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# Welfare Improvement Properties of an Allowance Market in a Production Economy

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## Abstract

This paper studies the welfare improvement properties of a market of allowances in an economy with a single type of externality. We show that thanks to the opening of such a market the Pareto optima can be decentralized as marginal pricing equilibria. However, the set of equilibria is much larger than this of Pareto optima. In order to discriminate the efficient equilibria we introduce a demand revealing mechanism tailored for this framework.

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**Key Words:** General Equilibrium Theory, Pareto Optimality, Externalities, Markets of Allowances.

**JEL Codes:** : D51, D62, Q54, Q58

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## 1 Introduction

It has long been recognized that the interactions between the economic activities and the environment have effects which are not properly reflected by the market prices. Those effects have thus been referred to as externalities. An abundant theoretic literature has focused on the means to overcome this failure of the market, that is to design economic models encompassing externalities whose equilibria have suitable Pareto optimality properties. A first branch of the literature pioneered by Arrow (2), proposed the creation of artificial commodities (hereafter called Arrobian commodities) which associate to every couple of agents and every commodity in the economy, the influence caused by the use of this commodity by the first agent on the second. This work and its extensions by Laffont (10), Bonnisseau (3) and others can be seen as a formalization of the Coase Theorem (see (5)) in a general equilibrium framework. On another hand, in (4), Boyd and Conley argue that externalities can be defined intrinsically and treated as public bads (or public goods in case of positive externalities). They then introduce markets of allowances for externalities and show that the associate Lindhal equilibria are Pareto optimal.

Contemplating the growing interest in “pollution permits” markets such as the European Union Emission Trading Scheme, one may assume that this literature has influenced the governmental policies dealing with environmental economic issues. Indeed the creation of those markets seem at first sight a direct application of this theoretic work. However a closer look brings to light that those markets actually do not feet in any of the models cited above. Indeed they are markets of allowances for public bads in the sense of Boyd and Conley, but in general the allowance is used only as a private good by the polluters. This is underlined by the fact that the justification for the creation of those markets is that they allow for reduction of the pollution at the least possible cost, not that they lead to Pareto optimality. Even when consumers have access to those markets, the free rider problem make it doubtful that a Lindhal equilibrium may be implemented. On the opposite, our approach is to underline the fact that when it creates an allowance market the government also creates a public good consisting in the difference between the situation that prevails under *laissez-faire* and the level of allowances it supplies to the economy. Our aim is to determine wether this particular type of public good provision may lead to Pareto optimal outcomes.

We focus on a general equilibrium model with a finite number of goods and agents. The firms production causes an external effect on consumers. The government forces by legal means the producers to use as input the quantity of allowances corresponding to the externality they cause and initially endow the agents with a certain quantity of allowances. Agents may then trade these on a market. We consider various possible structure for the allowance market:

the consumers may or may not have access to the market as buyers (i.e use the allowance as a public good), the consumption of allowances by consumers may be subsidized by the government.

Building upon a result of Bonnisseau (3), we show that Pareto optima can be decentralized as marginal pricing equilibria in such an economy independently of the market structure. Meanwhile the allowance market appears as a complex tool allowing at the same time to define a value for the environment, to influence the firms behavior and to implement wealth transfers.

However the analogous of the first welfare Theorem does not hold, that is marginal pricing equilibria are not necessarily Pareto optimal. In particular when the consumers have access to the allowance market as buyers, a necessary condition for an equilibrium to be Pareto optimal is paradoxically that no allowance is purchased by the consumers (as in Smith and al. (11)). Hence, the only way to obtain a Pareto optimal outcome is that the government chooses a proper initial allocation. In this respect, we provide a simple mechanism to implement optimality: the government diminishes its supply of allowances proportionally to the consumers purchase.

## 2 The Model

We consider an economy with a finite number  $L$  of commodities labeled by  $\ell = 1 \cdots L$ , lying within an environment which is characterized by a real parameter  $\tau$ .

There are  $n$  firms in the economy indexed by  $j = 1 \cdots n$  whose production possibilities are described by a closed set  $Y_j$ . As they produce, firms influence the environment. We measure according to the differentiable function  $f_j : \mathbb{R}^L \rightarrow \mathbb{R}_-$  the damage caused to the environment by firm  $j$ . The actual state of the environment when the firms choose a production scheme  $(y_j) \in \prod_{j=1}^n Y_j$  is  $\sum_{j=1}^n f_j(y_j)$ .

There are  $m$  consumers in the economy indexed by  $i = 1 \cdots m$ . They gain utility from the consumption of strictly positive quantities of commodities 1 to  $L$  and are sensitive to the state of the environment. Their preferences are represented by a quasi-concave and differentiable utility function  $u_i$  defined on  $\mathbb{R}_{++}^L \times \mathbb{R}$  which associates to a bundle of commodities  $x_i \in \mathbb{R}_{++}^L$  and to an environmental parameter  $\tau \in \mathbb{R}_-$  an utility level  $u_i(x_i, \tau)$ . We shall denote by  $P_i(x_i, \tau)$  the set of elements  $\{(x'_i, \tau') \in \mathbb{R}_{++}^L \times \mathbb{R}_- \mid u_i(x'_i, \tau') \geq u_i(x_i, \tau)\}$  corresponding to the pair of commodity bundle and state of the environment preferred to  $(x_i, \tau)$  by agent  $i$ . We shall assume in the remaining of the paper that the utility functions are monotone, locally non-satiated in commodities,

increasing with the environment, and that one of them is strictly monotone<sup>3</sup>. This will ensure in particular equilibrium prices are positive.

The initial resources of the economy are set equal to  $\omega \in \mathbb{R}_{++}^L$ .

The remaining of this paper is concerned with the Pareto optimal outcomes of this economy defined as:

**Definition 1** *An element  $((x_i), (y_j)) \in (\mathbb{R}_{++}^L)^m \times \prod_{j=1}^n Y_j$  is an attainable allocation if  $\sum_{i=1}^m x_i = \sum_{j=1}^n y_j + \omega$ .*

*An element  $((x_i), (y_j)) \in (\mathbb{R}_{++}^L)^m \times \prod_{j=1}^n Y_j$  is a Pareto optimum if it is an attainable allocation and if there exist no attainable allocation  $((x'_i), (y'_j))$  such that for all  $i$ ,  $u_i(x'_i, \sum_{j=1}^n f_j(y'_j)) \geq u_i(x_i, \sum_{j=1}^n f_j(y_j))$ , and for at least an  $i_0$ ,  $u_{i_0}(x'_{i_0}, \sum_{j=1}^n f_j(y'_j)) > u_{i_0}(x_{i_0}, \sum_{j=1}^n f_j(y_j))$ .*

A result of Bonnisseau (3) entails a general characterization of these Pareto optima:

**Theorem 1 (Theorem 3 in Bonnisseau (3))** *If  $((\bar{x}_i), (\bar{y}_j))$  is a Pareto optimum of  $\mathcal{E}$ , then there exist  $(\bar{p}, \bar{q}) \in \mathbb{R}_+^{L+1}$  such that*

- (1) *For all  $i$ , there exist  $\bar{q}_i$  such that  $(\bar{p}, \bar{q}_i) \in N_{P_i(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))}(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))$  and  $\sum_{i=1}^m \bar{q}_i = \bar{q}$*
- (2) *For all  $j$ ,  $\bar{p} + \bar{q} \nabla f_j(\bar{y}_j) \in N_{Y_j}(\bar{y}_j)$*

Building on this result, which extends the optimality properties of marginal pricing (see Guesnerie (7)) to a framework encompassing externalities, we focus on the possibility to decentralize Pareto optima as marginal pricing equilibria for appropriate market structures. Let us recall that marginal pricing equilibria coincide with the standard competitive equilibria under additional convexity assumptions but also allows to deal with increasing returns to scale in the production sector.

Now, when there exist markets for standard commodities only, a price equilibrium with marginal tariffication of the economy may be defined as:

**Definition 2** *An allocation  $(\bar{x}_i), (\bar{y}_j)$  in  $(\mathbb{R}_{++}^L)^m \times \prod_{j=1}^n Y_j$  is a marginal pricing equilibrium with transfers if there exist a price  $p \in \mathbb{R}_+^L$  and a wealth allocation  $(w_1, \dots, w_m) \in \mathbb{R}^m$  with  $\sum_{i=1}^m w_i = p \cdot (\omega + \sum_{j=1}^n y_j)$  such that:*

- (1) *For every  $i$ ,  $\bar{x}_i$  maximizes  $u_i(\cdot, \sum_{j=1}^n f_j(\bar{y}_j))$  among the feasible consumption plans  $\{x_i \in \mathbb{R}_{++}^L \mid p \cdot x_i \leq w_i\}$ ;*
- (2) *For all  $j$ ,  $\bar{p} \in N_{Y_j}(\bar{y}_j)$ ;*
- (3)  *$\sum_{i=1}^m \bar{x}_i = \sum_{j=1}^n \bar{y}_j + \sum_{i=1}^m \omega_i$ ;*

<sup>3</sup> So that one can apply Theorem 3 of Bonnisseau (3).

and it is well-known <sup>4</sup> that, when there are non-trivial environmental effects none of these equilibria is Pareto optimal. Indeed, at such an equilibrium the negative external effects of production on the environment are not reflected by the market prices.

### 3 Markets of allowances

A solution to this failure of standard markets is given by the Coase Theorem. It advocates the opening of allowance markets on which consumers sells to producers the right to deteriorate the environment against some financial compensation. However the first tentative implementations of the Coase Theorem in a general equilibrium framework by Arrow (2) and Laffont (10) requested the opening of one allowance market per commodity and per couple of agents. On those markets agent  $i$  and  $j$  were supposed to trade the influence on agent  $j$  of the use of the good  $l$  by agent  $i$ . As those authors themselves acknowledge, such a setting is far from realistic. In practice, in the US SO<sub>2</sub> market or in the European Union Emission Trading Scheme, a single market of allowances has been opened whose functioning can be summarized as follows.

Firms are forced by legal means to use as input in their production process a quantity of allowances corresponding to their actual influence on the environment. Namely, in order to produce  $y_j$ , firm  $j$  should use as input a quantity  $f_j(y_j)$  of allowances. Meanwhile the government supplies allowances to the economy by initially allocating a quantity  $A$  among consumers and producers. This leads to the opening of an allowance market on which firms may purchase from the other agents the quantity of allowances they need to set in motion their production plans.

This type of market, where the allowance bears on an externality whose essence is well defined, has been studied by Boyd and al. in (4) and Conley and al. in (6). These authors treat allowances symmetrically to public good and use Lindhal-like personalized prices in order to obtain decentralization results. Focusing on the symmetry with public goods, it seems to us these authors do not take in consideration an important particularity of the allowance market: by fixing the endowment in allowances the government freely supplies a public good to consumers: the difference between this endowment and the situation that prevails under *laissez-faire*. This particular way of providing public goods partly relax the free-riding problems. Hence, one may obtain decentralization results for simple market mechanisms, and it is not necessary to introduce personalized prices.

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<sup>4</sup> What one can also check by comparing the first order necessary conditions for equilibria with the characterization of Pareto optima given in Lemma 1.

## 4 Private Equilibria

Indeed, let us consider the simplest situation where the allowance is exchanged only as a private good among producers. The associated equilibrium concept is:

**Definition 3** *An allocation  $((\bar{x}_i), (\bar{y}_j, f_j(\bar{y}_j)))$  in  $(\mathbb{R}_{++}^L)^m \times \prod_{j=1}^n (Y_j \times \mathbb{R}_-)$  is a marginal pricing equilibrium with transfers and private use of the allowance <sup>5</sup> if there exist a price  $(\bar{p}, \bar{q}) \in \mathbb{R}_+^{L+1}$  and a wealth allocation  $(w_1, \dots, w_m) \in \mathbb{R}^m$  with  $\sum_{i=1}^m w_i = p \cdot (\omega + \sum_{j=1}^n \bar{y}_j)$  such that:*

- (1) *For every  $i$ ,  $\bar{x}_i$  maximizes  $u_i(\cdot, \sum_{j=1}^n f_j(\bar{y}_j))$  in the budget set  $\{x_i \in \mathbb{R}_{++}^L \mid p \cdot x_i \leq w_i\}$ ;*
- (2) *For all  $j$ ,  $\bar{p} + \bar{q} \nabla f_j(\bar{y}_j) \in N_{Y_j}(\bar{y}_j)$ ;*
- (3)  *$\sum_{i=1}^m \bar{x}_i = \sum_{j=1}^n \bar{y}_j + \omega$ ;*
- (4)  *$\sum_{j=1}^n f_j(\bar{y}_j) + A = 0$ .*

Using the characterization of Pareto optima in Lemma 1, it appears clearly that:

**Proposition 1** *For every Pareto optimum, there exist an initial allocation of allowances which allows to decentralize it as a private marginal pricing equilibrium with transfers.*

**Proof:** *Given the quasi-concavity of the utility function, a sufficient condition for an  $\bar{x}_i$  satisfying the budget constraint to solve the consumer problem at a private marginal pricing equilibrium is that there exist  $q_i \in \mathbb{R}$  such that  $(q_i, \bar{p}) \in N_{P_i(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))}(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))$ . This condition as well as (2) and (3) are clearly satisfied at a Pareto optimum according to Theorem 1. Hence given a Pareto optimum, it suffices to choose a wealth allocation letting each consumption plan satisfying the budget constraint and an initial allocation of allowances equal to the firms demand, in order to decentralize it as a marginal pricing equilibrium with transfers and private use of the allowance.  $\square$*

Hence whenever the environment acquires (through the allowance market) a value, Pareto optima may be decentralized as marginal pricing equilibria. Nevertheless there remains a huge indeterminacy on the allocations of allowances which entail Pareto optimality. *A priori* the probability to reach a Pareto optima is rather small, so to say, negligible: when the set of equilibria is a non-empty differentiable manifold, the optimality condition  $\sum_{i=1}^m q_i = q$  implies the set of Pareto optimal equilibria is a submanifold of codimension 1. To overcome this indeterminacy, we shall study refined notions of equilibria.

<sup>5</sup> which we will refer to as private marginal pricing equilibrium for short.

## 5 Public Equilibria

First, let us determine to which extent the opening of the allowance market to public use by the consumers can diminish the number of non-optimal equilibria. When the allowance market is opened to consumers they may purchase it as a public good in order to improve the state of the environment and their program is turned to maximize, given the other agents purchase of allowances  $(\bar{s}_k)_{k \neq i}$ , the utility  $u_i(x_i, -A + \sum_{k \neq i} \bar{s}_k + s_i)$  they get from the consumption bundle  $(x_i, s_i) \in \mathbb{R}_{++}^L \times \mathbb{R}_+$  of regular commodities and allowances. Accordingly the equilibrium concept is turned to:

**Definition 4** *An allocation  $(\bar{x}_i, \bar{s}_i), (\bar{y}_j, f_j(\bar{y}_j))$  in  $(\mathbb{R}_{++}^L \times \mathbb{R}_+)^m \times \prod_{j=1}^n (Y_j \times \mathbb{R}_-)$  is a public marginal pricing equilibrium with transfers <sup>6</sup> if there exist a price  $(\bar{p}, \bar{q}) \in \mathbb{R}_+^{L+1}$  and a wealth allocation  $(w_1, \dots, w_m) \in \mathbb{R}^m$  with  $\sum_{i=1}^m w_i = (\bar{p}, \bar{q}) \cdot (\omega + \sum_{j=1}^n \bar{y}_j, A + \sum_{j=1}^n f_j(\bar{y}_j))$  such that:*

- (1) *For every  $i$ ,  $(\bar{x}_i, \bar{s}_i)$  maximizes  $u_i(x_i, -A + \sum_{k \neq i} \bar{s}_k + s_i)$  among the feasible consumption plans  $\{(x_i, s_i) \in \mathbb{R}_{++}^L \times \mathbb{R}_+ \mid \bar{p} \cdot x_i + \bar{q} s_i \leq w_i\}$  ;*
- (2) *For all  $j$ ,  $\bar{p} + \bar{q} \nabla f_j(\bar{y}_j) \in N_{Y_j}(\bar{y}_j)$ ;*
- (3)  *$\sum_{i=1}^m \bar{x}_i = \sum_{j=1}^n \bar{y}_j + \omega$ ;*
- (4)  *$\sum_{i=1}^m \bar{s}_i = \sum_{j=1}^n f_j(\bar{y}_j) + A$ .*

At such a public equilibrium, the first-order conditions for the consumers program are given by the following Lemma:

**Lemma 1** *Let  $\bar{x}_i \in \mathbb{R}_{++}^L$  :*

- *The bundle  $(\bar{x}_i, \bar{s}_i)$  with  $\bar{s}_i > 0$  solves the consumer problem if and only if  $(\bar{p}, \bar{q}) \cdot (\bar{x}_i, \bar{s}_i) = w_i$  and  $(\bar{p}, \bar{q}) \in N_{P_i(\bar{x}_i, -A + \sum_{i=1}^m \bar{s}_i)}(\bar{x}_i, -A + \sum_{i=1}^m \bar{s}_i)$ .*
- *The bundle  $(\bar{x}_i, 0)$  solves consumer  $i$  program if and only if  $\bar{p} \cdot \bar{x}_i = w_i$  and there exist  $\bar{q}_i \leq \bar{q}$  such that  $(\bar{p}, \bar{q}_i) \in N_{P_i(\bar{x}_i, -A + \sum_{i=1}^m \bar{s}_i)}(\bar{x}_i, -A + \sum_{i=1}^m \bar{s}_i)$ .*

**Proof:** *Given the quasi-concavity of the utility function, first-order conditions are necessary and sufficient. Moreover, non-satiation implies the budgetary constraint is necessarily binding.*

*Now at a bundle  $(\bar{x}_i, \bar{s}_i) \in \mathbb{R}_{++}^{L+1}$ , no other constraint than the budgetary one may be binding so that the bundle is optimal if and only if  $(\bar{p}, \bar{q}) \cdot (\bar{x}_i, \bar{s}_i) = w_i$  and  $(\bar{p}, \bar{q}) \in N_{P_i(\bar{x}_i, -A + \sum_{i=1}^m \bar{s}_i)}(\bar{x}_i, -A + \sum_{i=1}^m \bar{s}_i)$ .*

*At a bundle  $(\bar{x}_i, 0)$  the constraint  $\bar{s}_i \geq 0$  is binding so that it is sufficient and necessary for optimality that  $\bar{p} \cdot \bar{x}_i = w_i$  and that there exist  $\mu_i \geq 0$  such that  $(\bar{p}, \bar{q}) \in N_{P_i(\bar{x}_i, -A + \sum_{i=1}^m \bar{s}_i)}(\bar{x}_i, -A + \sum_{i=1}^m \bar{s}_i) + \mu_i(0, 1)$ .*

<sup>6</sup> *in extenso:* marginal pricing equilibrium with transfers and public use of the allowances



The proof then proceeds easily.

This characterization yields that every Pareto optimum can be decentralized as a public equilibrium:

**Proposition 2** *For every Pareto optimum, there exist an initial allocation of allowances which allows to decentralize it as a public marginal pricing equilibrium with transfers.*

**Proof:** Let  $(\bar{x}_i), (\bar{y}_j)$  be a Pareto optimal allocation. According to Proposition 1 there exist a price  $(\bar{p}, \bar{q})$  and a wealth distribution  $(w_i)$  such that it can be decentralized as a private equilibrium. In order to show, that it may also be decentralized as a public equilibrium, it suffices to show that  $(\bar{x}_i, 0)$  solves the consumer program at a public equilibrium.

Now, from the proof of Proposition 1 one knows that there exist  $\bar{q}_i \in \mathbb{R}$  such that  $(\bar{p}, \bar{q}_i) \in N_{P_i(\sum_{j=1}^n f_j(\bar{y}_j), \bar{x}_i)}(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))$ . Using monotonicity of the utility functions, one in fact has  $\bar{q}_i \geq 0$  for all  $i$ . Using Pareto optimality of the equilibrium, one has  $\sum_{i=1}^m \bar{q}_i = \bar{q}$ , and hence  $\bar{q}_i \leq \bar{q}$ . Hence for all  $i$  there exist  $q_i := \bar{q}_i \leq \bar{q}$  such that  $(\bar{p}, q_i) \in N_{P_i(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))}(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))$ . Using clearance of the allowance market, and the fact that  $\bar{p} \cdot \bar{x}_i = w_i$ , sufficient condition for  $(\bar{x}_i, 0)$  to solve the consumers program are satisfied according to Lemma 1. The proof then proceeds as for Proposition 1.  $\square$

Moreover, Lemma 1 also provides a testable necessary condition for optimality. This condition already underlined by Smith and al. in (11) is that at an optimum, consumers are priced out of the allowance market. Indeed at an optimum the allowance price must be equal to the sum of marginal utilities for the environment and hence greater than each of these marginal utilities taken individually. For the consumer, such a situation is acceptable only if he has no allowances left to sale. In other words, a public equilibrium is Pareto optimal only if none of the consumers actually purchase allowances. Namely, one has

**Proposition 3** *A public marginal pricing equilibrium  $(\bar{x}_i, \bar{s}_i), (\bar{y}_j, f_j(\bar{y}_j))$  is Pareto optimal only if for all  $i$ ,  $\bar{s}_i = 0$ .*

**Proof:** Let  $(\bar{x}_i, \bar{s}_i), (\bar{y}_j, f_j(\bar{y}_j))$  be a public marginal pricing equilibrium and  $(\bar{p}, \bar{q})$  the corresponding equilibrium price. Let us assume that one of the  $s_i$  is positive. It implies according to Lemma 1 that  $(\bar{p}, \bar{q}) \in N_{P_i(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))}(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))$ . Now, the regularity of the utility functions imply that  $N_{P_i(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))}(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))$  is a half line whenever  $(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j)) \in \mathbb{R}_{++}^L \times \mathbb{R}$ . In other words, given  $\bar{p}$ , for all  $i$  there exist a single  $\bar{q}_i$ , such that  $(\bar{p}, \bar{q}_i) \in N_{P_i(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))}(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))$ . The strict monotonicity of the utility functions with regards to the environment imply that all those  $q_i$  are positive. According to the preceding, one of those is equal to  $\bar{q}$ . Therefore, one necessarily

has  $\sum_{i=1}^m q_i > q$  and the equilibrium can not be Pareto optimal.  $\square$

Paradoxically, the interest of opening the allowance market to consumers is to detect they do not participate in it.

On another hand, the set of public equilibria is a better approximation of the set of Pareto optima than the set of private equilibria as there are less public than private equilibria. Indeed while one can associate to every public equilibrium a private equilibrium by subtracting to every consumer endowment in allowances the amount it purchases at equilibrium, one can associate to a private equilibrium a public one only if at this private equilibrium every consumer marginal utility for the environment is lower than the allowance price.

To sum up, the opening of the allowance market to the consumers withdraw part of the indeterminacy on the optimality properties of the equilibria and provides a testable necessary condition for optimality, that consumers are priced out of the allowance market.

## 6 Subsidized Equilibria

Pursuing in the direction of refinement through public use of the allowance, let us now consider situations where the government amplifies the consumers demand in allowances by diminishing proportionally to the consumers purchase the level of allowances it supplies to the economy. Namely one considers  $(k_i)$ -amplified<sup>7</sup> equilibria at which the government announces a diminution of  $(k_i - 1)s_i$  of its supply of allowances whenever consumer  $i$  purchases  $s_i$  allowances. Therefore each consumer considers that when it purchases  $s_i$  allowances, the state of the environment is improved of  $k_i s_i$ .

However, consumers may anticipate that the amplification of their purchases by the government leads to a diminution of their own initial endowment in allowances and hence of their wealth. Taking this fact in consideration they may strategically reduce their purchase of allowances. Such a failure may be easily overcome. It suffices that the government announces it will amplify one agent purchase by diminishing only the other agents initial endowments. Such a mechanism can be related to the matching process described by Guttman in (9).

One can then define a  $(k_i)$ -amplified equilibrium as:

**Definition 5** *An allocation  $(\bar{x}_i, \bar{s}_i), (\bar{y}_j, f_j(\bar{y}_j))$  in  $(\mathbb{R}_{++}^L \times \mathbb{R}_+)^m \times \prod_{j=1}^n (Y_j \times$*

<sup>7</sup> In the following  $k_i > 1$

$\mathbb{R}_-$ ) is a  $(k_i)$ -amplified public marginal pricing equilibrium with transfers<sup>8</sup> if there exist a price  $(\bar{p}, \bar{q}) \in \mathbb{R}_+^{L+1}$  and a wealth allocation  $(w_1, \dots, w_m) \in \mathbb{R}^m$  with  $\sum_{i=1}^m w_i = (\bar{p}, \bar{q}) \cdot (\omega + \sum_{j=1}^n \bar{y}_j, A - \sum_{i=1}^m (k_i - 1)\bar{s}_i + \sum_{j=1}^n f_j(\bar{y}_j))$  such that:

- (1) For every  $i$ ,  $(\bar{x}_i, \bar{s}_i)$  maximizes  $u_i(x_i, -A + \sum_{h \neq i} k_h s_h + k_i s_i)$  among the feasible consumption plans  $\{(x_i, s_i) \in \mathbb{R}_+ \times \mathbb{R}_{++}^L \mid \bar{q}s_i + \bar{p} \cdot x_i \leq w_i\}$ ;
- (2) For all  $j$ ,  $\bar{p} + \bar{q}\nabla f_j(\bar{y}_j) \in N_{Y_j}(\bar{y}_j)$ ;
- (3)  $\sum_{i=1}^m \bar{x}_i = \sum_{j=1}^n \bar{y}_j + \omega$ ;
- (4)  $\sum_{i=1}^m \bar{s}_i = \sum_{j=1}^n f_j(\bar{y}_j) + A - \sum_{i=1}^m (k_i - 1)\bar{s}_i$ ;

At a  $(k_i)$ -equilibrium, the first-order conditions for the consumers program are given by the following Lemma:

**Lemma 2** Let  $\bar{x}_i \in \mathbb{R}_{++}^L$  :

- The bundle  $(\bar{x}_i, \bar{s}_i)$  with  $\bar{s}_i > 0$  solves the consumer problem if and only if  $(\bar{p}, \bar{q}) \cdot (\bar{x}_i, \bar{s}_i) = w_i$  and  $(\bar{p}, \frac{\bar{q}}{k_i}) \in N_{P_i(\bar{x}_i, -A + \sum_{i=1}^m k_i \bar{s}_i)}(\bar{x}_i, -A + \sum_{i=1}^m k_i \bar{s}_i)$ .
- The bundle  $(\bar{x}_i, 0)$  solves consumer  $i$  program if and only if  $\bar{p} \cdot \bar{x}_i = w_i$  and there exist  $\bar{q}_i \leq \bar{q}$  such that  $(\bar{p}, \frac{\bar{q}_i}{k_i}) \in N_{P_i(\bar{x}_i, -A + \sum_{i=1}^m k_i \bar{s}_i)}(\bar{x}_i, -A + \sum_{i=1}^m k_i \bar{s}_i)$ .

**Proof:** The proof is similar to this of Proposition 1 but for the  $\frac{1}{k_i}$  coefficient whose presence is due to the amplification of the consumers purchases.

This characterization yields that every Pareto optimum at which consumer  $i$  marginal utility for the environment<sup>9</sup>,  $\bar{q}_i$ , is lower than  $\frac{\bar{q}}{k_i}$  can be decentralized as a public equilibrium:

**Proposition 4** For every Pareto optimum at which consumer  $i$  marginal disutility,  $\bar{q}_i$ , is lower than  $\frac{\bar{q}}{k_i}$ , there exist an initial allocation of allowances which allows to decentralize it as a  $(k_i)$ -amplified equilibrium.

**Proof:** The proof is similar to this of Proposition 2 : one considers the corresponding private equilibrium and show that the necessary conditions for the consumers program given by Lemma 2 are satisfied when each consumer actually purchase 0 allowances.

While  $\sum_{i=1}^m \frac{1}{k_i}$  is greater than 1, one has a testable condition for optimality analogous to this given by Proposition 3, an amplified equilibrium is optimal only if at least one consumer is priced out of the allowance market:

**Proposition 5** Let  $(k_i)$  such that  $\sum_{i=1}^m \frac{1}{k_i} > 1$ , a  $(k_i)$ -equilibrium  $(\bar{x}_i, \bar{s}_i), (\bar{y}_j, f_j(\bar{y}_j))$  is Pareto optimal only if there exist  $i$  such that  $s_i = 0$ .

<sup>8</sup> *in extenso*: a marginal pricing equilibrium with transfers and  $k$  times amplification of the public use of allowances

<sup>9</sup> Here  $\bar{q}_i$  and  $\bar{q}$  are the elements given by Theorem 1.

**Proof:** *The proof is similar to this of Proposition 3.*

**Remark 1** *In fact, the preceding can be strengthened to: if  $I_1$  is a subset of consumers such that  $\sum_{i \in I_1} \frac{1}{k_i} > 1$ , it can not be that every agent in  $I_1$  purchases a positive level of allowances.*

According to Lemma 2, as  $\sum_{i=1}^m \frac{1}{k_i}$  decreases towards 1, the constraint bearing on the consumers allowances choices become tighter and tighter (as every agent marginal utility must be at least  $k_i$  times smaller than the allowance price). There are fewer and fewer equilibria and consequently the set of equilibria surround more and more closely the set of Pareto optima.

When  $\sum_{i=1}^m \frac{1}{k_i}$  reaches 1, the  $(k_i)$ -amplified equilibria finally satisfy sufficient conditions for optimality in the sense of:

**Proposition 6** *Let  $(k_i)$  such that  $\sum_{i=1}^m \frac{1}{k_i} = 1$ . At a  $(k_i)$ -amplified equilibrium  $(\bar{x}_i, \bar{s}_i), (\bar{y}_j, f_j(\bar{y}_j))$  such that for all  $i, \bar{s}_i > 0$ , the necessary conditions of Theorem 1 hold. If moreover, the production sets and the environmental damages functions are convex, such an equilibrium is Pareto optimal.*

**Proof:** *Indeed according to Lemma 2, at such an equilibrium one has  $(\bar{p}, \frac{\bar{q}}{k_i}) \in N_{P_i(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))}(\bar{x}_i, \sum_{j=1}^n f_j(\bar{y}_j))$ . Together with the other equilibrium conditions, it implies the necessary conditions of Theorem 1 are satisfied. As those conditions are sufficient for optimality under the additional convexity assumptions on the production, the proof is complete.*

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