



**HAL**  
open science

## A sparse version of the ridge logistic regression for large-scale text categorization

Sujeevan Aseervatham, Anestis Antoniadis, Éric Gaussier, Michel Burlet, Yves Denneulin

► **To cite this version:**

Sujeevan Aseervatham, Anestis Antoniadis, Éric Gaussier, Michel Burlet, Yves Denneulin. A sparse version of the ridge logistic regression for large-scale text categorization. *Pattern Recognition Letters*, 2011, 32 (2), pp.101-106. 10.1016/j.patrec.2010.09.023 . hal-00633629

**HAL Id: hal-00633629**

**<https://hal.science/hal-00633629>**

Submitted on 15 Oct 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A Sparse Version of the Ridge Logistic Regression for Large-Scale Text Categorization

Sujeewan Aseervatham<sup>a,\*</sup>, Anestis Antoniadis<sup>b</sup>, Eric Gaussier<sup>a</sup>, Michel Burlet<sup>c</sup>,  
Yves Denneulin<sup>d</sup>

<sup>a</sup> *LIG - Université Joseph Fourier, 385, rue de la Bibliothèque, BP 53, F-38041 Grenoble Cedex 9, France*

<sup>b</sup> *LJK - Université Joseph Fourier, BP 53, F-38041 Grenoble Cedex 9, France*

<sup>c</sup> *Lab. Leibniz - Université Joseph Fourier, 46 Avenue Félix Viallet, F-38031 Grenoble Cedex 1, France*

<sup>d</sup> *LIG - ENSIMAG, 51 avenue Jean Kuntzmann, F-38330 Montbonnot Saint Martin, France*

---

## Abstract

The ridge logistic regression has successfully been used in text categorization problems and it has been shown to reach the same performance as the Support Vector Machine but with the main advantage of computing a probability value rather than a score. However, the dense solution of the ridge makes its use unpractical for large scale categorization. On the other side, LASSO regularization is able to produce sparse solutions but its performance is dominated by the ridge when the number of features is larger than the number of observations and/or when the features are highly correlated. In this paper, we propose a new model selection method which tries to approach the ridge solution by a sparse solution. The method first computes the ridge solution and then performs feature selection. The experimental evaluations show that our method gives a solution which is a good trade-off between the ridge and LASSO solutions.

*Keywords:* Logistic Regression, Model Selection, Text Categorization, Large Scale Categorization

---

\*Corresponding author, Tel.: +33(0)476514515; Fax: +33(0)476446675

*Email addresses:* [Sujeewan.Aseervatham@imag.fr](mailto:Sujeewan.Aseervatham@imag.fr) (Sujeewan Aseervatham),  
[Anestis.Antoniadis@imag.fr](mailto:Anestis.Antoniadis@imag.fr) (Anestis Antoniadis), [Eric.Gaussier@imag.fr](mailto:Eric.Gaussier@imag.fr) (Eric Gaussier),  
[Michel.Burlet@imag.fr](mailto:Michel.Burlet@imag.fr) (Michel Burlet), [Yves.Denneulin@imag.fr](mailto:Yves.Denneulin@imag.fr) (Yves Denneulin)

## 1 1. Introduction

2 The automatic text categorization problem consists in assigning, according  
3 to its content, a textual document to one or more relevant predefined categories.  
4 Given a training dataset, where the documents have been manually labeled, the  
5 problem lies in inducing a function  $f$ , from the training data, which can then  
6 be used to classify documents. Machine learning algorithms are used to find  
7 the optimal  $f$  by solving a minimization problem which can be stated as the  
8 minimization of the cost of misclassification over the training dataset (Empirical  
9 Risk Minimization).

10 In order to use numerical machine learning algorithm, the Vector Space  
11 Model is commonly used to represent a textual documents by a simple term-  
12 frequency vector (Salton et al., 1975). This representation produces datasets in  
13 which 1) the number of features is often larger than the number of documents,  
14 2) the vectors are very sparse, i.e., a lot of features are set to zero and 3) the  
15 features are highly correlated (due to the nature of natural languages). More-  
16 over, real-life datasets tend to be larger and larger which makes the automatic  
17 categorization process complicated and leads to scalability problems. As long as  
18 the datasets only grow in terms of the number of observations, the problem can  
19 be tackled by distributing the computation over a network of processors (Chu  
20 et al., 2006). However, when the number of features becomes larger than the  
21 number of observations, machine learning techniques tend to perform poorly  
22 due to overfitting, i.e., the model performs well on the training set but poorly  
23 on any other set. To prevent overfitting, the complexity of the model must be  
24 controlled during the training process, through model selection techniques. In  
25 the Support Vector Machine (SVM) algorithm (Vapnik, 1995), the model com-  
26 plexity is given by the VC-dimension, which is the maximum number of vectors,  
27 for any combination of labels, that can be shattered by the model. SVMs rely  
28 on the Structural Risk Minimization (SRM) principle, which not only aims at  
29 minimizing the empirical risk (Empirical Risk Minimization - ERM) but also  
30 the VC-dimension. SVMs have been used for text categorization and their per-

31 formance is among the best ones obtained so far (Joachims, 1998).

32     The VC-dimension remaining unknown for many functions, the SRM is dif-  
33 ficult to implement. Another model selection, widely used, is to minimize both  
34 the ERM and a regularization term:  $\lambda\Omega[f]$  where  $\lambda$  is a penalty factor,  $\Omega[f]$   
35 a convex non-negative regularization term and  $f$  the model. For linear func-  
36 tions:  $f(\mathbf{x}) = \langle \mathbf{w}, \mathbf{x} \rangle + b$ , the regularization term is often defined as  $\Omega[f] = \|\mathbf{w}\|_p$   
37 where  $\|\cdot\|_p$  is the  $L_p$ -norm (Hoerl and Kennard, 1970; Tibshirani, 1994; Zou and  
38 Hastie, 2005). This has the effect of smoothing  $f$  and reducing its generaliza-  
39 tion error. The use of the  $L_2$ -norm is known as the ridge penalization, whereas  
40 the use of the  $L_1$ -norm as the LASSO penalization, which has the property of  
41 simultaneously doing shrinkage and feature selection.

42     In this paper, we focus on penalized logistic regression. Logistic regression  
43 has the main advantage of computing a probability value rather than a score,  
44 as for the SVM. Furthermore, the ridge logistic regression has been shown to  
45 reach the same performance as the SVM on standard text categorization prob-  
46 lems (Zhang and Oles, 2001). Nevertheless, it produces a dense solution which  
47 cannot be used for large scale categorization. In (Genkin et al., 2007), the  
48 LASSO logistic regression was used to obtain a sparse solution. However, when  
49 the number of features is larger than the number of observations and/or when  
50 the features are correlated, the ridge penalization performance dominates the  
51 LASSO one (Zou and Hastie, 2005). Taking into account these observations, we  
52 propose a new model selection which produces a sparse solution by approaching  
53 the ridge solution.

54     The rest of the paper is organized as follows: in the next section we discuss  
55 related works; we then describe, in section 3, our model selection approach  
56 before reporting, in section 4, our experimental results; section 5 concludes the  
57 paper.

## 58 2. Related work

59 In (le Cessie and van Houwelingen, 1992), the authors have shown how ridge  
60 penalization can be used to improve the logistic regression parameter estimates  
61 in the cases where the number of features is larger than the number of obser-  
62 vations or when the variables are highly correlated. They have applied ridge  
63 logistic regression on DNA data and have obtained good results with stable pa-  
64 rameters. More recently, the ridge logistic regression was used in (Zhang and  
65 Oles, 2001) on the text categorization problem where the data are sparse and  
66 the number of features is larger than the number of observations. The authors  
67 have proposed several algorithms, which take advantage of the sparsity of the  
68 data, to solve efficiently the ridge optimization problem. The experimental re-  
69 sults show that the  $L_2$  logistic regression reaches the same performance as the  
70 SVM. Although the ridge method allows to select a more stable model by doing  
71 continuous shrinkage, the produced solution is dense and thus not appropriate  
72 for large and sparse data such as textual data.

73 The LASSO regularization ( $L_1$ -norm) has been introduced in (Tibshirani, 1994).  
74 The author shows, for linear regression, that the  $L_1$  penalization can not only do  
75 continuous shrinkage but has also the property of doing automatic variable se-  
76 lection simultaneously which means that the  $L_1$  solution is sparse. In (Genkin  
77 et al., 2007), an optimization algorithm based on (Zhang and Oles, 2001) is  
78 presented for Ridge and LASSO logistic regressions in the context of text cate-  
79 gorization. According to their experiments, the lasso penalization gives slightly  
80 better results than the ridge penalization in terms of the macro-averaged- $F_1$   
81 measure (the micro-averaged results are not given). It has been shown in (Efron  
82 et al., 2004; Tibshirani, 1994; Zou and Hastie, 2005) that the performance of  
83 the LASSO is dominated by the ridge in the following cases (we denote by  $p$  the  
84 number of features and by  $n$  the number of observations):

- 85 •  $p > n$ : the LASSO will only select at most  $n$  features,
- 86 • the features are highly correlated: the LASSO will select only one feature  
87 among the correlated features.

88 To tackle the limitations of the LASSO, the Elastic net method has been pro-  
89 posed in (Zou and Hastie, 2005) which tries to capture the best of both  $L_1$  and  
90  $L_2$  penalizations. The Elastic net uses both  $L_1$  and  $L_2$  regularization in the lin-  
91 ear regression problem. The authors show that the  $L_2$  regularization term can  
92 be reformulated by adding  $p$  artificial input data such that each artificial data  $i$   
93 has only the  $i^{\text{th}}$  component non-null set to  $\sqrt{\lambda_2}$  where  $\lambda_2$  is the  $L_2$  regularization  
94 hyperparameter. This reformulation, which leads to a LASSO problem, relies  
95 on the particular form of the least square term, and cannot be extended to the  
96 logistic regression problem. Furthermore, as the  $L_1$  and  $L_2$  regularizations are  
97 done simultaneously, it is unclear how the solution of the Elastic net approaches  
98 the  $L_2$  solution. In (Zhao and Yu, 2006), the model consistency of LASSO is  
99 studied for linear regression and it is shown that the consistency of LASSO  
100 depends on the regularization parameter. In (Bach, 2008), the author proves  
101 that for a regularization parameter decay factor of  $\frac{1}{\sqrt{n}}$ , a consistent model can  
102 be obtained by applying LASSO on bootstrap samples and by selecting only  
103 the intersecting features. Nevertheless, using LASSO on bootstrap samples is  
104 a time consuming process. Moreover, since this method is based on LASSO, it  
105 also fails to induce a good model when the variables are correlated.

### 106 3. Selected Ridge Logistic Regression

107 The logistic regression model is part of the Generalized Linear Model (GLM)  
108 family (Hastie and Tibshirani, 1990; McCullagh and Nelder, 1989). The GLM  
109 is a family of models, parametrized by  $\beta$ , which associate a target variable  $y$  to  
110 an input data  $\mathbf{x}$  ( $x \in \mathbb{R}^p$ ) according to the relation  $\beta \cdot \mathbf{x} = g(y)$  where  $g$  is a link  
111 function and  $\beta \in \mathbb{R}^p$ . For simplicity, we represent any linear function  $\beta' \cdot \mathbf{x}' + \beta'_0$   
112 by  $\beta \cdot \mathbf{x}$ , where  $\mathbf{x}$  is  $\mathbf{x}'$  with an extra dimension set to 1, and  $\beta$  is  $\beta'$  with an  
113 extra dimension set to  $\beta'_0$ . The logistic regression model is obtained by using  
114 a logit function  $g(y) = \frac{P(y|\beta, \mathbf{x})}{1 - P(y|\beta, \mathbf{x})}$ . When  $y \in \{-1, 1\}$ , the logistic regression  
115 model can be written as:

$$P(y = 1|\beta, \mathbf{x}) = \frac{1}{1 + \exp(-\beta \cdot \mathbf{x})} \quad (1)$$

116  $\beta$  can be obtained by maximizing the log-likelihood over the training set  $\mathcal{D} =$   
 117  $\{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)\}$ . However, in order to obtain a strictly convex optimiza-  
 118 tion problem and to avoid overfitting, a Tikhonov regularization term (Hoerl  
 119 and Kennard, 1970) is added, leading to the following ridge logistic regression  
 120 problem:

$$\beta^* = \underset{\beta}{\operatorname{argmin}} \underbrace{\sum_{i=1}^n \log(1 + \exp(-y_i \beta \cdot \mathbf{x}_i)) + \lambda \|\beta\|_2^2}_{l(\beta)} \quad (2)$$

121 where  $\lambda$  is a strictly positive scalar. Adding a ridge regularization term is  
 122 equivalent, in a Bayesian framework, to using a Gaussian prior on each com-  
 123 ponent of  $\beta$ , under the assumption that the components are independent, i.e.  
 124  $P(\beta) = \prod_j P(\beta_j)$  with  $P(\beta_j) \sim \mathcal{N}(0, \frac{1}{2\lambda})$ .

125 Several algorithms have been proposed in the literature to solve the opti-  
 126 mization problem in 2 (Friedman et al., 2008; Minka, 2003). In (Genkin et al.,  
 127 2007), an efficient algorithm, based on the one presented in (Zhang and Oles,  
 128 2001), is proposed to solve problems with sparse data, such as text documents.  
 129 However, the ridge regression solution is a dense vector which can hardly be  
 130 used in large scale categorization where hundreds of thousand features are used.  
 131 The problem we face is thus the one of finding  $\hat{\beta}$  such that:

- 132 1.  $\hat{\beta}$  is close to  $\beta^*$  and thus behaves well, ie  $l(\hat{\beta}) \simeq l(\beta^*)$ ;
- 133 2.  $\hat{\beta}$  is a sparse solution and thus can be used on large datasets.

134 The second order Taylor series expansion on  $l(\beta)$  around  $\beta^*$  leads to:

$$\begin{aligned} l(\beta) &\simeq l(\beta^*) + (\beta - \beta^*)^T \nabla l(\beta^*) \\ &\quad + \frac{1}{2} (\beta - \beta^*)^T \mathbf{H}_l(\beta^*) (\beta - \beta^*) \\ &= l(\beta^*) + \frac{1}{2} (\beta - \beta^*)^T \mathbf{H}_l(\beta^*) (\beta - \beta^*) \end{aligned} \quad (3)$$

135 where  $\nabla l(\beta^*)$  and  $H_l(\beta^*)$  are respectively the gradient and the Hessian of  $l(\beta)$   
 136 at  $\beta^*$  and where the equality derives from the fact that for  $\beta^*$ , the ridge solution,  
 137 the gradient vanishes. Hence, obtaining a  $\hat{\beta}$  yielding a value for  $l$  close to the  
 138 one of  $\beta^*$  while being sparse can be achieved by solving the following strictly  
 139 convex optimization problem:

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} (\beta - \beta^*)^T H_l(\beta^*) (\beta - \beta^*) + \alpha \|\beta\|_1 \quad (4)$$

140 The  $L_1$  regularization term, used to ensure sparsity, corresponds, in the Bayesian  
 141 framework, to the Laplace distribution prior on the components of  $\beta$ :  $P(\beta_i) \sim$   
 142  $\operatorname{Laplace}(0, \frac{1}{\alpha})$  with  $\alpha$  a strictly positive scalar. We refer to the above approach  
 143 as the Selected Ridge Logistic Regression method.

144 The so-called bag-of-words representation used in most text classification  
 145 methods assumes independence between words in documents<sup>1</sup>. Such an inde-  
 146 pendence assumption naturally leads to assuming that the components of  $\beta$  are  
 147 independent of one another, and thus that the Hessian  $H_l(\beta^*)$  is diagonal. We  
 148 make this assumption in the remainder of the paper. In this case, an analytical  
 149 solution to equation 4 can be obtained. Indeed, equation 4 can be rewritten as:

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^p (\beta_i - \beta_i^*)^2 H_i(\beta^*) + \alpha \|\beta\|_1 \quad (5)$$

150 with

$$H_i(\beta) = \frac{\partial^2 l(\beta)}{\partial \beta_i^2} = \sum_{j=1}^n \frac{x_{ji}^2 \exp(-y_j \beta \cdot \mathbf{x}_j)}{(1 + \exp(-y_j \beta \cdot \mathbf{x}_j))^2} + 2\lambda \quad (6)$$

151 Thus, the overall optimization problem can be solved component by component:

$$\hat{\beta}_i = \underset{\beta_i}{\operatorname{argmin}} (\beta_i - \beta_i^*)^2 H_i(\beta^*) + \alpha |\beta_i| \quad (7)$$

152 and its solution is given by theorem 3.1.

---

<sup>1</sup>(Joachims, 2002) for example recommends to use linear kernels, and not polynomial or Gaussian ones, for text classification.

153 **Theorem 3.1.** *The solution,  $\hat{\beta}_i$ , of the minimization problem in 7 is given by:*

$$\hat{\beta}_i = \begin{cases} \beta_i^* - \frac{\alpha}{2H_i(\beta^*)} & \text{if } \beta_i^* > \frac{\alpha}{2H_i(\beta^*)} \\ \beta_i^* + \frac{\alpha}{2H_i(\beta^*)} & \text{if } \beta_i^* < -\frac{\alpha}{2H_i(\beta^*)} \\ 0 & \text{otherwise} \end{cases}$$

154 (note that  $\hat{\beta}_i = 0$  if  $H_i(\beta^*) = 0$ ).

155 **Proof** Let us assume that  $\beta_i^* \geq 0$  and let  $g(\beta_i) = (\beta_i - \beta_i^*)^2 H_i(\beta^*) + \alpha|\beta_i|$ .

156 We have:  $\forall \beta_i \geq 0, g(\beta_i) \leq g(-\beta_i)$ , so that  $\hat{\beta}_i \geq 0$ . Setting the derivative of the

157 strictly convex function  $g$  wrt  $\beta_i$  to 0, one gets:

$$\begin{aligned} \beta_i^+ &= \underset{\beta_i > 0}{\operatorname{argmin}} g(\beta_i) \\ &= \begin{cases} \beta_i^* - \frac{\alpha}{2H_i(\beta^*)} & \text{if } \beta_i^* > \frac{\alpha}{2H_i(\beta^*)} \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

158 In the case where

$$\beta_i^* > \frac{\alpha}{2H_i(\beta^*)}$$

159 let

$$\beta_i^* = \frac{\alpha}{2H_i(\beta^*)} + \epsilon$$

160 Then, we have:

$$g(0) = g\left(\beta_i^* - \frac{\alpha}{2H_i(\beta^*)}\right) + \epsilon^2 H_i(\beta^*) > g\left(\beta_i^* - \frac{\alpha}{2H_i(\beta^*)}\right)$$

161 This shows that  $\hat{\beta}_i = \beta_i^+$  when  $\beta_i^* \geq 0$ . The case  $\beta_i^* \leq 0$  is treated in exactly

162 the same way and completes the proof of theorem 3.1.

### 163 Automatic Setting of the penalty hyperparameter

164 In order to reduce the number of hyperparameters to estimate, one can set the

165 LASSO penalty  $\alpha$  to the universal penalty (or universal thresholding). Indeed,

166 the function to be minimized in 4 can also be interpreted as the penalized loss  
 167 of a Gaussian vector  $\beta$  with mean  $\beta^*$  and covariance matrix  $H_l^{-1}(\beta^*)$ . For  
 168  $H_l^{-1}(\beta^*)$  bounded in the vicinity of  $\beta^*$ , theorem 4 in (Antoniadis and Fan,  
 169 2001) applies and defines the universal penalty (or universal thresholding) to be  
 170  $\sqrt{\frac{2\log(p)}{p}}$ , a value which guarantees that the risk function of  $\hat{\beta}$  (solution of 4) is  
 171 within a factor of logarithmic order. This leads to the following property:

172 **Property 3.1.** *The universal penalty  $\alpha$  for minimizing 4 w.r.t.  $\beta$ , for  $H_l^{-1}(\beta^*)$   
 173 bounded in the vicinity of  $\beta^*$ , is  $\sqrt{\frac{2\log(p)}{p}}$ , with  $p$  the dimension of  $\beta$ .*

174 The algorithm associated with the above, overall approach can be described as  
 175 follows.

#### 176 **Summary of the approach**

177 The Selected Ridge Logistic Regression method is summarized in algorithm 1.

---

#### **Algorithm 1** Selected Ridge Logistic Regression

---

**Input:**  $\mathcal{D}$  - the training dataset

**Input:**  $\lambda$  - the ridge penalization factor

**Input:** *optionally*  $\alpha$  - the lasso penalization factor

**Output:**  $\hat{\beta}$  - the parameter of the model as defined in eq. 1

- 1: Compute  $\beta^*$  by solving eq. 2
  - 2: **if**  $\alpha$  is not given as an input argument **then**
  - 3:   Use property 3.1 to set  $\alpha$
  - 4: **end if**
  - 5: **for all**  $\hat{\beta}_i$  of  $\hat{\beta}$  **do**
  - 6:   Use theorem 3.1 to compute  $\hat{\beta}_i$
  - 7: **end for**
- 

178 Despite the fact that the Selected Ridge method involves the computation of  
 179 a ridge solution, it is important to note, as we will see in the experimental  
 180 section, that the training time of the Selected Ridge method is usually shorter  
 181 than that of the Ridge method. This is due to the fact that the optimal  $\lambda$  for  
 182 both methods are different and, especially in text categorization, the optimal

183  $\lambda$  for the Selected Ridge method is larger than the optimal  $\lambda$  for the Ridge  
 184 method. For a small  $\lambda$  close to zero, the training time of the Ridge method will  
 185 be important as more iterations will be needed to reach convergence.

186 **Relation to the Fisher Information Matrix**

187 The fisher information matrix  $I(\beta)$  is given, for each entry  $(i, j)$ , by the following  
 188 equation:

$$I_{i,j}(\beta) = -\mathbb{E}\left(\frac{\partial^2 \log P(y|\mathbf{x}, \beta)}{\partial \beta_i \partial \beta_j}\right) \quad (8)$$

189 Thus, using the empirical Fisher information matrix  $\hat{I}(\beta^*)$ , we have:

$$H_i(\beta^*) = \hat{I}_{i,i}(\beta^*) + 2\lambda \quad (9)$$

190 The Fisher information matrix summarizes the average amount of information  
 191 brought by the data on  $\beta$ . Hence according to theorem 3.1 and formula 9,  
 192 the more information the data brings on  $\beta_i^*$  (ie the higher  $\hat{I}_{i,i}(\beta^*)$ ), the higher  
 193  $H_i(\beta^*)$  will be and the closer  $\hat{\beta}_i$  will be to  $\beta_i^*$ . In other words, the value obtained  
 194 through the original ridge regression problem is almost not modified. On the  
 195 contrary, if the data brings little information on  $\beta_i^*$  (ie  $\hat{I}_{i,i}(\beta^*)$  is small), then  
 196  $H_i(\beta^*)$  will be small and  $\hat{\beta}_i$  will be set to zero for a large range of values of  $\beta_i^*$ .  
 197 Thus, *sparsity is obtained in the Selected Ridge Regression method by setting to*  
 198 *0 the dimensions of the ridge solution  $\beta^*$  which have small values and which*  
 199 *are not supported by the data, ie for which  $\hat{I}_{i,i}(\beta^*)$  is small.* This result reflects  
 200 the intuition that, in many text categorization problems, only a few words are  
 201 crucial and usually correspond to the dimensions for which the ridge values are  
 202 sufficiently large. The development provided here, in particular theorem 3.1,  
 203 shows that one should discard dimensions for which the ridge value is not larger  
 204 than, roughly, the inverse of the Fisher information. Thus, the ridge value is not  
 205 the only parameter one should consider. The information brought by the data  
 206 on this value plays indeed a crucial role: dimensions with small values strongly  
 207 supported by the data should be kept in the final solution.

208 **4. Experimental Results**

209 The proposed model selection method was evaluated over a set of three  
 210 well-known datasets and one large dataset. The first three datasets are Reuters  
 211 21578, Ohsumed and 20-NewsGroups (Hersh et al., 1994; Joachims, 1998, 2002).  
 212 All of these datasets have been widely studied in the text categorization litera-  
 213 ture. Reuters 21578<sup>2</sup> is a collection of news on different domains. Ohsumed<sup>3</sup> is a  
 214 collection of medical abstracts originally designed for content-based information  
 215 retrieval, and 20-NewsGroup<sup>4</sup> a collection of documents written in the context  
 216 of 20 different news groups. These collections are thus varied in terms of their  
 217 production and content. The last dataset is a subset of documents taken from  
 218 the DMOZ website<sup>5</sup>. This DMOZ dataset was collected in order to perform  
 219 Large-Scale text categorization experiments. The characteristics of the datasets  
 220 are reported in table 1. This last collection is a collection of web pages, and  
 221 contains documents of various types (scientific articles, business descriptions,  
 222 ...) on several domains.

Table 1: Datasets used for the experiments

Name	Train. size (n)	Test size	#Features (p)	#Categories	Case
Reuters-21578	7770	3019	6760	90	$p < n$ ( $\frac{p}{n} \approx 1$ )
Ohsumed	6286	7643	20520	23	$p > n$ ( $\frac{p}{n} \approx 3$ )
20-NewsGroups	12492	6246	51666	20	$p > n$ ( $\frac{p}{n} \approx 4$ )
DMOZ	20249	7257	133348	3503	$p > n$ ( $\frac{p}{n} \approx 6$ )

223 All the datasets have been pre-processed according to the setting defined  
 224 in (Joachims, 2002), which we briefly describe here. The Vector Space Model  
 225 (VSM) (Salton et al., 1975) is used to represent the textual documents in a vector

<sup>2</sup><http://www.daviddlewis.com/resources/testcollections/reuters21578/>

<sup>3</sup><http://ir.ohsu.edu/ohsumed/ohsumed.html>

<sup>4</sup><http://people.csail.mit.edu/jrennie/20Newsgroups/>

<sup>5</sup><http://www.dmoz.org>

226 space. The VSM is also known as the *Bag-of-Words* (BOW) representation in  
227 which a list of terms is used to define a vector space, each term defining an axis  
228 of the space. A textual document can then be represented as a vector, using for  
229 each axis, the corresponding term frequency value. In order to obtain an efficient  
230 vector representation, each document is pre-processed using the following steps:

- 231 1. Cleaning by removing non-Latin characters, numerical symbols and punc-  
232 tuation marks,
- 233 2. Segmenting terms separated by a white space into a list of words,
- 234 3. Removing stopwords (using a stopwords list),
- 235 4. Stemming each word using the Porter Stemming algorithm (Porter, 1980).

236 We also used the TF-IDF weighting scheme (Jones, 1988) to give more im-  
237 portance to terms that are frequent in a document (the TF part) and specific  
238 to a small number of documents (IDF part). Furthermore, we normalized all  
239 document vectors.

240 For multi-class categorization, we use the one-vs-the-rest strategy based on  
241 binary logistic regression models. To assign a document to a unique category in  
242 mono-label problems, we use the following decision function:  $\operatorname{argmax}_c P(y_i =$   
243  $+1|\beta_c, \mathbf{x}_i)$ ; if a document can be assigned to several categories (multi-label prob-  
244 lems), we assign it to each category  $c$  such that  $P(y_i = +1|\beta_c, \mathbf{x}_i) \geq 0.5$ .

245 In the experiments, the  $F_1$  measure (van Rijsbergen, 1979) is used to evaluate  
246 the performance of the classifiers. It is defined as:

$$F_1 = \frac{2 \times \text{TP}}{2 \times \text{TP} + \text{FP} + \text{FN}} \quad (10)$$

247 where TP stands for true positive, FP for false positive and FN for false negative.  
248 For multi-class datasets, we used the micro- $F_1$  and macro- $F_1$  measures. In the  
249 micro- $F_1$  measure, TP, FP and FN are summed over each category giving thus  
250 an equal weight to each document. In this case, this measure corresponds to  
251 the overall precision of the system and provides a measure of the accuracy of  
252 the classifier. The macro- $F_1$  is the arithmetic mean of  $F_1$  across the categories,

253 giving an equal weight to each category. If  $F_1^c$  denotes the  $F_1$  measure for  
254 category  $c$ , then the micro- $F_1$  is defined by:

$$\text{micro-}F_1 = \sum_{c=1}^K \frac{N_c}{N} F_1^c \quad (11)$$

255 where  $K$  denotes the number of categories,  $N_c$  corresponds to the number of  
256 documents in category  $c$ , and  $N$  is the total number of documents ( $N = \sum_c N_c$ ).  
257 The macro- $F_1$  is defined by:

$$\text{macro-}F_1 = \sum_{c=1}^K \frac{1}{K} F_1^c \quad (12)$$

258 We also report the degree of sparsity for each model. The sparsity is given  
259 by :

$$s = 1 - \frac{\text{avg \#features used in the model}}{\text{\#features in the dataset}} \quad (13)$$

260 A solution based on all the features will thus have a degree of sparsity of 0.

261 Moreover, it is important to note that the penalization parameter was fixed  
262 for each algorithm by cross-validation except for the DMOZ subset where the  
263 parameter was tuned using a validation set composed of 7256 documents. For  
264 the Selected Ridge method, the hyperparameter  $\alpha$  in equation 4 was automati-  
265 cally set using property 3.1.

266 To solve the LASSO and Ridge regression problems, we used the algorithm  
267 described in (Genkin et al., 2007). The training and prediction times are given  
268 as indications. Since, the calculations were distributed over a set of computers,  
269 the given times are the times spent on calculation plus the times consumed by  
270 the system (thread swapping, network transfer time, etc.).

271 It is also important to note that in all the results below, the training time  
272 of the Selected Ridge method is always shorter than that of the Ridge method.  
273 This can be confusing since the Selected Ridge involves the computation of a  
274 Ridge solution and, thus, one can expect its training time to be at least equal  
275 to that of the Ridge method. Actually, as we said above, this is due to the fact

276 that the ridge’s training time depends on the Ridge regularization parameter  $\lambda$ .  
 277 If  $\lambda$  is very close to 0 then the training time will be important as more iterations  
 278 will be needed to reach convergence. In all our experiments, the optimal  $\lambda$  for  
 279 the Ridge method was always smaller than that for the Selected Ridge method.  
 280 For example, for the DMOZ dataset in subsection 4.4, the optimal  $\lambda$  for Ridge  
 281 is 0.0001 (training time: 13299.43s) but the optimal  $\lambda$  for the Selected Ridge  
 282 is 0.001 (training time: 10996.80s). This difference can also be seen in table 6:  
 283 when the L1-parameter ( $\alpha$ ) is zero (Selected Ridge=ridge) then the optimal  $\lambda$   
 284 for the Selected Ridge method is 0.0001, but when  $\alpha$  is greater than 0 then the  
 285 optimal  $\lambda$  is always 0.001.

#### 286 4.1. Experiments on Reuters-21587

287 The Reuters-21587 dataset is a collection of newswire articles. Each docu-  
 288 ment was manually assigned to one or more categories, according to its subject.  
 289 In this collection, we used the standard “ModApte” split, which provides train-  
 290 ing and test sets. The results are reported in Table 2. The LASSO and the  
 291 ridge reach the same level of performance; however, the ridge method yields a  
 292 dense model whereas the LASSO one only selects 0.0043% of the features. The  
 293 feature selection method used on the ridge model (Selected Ridge Regression  
 294 method) allows to reach the same micro- $F_1$  performance than the ridge method,  
 295 but with a number of features reduced by 95%.

Table 2: Categorization Result on Reuters-21587(ModApte)

Algorithm	Micro $F_1$	Macro $F_1$	Sparsity	Training time (sec)	Pred. time (sec)
LASSO	0.8711	0.5167	0.9957	164.07	0.44
Ridge	0.8690	0.5099	0.0	257.96	13.20
Selected Ridge	0.8645	0.4563	0.9447	180.24	1.55

296 *4.2. Experiments on Ohsumed*

297 The Ohsumed corpus (Hersh et al., 1994) is a subset of the medical biblio-  
 298 graphic database MEDLINE. Each document is a reference of a medical article  
 299 published in a medical journal. Following the settings defined in (Joachims,  
 300 1998, 2002), we only kept the first 20,000 references which had abstracts and  
 301 were published in 1991. This set is split into a training set composed of the first  
 302 10,000 documents and a test set composed of the rest. Only abstracts are used  
 303 for the categorization task. After the pre-processing, the training set is reduced  
 304 to 6,286 unique documents and the test set to 7,643. Each document belongs  
 305 to one or more cardiovascular categories.

306 As shown in table 3, the LASSO method performs well on this dataset ; not  
 307 only it has the best performance in terms of micro and macro- $F_1$  but it also  
 308 gives a very sparse solution. The Selected Ridge method slightly improves the  
 309 micro- $F_1$  performance of the ridge method while removing 88% of its features.

Table 3: Categorization Result on Ohsumed

Algorithm	Micro $F_1$	Macro $F_1$	Sparsity	Training time (sec)	Pred. time (sec)
LASSO	0.6533	0.6053	0.9800	81.16	1.83
Ridge	0.6387	0.5897	0.0	144.06	31.20
Selected Ridge	0.6409	0.5802	0.8827	107.08	5.32

310 *4.3. Experiments on 20-Newsgroups*

311 The 20-NewsGroups is a collection of emails taken from the Usenet news-  
 312 groups. Each email is assigned to a unique category according to its topic. The  
 313 experiment on 20-newsgroups, reported in table 4, clearly shows that the ridge  
 314 penalization outperforms the LASSO method. In fact, the variable selection  
 315 of the LASSO is too aggressive and eliminates interesting features. However,

316 our variable selection method (Selected Ridge) achieves micro-F1 and macro-F1  
 317 scores similar to those obtained by the ridge, while relying on only 10% of the  
 318 features used in the ridge solution.

Table 4: Categorization Result on 20-NewsGroups

Algorithm	Micro $F_1$	Macro $F_1$	Sparsity	Training time (sec)	Pred. time (sec)
LASSO	0.8663	0.8644	0.9861	384.16	1.72
Ridge	0.9038	0.9018	0.0	157.96	71.25
Selected Ridge	0.8966	0.8939	0.9050	136.01	7.51

#### 319 4.4. Experiments on DMOZ

320 In order to assess the behavior of the different methods in a large scale cat-  
 321 egorization setting, we have collected 34,762 html documents from the DMOZ  
 322 website. DMOZ ([www.dmoz.org](http://www.dmoz.org)) is an open directory project that aims to  
 323 classify the whole web into categories. In the collected dataset, we only used  
 324 3,503 categories and we split the corpus into 3 parts: a training set composed  
 325 of 20,249 documents, a validation set composed of 7,256 documents and a test  
 326 set composed of 7,257 documents. The validation set is used to tune the hy-  
 327 perparameters. For the pre-processing of the documents, we removed the html  
 328 tags and the script parts to keep only the text and we applied the standard  
 329 pre-processing steps described above. For illustration, figure 1 shows a part of a  
 330 document from the corpus before and after the pre-processing. In this dataset,  
 331 each document belongs to a unique category.

332 As expected in the case where the number of features is largely greater than  
 333 the number of documents, the ridge method clearly outperforms LASSO as  
 334 shown in table 5. However, the ridge solution being dense, the categorization  
 335 of large sets is a time consuming process which makes the ridge solution inap-  
 336 propriate. The LASSO and the Selected Ridge methods both produce a sparse

<pre>[...] &lt;a href="../../Documents/PLDS210/ VolumeFront.html" target=" blank" class="NoFrames"&gt;(No Frames)&lt;/a&gt;&lt;/ td&gt;&lt;td class="text"&gt;John Morris&lt;/td&gt; &lt;td class="text"&gt;Complete data structures and algorithms course. This course is taught at the University Of Western Australia. The course teaches the following topics : Objects and ADTs, Constructors and destructors, Data Structure , Methods, Pre- and post-conditions C conventions [...]</pre>	<pre>[...] frame john morri complet data structur algorithm cours cours taught univers western australia cours teach follow topic object constructor destructor data structur method post condit convent [...]</pre>
<b>(a)</b>	<b>(b)</b>

Figure 1: A part of an html document taken from the DMOZ corpus (a) before and (b) after the pre-processing. The address of the document is <http://www.oopweb.com/Algorithms/Files/Algorithms.html> and it is referenced in DMOZ as “OOPWeb Algorithms Directory” at <http://www.dmoz.org/Computers/Algorithms>.

337 solution with a degree of sparsity of 99%. The selected ridge performs better  
338 than the LASSO in terms of the micro- $F_1$  measure, but however has a macro- $F_1$   
339 value slightly lower than the value obtained by LASSO.

Table 5: Categorization Result on DMOZ

Algorithm	Micro $F_1$	Macro $F_1$	Sparsity	Training time (sec)	Pred. time (sec)
LASSO	0.2936	0.1661	0.9999	9805.78	41.51
Ridge	0.3434	0.2020	0.0	13299.40	31084.90
Selected Ridge	0.3124	0.1586	0.9993	10996.80	42.52

340 In table 6, we report the performance of the Selected Ridge method according  
341 to the value of the  $L_1$  penalty term in equation 4. The results show that  
342 property 3.1 provides a good penalty value in terms of trade-off between micro-  
343  $F_1$  performance and sparsity. It is also interesting to note that with  $\alpha$  set to  
344  $10^{-7}$ , one obtains a method yielding results on a par with the ones obtained by  
345 the ridge (which provides the best results in terms of both micro- and macro- $F_1$ )  
346 while being twice sparser and almost four times faster.

Table 6: Performance of the Selected Ridge Method on DMOZ according to the penalty value in equation 4. The results corresponding to the optimal universal penalty value (property 3.1) are indicated in bold.

Penalty ( $\alpha$ )	Micro- $F_1$	Macro- $F_1$	Sparsity	Training time (sec)	Prediction time (sec)
1	0.1188	0.0353	0.9999	11103.9	8.33
0.1	0.2604	0.1209	0.9999	11065.8	32.81
0.05	0.2835	0.1343	0.9998	11325.1	39.82
0.03	0.2953	0.1436	0.9997	11013.4	39.34
0.02	0.3040	0.1524	0.9996	11164.6	49.07
<b>0.0133</b>	<b>0.3124</b>	<b>0.1586</b>	<b>0.9993</b>	<b>10996.8</b>	<b>42.52</b>
0.01	0.3156	0.1604	0.9992	10965.5	52.97
$10^{-7}$	0.3434	0.1949	0.5423	11090.2	8858.31
0	0.3434	0.2020	0.0	13299.40	31084.90

## 347 5. Conclusion

348 As pointed in (Zhao and Yu, 2006): *Sparsity or parsimony of statistical mod-*  
349 *els is crucial for their proper interpretations.* In this paper, we have proposed a  
350 model selection method to “sparsify” the ridge logistic regression solution. This  
351 method first solves the classic ridge logistic regression, then sets less informative  
352 features with low values to zero, while ensuring that the resulting sparse solution  
353 remains in the vicinity of the ridge solution. This latter property is obtained  
354 by using a Taylor expansion of the likelihood function around the solution of  
355 the ridge, penalized with the  $L_1$  norm. The experimental text categorization  
356 results obtained on well-studied datasets and on a large-scale dataset collected  
357 from *www.dmoz.org* show that our method produces a solution which offers a  
358 good trade-off between the performance of the ridge solution and the sparsity  
359 of the LASSO solution. In particular, when  $p > n$  (the number of features is

360 greater than the number of observations), our method leads to a sparse version  
361 of the ridge which is both accurate (in terms of both micro- and macro- $F_1$ ) and  
362 fast.

### 363 Acknowledgements

364 This work was partly supported by the LASCAR project, Univ. J. Fourier,  
365 Grenoble.

366 Antoniadis, A., Fan, J., 2001. Regularization of Wavelet Approximations. Jour-  
367 nal of the American Statistical Association 96, 939–967.

368 Bach, F. R., 2008. Bolasso: model consistent lasso estimation through the boot-  
369 strap. In: ICML '08: Proceedings of the 25th international conference on  
370 Machine learning. ACM, New York, NY, USA, pp. 33–40.

371 Chu, C.-T., Kim, S. K., Lin, Y.-A., Yu, Y., Ng, G. B. A. Y., Olukotun, K.,  
372 2006. Map-Reduce for Machine Learning on Multicore. In: Schölkopf, B.,  
373 Platt, J. C., Hoffman, T. (Eds.), NIPS. MIT Press, pp. 281–288.

374 Efron, B., Hastie, T., Johnstone, L., Tibshirani, R., 2004. Least angle regression.  
375 Annals of Statistics 32, 407–499.

376 Friedman, J., Hastie, T., Tibshirani, R., 2008. Regularization paths for gener-  
377 alized linear models via coordinate descent. Tech. rep., Dept. of Statistics,  
378 Stanford University.

379 Genkin, A., Lewis, D. D., Madigan, D., 2007. Large-Scale Bayesian Logistic  
380 Regression for Text Categorization. Technometrics 49, 291–304(14).

381 Hastie, T. J., Tibshirani, R. J., 1990. Generalized Additive Models. Chapman  
382 & Hall.

383 Hersh, W., Buckley, C., Leone, T. J., Hickam, D., 1994. OHSUMED: an in-  
384 teractive retrieval evaluation and new large test collection for research. In:

- 385 Proceedings of the 17th annual international ACM SIGIR. Springer-Verlag  
386 New York, Inc., New York, NY, USA, pp. 192–201.
- 387 Hoerl, A. E., Kennard, R. W., 1970. Ridge Regression: Biased Estimation for  
388 Nonorthogonal Problems. *Technometrics* 12 (1), 55–67.
- 389 Joachims, T., 1998. Text categorization with support vector machines: learning  
390 with many relevant features. In: *Proceedings of the 10th European Conference*  
391 *on Machine Learning (ECML)*. Springer Verlag, Heidelberg, DE, pp. 137–142.
- 392 Joachims, T., 2002. *Learning to Classify Text Using Support Vector Machines: Methods, Theory and Algorithms*. Kluwer Academic Publishers, Norwell, MA, USA.
- 395 Jones, K. S., 1988. A statistical interpretation of term specificity and its appli-  
396 cation in retrieval. *Document retrieval systems*, 132–142.
- 397 le Cessie, S., van Houwelingen, J. C., 1992. Ridge Estimators in Logistic Re-  
398 gression. *Applied Statistics* 41 (1), 191–201.
- 399 McCullagh, P., Nelder, J. A., 1989. *Generalized Linear Models*. Chapman &  
400 Hall, London, UK.
- 401 Minka, T. P., 2003. A Comparison of Numerical Optimizers for Logistic Regres-  
402 sion. Tech. rep., Dept. of Statistics, Carnegie Mellon University.
- 403 Porter, M. F., July 1980. An algorithm for suffix stripping. *Program* 14 (3).
- 404 Salton, G., Wong, A., Yang, C. S., 1975. A Vector Space Model for automatic  
405 indexing. *Communications of the ACM* 18 (11), 613–620.
- 406 Tibshirani, R., 1994. Regression Shrinkage and Selection Via the Lasso. *Journal*  
407 *of the Royal Statistical Society, Series B* 58, 267–288.
- 408 van Rijsbergen, C. J., 1979. *Information retrieval*, 2nd Edition. Butterworths,  
409 London, UK.

- 410 Vapnik, V. N., 1995. The nature of statistical learning theory. Springer-Verlag  
411 New York, Inc.
- 412 Zhang, T., Oles, F. J., 2001. Text Categorization Based on Regularized Linear  
413 Classification Methods. *Information Retrieval* 4, 5–31.
- 414 Zhao, P., Yu, B., 2006. On Model Selection Consistency of Lasso. *The Journal*  
415 *of Machine Learning Research* 7, 2541–2563.
- 416 Zou, H., Hastie, T., 2005. Regularization and variable selection via the elastic  
417 net. *Journal Of The Royal Statistical Society Series B* 67 (2), 301–320.