

# Dam break on a wet domain without friction in *SWASHES*: obtention of the velocity $c_m$ .

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We consider a dam break on a wet domain without friction, known as Stocker's solution. After the dam break, the solution is composed of a rarefaction wave and of a shock that connect at the height  $h_m$ , moving with the velocity  $c_m$ . For more details, we refer to [1, § 4.1.1].

The goal of this note is to detail the computation of the value of  $c_m$  and to explain the formula given in the code, see [https://sourcesup.renater.fr/frs/?group\\_id=876](https://sourcesup.renater.fr/frs/?group_id=876).

## 1 Obtention of the formula given in [1].

**First, starting from the Rankine-Hugoniot relations, we obtain relation (4.165) of [2].** Assuming the water going from the left part to the right part of the domain, the right-going shock is situated between a zone where the water height is  $h_m$  (with velocity  $u_m$ ), and the right part of the domain where  $h = h_r$  and  $u = u_r = 0$ . Consequently, the Rankine-Hugoniot relations read:

$$F(U_m) - F(U_r) = W(U_m - U_r) \text{ with } F(U) = \begin{pmatrix} hu \\ \frac{gh^2}{2} + hu^2 \end{pmatrix} \text{ and } U = \begin{pmatrix} h \\ hu \end{pmatrix}.$$

Developing the two expressions we get:

$$\begin{cases} h_m u_m - h_r u_r = h_m u_m = W(h_m - h_r), & (1) \\ \frac{gh_m^2}{2} + h_m u_m^2 - \frac{gh_r^2}{2} + h_r u_r^2 = W(h_m u_m - h_r u_r) = 2W(h_m u_m - h_r u_r) - W^2(h_m - h_r). & (2) \end{cases}$$

The first line (1) will be used in the following, while the second line (2) reads:

$$\begin{aligned} \frac{gh_m^2}{2} + h_m u_m^2 - 2W h_m u_m &= -W^2(h_m - h_r) + \frac{gh_r^2}{2}, \\ h_m(u_m - W)^2 + \frac{gh_m^2}{2} &= W^2 h_r + \frac{gh_r^2}{2}. \end{aligned} \quad (3)$$

**Next, we develop this expression to obtain an equation on  $c_m$ .** There is a left-going rarefaction wave between the left part (with  $h = h_l$  and  $u = u_l = 0$ ) and the zone where the water height is  $h_m$  with velocity  $u_m$ . Consequently, Riemann invariants give:  $u_m + 2c_m = u_l + 2c_l = 2c_l$ . In equality (3), we replace  $u_m$  by  $2(c_l - c_m)$  and  $W$  by  $\frac{h u_m}{h_m - h_r}$ , obtained from (1). Multiplying by  $2(h_m - h_r)^2$ , we get:

$$\begin{aligned} 2h_m(u_m h_m - u_m h_r - u_m h_m)^2 + gh_m^2(h_m - h_r)^2 &= 2h_r(u_m h_m)^2 + gh_r^2(h_m - h_r)^2, \\ 8h_m h_r^2(c_l - c_m)^2 + gh_m^2(h_m - h_r)^2 &= 8h_r h_m^2(c_l - c_m)^2 + gh_r^2(h_m - h_r)^2, \\ -8h_m h_r(c_l - c_m)^2(h_m - h_r) &= -g(h_m^2 - h_r^2)(h_m - h_r)^2. \end{aligned}$$

We simplify by  $h_m - h_r$ :

$$-8h_m h_r (c_l - c_m)^2 = -g (h_m + h_r) (h_m - h_r)^2,$$

which rewrites:

$$-8h_m h_r (c_l - c_m)^2 + g (h_m + h_r) (h_m - h_r)^2 = 0.$$

Multiplying by  $g^2$ , we have:

$$-8gh_m gh_r (c_l - c_m)^2 + (gh_m + gh_r) (gh_m - gh_r)^2 = 0,$$

that is

$$-8c_m^2 gh_r (\sqrt{gh_l} - c_m)^2 + (c_m^2 + gh_r) (c_m - gh_r)^2 = 0. \quad (4)$$

**Remark.** In the published version, there is a typo: one should read  $-8c_m^2 gh_r (\sqrt{gh_l} - c_m)^2$  instead of  $-8c_m^2 gh_r (gh_l - c_m^2)^2$ .

## 2 Obtention of the formula of the code

The previous relation (4) reads:

$$-8c_r^2 c_m^2 (c_l - c_m)^2 + (c_m^2 + c_r^2) (c_m^2 - c_r^2)^2 = 0.$$

We develop this equality and we obtain:

$$\begin{aligned} -8c_r^2 c_m^2 (c_l^2 - 2c_l c_m + c_m^2) + (c_m^2 + c_r^2) (c_m^4 - 2c_m^2 c_r^2 + c_r^4) &= 0, \\ -8c_r^2 c_m^2 c_l^2 + 16c_l c_r^2 c_m^3 - 8c_r^2 c_m^4 + c_m^6 - 2c_m^4 c_r^2 + c_m^2 c_r^4 + c_r^2 c_m^4 - 2c_m^2 c_r^4 + c_r^6 &= 0, \\ c_m^6 - 9c_r^2 c_m^4 + 16c_l c_r^2 c_m^3 - (c_r^2 - 8c_l^2) c_r^2 c_m^2 + c_r^6 &= 0. \end{aligned}$$

This last formulation corresponds to equation (4.166) of [2], and is the relation used in the code.

## References

- [1] Olivier Delestre, Carine Lucas, Pierre-Antoine Ksinant, Frédéric Darboux, Christian Laguerre, Thi Ngoc Tuoi Vo, François James, and Stéphane Cordier. *SWASHES: a compilation of shallow-water analytic solutions for hydraulic and environmental studies. International Journal for Numerical Methods in Fluids*, 72:269–300, 2013.
- [2] Jean-Michel Hervouet. *Hydrodynamics of free surface flows*. Wiley, 2007.