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Comment on "Quantum time-of-flight distribution for cold trapped atoms"

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Ali *et al* [1] have claimed that empirically testable differences exist between the quantum and semi-classical time of flight distribution for cold trapped atoms. We believe these claims to be misleading. For the particular case of non-interacting Bosonic particles released from a harmonic trapping potential, we show that the quantum and semi-classical calculations for the time-of-flight distribution are equivalent given a sufficiently long fall, provided one is not too close to the quantum degeneracy point.

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Empirical fits to the time-of-flight signal (TOF) that is observed when a cold atomic cloud is expanding following the switch-off of the trapping potential are commonly used to infer the thermal and quantum characteristics of the sample. Ali and co-workers [1] claim that the result of a "quantum calculation" for the TOF differs in an "empirically testable way" from the commonly used semi-classical description of the TOF signal. In particular they present results portraying striking differences for the TOF signal as the "size" of the initially trapped atomic cloud is varied. These results clearly differ from the semi-classical description which predicts that for a sufficiently long time of flight, the size of the expanding cloud is determined solely by the initial temperature of the sample. We believe however that the deviations from the semiclassical description predicted by Ali and co-workers are manifest only in extreme limits, when the simple thermal averaging they employed is not appropriate. The purpose of this comment is to point out several subtle yet important points that we believe were neglected by Ali and coworkers in their development.

The calculation of Ali *et al* is performed on non-interacting particles and is based on the quantum mechanical flux (which they call the *probability current*). Restricted to the one dimensional case, the proposed model starts by describing each atom in the cloud as a Gaussian wave packet centered at $z = 0$ and moving with a group velocity v , such that $\psi(z, t = 0) = (2\pi\sigma_0^2)^{-1/4} \exp\left(-\frac{z^2}{4\sigma_0^2}\right) \exp\left(i\frac{mv}{\hbar}z\right)$. The ballistic expansion is accomplished by computing the time evolution of the wave packets with the appropriate propagator for free fall under the influence of a constant gravitational field. Carrying this out, a v -dependent amplitude term appears in the evolved wave function $\psi(z, t)$. This term allows Ali and co-workers to compute a thermal average of the particle's flux $\hbar/m \text{Im}[\psi^*(z, t)\partial_z\psi(z, t)]$, with v distributed according to a Maxwell-Boltzmann factor. The

resulting TOF distribution is characterized by a width and a velocity of the center of the cloud. For a long propagation time, the width is given in their model by

$$\sigma_{(Ali \text{ et al.})} = \sqrt{\left(\frac{\hbar}{2m\sigma_0}\right)^2 + \frac{k_B T}{m}} t$$

This result is in contradiction with the semi-classical model [2]. Provided that the localization induced by trapping potential does not significantly alter the thermal velocity distribution, one can estimate the initial momentum distribution of a cloud at temperature T using the equi-partition theorem, $\langle \hat{p}^2 \rangle / 2m = k_B T / 2$, yielding $\sigma_p = \sqrt{\langle \hat{p}^2 \rangle} = \sqrt{mk_B T}$. Using Maxwell-Boltzmann statistics, the semi-classical approximation states thus that the size at large enough time t is

$$\sigma_{(s.c.)} = \sqrt{\frac{k_B T}{m}} t$$

The discrepancy, illustrated in fig.4 of Ref [1], happens when the term $\frac{\hbar}{2m\sigma_0}$ is comparable or larger than $\sqrt{\frac{k_B T}{m}}$. In their model Ali *et al* seem to be assuming that each velocity class of the atomic cloud starts out in a kind of minimum uncertainty wavepacket of size σ_0 . Very little is said about this size and it is treated by the authors as a free parameter which is independent of the temperature. But $\frac{\hbar}{2m\sigma_0}$ is nothing else than the momentum spread, due to the uncertainty principle. Hence, differences appear only when the characteristic energy associated with the strong uniform localization assumed by Ali *et al* is comparable to the thermal energy. This would seem to place them below the quantum degeneracy point for any realizable smoothly varying trapping potential with a reasonable number of atoms, a situation for which their use of a continuous Maxwell-Boltzmann velocity distribution to carry out the thermal averaging is certainly not adequate.

One of the most common trapping potential is the harmonic potential for which a fully quantum mechanical

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calculation can be carried out for the time of flight signal resulting from an initially thermalized cloud of noninteracting Bose atoms [3]. In this case the tof signal is the same as in the trap provided a rescaling of the coordinate is performed. Using the results of section B.4 of Ref [3], the exact root mean square value of the tof signal is given by :

$$\sigma_{(Q.)}^2 = (1 + \omega^2 t^2) \frac{\sigma^2}{2} \frac{\sum_{l=1}^{\infty} \frac{e^{\beta l \tilde{\mu}}}{(1 - e^{-\tau l})^3} \frac{1}{\tanh(\frac{\tau l}{2})}}{\sum_{l=1}^{\infty} \frac{e^{\beta l \tilde{\mu}}}{(1 - e^{-\tau l})^3}}$$

with $\sigma = \sqrt{\frac{\hbar}{m\omega}}$, $\tau = \hbar\omega/k_B T$, $\beta = 1/k_B T$ and $\tilde{\mu} = \mu - 3\hbar\omega/2$ where μ is the chemical potential and ω the oscillation frequency. For simplicity we have considered an isotropic harmonic potential. Using Bose-Einstein statistics, an improved semi-classical calculation valid close to degeneracy could be obtained under the approximation $\tau \ll 1$ and would give

$$\sigma_{(s.c. imp.)}^2 = (1 + \omega^2 t^2) \frac{\sigma^2}{\tau} \frac{g_4(e^{\beta \tilde{\mu}})}{g_3(e^{\beta \tilde{\mu}})}$$

$$\sigma_{(s.c. imp.)} = \sqrt{\frac{g_4(e^{\beta \tilde{\mu}})}{g_3(e^{\beta \tilde{\mu}})}} \sqrt{\frac{k_B T}{m}} t$$

where in the last line, only for simplicity and to clarify the final result, we have also approximate $\sqrt{1 + \omega^2 t^2} = \omega t$. In this expression we have also used the definition of the so-called Bose function $g_i(x) = \sum_{l=1}^{\infty} \frac{x^l}{l^i}$. The results are plotted in fig 1. For temperatures well above the quantum degeneracy $\sigma_{(Q.)}$ coincides with $\sigma_{(s.c.)}$ and $\sigma_{(s.c. imp.)}$ and is still very close to $\sigma_{(s.c. imp.)}$ near the degeneracy. Hence semi-classical approximations lead usually to accurate descriptions of tof signals provided one is not too close or below quantum degeneracy.

Finally we would like to mention that even for a non-interacting atomic cloud trapped within a harmonic potential typical semi-classical calculations can lead to results that deviate significantly from the quantum description above Bose-Einstein degeneracy. An interesting ex-

ample is the calculation of the atomic degeneracy parameter at the trap center [4]. Although the number of atoms in the ground-state energy level may be a small fraction of the total number of atoms, their density can still be comparable to that of the excited states population density due to the strong localization of the ground-state wave function. Since semi-classical calculations usually neglect the ground-state near to but above the quantum degeneracy temperature these models will fail to adequately describe the gas in this region.

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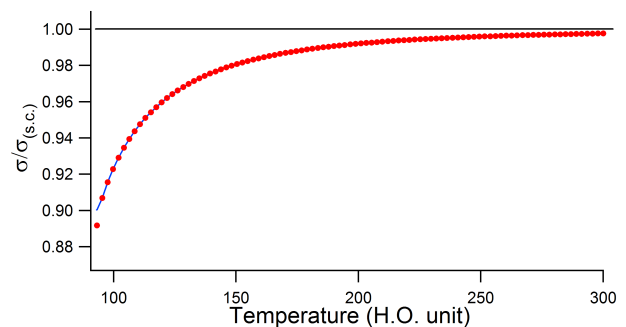


FIG. 1: (Color online) Width of the time of flight signal in function of the temperature (in units of $\hbar\omega$) for an expanding cloud initially confined in a harmonic potential of $N = 10^6$ atoms. The widths are normalized by $\sigma_{(s.c.)}$. The full quantum result (dots) is very close to the improved semi-classical results (solid line) above quantum degeneracy ($k_B T_c \approx 94\hbar\omega$). Both tend to $\sigma_{(s.c.)}$ at high temperature.

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