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Maintenance policy: degradation laws versus Hidden Markov Model availability indicator

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Abstract: Today, maintenance strategies and their analyses remain a worrying problem for companies. Socio economic stakes depending on competitiveness of each strategy are even more linked to activity and quality of maintenance interventions. A series of specific events can eventually warn the expert of an imminent breakdown. Our study aims at understanding such “signature” thanks to hidden Markov models. On that purpose, two methods for damage level estimation of maintained system are proposed to expert. The first one consists in using a non parametric and semi-parametric degradation laws (which will be used as references). The second one consists in using a markovian approach. All proposals are illustrated on two studies corresponding to two real industrial situations (continuous system for food processing and moulded products in aluminum alloys for the automotive industry).

Keywords: diagnosis, maintenance management, assessment, degradation laws, Hidden Markov Model

1 INTRODUCTION

Industrial processes need to be maintained to prevent breakdown. Some years ago, maintenance activities were only deployed to repair process after the problem occurs. Nowadays, in an international market context, companies need to improve their productivity. In this context, maintenance strategies are included in reliability engineering. This field includes technological aspects, personal organisation, logistic... Different kinds of maintenance policy can be applied: preventive or corrective according to manager strategies.

In case of preventive maintenance, different strategies should be used like planned preventive maintenance or condition-based maintenance. Planned actions would be program using feedback from experience which used statistical frequencies of defaults. Condition-based maintenance could use specific features extracted from the process like vibration sensors, oil analysers... Then according to these indicators, maintenance actions can be performed. In many cases, defaults are preceded by specific series of events. Black smoke behind a car informs that engine could stop. Dark clouds indicate it will rain... Events which precede default could inform about imminence of it.

Valdez-Florez *et al.*, [44] survey researches on model optimization for repair, replacement, and inspection of systems subject to stochastic deterioration. Simeu-Abazi *et al.*, [41] adopted a modular modelling approach, based on a cellular decomposition of the system. They use stochastic Petri nets and Markov chains to implement various maintenance strategies in complex production workshops. A parametric decision framework (multi-

threshold policy) is proposed to choose sequentially the best maintenance actions and to schedule future inspections, using on-line monitoring information on the system deterioration level [17], [25]. Deterioration of technical systems can often be classified into discrete states, and transitions between these states can be modelled using a Markov process. Instead of using an exponential distribution, it may be more realistic to assume that a general probability distribution describes staying time in one of these states [49]. Soro *et al.*, [42] proposed a model for evaluating availability, production rate and reliability function of multi-state degraded systems subject to minimal repairs and imperfect preventive maintenance. System status is considered to degrade with its use. These degradations may decrease system efficiency. It is assumed that the system can consecutively degrade into several discrete states, which are characterized by different performance rates, ranging from perfect functioning towards complete failure. Nevertheless, global performances are difficult to be controlled because system environment changes. Functioning modes are dependent on product flows, and ageing of components modifies continuously system characteristics. Thus today, most maintenance strategies are not well adapted to these requirements because purely reactive (fixing or replacing equipment after it fails) or time-scheduled [48]. As in these studies, we show that a degradation level of a process can be proposed to the expert (maintenance manager), from series of “field” events. In this study, we try to learn, without "a priori", this default signature. The originality of our work, is to use the maintenance activities as indicators (Fig. 1 and Fig. 2). Works presented in this paper take part of condition monitoring systems. Using observations provided by the process, we try to generate an availability indicator which can be used by maintenance manager to plan actions dynamically (Fig. 1 and Fig. 2). According to system availability, preventive maintenance could be scheduled to prevent uncontrolled stops of system.

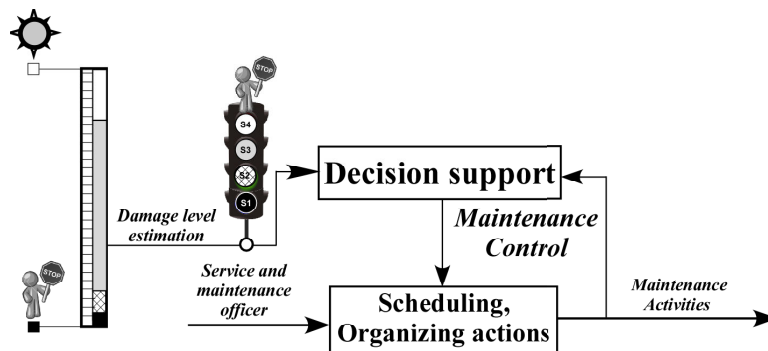


Fig. 1 Work goals

Hidden Markov Models (HMM) have been used, with success, to model sequences of events like, for example, in speech recognition. To improve results of these methods, model parameters should be adjusted to match event characteristics (states, topology ...). In this study, we use the same strategy to learn events which can be observed on an industrial process. Model topology is configured to provide an availability explanation to our model. When system is started, model will indicate a high level of availability. When system is stopped by defaults, model will be in the “off” state (red state: it is too late to prevent default). Our new estimator is compared with “classical” degradation laws. These degradation laws are used as references.

In the next part, we introduce maintenance strategies and our works are located in this context. In part 3, we recall some “classical” reliabilities laws. We give more details for Kaplan-Meier law and Cox model, which have been implemented. In part 4, our strategy to use HMM for availability indicator implementation is presented. In the last part, we compare results of “classical” degradation laws with our HMM availability indicator on two studies: a continuous food process and an aluminum alloys process for the automotive industry.

2 MAINTENANCE POLICIES

Maintenance strategy is a part of reliability engineering. Reliability is the ability of a system to perform its goals or functions. This can include technology, strategy, finance and humans [18]. Others explanations of maintenance should be find in [24] and [43]. In an international market context, maintenance strategies need to be improved with production tools being even more complex. Between the 40's, maintenance has grown between corrective actions towards predictive system which try to prevent failures [36].

Maintenance is defined by norm N. F. E. N. 13-306 [1]. It can be detailed as a list of actions which enables to a system or a service to perform its goals. In [39] authors split maintenance into two kinds of activities: technical activities and management activities. More details can be found in [50].

Fig. 2. shows different kinds of maintenance policies. This figure shows two keywords of maintenance definition: preventive and corrective. A review of different maintenance strategies can be found in [10] and [22]. In some specific cases, maintenance policy could be imposed, like it is the case for nuclear plant in France.

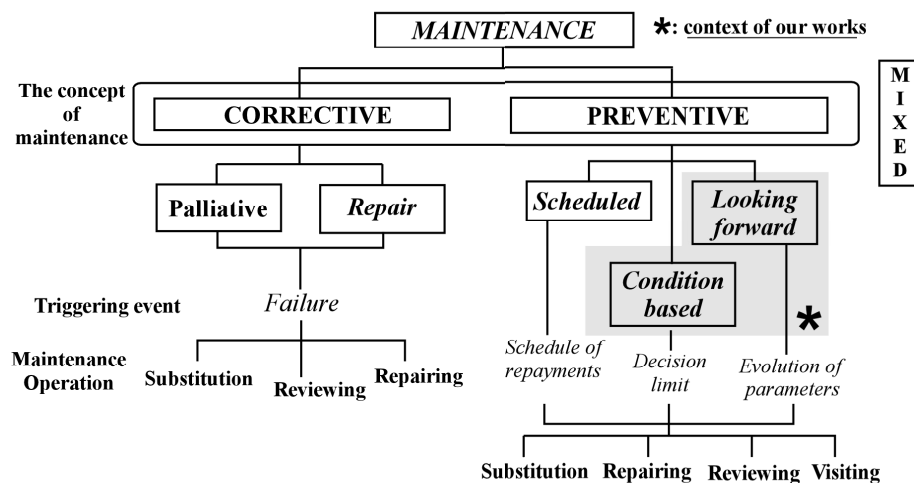


Fig. 2 Maintenance policies

We do not give details about corrective maintenance. The aim of the proposed method is to prevent failure to occur. It could be used to adjust preventive maintenance. This kind of maintenance can be applied by different ways and it is composed of:

- various tasks on systems, monitoring, calibration, ... It also includes lubrication and oil changes in order to reduce friction, wear, prevent galling and save energy.
- Auto-maintenance tasks by the operator. This kind of actions is included in Total Productive Maintenance (TPM) where machine operator performs much, and sometimes all, of routine maintenance tasks.
- Planned maintenance which is scheduled according to time or cycle. Visual inspection can be made or subsystems can be dismantled to inspect themes with accuracy. Tasks like calibration, cleaning, lubrication ... can be planned according to lifetime duration of parts.
- Systematic replacement based on time or cycles. This supposes an accurate counter of time or cycles and correct information about component lifetime. These parts should be systematically replaced.

- Condition-based maintenance uses particular information provided by sensors with, if necessary, specific signal processing. In this case, failure can be detected before it occurs using feedback from experience.

The replacement of components for which failure is thought to be imminent, can be performed when the component is strongly damaged according to different use criteria, or when it has reached a critical condition. The success of this approach depends on the ability to predict the remaining life of the component and when to perform the replacement [9], [14].

3 RELIABILITY STUDY OF MAINTAIN PROCESS

Reliability is often used by maintenance expert. In this section, some usual lifetime laws are presented. These laws can be used as well for medical studies as for industrial context. Main properties of these laws (probability density, reliability functions, failure rate) are defined and applied in reliability applications [27], [12]. Commonly, parametric models and nonparametric models based on proportional risk are used [11]. Different classical laws of degradations exist: exponential law, normal law (Laplace-Gauss), Log-normal law (or Galton law), Weibull. We do not present these laws in this paper.

Efficient maintenance is related to a pertinent estimation of components lifetime. This estimation is based on experience feedback. Lifetime study of each system can be split into two options which need a great analysis of this experience feedback:

- *System reliability is its ability to perform what it has to do in its usual conditions during a given time [3], [2]. Reliability expert try to plan new maintenance strategy using reliability evolution.*

- *Durability is the ability for a system to perform its goals, given using and maintaining conditions, until a limit state is reached [1]. Durability expert tasks should consist in estimation of remaining lifetime of working systems. This kind of study needs to take into account using conditions, parts replacement, to estimate system lifetime.*

3.1 Kaplan-Meier law

The Kaplan-Meier method, used in cross-disciplinary field ([16], [19]) provides an estimation of survival functions, with not necessarily regular time intervals, instead of actuarial tables¹. Survival curves can be used to analyse evolution of populations over the time. These techniques (also called product limit estimators) are used for analysis of survival data, whether for persons (e.g. cancer) or products (wear tools resistance). The first type of information is called "event data", while the second is called "censored data".

$S(t)$ is the survival function. According to datas (y_1, \dots, y_n) , we can provide unbiased empirical Kaplan-Meier estimator:

$$S_n(t) = \prod_{i \in \{1, \dots, n\}, y_i \leq t} \left(\frac{n-i}{n-i+1} \right)^{\delta_i} \quad n \neq i \quad (1)$$

Where $\delta_i = 1$ if y_i is an uncensored data (0 if censored data). Lo [34] and Bitouzé [13] propose a definition of this estimator for a measure of concentration of according $S(t)$ to real distribution, in a non-asymptotic context.

3.2 Cox model

Cox regression model is an useful method to study impact of variables on survival time of a process [32], of patients [15] (medical study)... It is applied to survival data, i.e., time variables, censored variables and

¹ These methods combine observations by random or predefined intervals. It enables to estimate and to obtain a hazard rate representation.

explanatory variables. This model is based on a maximum likelihood estimation, developed by [23]. Principle of Cox model is to link event happening to explanatory variables. For example, in medical field, we try to assess impact of a pre-treatment on the healing time of a patient. Cox model can be compared with classical regression models: events (modelled by date) should be linked with explanatory variables. Specificity of this approach is its ability to assess relationship between hazard and explanatory variables without assumptions on the shape of baseline hazard function. It contains real parameters and unknown functions (hence appearance of semi-parametric methods that take into account this double aspect). It is based on the proportional hazards assumption (instantaneous risk of event happening can be written as the product of a function that depends on time and a function that depends only on specimen features). It can be applied to any situation where event duration is studied. Cox is based on the assumption of proportional hazards. In proportional hazard models, the instantaneous risk is written:

$$h(t | \mathbf{z}, \theta) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} P(t < T \leq t + \Delta t | T > t, \mathbf{Z} = \mathbf{z}) = \alpha_0(t) f_\beta(\mathbf{z}) \quad (2)$$

With: $\mathbf{Z} = (\mathbf{Z}_1, \dots, \mathbf{Z}_p)^T$: a vector of covariates, α_0 : basis risk, unknown, independent of \mathbf{Z} , β : the regression parameter, unknown, $f_\beta(\mathbf{z})$: relative risk, independent of time.

For example: risk ratio for two individuals is independent of time. Regressors $\mathbf{Z}_1, \dots, \mathbf{Z}_p$, quantitative or qualitative are called prognostic factors (age, sex, treatment, ...). In the Cox model, the instantaneous risk for an individual i is written:

$$h(t | \mathbf{Z}_i) = \alpha_0(t) \exp(\beta_1 \mathbf{Z}_{i,1} + \dots, \beta_p \mathbf{Z}_{i,p}) = \alpha_0(t) \exp(\beta^T \mathbf{Z}_i) \quad (3)$$

$\alpha_0(t)$ is any function which depends only on time (basis risk is unknown and independent of \mathbf{Z}_i), and $\beta_1, \beta_2, \dots, \beta_p$ are constants with: $\alpha_0(t)$, $\beta = (\beta_1, \dots, \beta_p)^T$ is the unknown regression parameter. With: $\mathbf{Z}_i = (\mathbf{Z}_{i,1}, \dots, \mathbf{Z}_{i,p})$: a vector components. \mathbf{S}_0 is the basic survival function associated with α_0 . In this case, we have the following relationship: $\mathbf{S}(t | \mathbf{Z}_i) = [\mathbf{S}_0(t)] \exp(\beta^T \mathbf{Z}_i)$. This provides an estimate of \mathbf{S} knowing β the estimation vector. To estimate the components of the vector β from an ordered sample $(\mathbf{Y}_{(1)}, \dots, \mathbf{Y}_{(n)})$, we calculate the partial likelihood function of Cox (if no censored data):

$$L(\mathbf{y}_{(1)}, \dots, \mathbf{y}_{(n)}; \beta) = \prod_{i=1}^n \frac{\exp(\beta^T \mathbf{Z}_i)}{\sum_{k \in R(\mathbf{y}_{(i)})} \exp(\beta^T \mathbf{Z}_k)} \quad (4)$$

Note:

- $\exp(\beta_i)$ relative rate of subjects for which $X_j=1$ compared to those for which $X_j = 0$,
- $\exp(\beta_i) > 1$: harmful effect ; $\exp(\beta_i) = 1$: no effect ; $\exp(\beta_i) < 1$: positive effect.

4. MODELS OF RELIABILITY BASED ON MAINTENANCE

Failure of equipment can be characterized by a rate (λ) known as failure rate. This rate is also called the failure rate, hazard rate or mortality rate. It is defined as the conditional probability that equipment fails between time t and $t + \Delta t$ knowing that it has survived to time t . It can also be defined as components proportion that has

survived until time t . It also represents happening failure rate [5], [29]. Variable "time" could be considered as a unit of use. Indeed, in case of some devices, distance travelled, rounds number, requests number... could be considered as "time".

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \left(\frac{1}{\Delta t} \frac{R(t) - R(t + \Delta t)}{R(t)} \right) \quad (5)$$

With:

- $R(t)$: number of components that have survived until time t ,
- $R(t + \Delta t)$: number of components that have survived until time $t + \Delta t$.

This law is based on the fact that evolution of intensity function $\lambda(t)$ is determined by efficiency of maintenance operations. In practice, a maintenance activity is labelled:

- Minimal, if system state after repair is As Bad As Old: ABAO ($\lambda(t) = \lambda t; \forall t \geq 0$), (Fig. 3),
- Perfect, if maintenance returns the system As Good As New: AGAN

$$\lambda_t = \lambda(t - T_{N_t^-}) \quad (6)$$

with $T_{N_t^-}$ the left limit of stochastic process $N(t)$, (Fig. 3). This kind of problem, firstly studied by [28], has been the subject of many developments including several variants, [26], [37], [38],

- Imperfect (Proportional Age Reducing model). In this kind of works, Doyen *et al.*, [26] propose different alternatives to build stochastic models of the process of failures and repairs of various systems. A virtual age model is characterized by a sequence of positive random variables $\mathbf{A} = \{A_i\}_{i \geq 0}$ (ages staff) as $A_0 = 0$. These models are based on assumption that after lot of maintenance, system behaves as a new system that would operate for a period A_i without fail. A_i is virtual age of the system.

$$P(X_{i+1} > x \mid X_1, \dots, X_i, A_i) = P(Y > A_i + x \mid Y > A_i, A_i) \quad (7)$$

Y is a random variable equally distributed as X_1 .

From this relationship, model intensity can be written as:

$$\lambda_t(N, \mathbf{A}) = \lambda(t - T_{N_t^-} + A_{N_t^-}) \quad (8)$$

t is actual age of the system and virtual age, $V(t) = t - T_{N_t^-} + A_{N_t^-}$ is related to maintenance interventions. If we compare equations (6), (7) and (8), we find that ABAO model corresponds to $A_i = T_i$ and AGAN model corresponds to $A_i = 0$. Baxter *et al.*, [8] propose a simple model to characterize actual age. They assume that effect of the n^{th} maintenance is to reduce the virtual age just before failure, $A_{i-1} + X_i$ by a proportional amount of time elapsed since previous maintenance $B_i X_i$, with $B_i \in [0, 1]$ (equation (9)).

$$A_i = A_{i-1} + (1 - B_i) X_i \quad (9)$$

In this relationship, we can deduce easily the recurrence relation:

$$A_i = \sum_{j=1}^i (1-B_j)X_j \quad (10)$$

Related density function on this model is:

$$\lambda_t(N, B) = \lambda \left(t - T_{N_t^-} + \sum_{j=1}^{N_t^-} (1-B_j)X_j \right) \quad (11)$$

The most simple model of Kijima [33] is obtained assuming that efficiency of maintenance B_i is deterministic, constant and equal to ρ , called improvement factor. Taking into account this assumption, the intensity function becomes:

$$\lambda_t = \lambda(t - \rho T_{N_t^-}) \quad (12)$$

Depending on value of ρ , we can model any kind of maintenance:

- $\rho = 1$: maintenance is perfect (AGAN),
- $\rho \in]0, 1[$: maintenance is effective,
- $\rho = 0$: maintenance is minimal (ABAO),
- $\rho < 0$: maintenance is harmful.

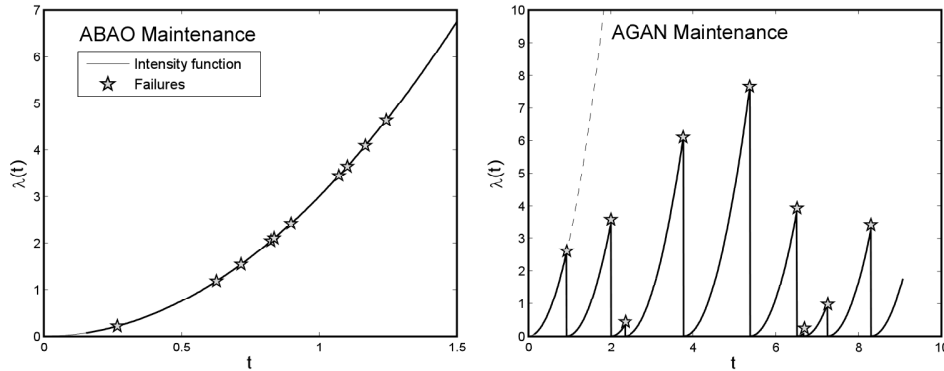


Fig. 3 Intensity functions for ABAO and AGAN maintenance [6]

5 DECISION SUPPORT BASED ON HIDDEN MARKOV MODELS

Stochastic models are representations of dynamic systems based on probabilities. Stochastic processes were firstly developed in the early 20th century by a Russian mathematician, Andrei Andreyevich Markov. His statistical study of language has led to the markovian hypothesis, which can be summarized as follows: "Future evolution of a system only depends on its present state". This hypothesis implies that current state of a system contains all information provided by its past. Therefore, it is a very important assumption. In practice, this condition is rarely satisfied. However, approximation by Markovian models could provide good modelling results (Hopp [30] proposes a maintenance model under Markovian deterioration is developed in which maintenance and replacement actions are permitted and states are completely observable). Meier-Hirmer [35] proposes a model used for the maintenance of railway tracks. In this paper, semi-regeneration properties at the

inspection times and associated Markov renewal techniques are used in order to compute the long-term mean costs.

Risk analysis of dynamical systems by classical markovian approaches considers only two states (On / Failure (Stop)). Between perfect working condition and complete failure state, industrial systems generally have a large set of degraded states in which system continues to provide service, even if it does not produce fully. "These degraded statements need to be taken into account to properly assess service level of industrial systems and this is especially true with regard to production systems [31]".

Hypothesis: events preceding a crash are often recurrent. Specific series of events should inform about the next failure. Some examples can illustrate this hypothesis.

- In mechanical systems, noises, vibrations precede failure. Loss of performances reflects failures or technical defaults,
- In computers, suspect pointer movements, loss of performances, application malfunctions like web browser may reflect virus presence on computer ...

Our approach tries to understand "this signature" using HMM. Hidden process will match system state (or subsystem state) and observations will be observable part of processes (Fig. 4). Our works (Vrignat *et al.*, 2010 ([47])) show that it is possible to model degradation levels of a "continuous" process. Hidden process will fit to system or subsystem states (Run, Degradation level 1, ..., Degradation level N, failure) and observations will be information which can be collected on the system (Fig. 4).

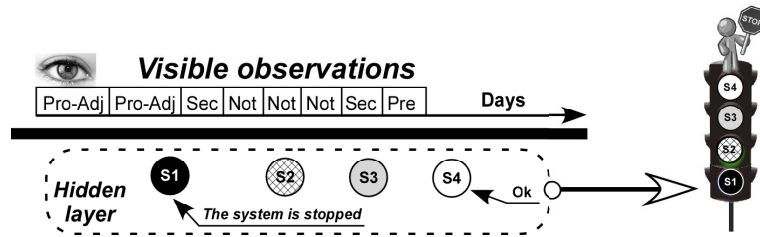


Fig. 4 Visible and hidden layers (system states)

5.1 HMM Approach

The aim of this paper is not to present exhaustively Hidden Markov Model. For readers interested in more details, we recommend to read papers [40] (which is already a good tutorial) and [4] which presents HMM general problems. In this paper, we use the same notation for models. A model $\lambda = (A, B, \Pi)$ is described by three matrices:

- $A = \{a_{ij} = P(S_j | S_i)\}; \sum_{j=1}^N a_{ij} = 1$ corresponding to transition probabilities between hidden states,
- $B = \{b_i(o_\tau) = P(o_\tau | S_i)\}; \sum_{j=1}^N b_i(o_j) = 1$ corresponding to probabilities of observations considering states,
- $\Pi = \{\pi_i = P(S_i)\}; \sum_{i=1}^N \pi_i = 1$ corresponding to initial state probabilities.

Learning a hidden Markov model consist in estimating the parameter vector λ on the basis of a set of observation sequences. The learning algorithm most commonly used is the Baum-Welch algorithm [7]. This algorithm is derived from the EM algorithm (Expectation-Maximization). The Baum-Welch algorithm solves the problem of learning with the criterion of Maximum Likelihood. For a sequence of observations \mathbf{O} , this criterion is to find the HMM λ^* which has the highest probability of generating the sequence \mathbf{O} that is to say, maximizing $P(\mathbf{O}=\mathbf{o}|\lambda)$. The Baum-Welch algorithm is a procedure which iteratively re-estimate matrices A , B and Π from an initial HMM. The Baum-Welch algorithm provides a local optimum of the likelihood function. By applying this learning with different initial models, it is possible to obtain either a global optimum or a near optimal model for the considered criterion.

Among all the criteria used for learning an HMM, the criterion of segmental K-means is different from others. For this criterion, we seek to maximize the probability $P(\mathbf{O}, \mathbf{S}=\mathbf{Q}^*|\lambda)$, with \mathbf{Q}^* corresponding to the sequence of hidden states that most likely generates the sequence as calculated by the Viterbi algorithm [45]. The segmental k-means algorithm can adjust the model parameters iteratively from an initial model. Just as the Baum-Welch algorithm, this algorithm could provide a local optimum result.

The two previous modes of learning have properties to preserve initial structure of models. When initial model probability is zero then the corresponding probability is zero in the learnt model. It is however important to note that a non-zero probability in the initial model may become zero in the learnt model. This phenomenon often occurs when certain symbols are not observable in learning sequences. A single occurrence of a “missing” symbol in a new sequence will cause non-recognition by the HMM: probability will be zero. To handle this problem, we introduce a smoothing step after the learning step. For each probability, not forced to zero by the model structure an epsilon is added. Constraints of stochastic matrices are obtained by normalizing sums to 1. This smoothing introduces a distortion of optimal learnt model. We therefore distinguish, in the following, non-smoothed and smoothed learning.

Once the model is characterized either by the Baum-Welch algorithm or the segmental k-means algorithm, with or without smoothing, we seek to estimate, the most likely sequence of statements on new observations sequences.

For the learning’s with the Baum-Welch algorithm, we estimate most probable states at a given time using Forward variables [40]. Let $\alpha_t(j)$ be the probability of generating the observation sequence $O = \{o_1, o_2, \dots, o_t\}$ and being in state q_t at time t , that is to say: $\alpha_t(j) = P(o_1, o_2 \dots o_t, Q_t = s_j | \lambda)$. The most probable state at time t is defined by $\mathit{argmax}_{j=1..k} \alpha_t(j)$.

For learnings with the segmental k-means, we consider the most probable state at a given time using the latest state of the optimal path given by Viterbi algorithm [45]. Considering the previous notation, the most probable state is defined by: $\delta_t(j) = \max_{q_1, \dots, q_{t-1}} P(Q_1 = q_1, \dots, Q_{t-1} = q_{t-1}, Q_t = S_j, o_1, \dots, o_t / \lambda)$.

5.2 Choice of topology

Before describing results of our tests, we need to argue the choice of topologies: How many significant and interpretable levels of vigilance can be chosen? In several fields, customs and practices use four vigilance levels:

- prevention policy of flood risk [20], anti-terrorist security plan [21], deterioration and maintenance models for components in hydropower plants [49], ...

For example, for flood risk [20], four vigilance levels are used:

- Green: no particular vigilance is required,

- Yellow: be attentive if practising activities exposed to meteorological risk, keep informed about weather development,
- Orange: be very vigilant; dangerous meteorological phenomena have been forecast,
- Red: Absolute vigilance is required; dangerous and exceptionally intense meteorological phenomena have been forecast.

Only three states model cannot be used. Which sense can be given at the median state? Glass is half full or half empty? We used an oriented topology (Fig. 5) which does not enable to jump one state from right to left (from high level of availability to lower). We need to go through all states (S2 and S3) to go from S4 state (system OK) to S1 state (System stopped). Symbols used are explained in introduction of Section 6. So we need at least four observations to go from S4 to S1. These are the minimum number of events to detect failure. We keep in mind, the sense we wanted to give to different states: S4 "everything is going well, " S3 "it can go, " S2 "things are bad, beware, " S1 "too late". With more states, we reduce sensitivity of our indicator. Then we choose a four states model. Training of our HMM model (Fig. 5) consists in estimating model parameters with a set of observations sequences.

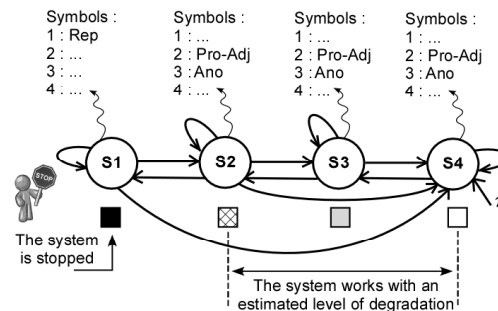


Fig. 5 Oriented topology with 4 states

6 CASE STUDIES

6.1 Industrial baker

In this part, we present a continuous process of bread production (Fig. 6). Industrial partner, which provide us these data, chose this part of the process because it was the most sensitive part of this production line. This plant works during all year without interruption with teams organized in shift work. For each team, there are available and informed databases in accordance with operational maintenance activities describes for the organization of the continuous process (Table 1). All details and simulations are presented in [46].

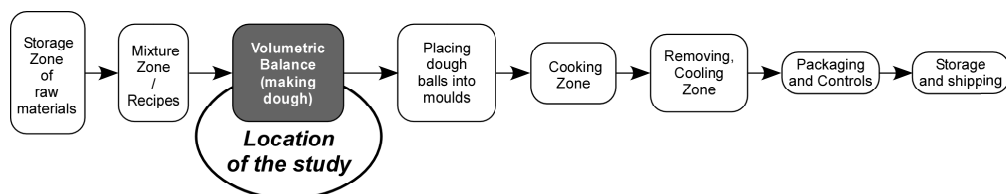


Fig. 6 Bakery continuous process

Entire production line can be stopped by a failure occurring on one of the subsystems of the line. Such situations could involve a loss of several production hours (some hundred kg of bakery products). To prevent such cases, preventive maintenance is scheduled. We have access to about two years of recording - from January 2005 to March 2007 - which represent about a thousand of events (Table 1). Model was trained using the “2005-2006” recording database (611 production records) while data of the year 2007 are used for our different tests.

The chosen sample time was the day (according to internal maintenance policy). To complete the data, a “Not” code stacking interventions was added when maintenance department was not requested (Fig. 7 (b)). Fig. 7 (a) lists different events that could be observed on recordings. Each of them is explained in the table. Some events can be grouped: maintenance actions (Pro-Adj, Pre, Pre-visit, Dis and Mak); observing actions (Obs, Ano, Sec); repairing action (Rep which means system is stopped); no action (Not).

Table 1 Example of logging events

Number in the list	NAME	TEAM	DATE	LINE	NAME OF THE MACHINE	WORK PERFORMED	OBS	CODING OF THE MAINTENANCE ACTIVITIES	Tl (min)	PARTS CHANGED	REF
1	M.Dupond	PM	03/01/2005	M2	Balance	The incharge request		Pro-Adj	20		
2	M.Dupond	PM	03/01/2005	M2	Balance	Regulate the cadence		Pro-Adj	20		

Table 2 is a sample of information collected during maintenance activities on the selected period (full uncensored data). From these data, we conducted a descriptive statistical study on subsystem. Definitions of headers of Table 2 are: CASE: repair situations, D_Not: production date OK, D_Rep: onset date of “Repair” situation, J_Rep: situation between two repairs (day), T_xxx (time spent in the activity in minutes) with xxx: Rep, Pro_Adj, Ano, Obs, Pre, Sec, Dis, Mak, Pre-visit, J_Not: time spent in Nothing (day), Time: month in year (month).

Table 2 Review of information related to maintenance activities in database

NUM	CASE	D_Not	D_Rep	J_Rep	T_Rep	T_Pro_Adj	T_Ano	T_Obs	T_Pre	T_Sec	T_Dis	T_Mak	T_Pre-visit	J_Not	Time
1	Rep1	03/01/2005	18/01/2005	15	40	40	0	0	60	70	0	0	120	8	JAN
2	Rep2	18/01/2005	24/01/2005	6	40	20	0	0	0	50	0	0	0	5	JAN

In our study of the system (sections 6.3 and 6.4), we consider situations where the volumetric balance was being repaired (no censure is adopted in data processing). After a descriptive analysis of the volumetric balance lifetimes, we will study the nonparametric and semi-parametric estimates of its reliability. Only operating periods were taken into account to study balance lifetime. Average duration between two “Repair” situations (system being stopped) estimated on database is 5.38 days. It includes, between two “Repair” situations, all other possible operational maintenance activities respecting internal symbolic coding. (Pro_Adj, Ano, Obs, Pre, Sec ... (Fig. 7(a)).

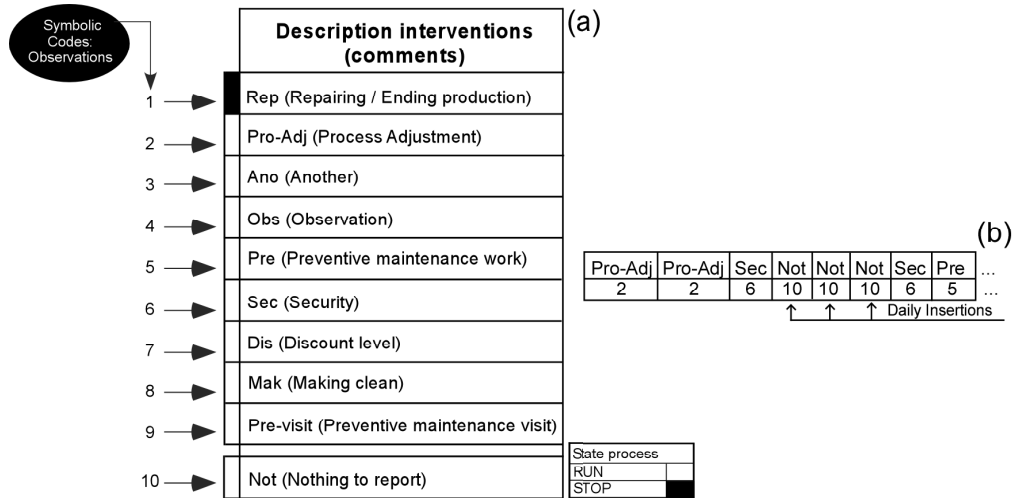


Fig. 7 (a): Symbolic coding of observations. **(b):** coding principle of observations sequence with “Not” insertion

In order to provide a daily evaluation by our indicator, we insert a specific symbol Not (Nothing to report) every day when no event is recorded (Fig. 7 (b)). If more than one symbol is observed during a day, it means that estimated state can change several times in the same day and consequently it increases sensibility of our indicator.

6.2 Foundry: new approach including Total Productive Maintenance framework

Fig. 8 shows this new approach including TPM activities.

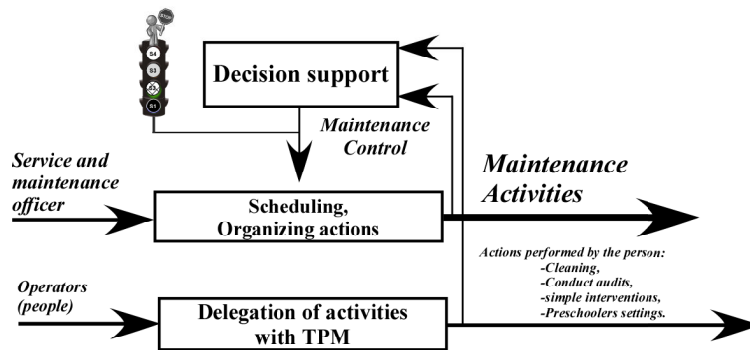


Fig. 8 Approach including TPM

The French industrial group is specialized in moulded products in aluminium alloys. Studied process, is a part of aluminium spares parts manufacturing (Fig. 9 (a) and (b)).

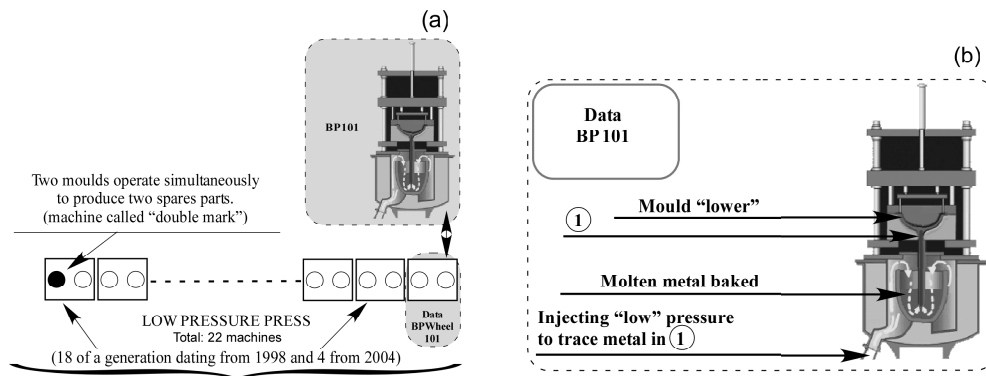


Fig. 9 Process description

The process involves filling piece with low pressure system. Operating principle of low pressure machine is based on four main elements:

- airtight oven to be under pressure,
- supply system between oven and mould,
- system controlling and supporting mould elements,
- mould.

Metal casting principle under low pressure consists in applying low pressure from 0.2 to 2 bar in airtight oven. This pressure can pull the liquid metal up to the mould. Liquid metal gets into link tubes before filling mould. Metal pressure extracts air out of the mould. Main advantage of low pressure process is that the filling method, which in a slow and down to up process, preserves alloy quality. Solidification naturally spreads from cold spots to hot zones. When the part is solidified, pressure is released from the oven and metal which remained liquid returns in the oven, thus separating part from supply system. This process is maintained by a maintenance service and by delegation of small tasks performed by operators according to TPM framework. This system works during all day teams organized in shift work. Our first task was to create database from all operator actions which were recorded on paper.

Table 3 Database samples

TEAM	N° mold	N°carcass	DATE	Lack of metal	TPM	Change Mold	Getting warm	Problem on the tube collar	Machine Problem	DEP
				401	850	606	202	112	405	
AM	7939	45	2/1/08				x			
	7935	47							x	
							x			
NIGHT	7939	45								
	7935	47								
PM	7906	39	3/1/08							
	7935	47				x				
Maintenance										

These datas, collected over six months, represent 2218 production records (Table 3). New codification is represented in Fig. 10 (a). Table 3 is a sample of informations collected during maintenance activities on the selected period (full uncensored data). From these datas, we conducted a descriptive statistical study. Definitions

of Table 4 headers are: CASE: repair situations, D_Not: production date OK, D_Rep: onset date of a “Repair” situation, J_Rep: situation between two repairs (day), T_xxx (time spent in the activity in minutes) with xxx: Rep, LC, CL,...No, Time: month in year (month).

Table 4 Review of informations relating to maintenance activities in the database

NUM	CASE	D_Not	D_Rep	J_Rep	T_Rep	T_LC	T_CL	T_BLC	T_CM	T_GW	T_B	T_S	T_N	T_J	T_TP	T_MP	T_RP	T_OP	T_M	T_Re	T_Pre	T_Not	T_No	Time
1	Rep1	02/01/08	03/01/08	1	30	0	0	0	90	190	0	25	0	0	120	105	0	45	0	0	0	0	0	JAN
2	Rep2	03/01/08	04/01/08	1	90	0	0	0	0	100	0	40	0	0	30	0	0	0	0	0	0	240	0	JAN
3	Rep3	04/01/08	06/01/08	2	60	0	0	0	0	260	0	115	0	0	0	0	0	0	0	0	0	240	240	JAN

In our study of lifetime system (sections 6.3 and 6.4), we consider situations where low pressure press was repaired (data are not censored). After a descriptive analysis of low pressure press lifetimes, we will study nonparametric and semi-parametric estimations of its reliability. Only operating periods were taken into account to study low pressure press. Average duration between two “Repair” situations (system is stopped) calculated on database is 3.05 days. It includes, between two “Repair” situations all other possible activities of operational maintenance respecting internal symbolic coding (LC, CL, BLC ..., (Fig. 10 (a)).

Like to the first process, symbol “Not” (Nothing to report) and No (No production) are inserted at least each eight hours when no event is recorded (Fig. 10 (b)). This sampling provides to estimate changes at least every eight hours or more times if more symbols are observed.

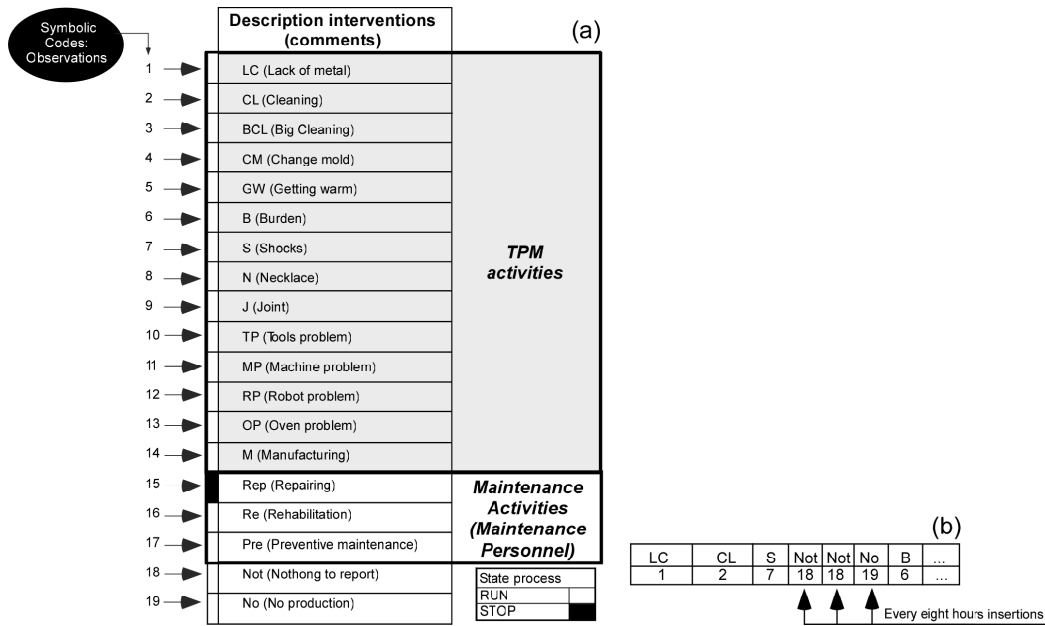


Fig. 10 (a): Symbolic coding of observations. **(b):** coding principle of observations sequence with “Not” and “no” insertion

6.3 Kaplan-Meier law applied on systems

Fig. 11 gives cumulative survival function with a confidence coefficient interval set at 95%. With this approach, the expert has evidence which indicates that degradation of system is fast (Fig. 11). For bakery continuous process studied, we find that after four days without breakdown, failure rate is up to 50% (median estimation Fig. 11 (a)).

In the case of low pressure press, we see that after two days without breakdown, failure rate is up to 53% chance (median estimation Fig. 11 (b)). Low pressure press process deteriorates faster than bakery continuous process previously studied.

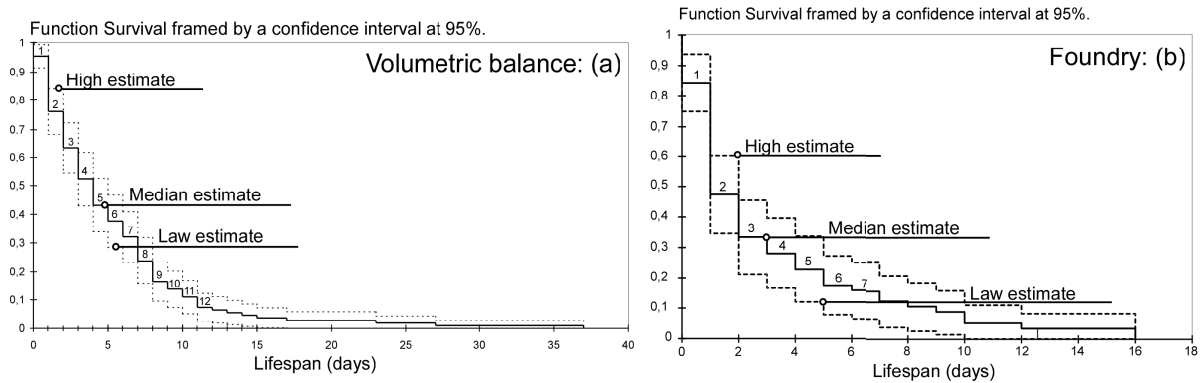


Fig. 11 Survival Functions by Kaplan-Meier

6.4 Cox law applied on systems

Proposed values in column "value" are numerical estimations of Cox model parameters. $P_r > \chi^2$ is a p-value for testing the null hypothesis H_0 (parameter = 0). If this p-value is smaller than 0.05 (less than 1 in 20 chance of being wrong), then parameter will be significantly different from 0. This test ($P_r > \chi^2$) is the threshold to reject the null hypothesis H_0 .

This value is defined in one hand by the Wald statistic (computed for each variable) and on the other hand by the critical value of Khi^2 . Values $\exp(\beta_i)$ are placed in column "Report risk, Report risk lower bound and Upper bound". Table 5 (a) shows that only variable T_SEC has an impact on survival time of process (time between two situations of repair). Coefficient $\exp(\beta_i)$, for the Report risk Upper bound (95%) is the only one < 1 associated with term ($P_r > \chi^2$) $< 0,0001$. We can conclude that more time spent in maintenance activity linked to security (SEC Code) will be "significant" for longer survival time.

According to values $\exp(\beta_i)$ of Table 5 (b), there is no event (maintenance activities) that affects survival time of system. Unlike study on volumetric balance, the expert does not have additional elements to help him.

Table 5 Regression coefficients of the proportional hazards: Cox model

Volumetric balance: (a)						Foundry: (b)		
Variable	Value	Standard deviation	Khi ² of Wald	Pr > Khi ²	Report risk	Report risk Lower bound (95%)	Report risk Upper bound (95%)	
T_Rep	0.001	0.001	4.843	0.028	1.001	1.000	1.003	
T_Pro-Adj	-0.004	0.005	0.586	0.444	0.996	0.987	1.006	
T_Ano	0.002	0.003	0.734	0.392	1.002	0.997	1.008	
T_Obs	-0.007	0.005	2.175	0.140	0.993	0.984	1.002	
T_Pre	0.000	0.001	0.064	0.801	1.000	0.998	1.001	
T_Sec	-0.037	0.009	18.976	< 0.0001	0.963	0.947	0.980	
T_Dis	-0.001	0.001	1.138	0.286	0.999	0.996	1.001	
T_Mak	-0.008	0.013	0.442	0.506	0.992	0.987	1.016	
T_Pre-visit	-0.002	0.002	1.132	0.287	0.998	0.995	1.001	
T_Rep	0.000	0.003	0.000	0.996	1.000	0.994	1.007	
T_LC	0.002	0.006	0.093	0.760	1.002	0.991	1.013	
T_CL	0.000	0.005	0.005	0.941	1.000	0.990	1.010	
T_BCL	0.002	0.002	2.509	0.113	1.002	0.999	1.005	
T_CM	0.001	0.002	0.552	0.458	1.001	0.998	1.005	
T_GW	-0.001	0.001	1.126	0.289	0.999	0.996	1.001	
T_B	-0.001	0.022	0.004	0.951	0.999	0.956	1.043	
T_S	-0.004	0.002	2.499	0.114	0.996	0.992	1.001	
T_N	-0.004	0.007	0.320	0.572	0.990	0.984	1.009	
T_J	0.005	0.004	1.399	0.237	1.005	0.997	1.013	
T_TP	-0.002	0.002	0.761	0.383	0.990	0.985	1.002	
T_MP	0.001	0.001	0.229	0.632	1.001	0.998	1.003	
T_RP	-0.002	0.013	0.027	0.870	0.998	0.972	1.024	
T_OP	0.001	0.001	0.896	0.344	1.001	0.999	1.002	
T_M	-0.009	0.009	0.927	0.336	0.991	0.973	1.009	
T_Re	-0.013	0.008	2.376	0.123	0.987	0.971	1.004	
T_Pre	0.003	0.010	0.107	0.744	1.003	0.983	1.024	

Fig. 12 describes cumulative survival function which takes into account explanatory variables. We can show that results of Cox tests provide more pessimistic values than survival function of Kaplan-Meier tests.

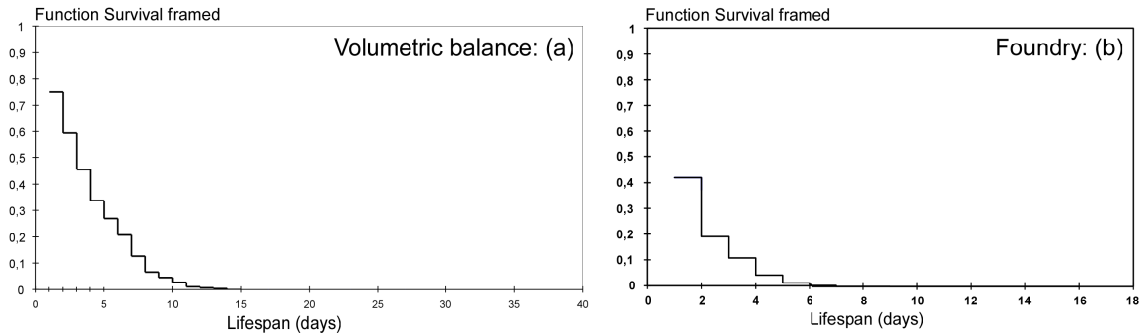


Fig. 12 Survival function estimator by Cox

Table 6 gives an overview of failure rate evolution according to time. From this first approach, the expert can adjust its level of vigilance and decide whether or not maintenance actions based on its return from experience and probability of failure between the last repair situations. Volumetric balance deteriorates slowly than BP Wheel.

Table 6 Failure probability for the two industrial systems

Volumetric balance: (a)		Probability of failure in different tests for the corpus of data for 2005 and 2006											
		at + 1 day	at + 2 days	at + 3 days	at + 4 days	at + 5 days	at + 6 days	at + 7 days	at + 8 days	at + 9 days	at + 10 days	at + 11 days	at + 12 days
Function Survival framed by a confidence interval at 95%. (Kaplan-Meier)	High estimate	15 %	27 %	38 %	48 %	53 %	59 %	68 %	76 %	79 %	83 %	88 %	90 %
	Law estimate	32 %	45 %	57 %	66 %	72 %	76 %	84 %	91 %	93 %	95 %	97 %	98 %
	Median estimate	23 %	36 %	47 %	57 %	62 %	67 %	76 %	83 %	86 %	89 %	92 %	93 %
	COX test	41 %	54 %	66 %	73 %	79 %	87 %	93 %	95 %	97 %	98 %	98 %	99 %

Foundry: (b)		Probability of failure in different tests for the corpus of data (first half 2008)											
		at + 1 day	at + 2 days	at + 3 days	at + 4 days	at + 5 days	at + 6 days	at + 7 days	at + 8 days	at + 9 days	at + 10 days	at + 11 days	at + 12 days
Function Survival framed by a confidence interval at 95%. (Kaplan-Meier)	High estimate	40 %	54 %	60 %	66 %	73 %	75 %	79 %	82 %	84 %	89 %	89 %	92 %
	Law estimate	66 %	79 %	84 %	88 %	92 %	94 %	96 %	97 %	98 %	100 %	100 %	100 %
	Median estimate	53 %	67 %	72 %	77 %	83 %	84 %	88 %	90 %	91 %	95 %	95 %	97 %
	COX test	81 %	89 %	96 %	99 %	100 %	100 %	100 %	100 %	100 %	100 %	100 %	100 %

6.5 First conclusion

We show that Kaplan-Meier model and Cox model (two nonparametric and semi-parametric statistical models) can provide help to the expert with a lot of elements. Without “a priori” information on the shape of the survival function, we first estimated the nonparametric model of Kaplan-Meier. Then, to introduce exogenous variables into the model, we studied a semi-parametric model: Cox. These tests show that, into two industrial case studies conducted, these systems needed to be "highly maintained".

However, models of Kaplan-Meier and Cox propose to the expert only two levels to define the system state (Fig. 13). There is no possible additional state and the main indicator is time that happens outside of any possible correlations.

We show, in case of volumetric balance, that a correlation between time spent in the activity "SEC" and survival time of the process exists. For the volumetric balance, with planning of maintenance activities being carried out by the day, help is provided to the expert for scheduling dynamically maintenance activities.

In case of BP Wheel, unscheduled interventions come "on the fly" through introduction of TPM. An operator can decide to perform tasks for which he is in charge.

Several problems arise:

- Is it possible to add intermediate states between "Stop" and "OK" (Fig. 13)?
- Is it then possible to prevent failure?
- How to choose the good frequency, the good sampling of collected information to estimate level of process degradation?
- Could all the collected information define a possible hidden state on the process?

We present in section 6.6, details of our approach using HMM. We will answer to issues outlined above. The two industrial cases, already presented, will support our various tests.

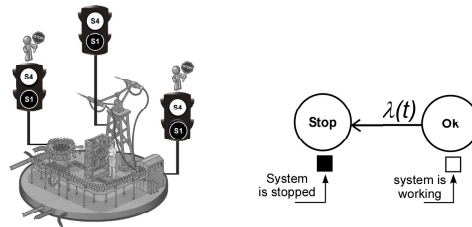


Fig. 13 Models of Kaplan-Meier and Cox associated with two states system

6.6 Hidden Markov Models applied on systems

Fig. 14 shows organization of the various tests adopted with four different algorithms. Sampling selected for the volumetric balance is the day (daily insertion of observations). Sampling selected for the BP Wheel is eight hours (shift work).

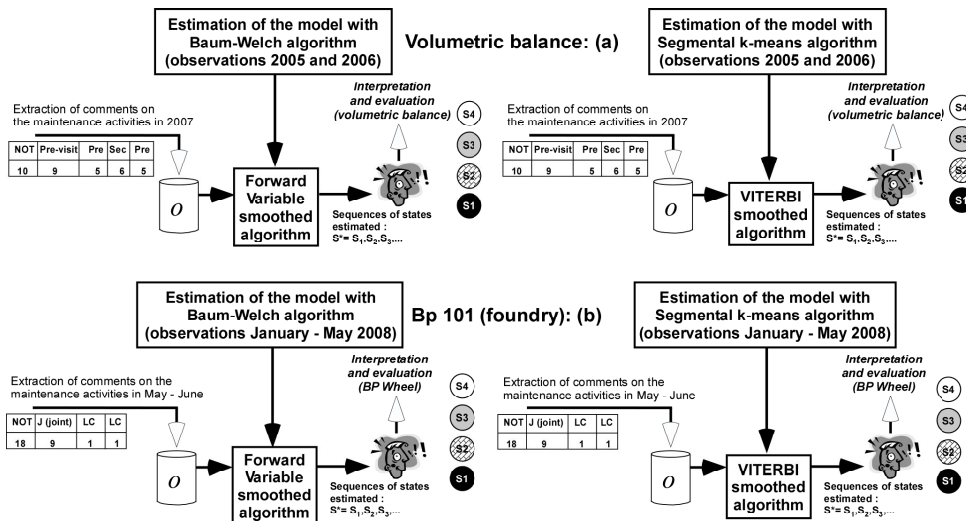
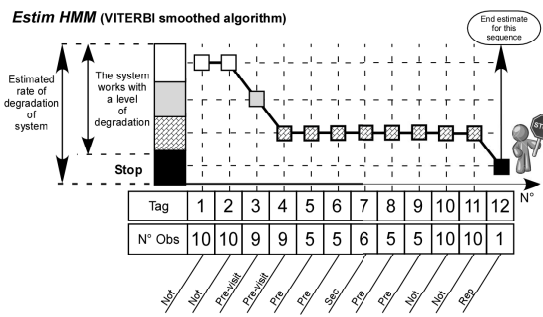


Fig. 14 Methods adopted for different tests

Our goal is to provide a S2 state which has a “high sense” for preventive maintenance (do not detect default, neither too late nor too early which could provide unnecessary or earlier interventions). Fig. 15 (a) and (b) shows that HMM can provide this kind of detection for S2 state if tools used are correctly chosen. In this figure (Prediction with Forward Variables Smoothed Probability Model / Baum-Welch algorithm), we show, for example, that our approach is efficient considering daily inclusion. S2 state is “on” only 2 days before default occurs. In the same context, with Viterbi algorithm smoothed, we can see that S2 state is on too early. This cannot provide correct clues for optimal planning of maintenance operations. Table 7 and Table 8 summarize results of tests with both algorithms used. In case of volumetric balance, after 3 days, failure is detected in 85.7% of cases (Table 7 (a)). This statistical information, being only one alone indicator, it can not meet requirements of the expert.

Volumetric balance: (a)

Sequence No. xxx				Prediction Variables Forward With Smoothed Model probas / Baum-Welch algorithm			
n°	DATE	CODE	Symbolic Codes	S1	S2	S3	S4
1	09/01/2007	Not	10	0,0%	0,0%	0,0%	100,0%
2	10/01/2007	Not	10	0,0%	0,0%	0,1%	99,9%
3	11/01/2007	Pre-v	9	0,0%	0,0%	81,8%	18,2%
4	11/01/2007	Pre-v	9	0,0%	17,5%	79,1%	3,4%
5	12/01/2007	Pre	5	0,0%	7,2%	91,2%	1,6%
6	12/01/2007	Pre	5	0,0%	6,2%	93,2%	0,6%
7	13/01/2007	Sec	6	0,0%	34,9%	64,0%	1,1%
8	13/01/2007	Pre	5	0,0%	10,2%	88,2%	1,7%
9	13/01/2007	Pr	5	0,0%	6,5%	92,7%	0,8%
10	14/01/2007	Not	10	0,0%	98,8%	0,0%	0,4%
11	15/01/2007	Not	10	0,0%	98,4%	0,0%	1,6%
12	16/01/2007	DEP	1	100,0%	0,0%	0,0%	0,0%



BP 101: (b)

Sequence No. xxx				Prediction Variables Forward With Smoothed Model probas / Baum-Welch algorithm			
n°	DATE	CODE	Symbolic Codes	S1	S2	S3	S4
1	19/06/2008	GW	5	0,0%	0,0%	0,0%	100,0%
2	19/06/2008	GW	5	0,0%	0,0%	0,0%	100,0%
3	20/06/2008	S	7	0,0%	0,0%	99,9%	0,1%
4	20/06/2008	Not	18	0,0%	1,8%	98,2%	0,0%
5	20/06/2008	S	7	0,0%	4,8%	95,2%	0,0%
6	20/06/2008	S	7	0,0%	5,4%	94,6%	0,0%
7	20/06/2008	S	7	0,0%	5,5%	94,5%	0,0%
8	20/06/2008	S	7	0,0%	5,5%	94,5%	0,0%
9	20/06/2008	LC	1	0,0%	12,1%	83,1%	4,8%
10	21/06/2008	S	7	0,0%	6,8%	93,2%	0,0%
11	21/06/2008	CL	2	0,0%	0,4%	1,4%	98,1%
12	21/06/2008	No	19	0,0%	0,0%	0,0%	100,0%
13	21/06/2008	No	19	0,0%	0,0%	0,0%	100,0%
14	21/06/2008	No	19	0,0%	0,0%	0,0%	100,0%
15	21/06/2008	No	19	0,0%	0,0%	0,0%	100,0%
16	22/06/2008	GW	5	0,0%	0,0%	0,0%	100,0%
17	22/06/2008	S	7	0,0%	0,0%	99,9%	0,1%
18	22/06/2008	MP	11	0,0%	92,0%	0,4%	7,6%
19	23/06/2008	MP	11	0,0%	97,9%	0,0%	2,1%
20	23/06/2008	Rep	15	100,0%	0,0%	0,0%	0,0%

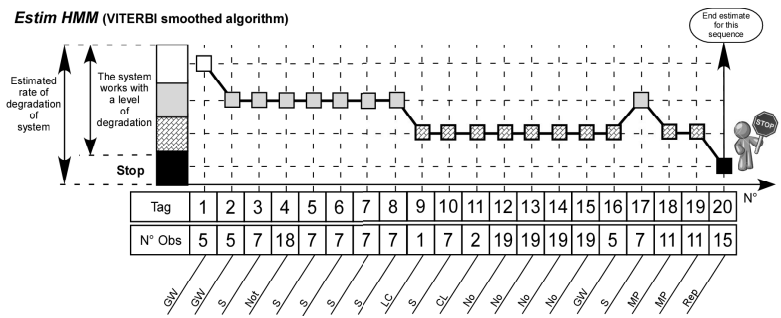


Fig. 15 Sample tests

Table 7 Failure probability for S2 state “on” - Volumetric balance (2007 database)

Volumetric balance: (a)

Prediction Viterbi smoothed	at + 1 day	at + 2 days	at + 3 days	at + 4 days	at + 5 days	at + 6 days	at + 7 days	at + 8 days	at + 9 days	at + 10 days	at + 11 days
		0,0%	9,1%	18,2%	27,3%	36,4%	36,4%	45,5%	45,5%	54,5%	63,6%
Prediction Variables Forward With Smoothed Model probas	at + 1 day	at + 2 days	at + 3 days	at + 4 days	at + 5 days	at + 6 days	at + 7 days	at + 8 days	at + 9 days	at + 10 days	at + 11 days
	28,6%	71,4%	85,7%	85,7%	85,7%	85,7%	100%	100%	100%	100%	100%

Table 8 Failure probability for S2 state “on” - BP Wheel (2008 database)

Foundry: (b)

Prediction Viterbi smoothed	at + 1 day	at + 2 days	at + 3 days	at + 4 days	at + 5 days
	12,5%	37,5%	50%	62,5%	100%
Prediction Variables Forward With Smoothed Model probas	at + 1 day	at + 2 days	at + 3 days	at + 4 days	at + 5 days
	25%	75%	100%	100%	100%

Explanation of Table 7 and Table 8: Proposed model always provides S1 state when a default occurs, but it is too late, because default occurs ... So to evaluate our method, we sum up occurrences of “S2 state at x days” before default occurs. For example, in Table 8 with forward variables at +2 days, 71,4% mean that the probability that a default occurs in two days is 71,4%. This supposes that “S2 state” always precedes “S1 state”.

In one hand, we show with these tables that learning with segmental k-means algorithm and testing with Viterbi algorithm, is not efficient to give good failure prediction (in both cases of volumetric balance and BP Wheel). S2 state is detected too early which cannot provide our goal.

On the other hand, Forward variables with Baum-Welch algorithm provide good results for default predictions. We can provide efficient predictions for S2 state, under statistical information (5.38 days). Fig. 16 (a) and (b) show additional information that is available to the expert. Once the S2 state is detected, the expert should react.

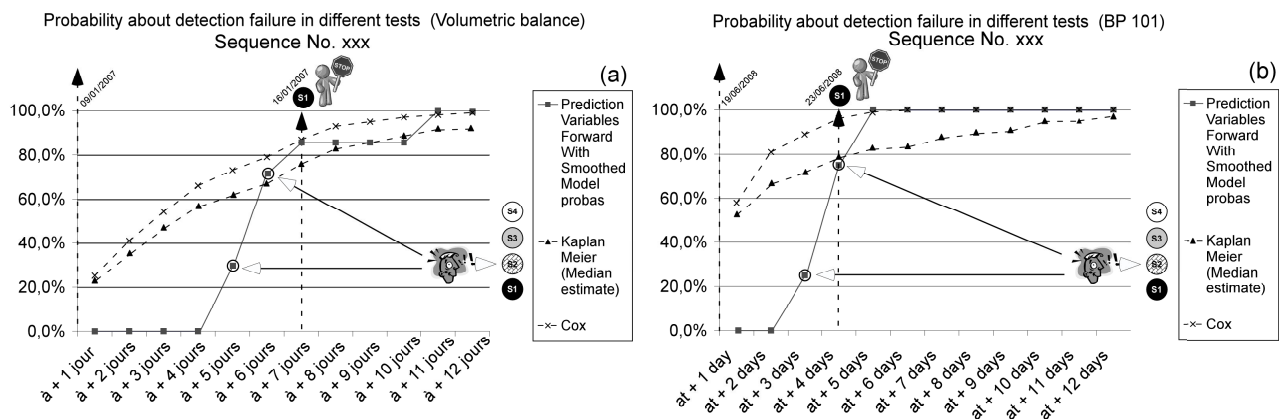


Fig. 16 Triggers of “S2 state”

7 CONCLUSION AND FUTURE WORKS

In this paper, after an overview of different maintenance approaches, we presented a nonparametric and semi-parametric evaluation of system state: Kaplan-Meier and Cox. Our approach based on HMM was then developed. Tests of these different methods were performed onto two real situations: volumetric balance and BP Wheel. Our approach shows that all results provided by this study could provide some help to expert decisions. Results show that our "decision support" provides a good estimation of repair probability when events (observations) are available.

Without events (observations) HMM method is less useful, then nonparametric and semi parametric laws could be used on foreground (Fig. 17). We also show that the expert with such a tool can try to qualify maintenance activities. Tests were carried out offline, so we cannot assess the effects of scheduling dynamically maintenance

activities. We will soon sign a contract with the foundry industrial group, for the On line implementation of these methods. Before considering implementation in real-time situation, we will explore possibilities of process simulation for which we could apply dynamic scheduling of maintenance.

Initial analysis of events provided by manufacturers, allowed them to consolidate their knowledge onto effects of specific maintenance activities (indicator can provide, in some cases, information about maintenance activities "quality"). For example, is a maintenance activity efficient or not or neutral for the system?

In future works, we wish to evaluate performances of the method on simulated data and perfectly controlled. This assessment will better evaluate performances of this availability indicator. The authors would like to thank the anonymous reviewers for their helpful comments and advices which contribute much to the improvements of this paper.

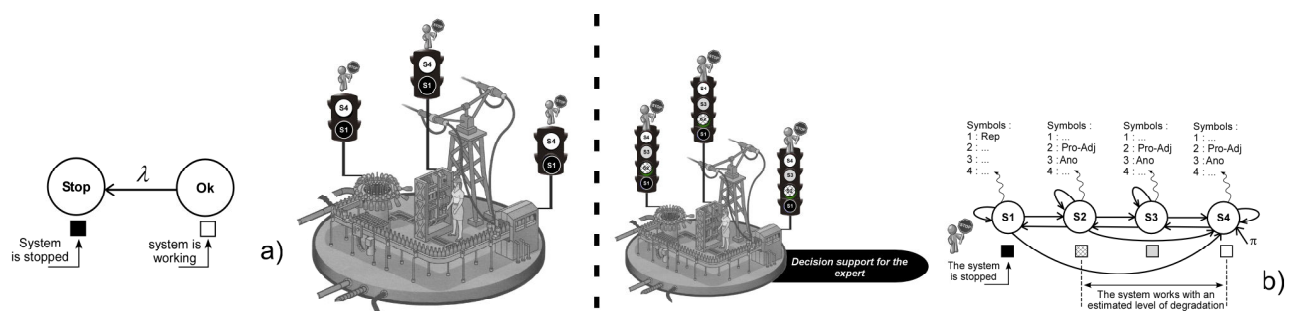


Fig. 17 Kaplan-Meier & Cox vs HMM

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