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Evolution equation for soft physics at high energy

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Abstract. Based on the non-linear logistic equation we study, in a qualitative and semi-quantitative way, the evolution with energy and saturation of the elastic differential cross-section in $pp(\bar{p}p)$ collisions at high energy. Geometrical scaling occurs at the black disk limit, and scaling develops first for small values of the scaling variable $|t|\sigma_{tot}$. Our prediction for $d\sigma/dt$ at LHC, with two zeros and a minimum at large $|t|$ differs, as far as we know, from all existing ones.

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Saturation phenomena are expected to dominate QCD physics at high energy and high matter density [1, 2], as it may happen at LHC and cosmic rays at ultra high energies. This is in fact an old problem related to unitarization and the need to reduce particle multiplicity and lowering total cross sections (see, for instance,[3]). Non linear differential equations include, in a natural way, saturation effects. This happens with the well known logistic equation[4]. See[5] and [6] for discussions on evolution and saturation.

We shall concentrate here in the evolution of the imaginary part of the impact parameter elastic amplitude, or the profile function $\Gamma(b, R) \equiv \text{Im } B(b, R)$, where b is the impact parameter, related to angular momentum ℓ by

$$b \simeq \frac{2}{\sqrt{s}}\ell, \quad (1)$$

where \sqrt{s} is the centre of mass energy, and R is an increasing with energy radial scale parameter. Partial wave unitarity constrains $\Gamma(b, R)$:

$$0 \leq \Gamma(b, R) \leq 1. \quad (2)$$

We now write two logistic equations, in R and b , respectively:

$$\frac{\partial \Gamma}{\partial R} = \frac{1}{\gamma}(\Gamma - \Gamma^2), \quad (3)$$

and

$$\frac{\partial \Gamma}{\partial b} = -\frac{1}{\gamma}(\Gamma - \Gamma^2), \quad (4)$$

where $\gamma > 0$ is practically a constant. From (3) one sees that $\partial \Gamma / \partial R > 0$ and that, for fixed b and $\Gamma > 0$, Γ reaches the black disk limit: $\Gamma = \Gamma^2 = 1$. From(4) one sees that in general Γ is a decreasing function of b and that, for large b , Γ decreases, as expected, exponentially ($\Gamma \sim \exp -b/\gamma$), saturation occurring first at small b .

A solution of (3) and (4), not the most general one, is:

$$\Gamma(b, R) = \frac{1}{\exp \frac{b-R}{\gamma} + 1}. \quad (5)$$

The total and elastic cross-section are written as

$$\sigma_{tot.}(s) = 2\pi \int \Gamma(b, R) db^2 \quad (6)$$

and, neglecting real part contributions,

$$\sigma_{el.}(s) = \pi \int |\Gamma(b, R)|^2 db^2, \quad (7)$$

respectively. The imaginary part of the elastic amplitude $\text{Im } F(t, R)$ is the Fourier-Bessel transform of $\Gamma(b, R)$ and the differential elastic cross-section is written as

$$\frac{d\sigma}{dt} = \frac{\sigma_{tot.}^2}{16\pi} \frac{\text{Im } F(t, R)^2}{\text{Im } F(0, R)^2}. \quad (8)$$

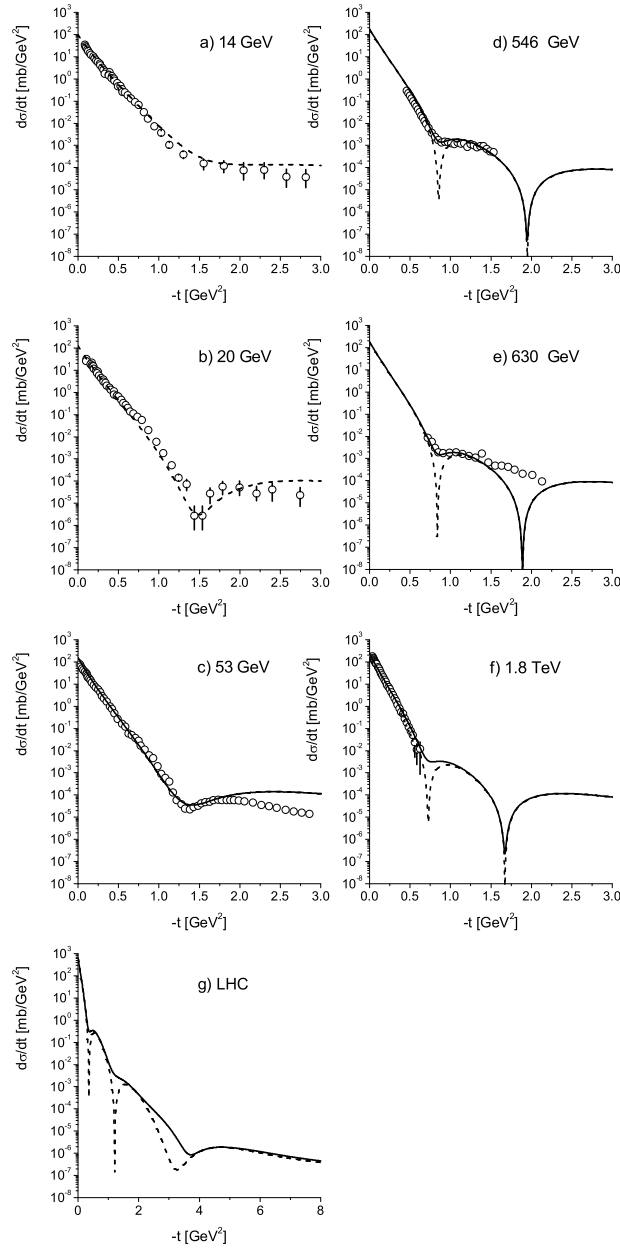


Figure 1. $d\sigma/dt$ as function of $-t$ at different energies, showing the sequence: no structure in a), one minimum in b) and c), two zeros in d) and e) and two zeros and one minimum in f) at LHC. Values of γ and R : a) $\gamma = 1.020$, $R = 1.972$; b) $\gamma = 1.026$, $R = 2.171$; c) $\gamma = 1.090$, $R = 2.259$; d) $\gamma = 1.000$, $R = 2.575$; e) $\gamma = 1.016$, $R = 2.496$; f) $\gamma = 1.078$, $R = 2.683$; g) $\gamma = 1$, $R = 3.770$. Data from [11]. Dashed line: only imaginary part contribution. Full line: the real part of the amplitude is included.

Taking (5) as a phenomenological model with two free parameters, γ and R , we have compared it with data on $d\sigma/dt$ from $\sqrt{s} = 14$ GeV up to LHC energies. An

unexpected qualitative agreement with data was found, Fig. 1.

In the evolution of the amplitude (5) with energy one finds three regimes (see Fig. 1):

i) $\sqrt{s} \lesssim 20$ GeV, Fig. 1.a).

This is the region corresponding to linear evolution, with Γ small and with exponential behavior, $d\sigma/dt$ being a monotonically decreasing function of $-t$.

ii) $20 \lesssim \sqrt{s} \lesssim 63$ GeV (ISR energies), Figs. 1.b) and c).

In this region a dip, which is a minimum, not a zero, appears at $-t \approx 1.4$ GeV² and slowly moves to the left as energy increases. Conventional wisdom says that the dip results from a zero: interference between one-Pomeron and two-Pomeron exchanges [12].

iii) $500 \lesssim \sqrt{s} \lesssim 1.8$ TeV, Figs. 1.d), e) and f).

In this region the minimum becomes negative, originating a pair of zeros. Instead of the clean second maximum of region ii) one has now a kind of shoulder, but with a cross-section higher by an order of magnitude.

In Fig. 1.g) we have also included our expectation for LHC (assuming $\sigma_{tot.} \approx 110$ mb - see [13] for expected range of values - and $\gamma = 1$ mb^{1/2}). Our LHC curve clearly shows how the evolution towards the black disk continues: from a monotonically decreasing curve at large $-t$ a minimum starts developing which at some stage generates a pair of zeros to join the previous pair. And so on! In the black disk limit we just have a sequence of pairs of zeros.

At high energy, $\sqrt{s} \gtrsim 60$ GeV, when $\sigma_{pp} \simeq \sigma_{\bar{p}p}$ is not difficult to use the derivative dispersion relations [14] to estimate the real part contribution to the differential cross section. In Figs. 1.c) to 1.g) we show, in full line, $d\sigma/dt$ including the real part correction. The real part contribution is important at the zeros of Γ .

Two conclusions can be drawn from our comparison with data in Fig. 1: 1) the parameter R is a monotonically growing function of energy, 2) the parameter γ can at high energy be taken as a constant, 1 mb^{1/2}

Coming back to (5) it should be noticed that in regions of energy where $\gamma/R \simeq const.$, as it happens at ISR energies ($20 \lesssim \sqrt{s} \lesssim 60$ GeV), $\Gamma(b, R)$ in (5) satisfies geometrical scaling [7],

$$\Gamma(b, R) \xrightarrow{\gamma/R \simeq const.} \Gamma(\beta), \quad (9)$$

with

$$\beta \equiv \frac{b}{R}, \quad (10)$$

and,

$$\frac{d\sigma}{dt} \sim R^2 (f(tR^2))^2. \quad (11)$$

As γ , contrary to R , does not show in general a consistent dependence on energy, only in the limit $R \rightarrow \infty$, $\gamma/R \rightarrow 0$ and one obtains again scaling [8],

$$\Gamma(b, R) \xrightarrow{R \rightarrow \infty} \Gamma(\beta) \equiv \begin{cases} 1, & 0 \leq \beta < 1 \\ 0, & \beta > 1 \end{cases}, \quad (12)$$

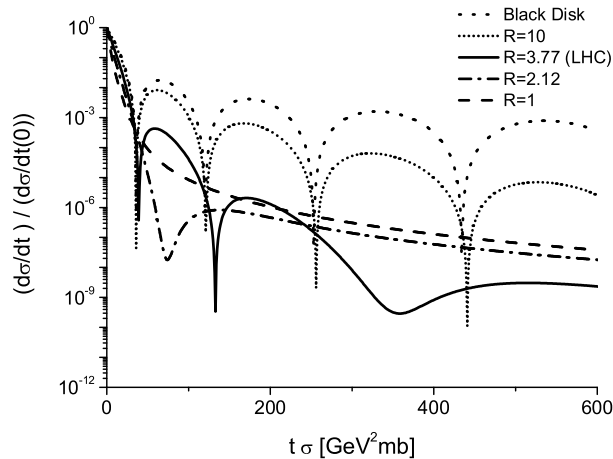


Figure 2. $d(\sigma/dt) / (d\sigma/dt(0))$ as a function of the scaling variable $|t|\sigma_{tot}$. showing the approach to black disk geometrical scaling from small to larger values of the scaling variable. Scaling only applies to the imaginary part of the amplitude. The parameter γ was put equal 1.

with $\sigma_{tot.} \sim \sigma_{el.} \sim R^2$ and $\sigma_{el.}/\sigma_{tot.} \rightarrow const. = 1/2$.

It should be also noticed that the parameter R in (5) separates the region of negative curvature, $b < R$, from the region of positive curvature, $b > R$. In fact, R plays the role of the angular momentum L , used in the proof of Froissart bound [9] by Martin and collaborators [10], that separates the region that contributes in a significant way to the total cross-section, $\ell < L$, from the region that is negligible, $\ell > L$:

$$R = \frac{2}{\sqrt{s}} L = \frac{1}{\sqrt{t_0}} \ln \left(\frac{s}{s_0} \right), \quad (13)$$

with $\sqrt{t_0} = 2m_\pi$. In fact γ , the parameter that controls the low density region, $b > R$, can be seen as a measure of the range of the interaction, with $\gamma \sim (2m_\pi)^{-1}$ in the Yukawa picture. The evolution of the cross-sections with the energy is controlled by the single parameter $R(s)$, the effective impact parameter radius.

In Fig. 2 we show the geometrical scaling plot (see (11)) of $\frac{d\sigma}{dt} / \frac{d\sigma}{dt}(t=0)$ as function of the variable $|t|\sigma_{tot.}$ for different values of R and for $\gamma = 1 \text{ mb}^{1/2}$, and the black disk limit. The way the approach to the scaling curve is achieved seems clear: as the energy, or R , increases scaling is satisfied for larger $|t|\sigma_{tot.}$ values. The LHC curve corresponds to $\sigma_{tot.} = 110 \text{ mb}$, $\sigma_{el.}/\sigma_{tot.} \simeq 0.28$.

In Fig. 3 we show the correlation between $\sigma_{el.}/\sigma_{tot.}$ and $\sigma_{tot.}$ in comparison with data. Note the transient geometrical scaling at ISR energies, $\sigma_{el.}/\sigma_{tot.} \simeq const.$.

It should be mentioned that the curves shown in Fig. 1 are not fits to data, but represent qualitative descriptions. This means that $\sigma_{el.}$ and $\sigma_{tot.}$, which are controlled by the first points in the low $|t|$ region, are not constrained. That is the reason why the values of $R(s)$ in Fig. 1 do not coincide with the values used in Fig. 3. For instance,

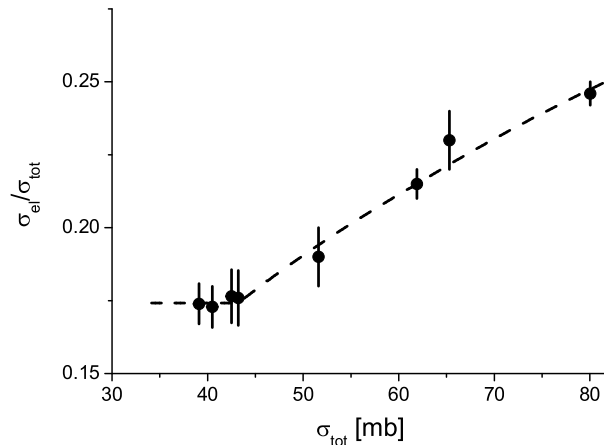


Figure 3. $\sigma_{el.}/\sigma_{tot.}$ as a function of $\sigma_{tot.}$ making use of (5). Data points, from left to right: $\sqrt{s} = 23.5$ GeV, $\gamma = 1.065$, $R = 1.710$; $\sqrt{s} = 30.6$ GeV, $\gamma = 1.090$, $R = 1.731$; $\sqrt{s} = 44.9$ GeV, $\gamma = 1.095$, $R = 1.805$; $\sqrt{s} = 52.8$ GeV, $\gamma = 1.107$, $R = 1.810$; $\sqrt{s} = 200$ GeV, $\gamma = 1.123$, $R = 2.100$; $\sqrt{s} = 540$ GeV, $\gamma = 1.080$, $R = 2.498$; $\sqrt{s} = 900$ GeV, $\gamma = 1.026$, $R = 2.662$; $\sqrt{s} = 1800$ GeV, $\gamma = 1.046$, $R = 3.050$. We expect for LHC $\sigma_{el.}/\sigma_{tot.} = 0.28$ and $\sigma_{tot.} = 110$ mb. Data from [11].

at ISR energies, in Fig. 1, $\sigma_{el.}$ and $\sigma_{tot.}$ are larger by a factor of the order of 15% relative to the true values (in Fig. 3).

Recently, in several papers [15, 16, 17] the questions of soft physics were addressed from different points of view. However, contrary to the present paper, no attempts were made to explain the qualitative aspects of $d\sigma/dt$ evolution with energy. Instead, interesting problems related to inelastic diffraction, as Higgs production, were addressed. The black disk perspective varied significantly from [15] to [16]: the black disk limit being already present at LHC energies in [15], but occurring at extremely high energies in [16]. For a discussion on the black disk limit see also [18].

The relevance or not of the Froissart bound (see [19] and [20]) and a dynamical interpretation of it (see [21] and [22]) are still matters open to discussion.

Finally, we summarize our work. Starting from the non-linear logistic equation we obtained a solution for the high-energy imaginary part of the amplitude, and we were able to describe in a qualitative and semi-quantitative way the essential features of the evolution of the differential elastic cross-section with energy, namely the sequence: no structure in $|t|$, one minimum, two zeros and so on. Our prediction for $d\sigma/dt$ at LHC energies is different from all the ones we are aware of (see, for instance, [23]).

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on zeros and minima in nuclear reactions

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