79260192530178496317997648945312500 T^2 \\
& - 3304693666049798324310954257812500, \quad \mathbf{g}_3 = 0, \quad \mathbf{g}_2 = 40068 T^{52} \\
& + 59556168 T^{50} + 34248844980 T^{48} + 8538182019360 T^{46} + 489064062475800 T^{44} \\
& - 249702181686342000 T^{42} - 69203448982691793000 T^{40} - 5213133014477381532000 T^{38} \\
& + 438714917019434320303500 T^{36} + 84931805893311124163955000 T^{34}
\end{aligned}$$

$$\begin{aligned}
& +4218956433098176838806237500 T^{32} + 31107276944713383532410600000 T^{30} \\
& +3271649949938812288254997050000 T^{28} + 201849235241947964535548861100000)T^{26} \\
& +4953109227394118516246184914250000T^{24} - 25630541428745966760074602575000000T^{22} \\
& -412192793582829745372981826294062500T^{20} - 4700092011184358352745951843198125000T^{18} \\
& +12509179325796087908708757937602187500T^{16} + 17173693466353903144949117196862500000T^{14} \\
& -36421201667691057461711715138515625000T^{12} + 130348386619893149350129124043656250000T^{10} \\
& +135527319688597478057052486959484375000T^8 + 12269848488887082318515323846312500000T^6 \\
& +4223241138846592268073003781757812500T^4 - 374846681549077124214699668671875000T^2 \\
& -1101564555349932774770318085937500, \quad \mathbf{g}_1 = 0, \quad \mathbf{g}_0 = 1540 T^{54} + 2791404 T^{52} \\
& +1916196156 T^{50} + 600986059380 T^{48} + 60893832396600 T^{46} - 14981298604738200 T^{44} \\
& -5448980120702995800 T^{42} - 677699093948317749000 T^{40} - 28109651584658009404500 T^{38} \\
& +2321568723921179741662500 T^{36} + 483153674937785992894792500 T^{34} \\
& +34432196292857178176426257500 T^{32} + 1329648591046758598027462050000 T^{30} \\
& +48030465435235520461569818550000T^{28} + 871144519174189671772071537750000T^{26} \\
& +862287173564925372614929829250000T^{24} + 37733399259532601896905168972937500T^{22} \\
& -97131347340613907133312877564687500T^{20} - 6464853953097864723190272724680937500T^{18} \\
& -13312771923417438005165293807457812500T^{16} - 21734401086635138140243391915653125000T^{14} \\
& +162602072233593709389730376910215625000T^{12} + 57203947835898442945082247296390625000T^{10} \\
& +95479747942182277597570867016671875000T^8 + 4823751187877355620719222898320312500T^6 \\
& -1973321762271862907337361717851562500T^4 - 20929726551648722720636043632812500T^2 \\
& +157366365049990396395759726562500,
\end{aligned}$$

$$\begin{aligned}
\mathbf{h}_{56} &= 0, \quad \mathbf{h}_{55} = 0, \quad \mathbf{h}_{54} = 28T, \quad \mathbf{h}_{53} = 0, \quad \mathbf{h}_{52} = 756 T^3 - 2268 T, \quad \mathbf{h}_{51} = 0, \quad \mathbf{h}_{50} = 9828 T^5 \\
& -181944 T^3 - 147420 T, \quad \mathbf{h}_{49} = 0, \quad \mathbf{h}_{48} = 81900 T^7 - 3660300 T^5 + 4630500 T^3 - 15214500 T, \quad \mathbf{h}_{47} = 0, \\
\mathbf{h}_{46} &= 491400 T^9 - 39160800 T^7 + 310428720 T^5 + 79380000 T^3 - 1631259000 T, \quad \mathbf{h}_{45} = 0, \\
\mathbf{h}_{44} &= 2260440 T^{11} - 275020200 T^9 + 5106674160 T^7 - 3889302480 T^5 + 392931000 T^3 - 148377889800 T, \\
\mathbf{h}_{43} &= 0, \quad \mathbf{h}_{42} = 8288280 T^{13} - 1398806640 T^{11} + 45284861160 T^9 - 207951559200 T^7 - 42200789400 T^5 \\
& +253204736400 T^3 - 8291708548200 T, \quad \mathbf{h}_{41} = 0, \quad \mathbf{h}_{40} = 24864840 T^{15} - 5449225320 T^{13} \\
& +264073498920 T^{11} - 2825524908360 T^9 + 1514969046360 T^7 + 6573185526600 T^5 + 137993987993400 T^3 \\
& +149047765713000 T, \quad \mathbf{h}_{39} = 0, \quad \mathbf{h}_{38} = 62162100 T^{17} - 16844335200 T^{15} + 1110003879600 T^{13} \\
& -20635793085600 T^{11} + 60341881030200 T^9 + 76117021596000 T^7 + 2171428651854000 T^5 \\
& +25299618434244000 T^3 + 62466943452952500 T, \quad \mathbf{h}_{37} = 0, \quad \mathbf{h}_{36} = 131231100 T^{19} \\
& -42306887700 T^{17} + 3541157207280 T^{15} - 98466907770000 T^{13} + 661042062865800 T^{11} \\
& -16499618011800 T^9 + 18885110233374000 T^7 + 314318361244734000 T^5 \\
& +1773658200690337500 T^3 + 2282181897675487500 T, \quad \mathbf{h}_{35} = 0, \quad \mathbf{h}_{34} = 236215980 T^{21} \\
& -87763321800 T^{19} + 8848450735740 T^{17} - 335891899617120 T^{15} + 3894124207743000 T^{13} \\
& -10086562817377200 T^{11} + 107633107470735000 T^9 + 2631781328889636000 T^7 \\
& +12651337028921629500 T^5 - 44712998532434925000 T^3 + 94738725025613707500 T, \quad \mathbf{h}_{33} = 0, \\
\mathbf{h}_{32} &= 365061060 T^{23} - 152080693380 T^{21} + 17658627372540 T^{19} - 861904089654300 T^{17} \\
& +14783141715388200 T^{15} - 91110040017935400 T^{13} + 397997325082942200 T^{11} \\
& +17756117603405181000 T^9 + 69992510515501612500 T^7 - 1798102986888203572500 T^5 \\
& -663927578365162072500 T^3 + 3221445733967484292500 T, \quad \mathbf{h}_{31} = 0, \quad \mathbf{h}_{30} = 486748080 T^{25} \\
& -221807355840 T^{23} + 28457171376480 T^{21} - 1718595944936640 T^{19} + 39120073399970640 T^{17} \\
& -415203708424982400 T^{15} + 817743548090145600 T^{13} + 98690112473697360000 T^{11} \\
& +525169842047690994000 T^9
\end{aligned}$$

$$\begin{aligned}
& -20729338406078319864000 T^7 + 10946551849110426780000 T^5 - 30430914937630182360000 T^3 \\
& + 57758643716593431150000 T, \quad \mathbf{h}_{29} = 0, \quad \mathbf{h}_{28} = 561632400 T^{27} - 273514978800 T^{25} \\
& + 37167025240800 T^{23} - 2726075908432800 T^{21} + 74609232551607600 T^{19} - 1241876967913890000 T^{17} \\
& + 2806282455935112000 T^{15} + 490091992910111592000 T^{13} + 4335307814553317910000 T^{11} \\
& - 150136948798788777210000 T^9 + 517137578894482137900000 T^7 - 221596077714765761700000 T^5 \\
& - 2219284940307326995950000 T^3 - 1317730766813851524750000 T, \quad \mathbf{h}_{27} = 0, \\
& \mathbf{h}_{26} = 561632400 T^{29} - 285683680800 T^{27} + 39160805609520 T^{25} - 3512223888264000 T^{23} \\
& + 102167332243376400 T^{21} - 2763398175005714400 T^{19} + 16607639793911166000 T^{17} \\
& + 1283509837177544592000 T^{15} + 33361595631780673950000 T^{13} - 845829484238863818540000 T^{11} \\
& + 5929023781997105418090000 T^9 - 2765051286836672017800000 T^7 - 27148916935817462743650000 T^5 \\
& - 115902479613006870052500000 T^3 - 104983925561520215883750000 T, \quad \mathbf{h}_{25} = 0, \\
& \mathbf{h}_{24} = 486748080 T^{31} - 252547369200 T^{29} + 32714359619760 T^{27} - 3765812506026480 T^{25} \\
& + 93108670306470000 T^{23} - 4900225415888602800 T^{21} + 73064176305750630000 T^{19} \\
& + 3317316129490661298000 T^{17} + 240092239695587184906000 T^{15} - 3820939255995521710050000 T^{13} \\
& + 36755694097751666247210000 T^{11} - 28748698362619854869250000 T^9 \\
& - 227283456202004553501150000 T^7 - 1415778815362819915057050000 T^5 \\
& - 2649841070338215122118750000 T^3 - 1615660724286293639204250000 T, \quad \mathbf{h}_{23} = 0, \\
& \mathbf{h}_{22} = 365061060 T^{33} - 188259180480 T^{31} + 20712348488160 T^{29} \\
& - 3475991858875200 T^{27} + 34258831092486000 T^{25} - 7053263233444209600 T^{23} \\
& + 254155505018359744800 T^{21} + 21963909822726159288000 T^{19} \\
& + 1097079878815670724975000 T^{17} - 13987261578728541022920000 T^{15} \\
& + 156374316828871749628020000 T^{13} - 233287710053247185295960000 T^{11} \\
& - 497446516059346978774650000 T^9 - 13650573914640909339093000000 T^7 \\
& - 22310053614024779723620500000 T^5 + 25842933923369284130409000000 T^3 \\
& - 25005235940076871386845437500 T, \quad \mathbf{h}_{21} = 0, \quad \mathbf{h}_{20} = 236215980 T^{35} - 117484137540 T^{33} \\
& + 8643874266720 T^{31} - 2885517466745760 T^{29} - 53721876349920240 T^{27} - 8070710526481402800 T^{25} \\
& + 703400025108222842400 T^{23} + 87738460958393897652000 T^{21} + 2207780690580728420157000 T^{19} \\
& - 47159498268521646014007000 T^{17} + 496567256795595289083780000 T^{15} \\
& - 978609893331396737190780000 T^{13} + 1437856090599426449801850000 T^{11} \\
& - 92732171219122521020284350000 T^9 - 240924172353273324602586900000 T^7 \\
& + 882173042531830165664047500000 T^5 + 1507151362901343475803361875000 T^3 \\
& - 245470559631182354558151562500 T, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = 131231100 T^{37} - 60628768200 T^{35} \\
& + 713083902300 T^{33} - 2235366203716800 T^{31} - 128502034840040400 T^{29} - 7113116608897068000 T^{27} \\
& + 1435474058223587382000 T^{25} + 179236025682574973112000 T^{23} \\
& + 584958278967735397725000 T^{21} - 114564525812965190232270000 T^{19} \\
& + 1365628046065395347760525000 T^{17} - 4320053654794030396643400000 T^{15} \\
& + 24178922285644163410723350000 T^{13} - 526170087460361614389619500000 T^{11} \\
& - 2187241005250348610938625250000 T^9 + 8215068646799644855790565000000 T^7 \\
& - 3234615439021971072725219062500 T^5 + 4466772809861963778344041875000 T^3 \\
& + 628766109312195978709523437500 T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 62162100 T^{39} - 25347791700 T^{37} \\
& - 2330970600420 T^{35} - 1619478410665500 T^{33} - 157961463444615600 T^{31} - 4857821543087528400 T^{29} \\
& + 2057113535504220270000 T^{27} + 201257821271491146162000 T^{25} - 4914048257261533415025000 T^{23} \\
& - 104146699011073208745255000 T^{21} + 3611762205877271065424805000 T^{19} \\
& - 6525946067600925299859525000 T^{17} - 90372116556342562197806550000 T^{15} \\
& - 1707727156947430436644076250000 T^{13} - 19914514153947633893074652250000 T^{11}
\end{aligned}$$

$$\begin{aligned}
&+60441504322604592785420072250000T^9 - 121990662905227310507903277187500T^7 \\
&+32024799656441660037917134687500T^5 + 133347159699666487023785273437500T^3 \\
&+50897855165920147555334226562500T, \quad \mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 24864840T^{41} - 8275528800T^{39} \\
&-2290606108560T^{37} - 1056111041351520T^{35} - 139995360430695000T^{33} - 3012968529441763200T^{31} \\
&+2016715929359309640000T^{29} + 122776919545138700688000T^{27} - 8724617561136462399414000T^{25} \\
&-8776215034264551673800000T^{23} + 5320414469496529196653860000T^{21} \\
&+34291992550412388290499000000T^{19} - 742197812663450900200994550000T^{17} \\
&-9969205728097481189275551600000T^{15} - 161880090490472685603186915000000T^{13} \\
&+248980392840144022927183314000000T^{11} - 818299264516747040989821234375000T^9 \\
&-476713419004836751657597177500000T^7 + 1758336091397966713046828163750000T^5 \\
&+1769350768557853190505858562500000T^3 + 280160068877261722604358028125000T \quad \mathbf{h}_{13} = 0, \\
\mathbf{h}_{12} = 8288280T^{43} - 1950296040T^{41} - 1303260734160T^{39} - 587448080326800T^{37} \\
&-96181467152740200T^{35} - 2333545862710847400T^{33} + 1303880857145840635200T^{31} \\
&+34549368376057348968000T^{29} - 6423142555199586204450000T^{27} + 84991830908599974348990000T^{25} \\
&+5196658815883842091604820000T^{23} + 71216267628306383393150340000T^{21} \\
&-537186993036056736281959050000T^{19} - 49680822174712697320494392250000T^{17} \\
&-510089192998572154824243387000000T^{15} + 648961140374231120423300313000000T^{13} \\
&-4595842673483238620153978392125000T^{11} - 2306917135543113677700059068125000T^9 \\
&+10465408425092291217199521618750000T^7 + 24940685779394420437367058843750000T^5 \\
&-1891607794892583614798379553125000T^3 + 1383666073198822939633454221875000T \\
\mathbf{h}_{11} = 0, \quad \mathbf{h}_{10} = 2260440T^{45} - 256299120T^{43} - 520433325720T^{41} - 265398457595040T^{39} \\
&-51963689173971960T^{37} - 2078330803534062000T^{35} + 504414194258306316600T^{33} \\
&+2354920000955531856000T^{31} - 1514616736300246258218000T^{29} + 131371987844966806057428000T^{27} \\
&+2424830356405951553429370000T^{25} + 74039904708285822523537320000T^{23} \\
&+996048278667469926140492250000T^{21} - 109841718054872507713652342700000T^{19} \\
&-527139316065717040229677183050000T^{17} + 948629826710126902238243346000000T^{15} \\
&-15738010488860491791083359360125000T^{13} - 21195405167566928689123351368750000T^{11} \\
&+14136303156631427798978019193125000T^9 + 216127929068602452779971891297500000T^7 \\
&-47561677461591404329676492409375000T^5 + 4018232176163239787982535556250000T^3 \\
&+2017953028228342458755361234375000T, \quad \mathbf{h}_9 = 0, \quad \mathbf{h}_8 = 491400T^{47} + 17098200T^{45} \\
&-150724589400T^{43} - 93307202742600T^{41} - 21786354176617800T^{39} - 1497011705550207000T^{37} \\
&+74558739462560199000T^{35} + 2282444312171889789000T^{33} + 1284486001836552906930000T^{31} \\
&+158764283968215852809190000T^{29} + 5961808009517232701540250000T^{27} \\
&+300170542939126568966807550000T^{25} + 3037790768826275206178984550000T^{23} \\
&-178058394981408498570197934750000T^{21} + 706918769910782227397813964750000T^{19} \\
&+3211142985153408253476494216250000T^{17} - 82873544921836068155438221914375000T^{15} \\
&+43774520932599987667701130441875000T^{13} - 150787474955869364748865826971875000T^{11} \\
&+2021475165637011479161476994659375000T^9 - 818085617975150180472289613765625000T^7 \\
&+557849276405116114837268303578125000T^5 - 77987440013694817376566123453125000T^3 \\
&-23844751123289021015776545234375000T, \quad \mathbf{h}_7 = 0, \quad \mathbf{h}_6 = 81900T^{49} + 17287200T^{47} \\
&-31138868880T^{45} - 24379010964000T^{43} - 6845248148092200T^{41} - 741093475985200800T^{39} \\
&-25555815521743122000T^{37} + 3158961025226410404000T^{35} + 1425135751337418094012500T^{33} \\
&+175549516977252515133960000T^{31} + 9472551469732244487395340000T^{29} \\
&+141872271768241531599713400000T^{27} - 2611527288780357895294497150000T^{25} \\
&-159876628457546422579162005000000T^{23} + 904821081291606171001080271500000T^{21}
\end{aligned}$$

$$\begin{aligned}
& +2911105425590491035120995781000000T^{19} - 57856971350556380128450774137187500T^{17} \\
& -597240785276937569608838354857500000T^{15} + 2056519193919467475565331182968750000T^{13} \\
& +12154683904194670162802727887137500000T^{11} + 13863900825388320276306901257843750000T^9 \\
& +1098571541450992569418057287187500000T^7 + 607947066875348083972181687343750000T^5 \\
& -947695220634386609405575242187500000T^3 - 35757135169692262292147626757812500T, \\
\mathbf{h}_5 = 0, \quad \mathbf{h}_4 = & 9828 T^{51} + 4101300 T^{49} - 4340604240 T^{47} - 4436265289680 T^{45} \\
& -1514282595903000 T^{43} - 234322521203647800 T^{41} - 15633540600684930000 T^{39} \\
& +1586816675525592126000 T^{37} + 603412183386569812909500 T^{35} \\
& +70495161343635553258387500 T^{33} + 3381977148695834843343420000 T^{31} \\
& +2314480249177743285393500000 T^{29} - 508001687496347812118539650000 T^{27} \\
& -45317179030459158057822637050000T^{25} - 639392657721512904650453512500000T^{23} \\
& -633453674783199881723813200500000T^{21} - 87308156097322179307308497039062500T^{19} \\
& -1355979332634292253868900567460312500T^{17} + 4411540235306765090271379701813750000T^{15} \\
& +10057603832362589442041629620843750000T^{13} + 26911295530554930144489745621340625000T^{11} \\
& +17125918398489101416653911676328125000T^9 + 18954282091963852165062854667093750000T^7 \\
& -6660318081890442429357638396906250000T^5 - 552093664050382974021790374023437500T^3 \\
& -34463233945947896810671380117187500T, \quad \mathbf{h}_3 = 0, \quad \mathbf{h}_2 = 756 T^{53} + 495432 T^{51} \\
& -362210940 T^{49} - 500636959200 T^{47} - 209842694598600 T^{45} - 42645497546559600 T^{43} \\
& -3498152086126744200 T^{41} + 377281024347463236000 T^{39} + 109988509720439880337500 T^{37} \\
& +7421060572419282448275000 T^{35} + 35933479335640711244227500 T^{33} \\
& -11661105491106988983093720000 T^{31} + 439741080879143818859780850000 T^{29} \\
& +19783711746209042465457739500000T^{27} - 5375663257081727637430152750000T^{25} \\
& -22493569190218996211616545271000000T^{23} - 186281502287073108403281941459812500T^{21} \\
& -1059084822334974004802067028318125000T^{19} + 6329153949445054314997820083110937500T^{17} \\
& -14918769495839465654360619394237500000T^{15} - 29373548513181426947900813984803125000T^{13} \\
& +152565680355947795987851167072956250000T^{11} + 54541012524300447784197649387171875000T^9 \\
& -62401428931462991825188978932187500000T^7 + 8198315520009349681029894474726562500T^5 \\
& -834776111135182389414040096171875000T^3 + 29742242994448184918798588320312500T, \\
\mathbf{h}_1 = 0, \quad \mathbf{h}_0 = & 28 T^{55} + 25956 T^{53} - 13362300 T^{51} - 26340465060 T^{49} - 13674573352440 T^{47} \\
& -3395567206168200 T^{45} - 366196973040498600 T^{43} + 12483830514741201000 T^{41} \\
& +8840223660767963656500 T^{39} + 1211740695196554052183500 T^{37} \\
& +95626598418360513349807500 T^{35} + 4134578042051998735362052500 T^{33} \\
& +79851486142884785720633550000 T^{31} + 664734262949553830334181650000T^{29} \\
& -97834567689265552651609176150000T^{27} - 3703009772457379493336677700250000T^{25} \\
& -14475894457788465791146209089437500T^{23} - 226810862867256551420326103376562500T^{21} \\
& -1684904275255975197128945251574062500T^{19} + 9448363637590584401034224408586562500T^{17} \\
& +28548596595041297866371213386578125000T^{15} + 146225151676068938646301685253571875000T^{13} \\
& +31326045499715628103678345844109375000T^{11} + 25092831591187433843822670758765625000T^9 \\
& -44918394960897223683724633522007812500T^7 - 7823888482100405864538916831992187500T^5 \\
& -57123990513146513891660780742187500T^3 + 5507822776749663873851590429687500)T,
\end{aligned}$$

$$\begin{aligned}
\mathbf{q}_{56} = 1, \quad \mathbf{q}_{55} = 0, \quad \mathbf{q}_{54} = & 28 T^2 + 28, \quad \mathbf{q}_{53} = 0, \quad \mathbf{q}_{52} = 378 T^4 - 2268 T^2 + 3402, \quad \mathbf{q}_{51} = 0, \\
\mathbf{q}_{50} = & 3276 T^6 - 62748 T^4 + 21924 T^2 + 483084, \quad \mathbf{q}_{49} = 0, \quad \mathbf{q}_{48} = 20475 T^8 - 749700 T^6
\end{aligned}$$

$$\begin{aligned}
&+2315250 T^4 + 5953500 T^2 + 66976875, \quad \mathbf{q}_{47} = 0, \quad \mathbf{q}_{46} = 98280 T^{10} - 5556600 T^8 + 48157200 T^6 \\
&+65091600 T^4 + 959704200 T^2 + 8132481000, \quad \mathbf{q}_{45} = 0, \quad \mathbf{q}_{44} = 376740 T^{12} - 29037960 T^{10} \\
&+479058300 T^8 - 442940400 T^6 + 9798270300 T^4 + 117272043000 T^2 + 765518890500, \quad \mathbf{q}_{43} = 0, \\
\mathbf{q}_{42} &= 1184040 T^{14} - 114123240 T^{12} + 2988021960 T^{10} - 13647803400 T^8 + 81493889400 T^6 \\
&+1166454966600 T^4 + 9193406607000 T^2 + 45889232697000, \quad \mathbf{q}_{41} = 0, \quad \mathbf{q}_{40} = 3108105 T^{16} \\
&-350020440 T^{14} + 13152128220 T^{12} - 117779757480 T^{10} + 606076420950 T^8 + 10517426904600 T^6 \\
&+74383606489500 T^4 + 157761403569000 T^2 + 1272788332979625, \quad \mathbf{q}_{39} = 0, \quad \mathbf{q}_{38} = 6906900 T^{18} \\
&-855924300 T^{16} + 43638210000 T^{14} - 571636018800 T^{12} + 4633520938200 T^{10} + 91699486263000 T^8 \\
&+670471524954000 T^6 - 2950507091070000 T^4 - 33200878716967500 T^2 + 74380367755732500, \\
\mathbf{q}_{37} &= 0, \quad \mathbf{q}_{36} = 13123110 T^{20} - 1685814900 T^{18} + 114021611550 T^{16} - 1773231490800 T^{14} \\
&+32095272062700 T^{12} + 488569383779400 T^{10} + 7285991633959500 T^8 - 37353863706390000 T^6 \\
&-712242268565231250 T^4 + 766523807513887500 T^2 + 4232772093141033750, \quad \mathbf{q}_{35} = 0, \\
\mathbf{q}_{34} &= 21474180 T^{22} - 2671057620 T^{20} + 242800796700 T^{18} - 3452383449900 T^{16} \\
&+173542141218600 T^{14} + 2616311024019000 T^{12} + 72996496011747000 T^{10} - 112701870219015000 T^8 \\
&-7619235080076427500 T^6 + 26397875963474977500 T^4 + 56266542474799387500 T^2 \\
&+207628508681964562500, \quad \mathbf{q}_{33} = 0, \quad \mathbf{q}_{32} = 30421755 T^{24} - 3341712780 T^{22} \\
&+434948276070 T^{20} - 2762553786300 T^{18} + 707628327869925 T^{16} + 16452498066820200 T^{14} \\
&+420040677679276500 T^{12} + 1905343386976953000 T^{10} - 55819031246158516875 T^8 \\
&+383292609490411582500 T^6 + 758856538412299923750 T^4 + 2764219001238915412500 T^2 \\
&+7818616326630954796875, \quad \mathbf{q}_{31} = 0, \quad \mathbf{q}_{30} = 37442160 T^{26} - 3107699280 T^{24} \\
&+676957508640 T^{22} + 7303633777440 T^{20} + 2213013639790800 T^{18} + 80237629739005200 T^{16} \\
&+2078146188047160000 T^{14} + 28199965799057976000 T^{12} - 257869432151320374000 T^{10} \\
&+3386896499825497890000 T^8 + 5008882358117664180000 T^6 + 35493664601967887700000 T^4 \\
&+79785466324073952750000 T^2 + 191031684895268043750000, \quad \mathbf{q}_{29} = 0, \quad \mathbf{q}_{28} = 40116600 T^{28} \\
&-1684897200 T^{26} + 943969761000 T^{24} + 36029988237600 T^{22} + 5459806605468600 T^{20} \\
&+266337649042542000 T^{18} + 8144921696066001000 T^{16} + 188833010171624280000 T^{14} \\
&-749375912196779175000 T^{12} + 26593887688740291150000 T^{10} + 13857652471353057675000 T^8 \\
&+490019806344263094900000 T^6 + 847091638120081055625000 T^4 + 76231684688082635250000 T^2 \\
&+2590948189562408935875000, \quad \mathbf{q}_{27} = 0, \quad \mathbf{q}_{26} = 37442160 T^{30} + 561632400 T^{28} \\
&+1204554985200 T^{26} + 88218653418000 T^{24} + 10930955933444400 T^{22} + 618649682178805200 T^{20} \\
&+19905633824246478000 T^{18} + 212421481759657170000 T^{16} - 7909388178373163790000 T^{14} \\
&+190838377838882403630000 T^{12} + 246648947765545606050000 T^{10} + 5106259955755665733950000 T^8 \\
&+14509298027190311210250000 T^6 - 16822775828173706102250000 T^4 - 57377741897262022755750000 T^2 \\
&+64893616962420115311750000, \quad \mathbf{q}_{25} = 0, \quad \mathbf{q}_{24} = 30421755 T^{32} + 2733277680 T^{30} \\
&+1411247917800 T^{28} + 156504460165200 T^{26} + 18220186626232500 T^{24} + 1063917237344598000 T^{22} \\
&+24341468738209623000 T^{20} - 1458407669898168990000 T^{18} - 38195651665360037598750 T^{16} \\
&+704274365043021222450000 T^{14} + 5599053528058930314375000 T^{12} \\
&+37978367503214552163750000 T^{10} + 257388209612512632466312500 T^8 \\
&-76827096194526390462750000 T^6 - 1281634404886971940179375000 T^4 \\
&+831668698074883802883750000 T^2 + 1475186709752105729050546875, \quad \mathbf{q}_{23} = 0, \\
\mathbf{q}_{22} &= 21474180 T^{34} + 3959508420 T^{32} + 1496491506720 T^{30} + 220136214564000 T^{28} \\
&+25819817141790000 T^{26} + 1464130513297998000 T^{24} + 431038469000412000 T^{22} \\
&-5567239710825100308000 T^{20} - 51972702340355822025000 T^{18} + 2111596128156974433975000 T^{16} \\
&+26110454746607837505900000 T^{14} + 370837660144520166097500000 T^{12}
\end{aligned}$$

$$\begin{aligned}
& +2211046006855432481484750000 T^{10} + 3768821426548444795278750000 T^8 \\
& -15057540757380336212497500000 T^6 + 24166290139115638826812500000 T^4 \\
& +22727364203519577125586562500 T^2 + 26036255163840613991361562500, \\
& \mathbf{q}_{21} = 0, \quad \mathbf{q}_{20} = 13123110 T^{36} + 3942989820 T^{34} + 1403651839590 T^{32} \\
& +254097412763040 T^{30} + 31521548873014200 T^{28} + 1794671024233626000 T^{26} \\
& -35581106620452393000 T^{24} - 8152518727359282276000 T^{22} + 80631055883822660716500 T^{20} \\
& +5932728449553959198145000 T^{18} + 82578636925153162005712500 T^{16} \\
& +1543335783106772812657500000 T^{14} + 14290988763259977767373375000 T^{12} \\
& +80369144237433402713612250000 T^{10} - 65257978237151882814757125000 T^8 \\
& +298324647301009641815002500000 T^6 + 423803283423874977134856093750 T^4 \\
& +357328418589665267175368437500 T^2 + 289950943789653252140878593750, \quad \mathbf{q}_{19} = 0, \\
& \mathbf{q}_{18} = 6906900 T^{38} + 3038504700 T^{36} + 1138241065500 T^{34} + 243867643658100 T^{32} \\
& +33248280773307600 T^{30} + 2138061788277030000 T^{28} - 18711013316805810000 T^{26} \\
& -3565489869883051110000 T^{24} + 375784427183868517815000 T^{22} + 8589634936729811329365000 T^{20} \\
& +177884937944352775261125000 T^{18} + 5022188934743990092458375000 T^{16} \\
& +34767700807474654377608250000 T^{14} + 346392882720924307150632750000 T^{12} \\
& +729840306804380479805847750000 T^{10} + 3349869712218751272882581250000 T^8 \\
& +5016164083568572942014820312500 T^6 + 6682149789566477649221235937500 T^4 \\
& +1308993528629685934471373437500 T^2 + 1438708282930546005298232812500, \quad \mathbf{q}_{17} = 0, \\
& \mathbf{q}_{16} = 3108105 T^{40} + 1879208100 T^{38} + 782694539550 T^{36} + 194946808587300 T^{34} \\
& +30068684997553125 T^{32} + 2434041863757781200 T^{30} + 79798925261816145000 T^{28} \\
& +8035953559797385530000 T^{26} + 754836005796200279201250 T^{24} \\
& +17125179786494377633035000 T^{22} + 613061169627775399022992500 T^{20} \\
& +13213975547514313000303875000 T^{18} + 82582615880709112432795781250 T^{16} \\
& -277980267954943276814841750000 T^{14} - 3948507840549888704785071375000 T^{12} \\
& +55686750696026949631741859250000 T^{10} + 42615372067309740929873220703125 T^8 \\
& +151368250313086725334355001562500 T^6 + 17141985927211128923407436718750 T^4 \\
& -42562069342332573564516923437500 T^2 + 14036118651669035253714022265625, \\
& \mathbf{q}_{15} = 0, \quad \mathbf{q}_{14} = 1184040 T^{42} + 942951240 T^{40} + 448820341200 T^{38} + 129061854306000 T^{36} \\
& +22950971378735400 T^{34} + 2420746857044901000 T^{32} + 189431470627899192000 T^{30} \\
& +19740165844212003960000 T^{28} + 1288418099780576436210000 T^{26} + 39634356458318874582330000 T^{24} \\
& +932948607189954436181100000 T^{22} + 24964784950300898037835500000 T^{20} \\
& +336813899817214828406648250000 T^{18} - 4277060110872501238036500750000 T^{16} \\
& -15793226076541858476785253000000 T^{14} + 182147996494954471930263315000000 T^{12} \\
& +957228897383337168450769033125000 T^{10} + 1659229465068162746724356015625000 T^8 \\
& +1250994378672604495344045881250000 T^6 - 1046218980520023853006332618750000 T^4 \\
& +233488960703366614557013828125000 T^2 + 114491186085844621649987015625000, \\
& \mathbf{q}_{13} = 0, \quad \mathbf{q}_{12} = 376740 T^{44} + 383173560 T^{42} + 211091556060 T^{40} + 69936531310800 T^{38} \\
& +14478829033214700 T^{36} + 1951093276439491800 T^{34} + 219781394892621292500 T^{32} \\
& +23753718553640461848000 T^{30} + 1737564922252746032985000 T^{28} + 85455658543552552907190000 T^{26} \\
& +3155694614350839848822535000 T^{24} + 83228531888006342213676300000 T^{22} \\
& +980904596645249105852397375000 T^{20} - 8069972179322250266601782250000 T^{18} \\
& +5852053736347304386211936625000 T^{16} + 470130559276041642828328479000000 T^{14}
\end{aligned}$$

$$\begin{aligned}
&+3826968310962656491781882545312500T^{12} + 14400168925110609944407683954375000T^{10} \\
&+35422981894412214489355285504687500T^8 - 8045800083267738229862592918750000T^6 \\
&+5267694033280469186100991223437500T^4 + 2241399641805817999957998421875000T^2 \\
&+580522174177198819681404351562500, \mathbf{q}_{11} = 0, \mathbf{q}_{10} = 98280T^{46} + 124671960T^{44} \\
&+79862456520T^{42} + 30469610860920T^{40} + 7363086516739800T^{38} + 1221618341096029800T^{36} \\
&+165380901580623267000T^{34} + 18523557951212005461000T^{32} + 1521418822317545771466000T^{30} \\
&+94208790464887079928870000T^{28} + 4142878394428875557173410000T^{26} \\
&+108603899031880110735226350000T^{24} + 1586101557851603861258940750000T^{22} \\
&+5626299013637437585429229250000T^{20} + 95942669081847030488951655750000T^{18} \\
&+1600756201260760979683508510250000T^{16} + 12940726819204628290681342633125000T^{14} \\
&+41771730297204236476700239561875000T^{12} + 118010611986482678310004663130625000T^{10} \\
&+154559704011983463621551783034375000T^8 + 53946862592799320363224126996875000T^6 \\
&+60136820917315236553097341828125000T^4 + 8188712841236219843772580734375000T^2 \\
&+824966108420796215602744640625000, \mathbf{q}_9 = 0, \mathbf{q}_8 = 20475T^{48} + 31789800T^{46} + 23681605500T^{44} \\
&+10399502698200T^{42} + 2927281902340950T^{40} + 577627006715451000T^{38} + 87402057712417255500T^{36} \\
&+9919859107719402765000T^{34} + 800327687122633415563125T^{32} + 42769550356395608294010000T^{30} \\
&+1015850709135001823928075000T^{28} - 9859626751919274812824650000T^{26} \\
&-12994083461579363825869687500T^{24} + 20016288702561441180144003750000T^{22} \\
&+679783469479385743524604426875000T^{20} + 2547659051955188113304398286250000T^{18} \\
&+23936520687149123724088500660703125T^{16} + 186593162117858937503104921003125000T^{14} \\
&-236735342801561911802080068295312500T^{12} - 413881814536318993632578613928125000T^{10} \\
&+2319699480672946592561913286558593750T^8 + 645133155249795867525146260359375000T^6 \\
&+272571033660634953096852335273437500T^4 - 34555655461929637202205397734375000T^2 \\
&+2590924796001627597801615498046875, \mathbf{q}_7 = 0, \mathbf{q}_6 = 3276T^{50} + 6129900T^{48} \\
&+5297821200T^{46} + 2674898024400T^{44} + 872848514676600T^{42} + 199686264011249400T^{40} \\
&+33197856289357914000T^{38} + 3837403641408383970000T^{36} + 276908778929881432372500T^{34} \\
&+9159559733209296842392500T^{32} - 131290592583932779023180000T^{30} \\
&-15032038313822098605428700000T^{28} + 271522692937305895882970250000T^{26} \\
&+23727100949644226008739114250000T^{24} + 768309999321268014688880302500000T^{22} \\
&+12902152525905099940011381352500000T^{20} + 120504248624518289819676115540312500T^{18} \\
&+645622592643611459396432157557812500T^{16} - 1119578306397850896579839825718750000T^{14} \\
&+10319330688733274219793888356250000T^{12} + 6342730065378280526865194003596875000T^{10} \\
&+12132603370280467942856600784046875000T^8 + 9468629274147888824201932240781250000T^6 \\
&-615966923109006853803313738593750000T^4 + 60481139634212975681436988242187500T^2 \\
&+5507822776749663873851590429687500, \mathbf{q}_5 = 0, \mathbf{q}_4 = 378T^{52} + 841428T^{50} \\
&+840568050T^{48} + 487096318800T^{46} + 183138162788700T^{44} + 47831592399129000T^{42} \\
&+8850859090368025500T^{40} + 1129359739079859210000T^{38} + 93052542964654276368750T^{36} \\
&+4403146806459838814107500T^{34} + 89202381643571262806643750T^{32} \\
&-1179658500799710888841500000T^{30} + 23432583245724520630079625000T^{28} \\
&+10578094500969856231695150750000T^{26} + 499339457548290824562364892625000T^{24} \\
&+10735921664568725480441788702500000T^{22} + 75790371288166572312787304136093750T^{20} \\
&+608524156666257484535247119442187500T^{18} + 2677685608955016886758750786536718750T^{16} \\
&+672486304831536963675354803606250000T^{14}
\end{aligned}$$

$$\begin{aligned}
&+9131740007447079465862895562023437500T^{12} + 45475192297272688169637000223265625000T^{10} \\
&+21369215838930001455459090380273437500T^8 + 13508958241351375588197597967031250000T^6 \\
&+348382904493170405887479407988281250T^4 + 146822818591641039837243824882812500T^2 \\
&+1652346833024899162155477128906250, \quad \mathbf{q}_3 = 0, \quad \mathbf{q}_2 = 28T^{54} + 73332T^{52} + 84306852T^{50} \\
&+55935513900T^{48} + 24073903614600T^{46} + 7132291668797400T^{44} + 1498506622541775000T^{42} \\
&+230871155311572237000T^{40} + 27771970845932873752500T^{38} + 2783359442956321295797500T^{36} \\
&+231023465050074485008027500T^{34} + 14304208964922571107875962500T^{32} \\
&+609392496844662576050832750000T^{30} + 17180550932698920283485680250000T^{28} \\
&+351839533333824978599094572250000T^{26} + 2915178549212598578022656292750000)T^{24} \\
&-32498957280378769107354302813437500T^{22} - 102838157202293944963833561572812500T^{20} \\
&+1615150792954957399277819919173437500T^{18} + 28742506839178493428111859797795312500T^{16} \\
&+72905835863993641253168075755228125000T^{14} + 158368618939599733643864923772109375000T^{12} \\
&-65146061199633257026804758778265625000T^{10} - 17057919476262103450025081489296875000T^8 \\
&+37949266119990300735095714833242187500T^6 + 5943570241573087281471449112539062500T^4 \\
&-76007954319145361459151947929687500T^2 + 1101564555349932774770318085937500, \quad \mathbf{q}_1 = 0, \\
&\mathbf{q}_0 = T^{56} + 3052T^{54} + 4019274T^{52} + 3044365884T^{50} + 1490175636075T^{48} + 501200329379400T^{46} \\
&+120293856369130500T^{44} + 21109132070495469000T^{42} + 2743830229192474739625T^{40} \\
&+264032656532001259402500T^{38} + 18705499355839478697033750T^{36} \\
&+967356830845193024425612500T^{34} + 36317543301591844669914796875T^{32} \\
&+1182862195373087251907172750000T^{30} + 43265271525507252011412727875000T^{28} \\
&+1146101963874736797501493362750000T^{26} + 18763596889105886868604052170546875T^{24} \\
&+156011886701245799140278339790312500T^{22} + 1621973055294513702812375078578593750)T^{20} \\
&+10334106828233210967813542074964062500T^{18} + 11130688397493816047672073880222265625T^{16} \\
&+53932911302618520117796345322953125000T^{14} + 171043092322485209840747207763351562500T^{12} \\
&+99160652131611586296198617527453125000T^{10} + 90541065978105828712959516510498046875T^8 \\
&+4768253316469725674256984968085937500T^6 + 1723869845940119797317349924628906250T^4 \\
&+40757888547947512666501769179687500T^2 + 39341591262497599098939931640625.
\end{aligned}$$