



**HAL**  
open science

# Gauged LL and different muon neutrino and anti-neutrino oscillations: MINOS and beyond

Julian Heeck, Werner Rodejohann

► **To cite this version:**

Julian Heeck, Werner Rodejohann. Gauged LL and different muon neutrino and anti-neutrino oscillations: MINOS and beyond. *Journal of Physics G: Nuclear and Particle Physics*, 2011, 38 (8), pp.85005. 10.1088/0954-3899/38/8/085005 . hal-00637339

**HAL Id: hal-00637339**

**<https://hal.science/hal-00637339>**

Submitted on 1 Nov 2011

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Gauged $L_\mu - L_\tau$ and different Muon Neutrino and Anti-Neutrino Oscillations: MINOS and beyond

Julian Heeck\* , Werner Rodejohann†

*Max-Planck-Institut für Kernphysik,  
Postfach 103980, D-69029 Heidelberg, Germany*

## Abstract

If a  $Z'$  gauge boson of a gauged  $L_\mu - L_\tau$  symmetry is very light, it is associated with a long-range leptonic force. In this case the particles in the Sun create via mixing of  $Z'$  with the Standard Model  $Z$  a flavor-dependent potential for muon neutrinos in terrestrial long-baseline experiments. The potential changes sign for anti-neutrinos and hence can lead to apparent differences in neutrino and anti-neutrino oscillations without introducing CP or CPT violation. This can for instance explain the recently found discrepancy in the survival probabilities of muon neutrinos and anti-neutrinos in the MINOS experiment. We obtain the associated parameters of gauged  $L_\mu - L_\tau$  required to explain this anomaly, and compare our scenario to usually considered non-standard interactions. When applied to MINOS, both approaches have difficulties with existing limits. The consequences for future long-baseline experiments and for the anomalous magnetic moment of the muon are discussed. The main feature is that atmospheric neutrino mixing has to be non-maximal in order to have an effect. Neutrino masses tend to have a mild hierarchy.

---

\*email: [julian.heeck@mpi-hd.mpg.de](mailto:julian.heeck@mpi-hd.mpg.de)

†email: [werner.rodejohann@mpi-hd.mpg.de](mailto:werner.rodejohann@mpi-hd.mpg.de)

# 1 Introduction

Additional gauged  $U(1)$  symmetries are a feature of many theories beyond the Standard Model (for a review, see e.g. Ref. [1]). A large amount of interesting phenomenology arises in such scenarios, including LHC physics, lepton flavor violation, dark matter, etc. Here we focus on a particularly interesting class of models, namely anomaly free  $U(1)$  symmetries under which the SM is invariant. It was observed long ago [2] that with the particle content of the Standard Model one can gauge one of the lepton flavor combinations  $L_e - L_\mu$ ,  $L_e - L_\tau$  or  $L_\mu - L_\tau$  without introducing anomalies. If the gauge bosons associated with this  $U(1)$  symmetry are very light, then long-range forces are introduced. In case the extra  $U(1)$  corresponds to  $L_e - L_\mu$  or  $L_e - L_\tau$ , the electrons in the Sun or the Earth generate a potential acting on the neutrinos in terrestrial experiments [3–5]. The flavor dependence of  $L_e - L_\mu$  or  $L_e - L_\tau$  induces modifications to the neutrino oscillations and therefore the coupling of the  $U(1)$  can be constrained. The lack of a significant amount of muons in the Sun or Earth lead to the fact that the oscillation phenomenology of gauged  $L_\mu - L_\tau$  with very light  $Z'$  was never studied, though this symmetry was analyzed with different phenomenology in mind [6–10]. The reason why  $L_\mu - L_\tau$  should be preferred over  $L_e - L_\mu$  or  $L_e - L_\tau$  is that the neutrino mass matrix in the symmetry limit has a very promising structure.

In the present paper we note that the unavoidable  $Z$ – $Z'$  mixing in models with gauged  $U(1)$  symmetries allows to put limits on the parameters associated with  $L_\mu - L_\tau$ . The flavor dependent potential generated by the  $Z'$  has different sign for neutrinos and anti-neutrinos and can therefore lead to seemingly different neutrino and anti-neutrino parameters. We apply this to the recently found discrepancy in the survival probabilities of muon neutrinos and anti-neutrinos by the MINOS collaboration [11]. In this long-baseline experiment, the results for the oscillation parameters in the neutrino and anti-neutrino running lead to different values, namely<sup>1</sup>

$$\begin{aligned} \Delta m^2 &= (2.35_{-0.08}^{+0.11}) \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta > 0.91, \\ \overline{\Delta m^2} &= (3.36_{-0.40}^{+0.45}) \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\bar{\theta} = 0.86 \pm 0.11, \end{aligned} \quad (1)$$

for neutrinos and anti-neutrinos, respectively [11]. We will use here the impact of a long-range force associated with the  $Z'$  of gauged  $L_\mu - L_\tau$  to explain this anomaly. We obtain the parameters ( $Z$ – $Z'$  mixing and gauge coupling) of the  $U(1)$  and discuss in addition consequences for future long-baseline neutrino oscillation experiments and the anomalous magnetic moment of the muon. An interesting feature of our proposal is that in order for gauged  $L_\mu - L_\tau$  to be the explanation of the MINOS results, atmospheric neutrino mixing needs to be non-maximal. We furthermore find an interesting correlation in what regards the sign of the differences between neutrino and anti-neutrino parameters. Neutrino masses tend to be quasi-degenerate.

The apparent difference of the neutrino and anti-neutrino parameters has motivated several explanation attempts, in the form of CPT violation [12], Non-Standard Interactions [13–15],

---

<sup>1</sup>This result is henceforth referred to as “MINOS anomaly”.

and sterile neutrinos plus gauged  $B - L$  [16]. As became clear [17, 18], each of the explanations put forward so far has problems with existing constraints on non-standard neutrino physics:<sup>2</sup> the standard three-neutrino picture is remarkably stable and robust. We will nevertheless present fits to the MINOS data and obtain the associated parameters of gauged  $L_\mu - L_\tau$  required to explain the anomaly. Comparing the results with Non-Standard Interactions (NSIs), we find that there are difficulties with existing limits on non-standard neutrino physics. There are however interesting differences to the usually considered standard NSIs. We furthermore check a variety of experimental observables which could be modified by the parameters of gauged  $L_\mu - L_\tau$  for consistency. These include the magnetic moment of the muon, Big Bang Nucleosynthesis, charge difference of electron and muon, electroweak precision data, and tests of the equivalence principle. The strongest constraints are and will be provided by neutrino oscillation experiments, which shows the remarkable sensitivity of neutrinos to new and interesting physics. Performing a GLOBES analysis, we finally obtain future limits on the parameters of  $L_\mu - L_\tau$ .

In Section 2 we outline the framework of gauged  $L_\mu - L_\tau$  symmetry including  $Z-Z'$  mixing, current constraints are described in Section 3. The results are applied to oscillation phenomenology and the MINOS results in Section 4, where we also study the impact on future neutrino oscillation experiments, the anomalous magnetic moment of the muon and neutrino masses. Section 5 summarizes our findings.

## 2 Gauged $L_\mu - L_\tau$ Symmetry

The most general Lagrangian after breaking the  $SU(3) \times SU(2) \times U(1)_Y \times U(1)_{L_\mu - L_\tau}$  symmetry can be written as [19]

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{Z'} + \mathcal{L}_{\text{mix}}, \quad (2)$$

where the relevant part of the Standard Model Lagrangian is

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}_{\mu\nu}^a\hat{W}^{a\mu\nu} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{\hat{e}}{\hat{c}_W}j_B^\mu\hat{B}_\mu - \frac{\hat{e}}{\hat{s}_W}j_W^{a\mu}\hat{W}_\mu^a, \quad (3)$$

and the hats denote that we are not in the mass eigenbasis. The currents  $j_B^\mu$  and  $j_W^{a\mu}$  are the usual Standard Model ones. The gauge coupling of the  $U(1)_{L_\mu - L_\tau}$  is denoted  $\hat{g}'$ . The  $Z'$  part in our case is

$$\mathcal{L}_{Z'} = -\frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} + \frac{1}{2}\hat{M}_Z'^2\hat{Z}'_\mu\hat{Z}'^\mu - \hat{g}'j'^\mu\hat{Z}'_\mu, \quad (4)$$

$$j'^\mu = \bar{\mu}\gamma^\mu\mu + \bar{\nu}_\mu\gamma^\mu P_L\nu_\mu - \bar{\tau}\gamma^\mu\tau - \bar{\nu}_\tau\gamma^\mu P_L\nu_\tau, \quad (5)$$

---

<sup>2</sup>An exception is probably CPT violation, if one is willing to abandon such an important cornerstone of modern physics.

with the projection operator  $P_L \equiv \frac{1}{2}(1 - \gamma_5)$ . The term  $\frac{1}{2}\hat{M}_Z^2 \hat{Z}'_\mu \hat{Z}'^\mu$  breaks the  $U(1)_{L_\mu-L_\tau}$  symmetry, and is generated by a vev of some Higgs sector (left unspecified here). Then there are terms associated with mixing of the field strength tensors and the two massive bosons:

$$\mathcal{L}_{\text{mix}} = -\frac{\sin \chi}{2} \hat{Z}'^{\mu\nu} \hat{B}_{\mu\nu} + \delta \hat{M}^2 \hat{Z}'_\mu \hat{Z}'^\mu \quad (6)$$

with the kinetic mixing angle  $\chi$ . The crucial mixing term  $\sin \chi$  can arise directly, or can be generated radiatively [20].

Diagonalizing [19] the kinetic terms (which gives fields denoted by  $B^\mu = \hat{B}^\mu + \sin \chi \hat{Z}'_\mu$  and  $Z'_\mu = \cos \chi \hat{Z}'_\mu$ ) and then the mass terms leads, besides the usual  $W$  bosons, to a massless photon field  $A^\mu = \hat{c}_W B^\mu + \hat{s}_W W^{3\mu}$  and two massive gauge bosons  $Z_1$  and  $Z_2$ . They are related to the original  $\hat{Z}$  and  $\hat{Z}'$  as

$$Z_1^\mu = \cos \xi \left( \hat{Z}^\mu - \hat{s}_W \sin \chi \hat{Z}'_\mu \right) + \sin \xi \cos \chi \hat{Z}'_\mu, \quad (7)$$

$$Z_2^\mu = \cos \xi \cos \chi \hat{Z}'_\mu - \sin \xi \left( \hat{Z}^\mu - \hat{s}_W \sin \chi \hat{Z}'_\mu \right), \quad (8)$$

where  $\xi$  is a new mixing angle defined by

$$\tan 2\xi = \frac{-2 \cos \chi (\delta \hat{M}^2 + \hat{M}_Z^2 \hat{s}_W \sin \chi)}{\hat{M}_{Z'}^2 - \hat{M}_Z^2 \cos^2 \chi + \hat{M}_Z^2 \hat{s}_W^2 \sin^2 \chi + 2\delta \hat{M}^2 \hat{s}_W \sin \chi}. \quad (9)$$

The above physical particles  $Z_1$  and  $Z_2$  are in the literature normally called  $Z$  and  $Z'$ . We will follow this notation from now on. Their masses are given by

$$M_{1,2}^2 = \frac{a+c}{2} \pm \sqrt{b^2 + \left(\frac{a-c}{2}\right)^2} \quad (10)$$

with

$$\begin{aligned} a &= \hat{M}_Z^2, & b &= \hat{s}_W \tan \chi \hat{M}_Z^2 + \frac{\delta \hat{M}^2}{\cos \chi}, \\ c &= \frac{1}{\cos^2 \chi} \left( \hat{M}_Z^2 \hat{s}_W^2 \sin^2 \chi + 2\hat{s}_W \sin \chi \delta \hat{M}^2 + \hat{M}_{Z'}^2 \right). \end{aligned} \quad (11)$$

The situation simplifies considerably if the  $Z'$  is much lighter than the  $Z$ , i.e., if  $\chi \ll 1$  and  $\delta \hat{M}^2 \ll \hat{M}_Z^2$  are very small. In this case we have for the masses

$$M_1^2 \simeq \hat{M}_Z^2, \quad M_2^2 \simeq c - \frac{b^2}{a-c}, \quad (12)$$

and the mixing angle is

$$\xi \simeq \frac{1}{\cos \chi} \left( \hat{s}_W \sin \chi + \frac{\delta \hat{M}^2}{\hat{M}_Z^2} \right) \simeq \hat{s}_W \chi + \frac{\delta \hat{M}^2}{\hat{M}_Z^2}. \quad (13)$$

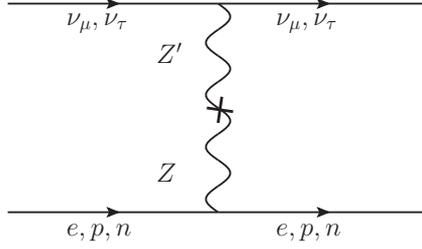


Figure 1: Long-range  $\nu_{\mu,\tau}-(e, p, n)$  interaction through  $Z-Z'$ -mixing.

With this approximation the Lagrangians for the physical particles are<sup>3</sup>

$$\begin{aligned}\mathcal{L}_A &= -e (j_{\text{EM}})_\mu A^\mu, \\ \mathcal{L}_{Z_1} &= - \left( \frac{e}{s_W c_W} ((j_3)_\mu - s_W^2 (j_{\text{EM}})_\mu) + g' \xi (j')_\mu \right) Z_1^\mu, \\ \mathcal{L}_{Z_2} &= - \left( g' (j')_\mu - (\xi - s_W \chi) \frac{e}{s_W c_W} ((j_3)_\mu - s_W^2 (j_{\text{EM}})_\mu) - e c_W \chi (j_{\text{EM}})_\mu \right) Z_2^\mu.\end{aligned}\tag{14}$$

The Lagrangian for the  $A^\mu$  field is the canonical one and hence  $\hat{e} = e$ . The other gauge coupling  $g'$  is simply  $\hat{g}'$ .

If we take the mass of the  $Z'$  to be  $M_2 < 1/R_{\text{A.U.}} \simeq 10^{-18}$  eV ( $R_{\text{A.U.}} \simeq 7.6 \times 10^{26}$  GeV<sup>-1</sup> denotes an astronomical unit) we obtain for particles on Earth a static potential generated by particles in the Sun. This has been studied for the  $U(1)_{L_e-L_\mu}$  and the  $U(1)_{L_e-L_\tau}$  gauge bosons, for which the electrons in the Sun generate a potential

$$V = \alpha_{e\beta} \frac{N_e}{R_{\text{A.U.}}} \simeq 1.3 \cdot 10^{-11} \left( \frac{\alpha_{e\beta}}{10^{-50}} \right) \text{eV}\tag{15}$$

for the electron neutrinos  $\nu_e$  on Earth. For  $\bar{\nu}_e$  and  $\nu_\beta$  the sign of the potential changes. Here  $\alpha_{e\beta} = g'^2/(4\pi)$  is the “fine-structure constant” of the  $U(1)_{L_e-L_\beta}$  and  $N_e$  is the number of electrons in the Sun. The constraints from solar neutrino and KamLAND data are  $\alpha_{e\mu} < 3.4 \times 10^{-53}$  and  $\alpha_{e\tau} < 2.5 \times 10^{-53}$  at  $3\sigma$  [3–5]. The lack of muons and taus seems to forbid analogous studies of  $L_\mu - L_\tau$ , since its  $Z'$  does not couple directly to protons, neutrons or electrons. Consequently, to the best of our knowledge, there is no limit on  $\alpha_{\mu\tau}$  from oscillation experiments.

However, there is an indirect effect due to the  $Z-Z'$  mixing (see Fig. 1). For a neutral and unpolarized Sun the final result for the potential is (see the Appendix for details)

$$V_{\mu,\tau} = \pm g' (\xi - s_W \chi) \frac{e}{4 s_W c_W} \frac{N_n}{4\pi R_{\text{A.U.}}},\tag{16}$$

<sup>3</sup>Here we defined the physical Weinberg angle as  $s_W^2 c_W^2 = \frac{\pi \alpha(M_1)}{\sqrt{2} G_F M_1^2}$ . This gives the identity  $s_W c_W M_1 = \hat{s}_W \hat{c}_W \hat{M}_Z$  and the neutral current coupling constant becomes  $e/(\hat{s}_W \hat{c}_W) \simeq e/(s_W c_W) (1 - \xi^2/2)$ .

where the plus (minus) sign holds for a muon (tauon) neutrino. The potential for the corresponding anti-neutrinos can be obtained via  $g' \rightarrow -g'$ . Looking at Fig. 1, the main features of this potential can be understood as  $g'$  and  $e/(s_W c_W)$  arising from the vertices and  $(\xi - s_W \chi)$  from the  $Z$ - $Z'$  mixing (see Eq. (14)). The contributions of the electrons and protons cancel each other, so that finally only the neutrons generate the potential. Their total number in the Sun is about  $N_n \simeq N_e/4 \simeq 1.5 \times 10^{56}$ . The Earth also generates a comparable potential, approximating a static potential at the surface, we get

$$\frac{V_{\text{earth}}}{V_{\text{sun}}} = \frac{N_{n,\text{earth}}}{N_{n,\text{sun}}} \frac{R_{\text{A.U.}}}{R_{\text{surface}}} \simeq \frac{1.8 \times 10^{51}}{1.5 \times 10^{56}} \frac{1.5 \times 10^8}{6380} \simeq 0.28. \quad (17)$$

Our full potential at the surface of the Earth is therefore:

$$V_{\mu,\tau} \equiv \pm V = \pm 3.60 \times 10^{-14} \text{ eV} \left( \frac{\alpha}{10^{-50}} \right) \text{ with } \alpha \equiv g' (\xi - s_W \chi). \quad (18)$$

For anti-neutrinos, the sign of  $V$  changes. We stress here that the parameter  $\alpha$  that we have defined is not a ‘‘fine-structure constant’’ as for the  $L_e - L_\mu$  or  $L_e - L_\tau$  potentials, but a combination of coupling and mixing parameters. It can in particular be either positive or negative. Note further that due to the various factors in  $V$  the scale for  $\alpha = 10^{-50}$  is different than for  $\alpha_{e\beta} = 10^{-50}$  in the cases of gauged  $L_e - L_\mu$  or  $L_e - L_\tau$  in Eq. (15).<sup>4</sup> We will use in the following the value give in Eq. (18) for a long-range force according to the Earth-Sun distance. In order not to completely spoil the successful oscillation phenomenology,  $V$  should not become too close to  $\Delta m^2/E \simeq 2.9 \times 10^{-12} (\text{GeV}/E) \text{ eV}$ , where we took for  $\Delta m^2$  the mean of the two mass-squared differences from Eq. (1).

The crucial  $Z$ - $Z'$  mixing, and consequently the potential (16), can only be avoided if for the Lagrangian in Eq. (6)  $\mathcal{L}_{\text{mix}} = 0$  holds, i.e., if both  $\chi$  and  $\delta \hat{M}^2$  vanish. As can be seen from Eq. (13),  $\alpha$  would vanish for  $\delta \hat{M}^2 = 0$ . In that case, however, one can show that the next order term for  $\xi$  would generate non-zero  $\alpha \simeq g' s_W (M_{Z'}/M_Z)^2 \chi$ , which is however too small for our purposes, as we will see later. In the case  $\chi = 0$ , the mixing angle is given by  $\tan 2\xi = \frac{2\delta \hat{M}^2}{M_Z^2 - M_{Z'}^2}$ , and  $\alpha$  looks as before.

If  $M_2 < 1/R_{\text{gal}} \simeq 10^{-27} \text{ eV}$ , with  $R_{\text{gal}}$  the distance between the Sun and the core of the galaxy ( $R_{\text{gal}} \simeq 1.6 \times 10^9 R_{\text{A.U.}}$ ), we would obtain a potential

$$\frac{V_{\text{gal}}}{V_{\text{sun}}} = \frac{(1 - 4) \times 10^{11}}{1.6 \times 10^9} \simeq 60 - 240, \quad (19)$$

(with 100 – 400 billion stars) which would dominate over the Earth and Sun potentials. Depending on the range of the  $U(1)$  force the results which we obtain in the following can be easily rescaled.

---

<sup>4</sup>We note that while small values for our coupling constant and mixing angles are natural in the sense of 't Hooft (setting them to zero leads to exact  $Z'$ -number and  $L_\mu - L_\tau$  conservation, respectively), a discussion of radiative corrections to quantify fine-tuning would require a specific Higgs-sector and lies outside the realm of this work.

### 3 Current bounds on $L_\mu - L_\tau$ parameters

In this Section we will discuss the current bounds on the parameters of  $L_\mu - L_\tau$ . They arise from gravitational fifth force searches, electroweak precision observables, fermion charge universality and cosmological considerations.

In principle our model violates the equivalence principle because it adds a lepton number dependent force to gravitation. The bounds on such forces are very strict [21] but are not directly applicable here since they are based on lunar ranging and torsion balance experiments, which are only sensitive to the electron and baryon content. The only effect comes once again from mixing; as shown in the Appendix, the potential corresponding to  $Z'$  generated by a massive body depends on its neutron number  $N_n$ :

$$V(r) = \frac{e(\xi - s_W \chi)}{4 s_W c_W} N_n \frac{e^{-rM_2}}{4\pi r}. \quad (20)$$

The gravitational potential between two bodies with masses  $m_1$  and  $m_2$  and neutron content  $N_{n_1}$  and  $N_{n_2}$  is therefore changed to

$$V_{\text{grav}}(r) = -G_N \frac{m_1 m_2}{r} \left( 1 - \left( \frac{e(\xi - s_W \chi)}{4 s_W c_W} \right)^2 \frac{N_{n_1} N_{n_2}}{m_1 m_2} \frac{1}{4\pi G_N} e^{-rM_2} \right), \quad (21)$$

which can be visualized in a similar way as Fig. 1, but with two mixing ‘‘vertices’’. The 95% C.L. limits for a neutron dependent fifth force as a function of its range are given in [21] (see references therein for a description of the experiments), where the effect of new light vector or scalar bosons is parameterized as

$$V_{\text{grav}}(r) = -G_N \frac{m_1 m_2}{r} \left( 1 + \tilde{\alpha} \frac{N_{n_1} N_{n_2}}{\mu_1 \mu_2} e^{-r/\lambda} \right), \quad (22)$$

$\mu$  being a test body mass in units of atomic mass unit  $u$  and  $\tilde{\alpha} = \pm \tilde{g}^2 / (4\pi G_N u^2)$  (the sign distinguishes between vector and scalar interaction). Comparison with Eq. (21) gives the translation into our parameters

$$|\tilde{\alpha}| \equiv \frac{1}{4\pi G_N u^2} \left( \frac{e(\xi - s_W \chi)}{4 s_W c_W} \right)^2, \quad \lambda \equiv \frac{1}{M_2}. \quad (23)$$

For Earth-Sun range we take the bound  $|\tilde{\alpha}| < 10^{-11}$ , given in [21], corresponding to

$$|\xi - s_W \chi| < 5 \times 10^{-24}, \quad (24)$$

whereas the limit for an Earth range force is given as  $|\tilde{\alpha}| < 5 \times 10^{-9}$ , corresponding to

$$|\xi - s_W \chi| < 10^{-22}. \quad (25)$$

These are the strongest constraints on the mixing angles.

The parameters are however also constrained through precision data from electroweak observables. Measurements around the  $Z$ -pole examine the mass-eigenstate  $Z_1$  with mass (see Eqs. (10,11))  $M_1^2 \simeq a(1 + b^2/a^2)$ , while measurements on  $W$ -bosons give values for  $M_W = \hat{M}_Z c_W$ . Therefore the mixing changes the  $\rho$ -parameter of the Standard Model from  $\rho = M_W^2/(M_Z^2 c_W^2)$  to

$$\rho_{\text{mix}} = \left( \frac{M_W}{M_1 c_W} \right)^2 = \rho \frac{1}{1 + b^2/a^2} \simeq \rho(1 - \xi^2). \quad (26)$$

The current value [22] is  $\rho = 1.0008_{-0.0007}^{+0.0017}$  which constrains  $\xi \lesssim 10^{-2}$ . Stronger limits arise from the modified vector/axial couplings of the tauon, which can be read off from Eq. (14):

$$g_V^\tau \rightarrow 2 s_W^2 - \frac{1}{2} - 2 \frac{s_W c_W}{e} g' \xi, \quad g_A^\tau \rightarrow -\frac{1}{2}, \quad (27)$$

where  $2 s_W^2 - \frac{1}{2}$  stems from the SM neutral current  $j_3^\mu - s_W^2 j_{\text{EM}}^\mu$ . The asymmetry parameter  $A^\tau \equiv 2 g_V^\tau g_A^\tau / ((g_V^\tau)^2 + (g_A^\tau)^2)$  becomes approximately

$$A^\tau \rightarrow A_{\text{SM}}^\tau \left( 1 + \frac{4 s_W c_W}{1 - 4 s_W^2} \frac{g' \xi}{e} \right) \equiv A_{\text{SM}}^\tau + \Delta A^\tau(g' \xi), \quad (28)$$

where  $A_{\text{SM}}^\tau = (1 - 4 s_W^2) / (1 - 4 s_W^2 (1 - 2 s_W^2))$  is the value without any new physics. This quantity is measured to be  $A^\tau = 0.143 \pm 0.004$  (Ref. [22]), while with the central value  $\sin^2 \theta_W(M_Z) = 0.23116$  one expects  $A_{\text{SM}}^\tau = 0.1499$ . Since the measured  $A^\tau$  and  $A^\mu$  are of the same order while a nonzero  $g' \xi$  shifts them in different directions, we will require  $\Delta A^\tau(g' \xi)$  to be within the measured error, i.e.  $\Delta A^\tau(g' \xi) < 0.004$ . This restricts  $g' \xi$  to values

$$g' \xi < 3.6 \times 10^{-4}. \quad (29)$$

This limit is stronger than e.g. from the  $Z$ -coupling to  $\nu_\mu$  or the ratio  $\Gamma(Z \rightarrow \mu^+ \mu^-) / \Gamma(Z \rightarrow e^+ e^-)$ , where

$$\Gamma(Z \rightarrow \ell \bar{\ell}) = \frac{\alpha_{\text{EM}} M_Z}{12 s_W^2 c_W^2} ((g_V^\ell)^2 + (g_A^\ell)^2) \quad (30)$$

at tree-level, ignoring lepton masses.

The mixing also changes the electromagnetic behavior, as can be seen from the Lagrangian (14), slightly rewritten and shown only for negatively charged muons ( $\mu$ ), electrons ( $e$ ) and positrons ( $e^+$ ):

$$\begin{aligned} \mathcal{L}_{Z_2} = & - \left\{ \left[ g' + e c_W \chi - (\xi - s_W \chi) \frac{e}{s_W c_W} \left( s_W^2 - \frac{1}{4} \right) \right] \bar{\mu} \gamma_\beta \mu \right. \\ & - \left[ e c_W \chi - (\xi - s_W \chi) \frac{e}{s_W c_W} \left( s_W^2 - \frac{1}{4} \right) \right] \bar{e}^+ \gamma_\beta e^+ \\ & \left. + \left[ e c_W \chi - (\xi - s_W \chi) \frac{e}{s_W c_W} \left( s_W^2 - \frac{1}{4} \right) \right] \bar{e} \gamma_\beta e \right\} Z_2^\beta, \end{aligned} \quad (31)$$

In muonium the coupling between positive muons and electrons is modified because there is not only photon exchange, but also photon- $Z'$  mixing. In direct analogy to the derivation of the neutrino potential given in the Appendix, one finds an effective potential

$$V_{\mu^+e^-}(r) = -\frac{e^2}{4\pi} \left( 1 - \frac{g'}{e} \tilde{Q}_P e^{-rM_2} \right) \frac{1}{r}. \quad (32)$$

where  $\tilde{Q}_P \equiv -(\xi - s_W \chi)(1/4 - s_W^2)/(s_W c_W) - c_W \chi$ . Hence, the result is an effective change of the fine-structure constant in systems involving muons (or tauons).<sup>5</sup> On atomic scales the factor  $e^{-rM_2}$  can be omitted. By comparing the above potential with the potential for positronium we find the ratio of the  $\mu^+$  and positron charge

$$\frac{Q(\mu^+)}{Q(e^+)} = \frac{\frac{e^2}{4\pi} \left( 1 - \frac{g'}{e} \tilde{Q}_P \right)}{\frac{e^2}{4\pi}} \simeq 1 - \frac{g'}{e} \tilde{Q}_P. \quad (33)$$

This ratio has been measured via the muonium hyperfine-structure [27] to be 1 with an accuracy of  $10^{-7}$ , corresponding to a limit

$$g' (3 s_W \chi + (1 - 4 s_W^2) \xi) < 5 \times 10^{-8}. \quad (34)$$

Note that, as it should, there is no effect in case of  $\chi = \xi = 0$ , i.e., when there is no photon- $Z'$  mixing. In case di-muonium (a bound state of  $\mu^-$  and  $\mu^+$  [28]) would be produced, one could test the  $Z'$  even in the limit of no mixing.

Another effect the new light  $Z'$  would have is a contribution to the effective number of degrees of freedom, potentially threatening for instance the success of Big Bang Nucleosynthesis (BBN). Recent BBN measurements as well as other cosmological probes are compatible with about one extra degree of freedom [23]. Let us demand that the  $Z'$  does not contribute. This means for the case of BBN that it should enter equilibrium after weak interactions freeze out ( $T \simeq \text{MeV}$ ), and requires to consider the process  $Z' Z' \rightarrow \nu_{\mu,\tau} \nu_{\mu,\tau}$ , whose rate goes as  $(g'^2/(4\pi))^2 T$ . Comparing this to the Hubble rate  $H \simeq T^2/M_{\text{Pl}}$  gives the requirement  $g'^2/(4\pi) \lesssim 10^{-11}$  [24]. A constraint of similar size has been estimated from Supernova 1987a [25]. An upper limit of  $g'^2/(4\pi) \lesssim 10^{-18}$  can be obtained with the process  $\gamma \mu \rightarrow Z' \mu$ , going with  $g'^2/(4\pi) \propto T$ , and demanding that  $Z'$  is not in equilibrium at  $T = m_\mu$  [26].

As expected, the largest constraints stem from the equivalence principle and BBN. However, the small values of the  $L_\mu - L_\tau$  parameters required in order to give observable effects in oscillation experiments are compatible with these limits.

---

<sup>5</sup>There is an effect quadratic in  $\tilde{Q}_P$  due to two mixings in systems like positronium or hydrogen, which is however way to small to be observable.

## 4 MINOS and Beyond

The potential  $V$  in Eq. (18) generated by  $L_\mu - L_\tau$  is flavor dependent, acts on the  $\mu$ - $\tau$  part of the system, and has a different sign for neutrinos and anti-neutrinos. Consequently it is a good candidate for an explanation of the MINOS results, which seemingly give different mixing parameters in the muon neutrino and anti-neutrino oscillations. In a 2-flavor approach, the Schrödinger-like equation for neutrinos is (note that we start in the mass basis)

$$i \frac{d}{dt} \vec{\nu}_M = \frac{1}{2E} \begin{pmatrix} m_2^2 & 0 \\ 0 & m_3^2 \end{pmatrix} \vec{\nu}_M + V U^\dagger \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} U \vec{\nu}_M, \quad (35)$$

where  $\Delta m^2 \equiv m_3^2 - m_2^2$  is the atmospheric mass-squared difference and  $\vec{\nu}_M = (\nu_2, \nu_3)^T$  are the mass eigenstates which are connected to the flavor states  $\vec{\nu}_{\text{flavor}} = (\nu_\mu, \nu_\tau)^T = U \vec{\nu}_M$  via the matrix

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (36)$$

Here  $\theta = \theta_{23}$  is the atmospheric mixing angle. The Schrödinger-like equation (35) thus contains the Hamiltonian:

$$H_V = \frac{1}{2E} \begin{pmatrix} m_2^2 + 2EV \cos 2\theta & 2EV \sin 2\theta \\ 2EV \sin 2\theta & m_3^2 - 2EV \cos 2\theta \end{pmatrix} = \frac{1}{2E} U_V \begin{pmatrix} m_{2,V}^2 & 0 \\ 0 & m_{3,V}^2 \end{pmatrix} U_V^\dagger. \quad (37)$$

As we have indicated,  $H_V$  is diagonalized by the rotation matrix

$$U_V = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}, \quad \text{with } \tan 2\phi = \frac{2\eta \sin 2\theta}{1 - 2\eta \cos 2\theta}. \quad (38)$$

We have introduced  $\eta \equiv \frac{2EV}{\Delta m^2}$ . The new mass eigenvalues  $m_{2,V}^2$  and  $m_{3,V}^2$  are associated to the new mass eigenstates  $\vec{\nu}_{M,V} = (\nu_{2,V}, \nu_{3,V})^T$  via

$$\vec{\nu}_{M,V} = U_V^\dagger \vec{\nu}_M = U_V^\dagger U^\dagger \vec{\nu}_{\text{flavor}}. \quad (39)$$

Thus, in the presence of the potential  $V$ , the mixing angle between flavor and mass eigenstates becomes  $\theta + \phi$  and  $\Delta m^2$  changes to  $\Delta m_V^2 \equiv m_{3,V}^2 - m_{2,V}^2$ . The exact results for the parameters are

$$\sin^2 2\theta_V = \frac{\sin^2 2\theta}{1 - 4\eta \cos 2\theta + 4\eta^2}, \quad (40)$$

$$\Delta m_V^2 = \Delta m^2 \sqrt{1 - 4\eta \cos 2\theta + 4\eta^2} = \Delta m^2 \sqrt{\frac{\sin^2 2\theta}{\sin^2 2\theta_V}}. \quad (41)$$

For  $V = 0$  the vacuum results  $\sin^2 2\theta$  and  $\Delta m^2$  are obtained. For anti-neutrinos, the potential  $V$  and hence  $\eta$  changes sign, thereby an apparent difference between the oscillation

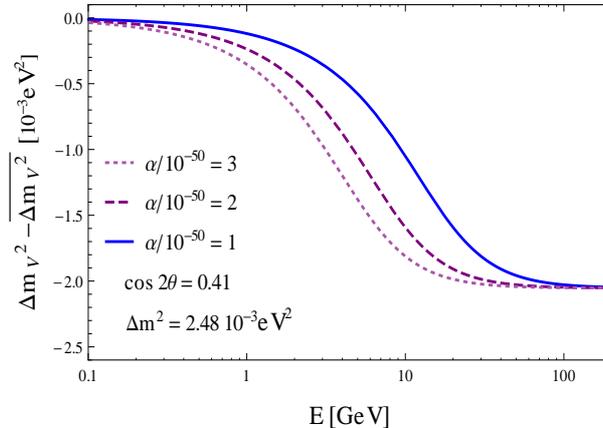


Figure 2: Difference between the mass-squared differences of neutrinos and anti-neutrinos (choosing initial values of  $\Delta m^2 = 2.48 \times 10^{-3} \text{ eV}^2$  and  $\cos 2\theta = 0.41$ ) for different values of  $\alpha$  as a function of energy.

parameters of neutrinos ( $\Delta m_V^2, \theta$ ) and anti-neutrinos ( $\overline{\Delta m_V^2}, \bar{\theta}$ ) could arise. Fig. 2 shows the difference between the mass-squared differences of neutrinos and anti-neutrinos (choosing an initial value of  $\Delta m^2 = 2.48 \times 10^{-3} \text{ eV}^2$ ) for different values of  $\alpha$  as a function of energy. We note here some important properties following from Eqs. (40, 41):

- first, the effect goes with  $\eta \cos 2\theta$ , and therefore it is absent if  $\theta$  is maximal. In this case the oscillation parameters  $\theta$  and  $\Delta m^2$  would be the same for neutrinos and anti-neutrinos, but with a common offset compared to their values for  $V = 0$ . If the long-range force mediated by  $L_\mu - L_\tau$  is responsible for the MINOS anomaly, then the necessary  $\theta \neq \pi/4$  is a possibility to disentangle it from other proposed explanations;
- the second point is that the corrections to the mixing angle and the mass-squared difference are correlated. For positive  $\Delta m^2$  and  $\alpha$  the correction for  $\sin^2 2\theta$  goes in the opposite direction as the correction of the  $\Delta m^2$ . Recalling that MINOS finds  $\overline{\Delta m^2} > \Delta m^2$  we therefore predict for positive  $\Delta m^2$  and  $\alpha$  that  $\sin^2 2\theta > \sin^2 2\bar{\theta}$ , which is compatible with the MINOS results (see Eq. (1)), and can be checked with higher statistics data sets. For negative  $\Delta m^2$  and positive  $\alpha$  the correction goes in the same direction, and hence  $\sin^2 2\theta < \sin^2 2\bar{\theta}$ ;
- the third point is that the relative effect is expected to be slightly larger for  $\sin^2 2\theta$  than for  $\Delta m^2$ ;
- though the effect looks like a diagonal NSI, we note that the potential does not depend on the matter density and therefore even for vacuum oscillations there is an effect.

We stress that the first three points given above are directly related to the form of the potential (35) in flavor space and hence are in general different for usual NSI; however,

in the 2-flavor framework used here these points hold for all flavor-diagonal potentials. Considering on the other hand an off-diagonal NSI parameter  $\epsilon_{\mu\tau}$  maximizes the difference  $|\Delta m^2 - \overline{\Delta m^2}|$  for  $\theta = \pi/4$ , while the relative effect for  $\sin^2 2\theta$  is suppressed by  $2 \cos 2\theta \cot 2\theta$  compared to  $\Delta m^2$ .

We can estimate the magnitude of the parameter  $\eta$  as

$$\eta \simeq 0.025 \left( \frac{\alpha}{10^{-50}} \right) \left( \frac{E}{\text{GeV}} \right), \quad (42)$$

which allows for not too high energies (note that at MINOS the oscillation dip occurs at around  $E \sim 1$  GeV) and for  $\alpha$  around  $10^{-50}$  (see the discussion after Eq. (18)),  $\eta$  is small and can be used as an expansion parameter. As can be seen from (40) and (41) the relative difference of the mass-squared differences is in this case obtained as

$$\frac{\Delta m_V^2 - \overline{\Delta m_V^2}}{\Delta m^2} \simeq -4 \eta \cos 2\theta, \quad (43)$$

while for the mixing angle the result is:

$$\frac{\sin^2 2\theta_V - \sin^2 2\overline{\theta}_V}{\sin^2 2\theta} \simeq 8 \eta \cos 2\theta. \quad (44)$$

These expressions nicely confirm the three points mentioned above. The muon neutrino and anti-neutrino survival probabilities are

$$P \equiv P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_V \sin^2 \frac{\Delta m_V^2}{4E} L, \quad (45)$$

$$\overline{P} = P(\overline{\nu}_\mu \rightarrow \overline{\nu}_\mu) = P(\nu_\mu \rightarrow \nu_\mu)(\alpha \leftrightarrow -\alpha), \quad (46)$$

which are subject to the following degeneracies

$$P(\theta, \Delta m^2, \alpha) = P(\theta, -\Delta m^2, -\alpha) = P(\theta + \pi/2, \Delta m^2, -\alpha) = P(\theta + \pi/2, -\Delta m^2, \alpha). \quad (47)$$

While the part discussed so far was rather general, we continue by applying the formalism to the recently found MINOS results [11]. We have performed with the expressions (45, 46) a  $\chi^2$ -fit to the MINOS data (given in bins of energy  $E_i$ ) on the ratio of observed events divided by the expectation for no oscillations. This data was taken, as in Ref. [13], from the slides of the talk referred to in our Ref. [11]. In case of asymmetric errors, the largest one was used and inserted in the  $\chi^2$ -function

$$\chi^2(\theta, \Delta m^2, \alpha) = \sum_i \left( \frac{P(\theta, \Delta m^2, \alpha, E_i) - R_i}{\sigma_i} \right)^2 + \sum_i \left( \frac{\overline{P}(\theta, \Delta m^2, \alpha, E_i) - \overline{R}_i}{\overline{\sigma}_i} \right)^2, \quad (48)$$

where  $P$  ( $\overline{P}$ ) is the survival probability  $P(\nu_\mu \rightarrow \nu_\mu)$  from Eq. (45) (from Eq. (46)),  $R_i$  ( $\overline{R}_i$ ) the ratio of observed events relative to the no-oscillation expectation, and  $\sigma_i$  ( $\overline{\sigma}_i$ ) the

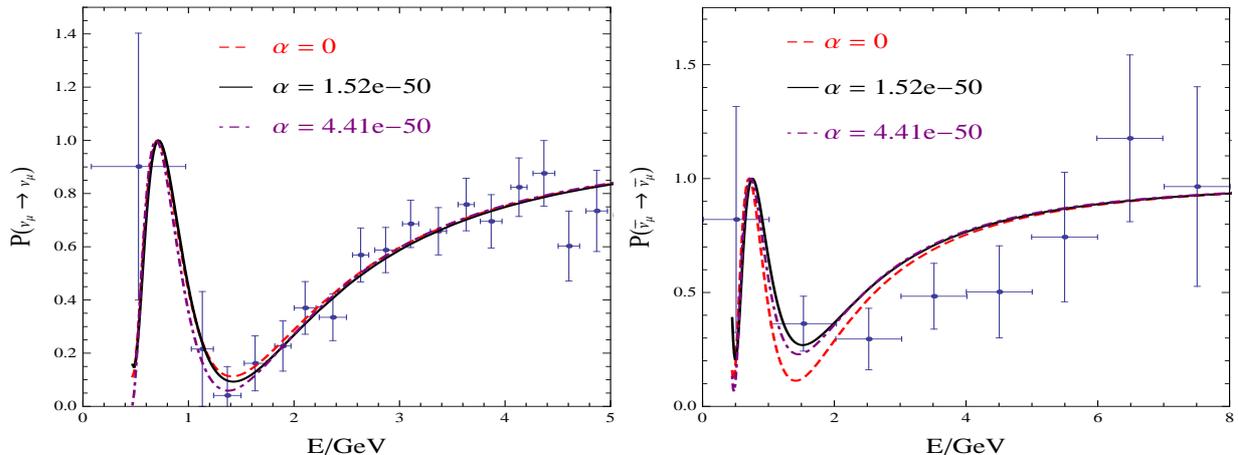


Figure 3: The oscillation probabilities for the best-fit values from Eq. (49) for neutrinos and anti-neutrinos superimposed on the MINOS data. Also plotted are the cases  $\alpha = 0$  and the value for the second, local  $\chi^2$ -minimum.

error for the neutrino (anti-neutrino) data set. The result of our fit after marginalizing over  $\Delta m^2$  and  $\theta$  is<sup>6</sup>

$$\sin^2 2\theta = 0.83 \pm 0.08, \quad |\Delta m^2| = (2.48 \pm 0.19) \times 10^{-3} \text{ eV}^2, \quad |\alpha| = (1.52_{-1.14}^{+1.17}) \times 10^{-50}, \quad (49)$$

with  $\chi^2_{\text{min}}/N_{\text{dof}} = 47.77/50 \simeq 0.96$ . The absolute values stem from the degeneracies listed in Eq. (47), in the following we will w.l.o.g. use positive values. In Fig. 3 we show the experimental data together with the results of our fit. One can see that the non-zero value of  $\alpha$  puts in particular the data points at the oscillation minimum in better agreement with the curves. From the plot of the  $\chi^2$ -function in Fig. 4 one sees that there is a second (local) minimum, corresponding to  $\sin^2 2\theta = 0.98$ ,  $\Delta m^2 = 2.36 \times 10^{-3} \text{ eV}^2$  and  $\alpha = 4.41 \times 10^{-50}$ , with  $\chi^2_{\text{min}}/N_{\text{dof}} = 48.73/50 \simeq 0.97$ . The curves for this point are also plotted in Fig. 3. The goodness of fit is not particularly worse for the absence of new physics, which has been noted also in Ref. [13].

The uncertainty of  $\alpha$  will decrease once higher statistics is available. To get a feeling for the improvement stemming from future data, we performed the fits with naively doubling the statistics. Doubled statistics in the  $\bar{\nu}$ -channel can reduce the relative error of  $\alpha$  from 77% to 61%. Including also an additional  $\nu$ -run, the error goes down to roughly 54%. We continue by discussing the consequences of the implied value of  $\alpha$  in future neutrino oscillation experiments. We have modified the commonly used GLoBES software [29] to include the potential  $V$  from Eq. (16). Using the pre-defined packages (“AEDL files”) for

<sup>6</sup>We have checked our analysis by setting  $\alpha = 0$  and have obtained the best-fit values  $\Delta m^2 = 2.28 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta = 0.94$  for the neutrino data set, and  $\Delta m^2 = 3.38 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta = 0.81$  for the anti-neutrinos, in good agreement with the MINOS results. A fit to the total data set yields  $\Delta m^2 = (2.38_{-0.17}^{+0.20}) \times 10^{-3} \text{ eV}^2$  and  $\sin^2 2\theta = 0.89_{-0.07}^{+0.08}$ , with  $\chi^2_{\text{min}}/N_{\text{dof}} = 49.43/51 \simeq 0.97$ .

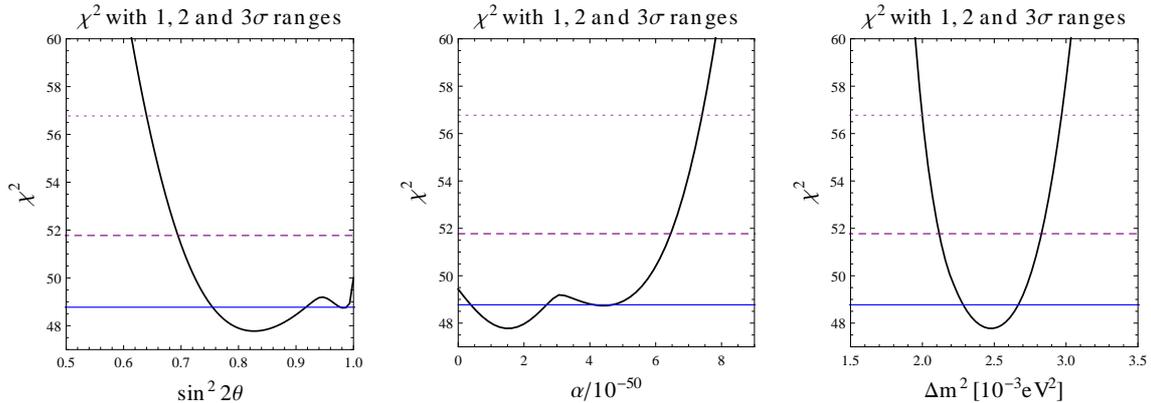


Figure 4: The  $\chi^2$ -function from Eq. (48) as a function of the fit parameters.

the most frequently discussed future experiments, we analyzed future neutrino experiments, as listed in Table 1, to obtain future constraints on  $\alpha$ . The oscillation parameters we use are listed in Table 2. The results are given in Table 3. The  $\chi^2$ -functions generated by GLOBES are shown in Fig. 5 (left) for some examples.

Setting the true parameter values of  $\alpha$ ,  $\theta$  and  $\Delta m^2$  (and their errors) to our best-fit values from Eq. (49), we can see how the “precision” on  $\alpha$  can be improved. From the plots of  $\chi^2$  in Fig. 5 (right) one sees that NO $\nu$ A would give  $\alpha = (1.52 \pm 0.27) \times 10^{-50}$ , T2K would yield  $\alpha = (1.52 \pm 0.46) \times 10^{-50}$  and NuFact would determine very precisely  $\alpha = (1.52^{+0.11}_{-0.21}) \times 10^{-50}$ .

As mentioned above, long-range forces generated by  $L_e - L_{\mu,\tau}$  have been discussed before. Ref. [3] bounds  $\alpha_{e\mu,\tau}$  by analyzing  $\nu_\mu$  and  $\nu_\tau$  oscillations and using atmospheric neutrino data. It is easy to see that in a two-flavor framework, the potential  $V_{e\tau} = \alpha_{e\tau} N_e/R_{\text{A.U.}}$  corresponds to  $2V_{\mu\tau}$ . Likewise,  $V_{e\mu}$  corresponds to  $-2V_{\mu\tau}$ . Therefore, the limit of  $\alpha_{e\tau} < 6.4 \times 10^{-52}$  obtained in Ref. [3] corresponds to  $\alpha = g'(\xi - s_W \chi) < 8.9 \times 10^{-50}$ , not in

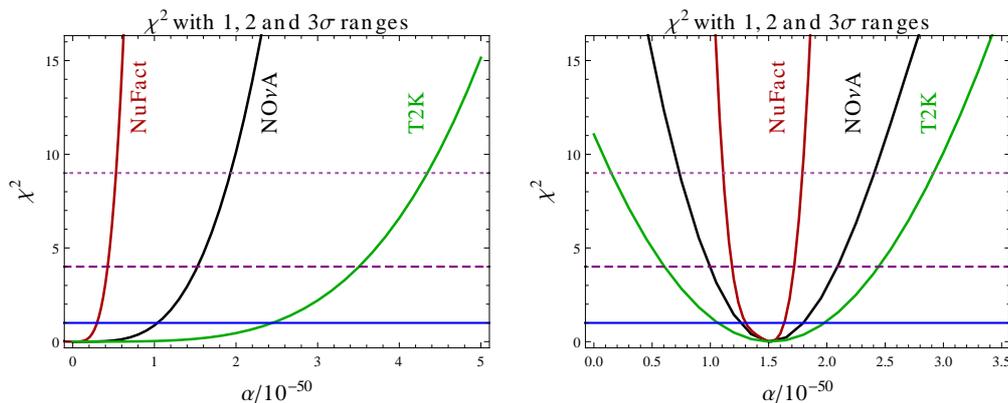


Figure 5: The 1, 2 and  $3\sigma$  limits which can be obtained by T2K, NO $\nu$ A and a neutrino factory if  $\alpha = 0$  (left) and if  $\alpha = (1.52^{+0.11}_{-0.21}) \times 10^{-50}$  (right).

conflict with our fit-result from Eq. (49). In turn, this means that not only  $L_\mu - L_\tau$  could be the origin of the MINOS anomaly, but also  $L_e - L_\mu$  or  $L_e - L_\tau$ , for which  $\alpha = 1.52 \times 10^{-50}$  translates into  $\alpha_{e\mu,\tau} = 1.1 \times 10^{-52}$ . If we take the  $3\sigma$ -bound  $\alpha_{e\tau} < 2.5 \times 10^{-53}$  from solar neutrino and KamLAND data [4] and treat it like in the 2-flavor case we obtain  $\alpha < 3.5 \times 10^{-51}$ . However, the interplay of the other limits on long-range forces, and also the impact of stronger bounds on  $\alpha_{e\mu,\tau}$  using solar and KamLAND data [4], can not be used without doing a full 3-flavor fit to all data. In general, we note that the different flavor structures of the potentials arising from  $L_e - L_\mu$ ,  $L_e - L_\tau$  and  $L_\mu - L_\tau$ ,

$$\begin{pmatrix} V & 0 & 0 \\ 0 & -V & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} V & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -V \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & V & 0 \\ 0 & 0 & -V \end{pmatrix}, \quad (50)$$

render it difficult to translate existing bounds on  $L_e - L_\mu$  or  $L_e - L_\tau$  into constraints on  $L_\mu - L_\tau$ , in particular if in addition a matter potential is present in  $V_{ee}$ . We would like to stress though that the solar neutrino oscillations should really be fitted specifically for this model, since the electron and neutron densities in the Sun are not exactly proportional. However, it would admittedly be surprising if a full 3-flavor solar neutrino analysis of our scenario would give a limit below the  $3\sigma$ -bound  $\alpha < 3.5 \times 10^{-51}$ , which was obtained from translating the limit from solar neutrino and KamLAND data, as discussed above. We emphasize that in a model of shorter range, e.g.  $M_2 \sim 1/R_\oplus$ , the limits from solar neutrinos are even harder to translate and might be completely invalid. However, even though this solar neutrino limit and the NSI limits given below look not promising for our explanation of the MINOS anomaly (which is true for every solution of the results put forward so far), our scenario has advantages over the other proposed solutions for the MINOS results, as we will discuss in what follows.

It should be clear that our scenario is different from the standard lore of NSIs. However, adding the potential to the Hamiltonian then looks like a typical NSI Hamiltonian, for which limits have of course been derived already [31] (also pointed out in Ref. [17]). In particular, in a 2-neutrino framework the relation  $2V = V_m \epsilon_{\mu\mu}$  holds, and our range of  $\alpha$  would correspond to  $\epsilon_{\mu\mu} \gtrsim 0.25$ , to be compared with the 90 % C.L. limit [31]  $|\epsilon_{\mu\mu}| \leq 0.068$ . Saturating this limit would correspond to  $\alpha = 1.04 \times 10^{-51}$ . A fit to the data fixing it to this value yields a bad fit of  $\sin^2 2\theta = 0.88_{-0.07}^{+0.08}$  and  $\Delta m^2 = (2.39_{-0.17}^{+0.20}) \times 10^{-3} \text{ eV}^2$ , with  $\chi_{\min}^2/N_{\text{dof}} = 49.25/51 \simeq 0.97$ .

At this point it is important to note that the NSI limits we are concerned with stem predominantly from atmospheric neutrinos, which travel through all the different layers of the Earth. Hence, and this is especially important for forces of shorter range also covered by our model, the position dependence of the potentials has to be taken into account (while it can be ignored for LBL experiments like MINOS). Since the NSI potential follows the highly discontinuous electron density while  $V_{\mu,\tau}(r)$  smoothly satisfies a Poisson equation  $\Delta V_{\mu,\tau}(r) \sim n_n(r)$ , the neutrino oscillation behaviour will be different and make a full new

fit to atmospheric neutrino data necessary. Of course this will most likely produce limits of a similar order as those for NSI.

Note that recent works explaining the MINOS result with NSIs require values of  $|\epsilon_{\mu\mu}|$  or  $|\epsilon_{\mu\tau}|$  way above 0.1 [13, 14]. We conclude that all solutions to the MINOS anomaly using CPT conservation are in conflict with existing limits, though one cannot completely rule them out, given the (admittedly somewhat unlikely) possibilities of cancellations, 3-flavor effects, additional symmetries etc.

However, there is one important difference between the scenario presented here and standard NSIs: in a gauge invariant framework the  $\epsilon$  parameters of the neutrino NSIs are responsible also for charged lepton decays, which are subject to stringent constraints and improve the bounds by typically one or two orders of magnitude. This can be evaded only by an additional symmetry protecting the charged leptons, or highly fine-tuned cancellations of different higher order terms [32]. To illustrate the problems of NSIs, consider the Lagrangian<sup>7</sup>

$$\mathcal{L}_{\text{CC}}^{\text{NSI}} \supset -2\sqrt{2} G_F \epsilon_{\tau\mu}^d V_{ud} [\bar{u} \gamma^\mu d] [\bar{\mu} \gamma_\mu P_L \nu_\tau],$$

which leads to interference between  $\nu_\mu$  CC events and events in which  $\nu_\mu$  oscillate into  $\nu_\tau$ , subsequently creating muons via  $\epsilon_{\tau\mu}^d$ . For anti-neutrinos,  $\epsilon_{\tau\mu}^d \rightarrow (\epsilon_{\tau\mu}^d)^*$ , and hence different neutrino and anti-neutrino parameters arise. Values of  $|\epsilon_{\tau\mu}^d|$  around 0.1 are enough to explain the MINOS results [14]. However, the Lagrangian written in a gauge invariant way induces the tree-level decay  $\tau \rightarrow \mu \pi^0$ , from which a limit of  $|\epsilon_{\tau\mu}^d| \lesssim 10^{-4}$  is derived [33]. We note here that the scenario of gauged  $L_\mu - L_\tau$  discussed here does not suffer from such problems (the reason being diagonal and small couplings to leptons), and does not require strong and fine-tuned cancellations or extra symmetries protecting charged leptons.

It is worth discussing the anomalous magnetic moment of the muon, where since many years a conflict between theory (i.e., its Standard Model calculation) and experiment exists [22]. The current experimental value of  $a_\mu$  differs by  $3.2\sigma$  from the Standard Model prediction, although there is some uncertainty in the hadronic contributions. Nevertheless, since the  $Z'$  couples to the muon, it contributes to  $\Delta a_\mu$  [6]. In the limit of  $M'_Z \ll m_\mu$ , the contribution is

$$\Delta a_\mu = \frac{g'^2}{8\pi^2}, \quad (51)$$

which in our light case translates into a constraint on the coupling  $g'$ . From the constraint  $\Delta a_\mu \lesssim 255 \times 10^{-11}$  it follows that  $g' \lesssim 4.49 \times 10^{-4}$ . This would imply  $|\xi - s_W \chi| \gtrsim 3.3 \times 10^{-47}$  in order to explain the MINOS anomaly.

Turning to neutrino masses, the conservation of  $L_\mu - L_\tau$  dictates the effective neutrino Majorana mass matrix to be [9, 10]

$$m_\nu = \begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}, \quad (52)$$

---

<sup>7</sup>This is a charged current (CC) NSI, because neutral current NSIs required to explain the MINOS data are at least of order 0.1 and hence in conflict with bounds obtained from neutrino data alone [13–15].

regardless of its origin, such as some form of see-saw. It would result in neutrino masses  $a$  and  $\pm b$ , hence one expects (close to) quasi-degenerate masses. Though the mass matrix is  $\mu$ - $\tau$  symmetric, and hence implies  $\theta_{13} = 0$  and  $\theta_{23} = \pi/4$ , it is too simple and can not reproduce all data. Breaking  $L_\mu - L_\tau$  is achieved by introducing extra Higgs particles  $\Phi'$ , which obtain a vev. Necessarily, the implied scale of the  $Z'$  mass (which is generated by breaking of  $L_\mu - L_\tau$ ) and the additional entries in  $m_\nu$  are correlated via  $m'_{Z'} \sim g' \langle \Phi' \rangle$  and  $(m_\nu)_{\alpha\beta} \lesssim \langle \Phi' \rangle$  if it is a weak triplet,  $(m_\nu)_{\alpha\beta} \lesssim v_{\text{wk}} \langle \Phi' \rangle / \Lambda$  if it is a doublet and couples to the SM Higgs, or  $(m_\nu)_{\alpha\beta} \lesssim \langle \Phi' \rangle^2 / \Lambda$  if it does not. Here  $\Lambda$  denotes the high energy scale which acts as the necessary suppression of the neutrino mass. Simultaneous ultra-light  $Z'$  of order  $10^{-19}$  eV and sizable  $(m_\nu)_{\alpha\beta} \simeq 0.1$  eV implies for, say,  $\Lambda = 10^{15}$  GeV that for doublets  $\langle \Phi' \rangle$  is of order  $10^3$  GeV and hence  $g' \sim 10^{-30}$ , while  $g' \sim 10^{-17}$  for triplets. Hence, the  $Z'$  will essentially not contribute to the anomalous magnetic moment of the muon.

The effective mass parameter for neutrinoless double  $\beta$ -decay will to zeroth order be given by  $m_{\beta\beta} \simeq |a|$ , modified by  $L_\mu - L_\tau$ -breaking terms. Due to the quasi-degeneracy of the neutrino masses, we expect  $m_{\beta\beta}$  to be large enough to be observable in future  $0\nu\beta\beta$ -experiments. Concrete limits and implications due to the form of  $(m_\nu)_{\alpha\beta}$  are, of course, highly dependent on the specified Higgs-sector/see-saw model and lie outside the realm of this work.

## 5 Conclusions

Long-range forces mediated by the  $Z'$  boson associated with gauged  $L_\mu - L_\tau$  can lead to interesting and largely unexplored phenomenology. Effects from gauged  $L_e - L_\tau$  and  $L_e - L_\mu$  have been studied before, but suffer from unsuccessful neutrino mass matrices. In contrast, the mass matrix for conservation of  $L_\mu - L_\tau$  is very promising. We succeeded here to apply gauged  $L_\mu - L_\tau$  to neutrino oscillation phenomenology by noting the unavoidable  $Z$ - $Z'$  mixing. Neutrons in the Sun generate via this mixing a flavor-dependent potential for terrestrial muon and tau neutrinos. This potential changes sign for anti-neutrinos, and hence can lead to apparent differences in neutrino and anti-neutrino oscillations. It is not necessary to introduce CP or CPT violation. Applying this new finding to the recently found MINOS anomaly implies a value of around  $\alpha \simeq 10^{-50}$ , where  $\alpha = g' (\xi - s_W \chi)$  is the product of the new gauge coupling and the parameters quantifying the  $Z$ - $Z'$  mixing. An interesting correlation between the atmospheric neutrino parameters  $\Delta m^2$  and  $\theta$  is found. The latter is required to be non-maximal, which is one of the handles to probe this explanation of the anomaly. We have checked that the necessary values are not in conflict with Big Bang Nucleosynthesis, charge differences of electron and muon, electroweak precision data, or tests of the equivalence principle. However, all solutions to the MINOS result which conserve CPT are in some trouble with existing data. Because the scenario is different from the standard lore of non-standard interactions (e.g., we avoid the problem of charged lepton decays due to  $SU(2)_L$  gauge invariance, the effect is present

even for vacuum oscillations), we nevertheless feel that our analysis is worthwhile. By making use of the GLoBES software we have discussed future constraints on  $\alpha$ .

Time will show whether the discrepancy in the MINOS results survives. Nevertheless, many new physics effects imply different neutrino and anti-neutrino behavior, which underlines the importance of analyzing them separately. The new effect arising from  $L_\mu - L_\tau$  (via  $Z-Z'$  mixing) noted in the present paper is one more example for this, and we have given estimates for future constraints.

It would be interesting to discuss a similar approach for other “anomalous” oscillation results in which apparent differences of neutrinos and anti-neutrinos are found, such as the recent MiniBooNE excess in a  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  search [34], or the slightly larger  $\theta_{12}$  found in solar neutrino analyses with respect to the  $\theta_{12}$  in reactor anti-neutrino experiments.

### Acknowledgments

We thank Borut Bajc and Osamu Yasuda for helpful and interesting discussions. This work was supported by the ERC under the Starting Grant MANITOP and by the DFG in the project RO 2516/4-1 as well as in the Transregio 27.

Experiment	Baseline	Running-time [years]	Beam-energy [GeV]	mass
T2K	295 km	5 $\nu$ + 5 $\bar{\nu}$	0.2 – 2	22.5 kt
T2HK	295 km	4 $\nu$ + 4 $\bar{\nu}$	0.4 – 1.2	500 kt
SPL	130 km	2 $\nu$ + 8 $\bar{\nu}$	0.01 – 1.01	500 kt
NO $\nu$ A	812 km	3 $\nu$ + 3 $\bar{\nu}$	0.5 – 3.5	15 kt
Nufact	3000 km	4 $\nu$ + 4 $\bar{\nu}$	4 – 50	50 kt

Table 1: Parameters of long-baseline oscillation experiments simulated by the GLoBES software [29].

$\theta_{12}$	$\arcsin \sqrt{0.318} \pm 0.02$ (3%)
$\theta_{13}$	$0 \pm 0.2$
$\theta_{23}$	$\arcsin \sqrt{0.500} \pm 0.07$ (9%)
$\delta_{\text{CP}}$	$\in [0, 2\pi]$
$\Delta m_{21}^2$ [ $10^{-5}$ eV $^2$ ]	$7.59 \pm 0.23$ (3%)
$\Delta m_{31}^2$ [ $10^{-3}$ eV $^2$ ]	$2.40 \pm 0.12$ (5%)

Table 2: Oscillation parameters [30] used as input to the GLoBES simulation.

## A Derivation of the Potential

For the sake of completeness, let us give here a derivation of the static potential which the particles in the Sun generate for terrestrial neutrinos. The potential (15) for gauged

Experiment	Sensitivity to $\alpha/10^{-50}$
T2K ( $\nu$ -run)	11.8
T2K	4.3
T2HK	1.7
SPL	7.5
NO $\nu$ A	1.9
Combined Superbeams	1.4
Nufact	0.53

Table 3: Future constraints on  $\alpha$  at 99.73% C.L., obtained with GLoBES.

$L_e - L_\mu$  or  $L_e - L_\tau$  can also be derived in this fashion. From Eq. (14) we consider the time-like components, note that  $j_{\text{EM}}^0 = 0$  and have that

$$j_3^0 = -\frac{1}{2} \bar{e}_L \gamma^0 e_L + \frac{1}{2} \bar{p}_L \gamma^0 p_L - \frac{1}{2} \bar{n}_L \gamma^0 n_L = -\frac{1}{4} (n_e - n_p + n_n) = -\frac{n_n}{4}, \quad (\text{A1})$$

since the axial-part will result in a spin-operator in the non-relativistic limit and we assume the Sun is not polarized. The equation of motion for  $Z_2^0$ , following from the Euler-Lagrange equation

$$\partial_\nu \frac{\delta}{\delta(\partial_\nu Z_{2\mu})} \left( -\frac{1}{4} Z_{2\alpha\beta} Z_2^{\alpha\beta} \right) - \frac{\delta}{\delta Z_{2\mu}} \left( \frac{1}{2} M_2^2 Z_{2\alpha} Z_2^\alpha + \mathcal{L}_{Z_2} \right) = 0, \quad (\text{A2})$$

is therefore

$$(\partial^2 + M_2^2) Z_2^0 = (\xi - s_W \chi) \frac{e}{s_W c_W} \frac{n_n}{4}. \quad (\text{A3})$$

In the static case outside of the Sun this is ( $n_n(\vec{x}) = N_n \delta^{(3)}(\vec{x})$ ):

$$(\Delta - M_2^2) Z_2^0 = -(\xi - s_W \chi) \frac{e}{s_W c_W} \frac{1}{4} N_n \delta^{(3)}(\vec{x}) \quad (\text{A4})$$

with the well-known solution

$$V(r) = Z_2^0 = (\xi - s_W \chi) \frac{e}{s_W c_W} \frac{1}{4} N_n \times \frac{e^{-rM_2}}{4\pi r}. \quad (\text{A5})$$

In the limit  $M_2 \rightarrow 0$  the potential for  $\nu_\mu$  on Earth is:<sup>8</sup>

$$V_\mu = g' (\xi - s_W \chi) \frac{e}{4 s_W c_W} \frac{N_n}{4\pi R_{\text{A.U.}}} + \mathcal{O}(\xi^2, \chi^2, \xi\chi), \quad (\text{A6})$$

while for  $\nu_\tau$  the sign changes ( $V_\tau = -V_\mu$ ). The potential for anti-neutrinos can be obtained from Eq. (A6) by the transition  $g' \rightarrow -g'$ .

---

<sup>8</sup>We assume that the mixing angles are somewhat smaller than  $g'$  so we can drop the  $\mathcal{O}(\xi^2, \chi^2, \xi\chi)$  terms against  $\mathcal{O}(g'\chi, g'\xi)$ . In the actual neutrino oscillation the terms without  $g'$  will be generation independent and therefore drop out.

## References

- [1] P. Langacker, Rev. Mod. Phys. **81**, 1199 (2008) [arXiv:0801.1345 [hep-ph]].
- [2] R. Foot, Mod. Phys. Lett. A **6**, 527 (1991); X. G. He, G. C. Joshi, H. Lew and R. R. Volkas, Phys. Rev. D **44**, 2118 (1991); R. Foot, X. G. He, H. Lew and R. R. Volkas, Phys. Rev. D **50**, 4571 (1994) [arXiv:hep-ph/9401250].
- [3] A. S. Joshipura and S. Mohanty, Phys. Lett. B **584**, 103 (2004) [arXiv:hep-ph/0310210].
- [4] A. Bandyopadhyay, A. Dighe and A. S. Joshipura, Phys. Rev. D **75**, 093005 (2007) [arXiv:hep-ph/0610263].
- [5] J. A. Grifols and E. Masso, Phys. Lett. B **579**, 123 (2004) [arXiv:hep-ph/0311141]; M. C. Gonzalez-Garcia *et al.*, JCAP **0701**, 005 (2007) [arXiv:hep-ph/0609094]; A. Samanta, arXiv:1001.5344 [hep-ph].
- [6] S. Baek, N. G. Deshpande, X. G. He and P. Ko, Phys. Rev. D **64**, 055006 (2001) [arXiv:hep-ph/0104141].
- [7] S. Baek and P. Ko, JCAP **0910**, 011 (2009) [arXiv:0811.1646 [hep-ph]].
- [8] N. F. Bell and R. R. Volkas, Phys. Rev. D **63**, 013006 (2001) [arXiv:hep-ph/0008177]; E. Ma, D. P. Roy and S. Roy, Phys. Lett. B **525**, 101 (2002) [arXiv:hep-ph/0110146].
- [9] S. Choubey and W. Rodejohann, Eur. Phys. J. C **40**, 259 (2005) [arXiv:hep-ph/0411190].
- [10] W. Rodejohann and M. A. Schmidt, Phys. Atom. Nucl. **69**, 1833 (2006) [arXiv:hep-ph/0507300]; B. Adhikary, Phys. Rev. D **74**, 033002 (2006) [arXiv:hep-ph/0604009]; T. Ota and W. Rodejohann, Phys. Lett. B **639**, 322 (2006) [arXiv:hep-ph/0605231]; E. J. Chun and K. Turzyski, Phys. Rev. D **76**, 053008 (2007) [arXiv:hep-ph/0703070].
- [11] Talk given by P. Vahle at 24th International Conference On Neutrino Physics And Astrophysics (Neutrino 2010), June 14, 2010, slides can be found at <http://indico.cern.ch/materialDisplay.py?contribId=201&sessionId=1&materialId=slides&confId=73981>
- [12] G. Barenboim and J. D. Lykken, Phys. Rev. D **80**, 113008 (2009); D. Choudhury, A. Datta, A. Kundu, arXiv:1007.2923 [hep-ph]; C. M. Ho, arXiv:1012.1053 [hep-ph]; D. Fargion, D. D'Armiento, P. Desiati and P. Paggi, arXiv:1012.3245 [hep-ph].
- [13] W. A. Mann, D. Cherdack, W. Musial, T. Kafka, Phys. Rev. **D82**, 113010 (2010).
- [14] J. Kopp, P. A. N. Machado, S. J. Parke, Phys. Rev. **D82**, 113002 (2010).
- [15] E. Akhmedov, T. Schwetz, JHEP **1010**, 115 (2010).

- [16] N. Engelhardt, A. E. Nelson and J. R. Walsh, Phys. Rev. D **81**, 113001 (2010).
- [17] O. Yasuda, arXiv:1012.3478 [hep-ph].
- [18] J. Heeck and W. Rodejohann, arXiv:1012.2298 [hep-ph].
- [19] K. S. Babu, C. F. Kolda and J. March-Russell, Phys. Rev. D **57**, 6788 (1998) [arXiv:hep-ph/9710441].
- [20] B. Holdom, Phys. Lett. B **166**, 196 (1986).
- [21] S. Schlamminger, K. Y. Choi, T. A. Wagner, J. H. Gundlach and E. G. Adelberger, Phys. Rev. Lett. **100**, 041101 (2008) [arXiv:0712.0607 [gr-qc]]; E. G. Adelberger, J. H. Gundlach, B. R. Heckel, S. Hoedl and S. Schlamminger, Prog. Part. Nucl. Phys. **62**, 102 (2009).
- [22] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **G37**, 075021 (2010).
- [23] For a recent review, see S. Hannestad, arXiv:1007.0658 [hep-ph].
- [24] J. A. Grifols and E. Masso, Phys. Lett. B **396**, 201 (1997) [arXiv:astro-ph/9610205]; L. B. Okun, Mod. Phys. Lett. A **11**, 3041 (1996) [arXiv:hep-ph/9611360].
- [25] S. N. Gninenko, Phys. Lett. B **413**, 365 (1997) [arXiv:hep-ph/9708465].
- [26] A. D. Dolgov, Phys. Rept. **320**, 1 (1999).
- [27] X. Fei, Phys. Rev. A **49**, 1467 (1994).
- [28] S. J. Brodsky and R. F. Lebed, Phys. Rev. Lett. **102**, 213401 (2009) [arXiv:0904.2225 [hep-ph]].
- [29] P. Huber, M. Lindner and W. Winter, Comput. Phys. Commun. **167**, 195 (2005) [arXiv:hep-ph/0407333]; P. Huber, J. Kopp, M. Lindner, M. Rolinec and W. Winter, Comput. Phys. Commun. **177**, 432 (2007) [arXiv:hep-ph/0701187].
- [30] T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **10**, 113011 (2008) [arXiv:0808.2016v3 [hep-ph]].
- [31] C. Biggio, M. Blennow and E. Fernandez-Martinez, JHEP **08**, 090 (2009) [arXiv:0907.0097v3 [hep-ph]].
- [32] M. B. Gavela, D. Hernandez, T. Ota, W. Winter, Phys. Rev. **D79**, 013007 (2009) [arXiv:0809.3451 [hep-ph]].
- [33] B. Gavela, talk at NOW 2010, September 2010; S. Antusch, M. Blennow, E. Fernandez-Martinez, T. Ota, JHEP **1006**, 068 (2010) [arXiv:1005.0756 [hep-ph]].
- [34] A. A. Aguilar-Arevalo *et al.* [The MiniBooNE Collaboration], Phys. Rev. Lett. **105**, 181801 (2010) [arXiv:1007.1150 [hep-ex]].