A Tool for Aggregation with Words

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Abstract

The need of computing with words has become an important topic in many areas dealing with vague information. The aim of this paper is to present different tools which support computing with words. Especially, we are concerned with the weighted aggregation of linguistic term sets, without using fuzzy concepts.

We propose a new aggregation operator, referred to as the *symbolic weighted median* that computes the most representative element from an ordered collection of weighted linguistic terms. This operator aggregates the linguistic labels such that its result is expressed in terms of the initial linguistic term set though is modified by using dedicated tools called the *generalized symbolic modifiers*. One advantage of this proposal is that the expression domain does not change: we increase or decrease the granularity only where it becomes necessary. Additionally this new operator exhibits several interesting mathematical properties.

Key words: Linguistic tools, symbolic modifiers, decision making, median aggregation operator, weighted information

1 Introduction

The problem raised in this paper is the weighted aggregation of linguistic statements [2,18–20]. It is a part of the Computing with Words (CW) paradigm

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proposed by Zadeh [21] and recently discussed e.g., in [22]. The fuzzy logic framework and especially fuzzy sets themselves [21] that underlie CW are not always easy to obtain from the linguistic term sets. That is why we choose to keep the words themselves — called linguistic symbols — without going through a fuzzy modeling.

An important point in the CW is the granularity of information that allows for a better approximation of the concepts when it is needed [6]. In [1,14] we introduced *linguistic modifiers* that offer refinements of a linguistic symbol. In such a way, data can be represented at the most appropriate level of precision. Linguistic modifiers associate linguistic terms with functions.

In this paper, we focus interest on the mean, the median [20] and others operators for linguistic information [7,8]. As a result of the aggregation of linguistic symbols, the aim of the approach is to obtain a linguistic symbol, which *more or less* resembles another symbol coming from the initial set. The resemblance is expressed by means of linguistic modifiers and the proposed process allows for the use of the same linguistic term set by being only extended by these modifiers. This approach is quite convenient and interesting since experts, users, decision makers involved in the problem defined in the linguistic framework do not have to deal with new terms nor with an artificial expression domain.

The paper is organized as follows: in Section 2, we focus on some interesting operators that deal with weighted linguistic information. We also present the basic aggregation operators like means or medians. In Section 3, we then introduce our tools which allow us to modify values in a linguistic context: the *generalized symbolic modifiers*. Section 4 details our proposal, i.e. a *median for weighted linguistic values*. Finally, Section 5 concludes this study.

2 Existing aggregation operators

2.1 Operators for weighted linguistic information

Some authors like Herrera & Herrera–Viedma have proposed aggregation operators dealing with weighted linguistic information [7]. These operators are useful when there are various information sources providing linguistic information that is not equally relevant. The authors propose three aggregation operators: the linguistic weighted disjunction (LWD), the linguistic weighted conjunction (LWC), and the linguistic weighted averaging (LWA). According to them, aggregation comprises two operations: (1) the aggregation of weights and (2) the aggregation of information combined with the weights. To accom-

plish step (1) different operations such as LWD, LWC and LWA operators based on the LOWA (linguistic ordered weighted averaging) operator [11] can be used. For step (2), they propose a different function for each aggregation operator based on a min, max or a LOWA operator.

Another aggregation operator has been introduced in [10,13] in order to deal with multiple linguistic scales. The result of the aggregation is determined by using linguistic hierarchies and their computational model. A linguistic hierarchy is a set of levels, where each level is a linguistic term set coming at a certain level of granularity. The authors also introduced the concept of *2-tuple* [12] composed of a linguistic term in a certain hierarchy and a *symbolic translation* that mathematically expresses a reinforcement or a weakening of the term. The linguistic information (converted into 2-tuples) is aggregated using an arithmetic means that gives a new 2-tuple [9,12]. It is to notice that in [10] they consider the use of linguistic term sets that are *non uniformly distributed* on the given scale.

Other authors like Valls & Torra use clustering techniques to aggregate data [17]. They consider heterogeneous data, often involved in multi criteria decision making, and propose a method to classify the alternatives according to criteria. The authors study each alternative in relation to the others. They give a result in linguistic terms as defined by one of the experts. We will see that the approach proposed in this paper also aims at giving a linguistic answer obtained from a dictionary.

2.2 Means and medians

Let us present now the three usual forms of the median: let $x_1, x_2 \dots x_n$ be n arguments, with $x_1 < x_2 < \dots < x_n$. The pessimistic (i), optimistic (ii) and "middle" (iii) medians $\mathcal{A}$ are defined as:

$$\text{(i)}\ \ \mathcal{A}(x_1, x_2, \dots, x_n) = \begin{cases} x_{\frac{n+1}{2}} & \text{if } n \text{ is odd} \\ x_{\frac{n}{2}} & \text{if } n \text{ is even} \end{cases}$$

$$\text{(ii)}\ \ \mathcal{A}(x_1, x_2, \dots, x_n) = \begin{cases} x_{\frac{n+1}{2}} & \text{if } n \text{ is odd} \\ x_{\frac{n}{2}+1} & \text{if } n \text{ is even} \end{cases}$$

$$\text{(iii)}\ \ \mathcal{A}(x_1, x_2, \dots, x_n) = \begin{cases} x_{\frac{n+1}{2}} & \text{if } n \text{ is odd} \\ \frac{1}{2}(x_{\frac{n}{2}} + x_{\frac{n}{2}+1}) & \text{if } n \text{ is even} \end{cases}$$

Considering that the elements may not always be equally important, Yager has proposed a weighted median with the condition that the weights are ordered before the computation [20]:

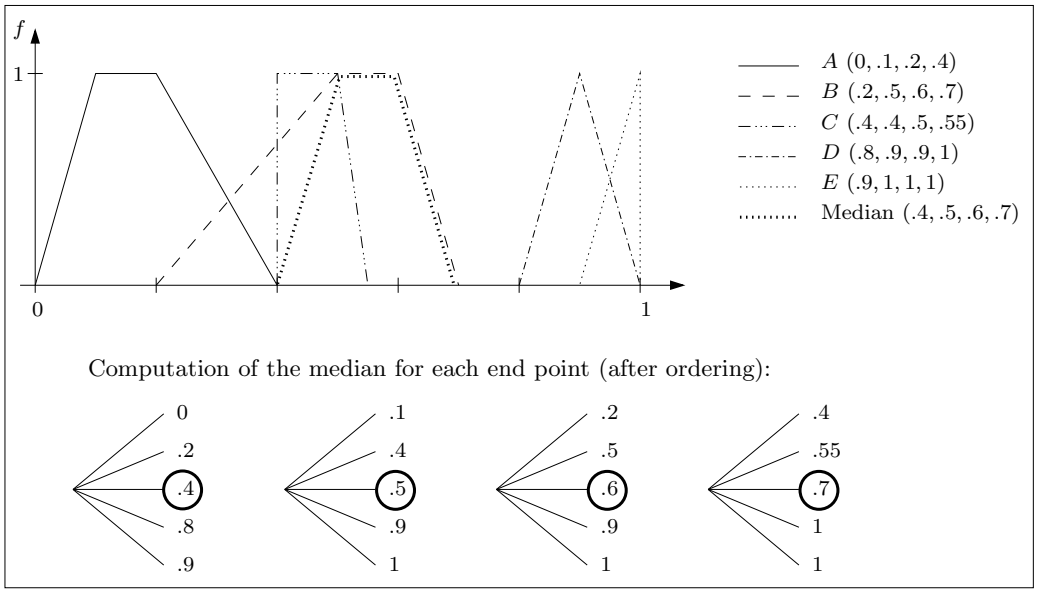

Fig. 1. Median for fuzzy subsets.

Let $(x_1, w_1), (x_2, w_2), \ldots, (x_n, w_n)$ be the elements to aggregate, with $w_i \in [0, 1]$ and $\sum w_i = 1$. Let $T_j = \sum_{j=1}^{i} w_j$ be the sum of the first i weights. The weighted median is x_k where k is defined by: $T_{k-1} < 0.5$ and $T_k \geq 0.5$.

However, operators such as medians are usually used to aggregate numbers. Aggregating fuzzy numbers (representing linguistic statements) with a median is very interesting. Let us suppose that the information is represented by means of trapezoidal fuzzy subsets. They can be characterized by using the *end points* of their core and support (cf. Figure 1). We propose to compute one median per type of end point, i.e. we obtain four medians. The end points of the same kind (i.e. left support limits, or left core limits...) are grouped together and ordered (cf. definition of the median).

Figure 1 shows an example where the median is not equivalent to an initial subset: we can say that the result is composed of "some" B and "a little" C. The problem is now about a suitable representation of this median. Applying fuzzy modifiers [3–5] on the initial subsets shall provide good results.

In this paper, the approach is similar to that one but the median deals with linguistic symbols directly, not with numeric values or fuzzy numbers. The median we define is expressed by the initial symbols after having applied a certain modification to them. This modification is performed by specific tools defined in [1] that is the *generalized symbolic modifiers*.

3 Generalized symbolic modifiers

The truth of a proposition can be evaluated by means of adverbs that are represented on a scale of linguistic degrees or linguistic symbols. In [1], we have proposed tools to combine such degrees. In particular these tools are useful to measure differences between linguistic symbols. They allow us to express the *modification* that a linguistic symbol must undergo to resemble or to become another linguistic symbol: they are called the *linguistic modifiers* or *generalized symbolic modifiers*. Only one condition must be satisfied: the linguistic symbols have to be totally ordered.

A generalized symbolic modifier (GSM) is a mapping from an initial pair (a, b) to a new pair (a', b'). A pair is composed of a symbol a (also called degree) and an integer b (corresponding to the total number of symbols). Using a certain radius ρ — considered as a strength — the new pair is more or less close to the initial pair: the higher the radius, the less the pairs are close. The position of a degree a in a scale is denoted $p(a)$, with $p(a) \in \mathbb{N}$. A general definition of a GSM is the following:

Definition 1 *Let $\mathcal{L}_b$ be a collection of b linguistic terms, with $b \in \mathbb{N}^* \setminus \{1\}$. A GSM m_ρ is defined as:*

$$m_\rho : \mathcal{L}_b \to \mathcal{L}_{b'}$$

$$a \mapsto a'$$

i.e. $m_\rho(a) = a'$, with $b' \in \mathbb{N}^* \setminus \{1\}$, $p(a) < b$, $p(a') < b'$ and $\rho \in \mathbb{N}^*$.

A proportion or an intensity rate is associated with each linguistic degree on the considered scale; this rate is expressed as the ratio $\text{Prop}(a) = \dfrac{p(a)}{b-1}$

For example, if we consider a collection $\mathcal{L}_5$ with $\mathcal{L}_5$={"very bad"; "bad"; "average"; "good"; "very good"}, then Prop("very bad") = 0.

Comparing the proportions between $\text{Prop}(a)$ and $\text{Prop}(a')$, we will define three families of modifiers: weakening, reinforcing and central modifiers. The definitions of the weakening and reinforcing GSMs are given in Table 1 and EC$'$ and DC$'$ are central GSMs recalled in definitions 2 and 3 [16].

Reinforcing and weakening GSMs increase or decrease the Prop of the initial pair while central GSMs (EC, DC, EC$'$ and DC$'$) act like a zoom on the initial pair, keeping the Prop unchanged. There is a link between EC and DC that erode or dilate the scale, and the 2-tuples (and the linguistic hierarchies) of Herrera & Martínez [12,13] which also offer a multigranular context when representing the knowledge. Indeed it is possible to define EC and DC with 2-tuples. An example of the usefulness of central GSMs becomes apparent when

Table 1. Definitions of weakening and reinforcing GSMs.

MODE / **NATURE**	**Weakening**	**Reinforcing**
Erosion	$p(a') = \max(0, p(a) - \rho)$ $b' = \max(2, b - \rho)$ **EW**(ρ)	$p(a') = p(a)$ $b' = \max(p(a) + 1, b - \rho)$ **ER**(ρ)
		$p(a') = \min(p(a) + \rho, b - \rho - 1)$ $b' = \max(1, b - \rho)$ **ER′**(ρ)
Dilation	$p(a') = p(a)$ $b' = b + \rho$ **DW**(ρ)	$p(a') = p(a) + \rho$ $b' = b + \rho$ **DR**(ρ)
	$p(a') = \max(0, p(a) - \rho)$ $b' = b + \rho$ **DW′**(ρ)	
Conservation	$p(a') = \max(0, p(a) - \rho)$ $b' = b$ **CW**(ρ)	$p(a') = \min(p(a) + \rho, b - 1)$ $b' = b$ **CR**(ρ)

a teacher has to switch from a certain scale of marks to another one.

Definition 2 *Let (a, b) be a pair and $\rho \in \mathbb{N}^* \setminus \{1\}$. The GSM* $\mathrm{EC}'(\rho)$ *gives a new pair (a', b'), such that:*

$$p(a') = \begin{cases} \dfrac{p(a)}{b-1}\left(\dfrac{b}{\rho} - 1\right) & \text{if} \quad \dfrac{p(a)}{b-1}\left(\dfrac{b}{\rho} - 1\right) \in \mathbb{N} \\ \text{or} \quad \left\lfloor \dfrac{p(a)}{b-1}\left(\dfrac{b}{\rho} - 1\right) \right\rfloor \ \textit{(pessimistic)} & \\ \text{or} \quad \left\lfloor \dfrac{p(a)}{b-1}\left(\dfrac{b}{\rho} - 1\right) \right\rfloor + 1 \ \textit{(optimistic)} & \text{otherwise} \end{cases}$$

$$b' = \begin{cases} \dfrac{b}{\rho} & \text{if} \quad \dfrac{b}{\rho} \in \mathbb{N} \\ \text{or} \quad \left\lfloor \dfrac{b}{\rho} \right\rfloor \ \textit{(pessimistic)} & \\ \text{or} \quad \left\lfloor \dfrac{b}{\rho} \right\rfloor + 1 \ \textit{(optimistic)} & \text{otherwise} \end{cases}$$

Definition 3 *Let (a, b) be a pair and $\rho \in \mathbb{N}^* \setminus \{1\}$. The GSM* $\mathrm{DC}'(\rho)$ *gives a new pair (a', b'), such that:*

$$p(a') = \begin{cases} a\left(\dfrac{b\rho - 1}{b-1}\right) & \text{if} \quad \dfrac{b\rho - 1}{b-1} \in \mathbb{N} \\ \text{or} \quad \left\lfloor a\left(\dfrac{b\rho - 1}{b-1}\right) \right\rfloor \ \textit{(pessimistic)} & \\ \text{or} \quad \left\lfloor a\left(\dfrac{b\rho - 1}{b-1}\right) \right\rfloor + 1 \ \textit{(optimistic)} & \text{otherwise} \end{cases}$$

$$b' = b\rho$$

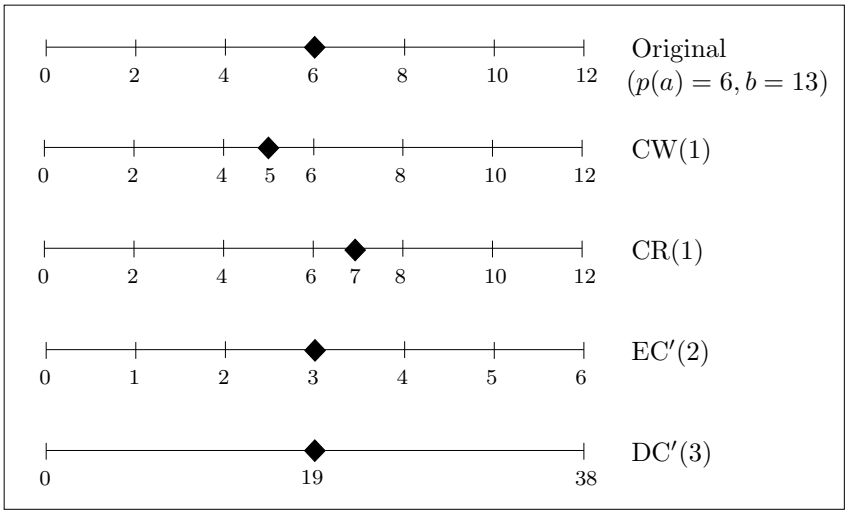

Fig. 2. Examples of GSMs.

Figure 2 shows examples of GSMs. For instance, if 6 corresponds to the symbol "interesting", then applying a CW(1) it results in the expression "a bit less than interesting". Applying EC′(2) can produce "more or less interesting" and when applying a DC′(3) we obtain "very precisely interesting".

The proportions computed for initial and final degrees allow us for a comparison between the GSMs. Let $(a, b), (a'_1, b'_1)$ and (a'_2, b'_2) be an initial pair and two modified pairs obtained using GSMs $m_{1,\rho}$ and $m_{2,\rho}$, respectively. For a given ρ and for any pair (a, b), if $\text{Prop}(a'_1) < \text{Prop}(a'_2)$ then $m_{1,\rho}$ is *weaker* than $m_{2,\rho}$. The GSMs are thus ordered and a lattice can be established [16] (cf. Figure 3).

Another interesting result concerns the *composition* of the GSMs [14]. For example, composing a modifier ER with a modifier DR consists in applying (on an initial pair) first a modifier DR and then a modifier ER. Two kinds of compositions have to be distinguished: *homogeneous* and *heterogeneous* ones. Homogeneous compositions are compositions of modifiers from the same family with the same nature, same mode, same name but not necessarily the same radius. Any other form of composition is heterogeneous, including compositions of GSMs from different families. These compositions can reach any degree on any scale if necessary. Moreover, the linguistic counterpart can be expressed as combinations of adverbs, such as "very very" corresponding to $\text{CR}(\rho) \circ \text{CR}(\rho)$. The following two theorems can be then proved easily [14]:

Theorem 1 *The result of the composition of generalized symbolic modifiers is also a generalized symbolic modifier: when composing n generalized symbolic modifiers (of any kind), a valid pair degree/scale is always obtained.*

Theorem 2 *If m_{ρ_1} is any weakening or reinforcing GSM with a radius ρ_1, and m_{ρ_2} is any GSM of the same family than m_{ρ_1} with a radius ρ_2, ... and*

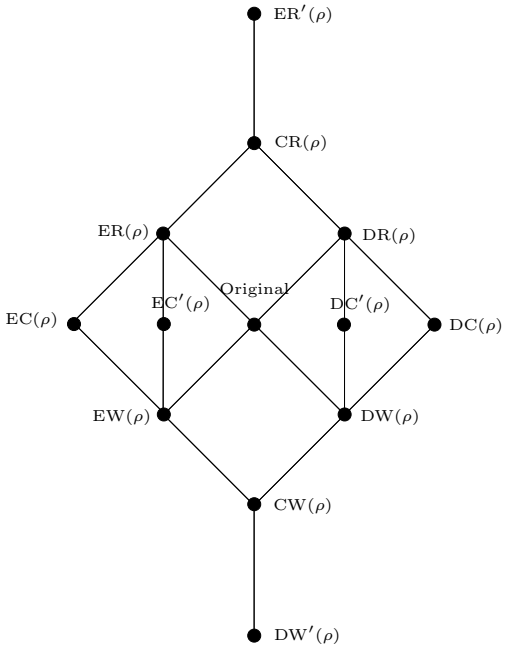

Fig. 3. Lattice for the GSMs.

m_{ρ_n} is any GSM of the same family than m_{ρ_1} with a radius ρ_n, then $m_{\rho_s} = m_{\rho_1} \circ m_{\rho_2} \circ \ldots \circ m_{\rho_n}$ is a GSM of the same mode than m_{ρ_1}, with a radius ρ_s equal to the sum of the radii.

For example, $\forall \rho_1, \rho_2, \ldots \rho_n \in \mathbb{N}^*$, $\mathrm{DR}(\rho_1) \circ \mathrm{DR}(\rho_2) \circ \ldots \circ \mathrm{DR}(\rho_n) = \mathrm{DR}(\rho_1 + \rho_2 + \ldots + \rho_n)$.

4 A new aggregation operator for linguistic weighted terms

4.1 Definition

Let us consider the problem of a questionnaire with weighted answers obtained through an opinion poll.

Definition 4 *Let $\mathcal{L}_b = \{a_{0,b-1}, a_{1,b-1}, \ldots, a_{b-1,b-1}\}$ be a collection of b ordered elements a_i. The collection of b weighted ordered elements is denoted $\langle a_{0,b-1}^{w_0}, a_{1,b-1}^{w_1}, \ldots, a_{b-1,b-1}^{w_{b-1}} \rangle \in \mathcal{B}^{\mathcal{L}_b}$ (set of these collections) such that $\sum w_i = 1$.*

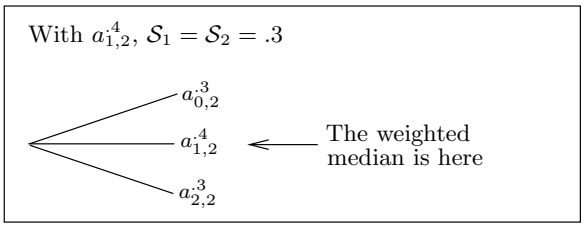

Fig. 4. Median: first example.

The symbolic weighted median $\mathcal{M}$ *is defined as:*

$$
\begin{aligned}
\mathcal{M}: \quad \mathcal{B}^{\mathcal{L}_b} &\to \mathcal{L}_{b'} \\
\langle a^{w_0}_{0,b-1}, a^{w_1}_{1,b-1}, \ldots, a^{w_{b-1}}_{b-1,b-1} \rangle &\mapsto \mathcal{M}(\langle a^{w_0}_{0,b-1}, a^{w_1}_{1,b-1}, \ldots, a^{w_{b-1}}_{b-1,b-1} \rangle) \\
&= a'^{w'_i}_{i,b'-1} \textit{ such that: } \left| \sum_{p=0}^{i-1} w'_p - \sum_{p=i+1}^{b'-1} w'_p \right| < \varepsilon \\
&= m(a^{w_j}_{j,b-1}) \textit{ with } w_j = 1 \\
&= m(a_{j,b-1})
\end{aligned}
$$

with $m(a_{j,b-1})$ *a GSM applied to an element of the initial collection* $\mathcal{L}_b$. $\sum_{p=0}^{i-1} w'_p$ *(resp.* $\sum_{p=i+1}^{b'-1} w'_p$*) is the sum* $\mathcal{S}_1$ *(resp.* $\mathcal{S}_2$*) of the weights of the elements that are before — remember that the collection is ordered — (resp. after) the element* $a'^{w'_i}_{i,b'-1}$.

Note that $\mathcal{M}$ does not have any weight, as it is the case for the classical aggregation operators.

In order to obtain a correct median (i.e. a small ε or $\varepsilon = 0$), a method is to split the element (with a weight w) into $w * 10$ "sub-elements" with a weight of .1 if w is odd, and into $w * 5$ "sub-elements" with a weight of $w/2$ if w is even. This way, a new collection is obtained and the sums $\mathcal{S}$ can be computed with this new collection. Thus the median is either an initial element (taken directly from $\mathcal{L}_b$) or a *sub-element* [15].

Figure 4 shows a first example. The median $\mathcal{M}$ is an existing element because $\mathcal{S}_1 = \mathcal{S}_2$ (i.e. $\varepsilon = 0$).

In the second example (cf. Figure 5) when computing the sums $\mathcal{S}$ (for each original element), the difference between $\mathcal{S}_1$ and $\mathcal{S}_2$ in both cases is too important (ε would be too high). That is why the division is performed. Equality between the sums $\mathcal{S}$ is obtained and the median is a *sub-element*. We denote $a^4_{1,2}$ the *parent element* of the median $a'^{.2}_{3,5}$.

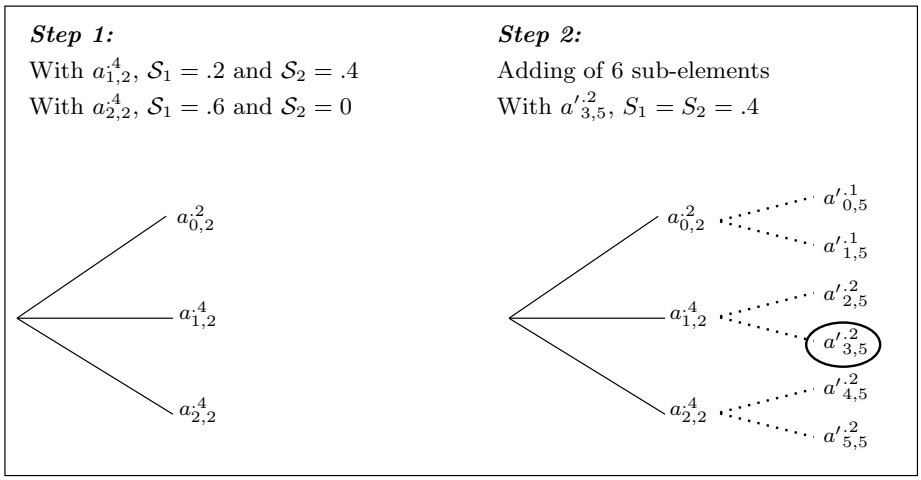

Fig. 5. Median: second example.

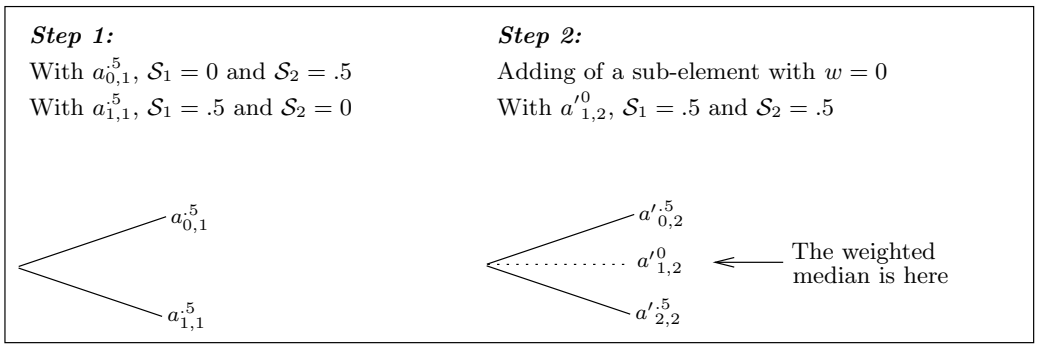

Fig. 6. Median: third example.

Another situation is shown in Figure 6: when computing the sums $\mathcal{S}$ for each initial element, we obtain either $\mathcal{S}_1 = \sum w_i/2 = .5$ (and $\mathcal{S}_2 = 0$) or $\mathcal{S}_2 = \sum w_i/2 = .5$ (and $\mathcal{S}_1 = 0$). In this case, the element supporting the division is a virtuel one, not an initial element. The weights associated to these new sub-elements are equal to zero since they don't correspond to real answers given by people in the opinion poll.

In all cases (except in Figure 4) we can consider that the elements are presented as a tree — an *initial* tree — and the median is presented as an element coming from another tree — a *derived* tree.

The accuracy of the median depends on the value of ε. A compromise has to be made between computation time and accuracy.

4.2 Properties of the symbolic weighted median

The symbolic weighted median satisfies the following properties that were proved in [14]:

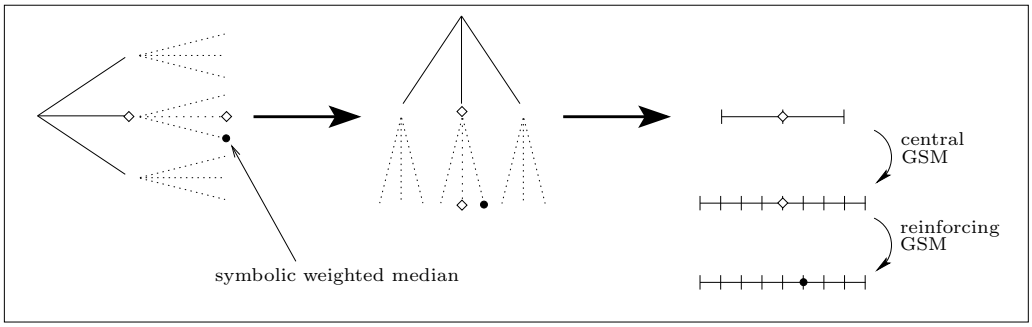

Fig. 7. From the symbolic weighted median towards the GSMs.

- Identity, monotonicity, idempotence, compensation.
- Boundary conditions: this property means that p times the aggregation of the lowest element of the tree is *the element itself*. Similarly, p times the aggregation of the highest element of the tree is *the element itself*.
- Continuity (adapted in this case to discrete elements): when the elements of the tree change slightly, the aggregation operator gives a result slightly different from the original one.
- Counterbalancing: adding weights on leaves placed above the symbolic weighted median on the tree, will decrease the final result. And, conversely, adding weights on leaves below the symbolic weighted median on the tree, will increase the final result.

4.3 Linguistic counterpart of the symbolic weighted median

After we have provided the definition and the algorithm to compute the median, we have to express a linguistic counterpart of the median. Looking carefully at what is done during the computation, we notice that it looks like applying modifiers to one of the initial elements (cf. Figure 7). In the example a central modifier is used, followed by a reinforcing one.

We propose to define which modifier(s) is (are) applied to the initial value when the symbolic weighted median is computed. To do this, two proportions will be considered: proportion PM of the element corresponding to the median and proportion PPM of the element corresponding to the parent element of the median. We denote $PM = \text{Prop}(a'_{i',j'})$ and $PPM = \text{Prop}(a_{i,j})$ where $a'_{i',j'}$ is an element of $\mathcal{L}_{b'}$ representing the weighted symbolic median and where $a_{i,j}$ is an element of $\mathcal{L}_b$ representing the parent element of $a'_{i',j'}$.

In the example shown in Figure 7, $PM = 5/8$ and $PPM = 1/2$. In some other cases, the computation of PPM is not that easy: in Figure 6, for instance, the parent element is either $a'^{.5}_{0,2}$ (pessimistic case) or $a'^{.5}_{2,2}$ (optimistic case).

By using the sign of the difference $PM - PPM$, the correspondence between the

Table 2. Correspondence between the sign of $PM - PPM$ and GSMs.

Sign of $PM - PPM$	**GSM(s) to apply**
< 0	$\mathrm{CW}(\rho_1) \circ \mathrm{DC}'(\rho_2)$
$= 0$	$\mathrm{DC}'(\rho)$
> 0	$\mathrm{CR}(\rho_1) \circ \mathrm{DC}'(\rho_2)$

modifiers and the symbolic weighted median can be carried out (cf. Table 2). The radii are computed using PM and PPM.

In the example shown in Figure 7, $PM - PPM = 1/8$, so the GSMs to apply are $\mathrm{CR}(1) \circ \mathrm{DC}'(3)$.

The last step is to find an adequate linguistic equivalence of the symbolic weighted median.

The idea is to use the GSMs since they can be associated to words, given an dictionary [16]. For example, and according to the context, the GSM $\mathrm{DC}'(2)$ can be associated to the word "precisely" (remember that $\rho = 1$ is not valid for DC') and $\mathrm{DC}'(3)$ to "very precisely", $\mathrm{CR}(1)$ to "a little more than", $\mathrm{CR}(2)$ to "rather more than", $\mathrm{CR}(3)$ to "more than" and $\mathrm{CR}(4)$ to "much more than", $\mathrm{CW}(1)$ to "a little less than", $\mathrm{CW}(2)$ to "rather less than", $\mathrm{CW}(3)$ to "more less" and $\mathrm{CW}(4)$ to "much less than". As the GSMs, words can also be composed with each other: this depends on the application context, on the language, etc. Given the above dictionary, in Figure 5, the median will be a $\mathrm{CR}(\rho_1) \circ \mathrm{DC}'(\rho_2)$ composition, with $\rho_1 = 1$ and $\rho_2 = 2$. Linguistically, the answer will be *"precisely a little more than* $a^4_{1,2}$*"*.

Figure 8 summarizes the construction of the symbolic weighted median.

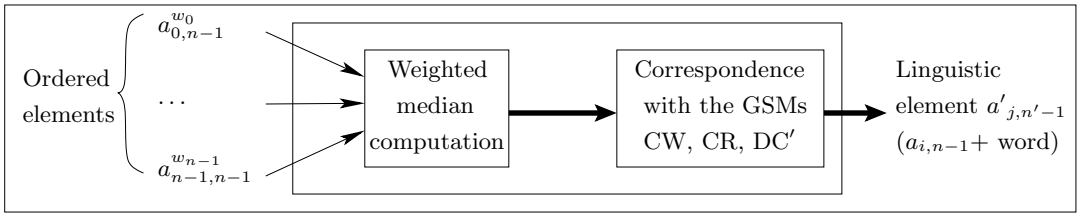

Fig. 8. General diagram of our aggregation operator.

5 Conclusions

In this paper we have introduced a new aggregation operator such as the symbolic weighted median. This operator deals with linguistic information modelled by means of linguistic terms. The only assumption required to compute the weighted median is to consider a total order defined on the linguistic term

set used to assess the linguistic information. It receives, as an input, several weighted linguistic terms from a linguistic term set and, as the output, the expression of the result is a modified linguistic term, taken from the initial set. Thus the aggregated answer is always equal to one of the initial symbols (as it is the case with a usual median), but the symbol may have been weakened or reinforced. The weights will determine the strength with which the element is weakened or reinforced. The expression of the aggregation is done with the use of weakening and reinforcing modifiers applied to the corresponding element. The result given by the median is more or less accurate, depending on the computation time.

In further works it could be interesting to use the GSMs, or other linguistic tools, for the construction of operators such as means, for example, or new kinds of aggregation operators, in order to offer a large set of linguistic statement aggregation operators.

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