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Spin waves in magnetic quantum wells with Coulomb interaction and sd exchange coupling

F. Perez

*Institut des NanoSciences de Paris, CNRS/Université Paris 6,
140 rue de Lourmel, 75015 Paris, France*

J. Cibert

Institut Néel, CNRS/UJF, BP166, 38042 Grenoble cedex 9, France

M. Vladimirova and D. Scalbert

*Groupe d'Etude des Semiconducteurs,
UMR 5650 CNRS, Université Montpellier 2,
Place Eugène Bataillon, 34095 Montpellier cedex, France*

Abstract

We theoretically describe the spin excitation spectrum of a two dimensional electron gas embedded in a quantum well with localized magnetic impurities. Compared to the previous work, we introduce equations that allow to consider the interplay between the Coulomb interaction of delocalized electrons and the sd exchange coupling between electrons and magnetic impurities. Strong qualitative changes are found : mixed waves propagate below the single particle continuum, an anticrossing gap is open at a specific wavevector and the kinetic damping due to the electron motion strongly influences the coupling strength between electrons and impurities spins.

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I. INTRODUCTION

Collective spin dynamics in dilute magnetic semiconductors (DMS) has recently drawn lots of attention.^{1–5} This field provides an insight into the origins of carrier-induced ferromagnetism in semiconductors^{6–8} and particular features in the spin excitation spectrum^{8,9} due to the presence of two spin sub-systems that are dynamically coupled by Coulomb-exchange interaction: that of the itinerant carrier and that of the localized magnetic impurities. The transverse spin excitation spectrum has been theoretically found to be composed of three types of excitations. These are: two collective spin waves corresponding to itinerant and localized spins precessing in phase or out of phase to each other, and single-particle (or Stoner-like) excitations of the itinerant carriers.^{4,8–10} If the DMS is in the ferromagnetic state, the in-phase spin wave (IPW) becomes the Goldstone-like mode with an acoustic type dispersion responsible for long-range spin order in the ground state. The out of phase spin wave (OPW) develops an optical branch, with a zone-center energy determined by the strength of Coulomb-exchange interaction between carriers and the spins of magnetic impurities.

Experimental evidence of the entire spectrum in a ferromagnetic DMS like GaMnAs is not available. What has been reported so far are features related to the zone-center IPW, dominated by the Mn spin precession, its dynamics.^{2,11,12} and its ferromagnetic resonance.¹³ We find no experimental data available for the out of phase mode. Indeed, ferromagnetism in GaMnAs systems requires a high Mn concentration, which destroys the periodicity of the crystal potential and smooths out all optical resonances.

More insight into the DMS spin excitation spectrum has been gained in CdMnTe doped quantum wells (QW), which constitute a clean test-bed system, appropriate to capture general properties of the collective spin dynamics in DMS materials. Evidence for carrier-induced ferromagnetism has been found in CdMnTe quantum wells doped with holes¹⁴. When doped with electrons, due to the very low Curie temperature, only the paramagnetic phase is available to most experiments. The OPW mode dispersion and single-particle excitations have been probed by Raman measurements in the paramagnetic state^{15,16}. The mixed nature of the IPW and OPW waves has been evidenced in the frequency¹⁷ and time domain¹⁴. Nevertheless, there is a lack of a full theoretical description of the spin excitation spectrum in CdMnTe QW. Indeed, so far two approaches were followed to describe the spin

excitations : in the first one¹⁰, the *sd*-exchange dynamical coupling between Mn and electron spins was considered, but the Coulomb interaction between electrons was dropped out. In the second¹⁸, the reverse point of view was adopted : spin resolved Coulomb interaction between electrons was taken into account, but only the static mean-field *sd* contribution from the Mn was kept to form a highly spin-polarized two dimensional electron gas (SP2DEG).

This work fills the gap between the two theoretical approaches, by solving the spin dynamical equations in presence of both the *sd*-exchange dynamical coupling between Mn and electron spins and the Coulomb interaction between electrons. Starting from the full DMS Hamiltonian, the approach combines exact commutation rules and standard generalized Random Phase Approximation (RPA). We also include the intrinsic damping of the pure electron spin waves due to the delocalized character of the electrons^{19,20}. We show that the introduction of Coulomb electron-electron interaction induces strong qualitative changes in the spectrum compared to the approach of Ref.¹⁰ and that inclusion of the intrinsic damping diminishes the strength of the coupling between the two spin subsystems for non-zero wavevectors. Generalization of this model to hole systems might be considered : then one should take into account the fact that the hole spin states are not isotropic.

The paper is divided as follows : in Sec. II, we detail the Hamiltonian of the system and rewrite it in terms of collective variables, in Sec. III, we use transverse spin dynamics equations to derive spin response functions, and in the last section, we study the spectrum of spin mixed electron-Mn modes.

II. THE 2DEG DMS HAMILTONIAN UNDER STATIC FIELD

We consider a QW of width w containing $x_{eff}N_0$ unpaired²¹ Mn spins per unit volume. The first subband is populated by n_{2D} electrons per unit surface. The 2DEG-DMS Hamiltonian under the influence of a static magnetic field $\mathbf{B} = B\mathbf{e}_z$ applied in the plane of the QW writes :

$$\begin{aligned}
\hat{H} &= \hat{H}_{Kin} + \hat{H}_{Coulomb} + \hat{H}_{s-d} + \hat{H}_{Zeeman} \\
\hat{H}_{s-d} &= -\alpha \iiint \hat{\mathbf{S}}(\mathbf{r}) \cdot \hat{\mathbf{M}}(\mathbf{r}) d^3r \\
\hat{H}_{Zeeman} &= g_e \mu_B \iiint \hat{\mathbf{S}}(\mathbf{r}) \cdot \mathbf{B} d^3r + g_{Mn} \mu_B \iiint \hat{\mathbf{M}}(\mathbf{r}) \cdot \mathbf{B} d^3r
\end{aligned} \tag{1}$$

where α is the exchange coupling between conduction electrons and Mn spins ($\alpha > 0$), and g_e and g_{Mn} are normal g-factor of, respectively, conduction electrons and Mn electrons. In the convention where $\mu_B > 0$, we have $g_e \simeq -1.44$ and $g_{Mn} \simeq 2.00$. We have introduced two vector operators : $\hat{\mathbf{S}}(\mathbf{r}) = \chi^2(y) \sum_i \hat{\mathbf{s}}_i \delta(\mathbf{r}_{//} - \mathbf{r}_{i//})$ is the 3D electron spin density in a splitted coordinates frame $\mathbf{r} = (\mathbf{r}_{//}, y)$ with $\mathbf{r}_{//}$, the in-plane position and y the out of plane coordinate. $\chi(y)$ is the electron envelope-function of the first subband of the QW. The i index accounts for the i -th electron of the 2DEG, its spin $\frac{1}{2}$ is described by the operator $\hat{\mathbf{s}}_i$ and its position is $\mathbf{r}_{i//}$. $\hat{\mathbf{M}}(\mathbf{r}) = \sum_j \hat{\mathbf{I}}_j \delta(\mathbf{r} - \mathbf{R}_j)$ is the Mn 3D spin density. The j -th $\frac{5}{2}$ -spin $\hat{\mathbf{I}}_j$ of a single Mn impurities is localized on the cation site $\mathbf{R}_j$. In the equilibrium state at temperature T , each Mn spin has the average value $\langle \hat{I}_z \rangle(B, T)$, which is the thermodynamic average over the five occupied states of the Mn atom d-shell, given by the modified Brillouin function²¹. The 2DEG has the equilibrium spin polarization $\zeta = (n_{\uparrow} - n_{\downarrow})/n_{2D}$.

We, now, rewrite the $s-d$ Hamiltonian using the electron (and Mn) spin fluctuations operators at in-plane wave-vector $\mathbf{q}$, respectively, $\hat{\mathbf{S}}_{\mathbf{q}} = \iiint \hat{\mathbf{S}}(\mathbf{r}) e^{-i\mathbf{q} \cdot \mathbf{r}_{//}} d^2r_{//} dy = \sum_i \hat{\mathbf{s}}_i e^{-i\mathbf{q} \cdot \mathbf{r}_{i//}}$ and $\hat{\mathbf{M}}_{\mathbf{q}} = \sum_j \hat{\mathbf{I}}_j e^{-i\mathbf{q} \cdot \mathbf{R}_{j//}}$. Due to the 2D and 3D characters of, respectively, the conduction electron and Mn spins subsystems, the electron spin-degrees of freedom naturally couples to Mn spin profile weighted by the squared electron wave-function. For later convenience, we introduce the following n -profile Mn spin fluctuations operators :

$$\hat{\mathbf{M}}_{\mathbf{q}}^{(n)} = w^n \iiint \chi^{2n}(y) \hat{\mathbf{M}}(\mathbf{r}) e^{-i\mathbf{q} \cdot \mathbf{r}_{//}} d^2r_{//} dy = w^n \sum_j \chi^{2n}(y_j) \hat{\mathbf{I}}_j e^{-i\mathbf{q} \cdot \mathbf{R}_{j//}}$$

Hence, $\hat{\mathbf{M}}_{\mathbf{q}} = \hat{\mathbf{M}}_{\mathbf{q}}^{(0)}$. $\hat{\mathbf{M}}_{\mathbf{q}}^{(n)}$ are vector operators verifying the following commuting relations

:

$$[\hat{M}_{\alpha, \mathbf{q}}^{(n)}, \hat{M}_{\beta, \mathbf{q}'}^{(p)}] = i \epsilon_{\alpha, \beta, \gamma} \hat{M}_{\gamma, \mathbf{q} + \mathbf{q}'}^{(n+p)} \tag{2}$$

where $\alpha, \beta, \gamma = x, y, z$ and $\epsilon_{\alpha, \beta, \gamma}$ is the Levi-Cevita tensor. It follows :

$$\hat{H}_{s-d} = -\frac{\alpha}{wL^2} \sum_{\mathbf{q}} \hat{\mathbf{S}}_{\mathbf{q}} \cdot \hat{\mathbf{M}}_{-\mathbf{q}}^{(1)} \quad (3a)$$

$$= -\tilde{\alpha} \left\{ \begin{aligned} &\delta \hat{S}_{z,\mathbf{q}=0} \cdot \langle \hat{M}_{z,\mathbf{q}=0}^{(1)} \rangle_0 + \langle \hat{S}_{z,\mathbf{q}=0} \rangle_0 \cdot \delta \hat{M}_{z,\mathbf{q}=0}^{(1)} + \delta \hat{S}_{z,\mathbf{q}=0} \cdot \delta \hat{M}_{z,\mathbf{q}=0}^{(1)} \\ &+ \frac{1}{2} \hat{S}_{+,\mathbf{q}=0} \cdot \hat{M}_{-,\mathbf{q}=0}^{(1)} + \frac{1}{2} \hat{S}_{-,\mathbf{q}=0} \cdot \hat{M}_{+,\mathbf{q}=0}^{(1)} \\ &+ \sum_{\mathbf{q} \neq 0} \hat{\mathbf{S}}_{\mathbf{q}} \cdot \hat{\mathbf{M}}_{-\mathbf{q}}^{(1)} \end{aligned} \right\} \quad (3b)$$

In Eq. (3a), we have defined the exchange coupling constant $\tilde{\alpha} = \alpha/wL^2$, equilibrium averaging $\langle \rangle_0$ and fluctuation operators : $\delta \hat{A} = \hat{A} - \langle \hat{A} \rangle_0$.

Finally, we get :

$$\hat{H}_{s-d} + H_{Zeeman} = Z_e(B) \cdot \delta \hat{S}_{z,\mathbf{q}=0} + g_{\text{Mn}} \mu_B B \cdot \delta \hat{M}_{z,\mathbf{q}=0}^{(0)} + K \cdot \delta \hat{M}_{z,\mathbf{q}=0}^{(1)} \quad (4a)$$

$$- \tilde{\alpha} \frac{1}{2} \left(\hat{S}_{+,\mathbf{q}=0} \cdot \hat{M}_{-,\mathbf{q}=0}^{(1)} - \hat{S}_{-,\mathbf{q}=0} \cdot \hat{M}_{+,\mathbf{q}=0}^{(1)} \right) \quad (4b)$$

$$- \tilde{\alpha} \delta \hat{S}_{z,\mathbf{q}=0} \cdot \delta \hat{M}_{z,\mathbf{q}=0}^{(1)} \quad (4c)$$

$$- \tilde{\alpha} \sum_{\mathbf{q} \neq 0} \hat{\mathbf{S}}_{\mathbf{q}} \cdot \hat{\mathbf{M}}_{-\mathbf{q}}^{(1)} \quad (4d)$$

with the total bare Zeeman energy of conduction electrons given by :

$$Z_e(B) = \underbrace{\tilde{\alpha} \gamma_1 N_{\text{Mn}} \langle \hat{I}_z \rangle (B, T)}_{\Delta} - |g_e| \mu_B B \quad (5)$$

where $\gamma_1 = \int_0^w \chi^2(y) dy$ is the probability to find the electron in the QW, $N_{\text{Mn}} = x_{eff} N_0 w L^2$ is the number of Mn spins available in the QW. In Eq. (5), we evidence the "Overhauser shift" Δ and the normal Zeeman contribution, opposite to Δ (the sd contribution). Indeed, as $g_e < 0$, $g_{\text{Mn}} > 0$ and $\alpha > 0$, the Mn spins are anti-parallel to the field, thus, the sd coupling aligns the electron spins anti-parallel to the field, while the normal Zeeman aligns the electron spin parallel to the field. When the sd coupling dominates over the normal Zeeman contribution and $B > 0$, both Mn and electron spins are $\downarrow : \langle \hat{I}_z \rangle < 0$ and the 2DEG spin polarization degree ζ is also negative.

The first line, Eq. (4a) gives the mean-field "effective Zeeman" Hamiltonian, where non-dynamical coupling between electrons and Mn spins appears. The mean-field Hamiltonian

naturally introduces the "Knight shift", due to the equilibrium electron spin polarization $\langle S_{z,\mathbf{q}=\mathbf{0}} \rangle_0 = n_{2D} L^2 \zeta / 2$, which shifts the Mn spin precession energy:

$$K = -\tilde{\alpha} \langle S_{z,\mathbf{q}=\mathbf{0}} \rangle_0 = \frac{1}{2} \frac{\alpha}{w} n_{2D} |\zeta| \quad (6)$$

Eq. (4b) and Eq. (4d) give rise to a first order *sd*-dynamical coupling between transverse spin degrees of freedom and induce spin-mixed electron-Mn modes of precession. They also contain higher orders correlation terms which have been indentified to be responsible for a damping⁵. The effects of the correlations contained in Eqs. (4b)-(4d) are out of the scope of the present work and will be neglected when they appear.

III. TRANSVERSE SPIN DYNAMICS

A. SP2DEG dynamics without the s-d dynamical coupling

In this paragraph, we take into account only the first line of Eqs.(4a)-(4d). Hence conduction electron and Mn dynamics are independent. The 2DEG is polarized by the static exchange field of Mn spins, this forms a spin-polarized 2DEG (SP2DEG) as described in Ref.¹⁸. The Hamiltonian which rules the electron dynamics in the SP2DEG reduces to :

$$\hat{H}_{SP2DEG} = \hat{H}_{Kin} + \hat{H}_{Coulomb} + Z_e (B) \hat{S}_{z,\mathbf{q}=\mathbf{0}} \quad (7)$$

Introducing the the electron creation-anihilation operators, a spin-flip single particle excitation (SF-SPE) is described by a single electron-hole pair operator $c_{\mathbf{k}-\mathbf{q},\uparrow}^+ c_{\mathbf{k},\downarrow}$, and electrons spin-wave operators, introduced above, are given by $\hat{S}_{+,\mathbf{q}} = \hat{S}_{x,\mathbf{q}} + i\hat{S}_{y,\mathbf{q}} = \sum_{\mathbf{k}} c_{\mathbf{k}-\mathbf{q},\uparrow}^+ c_{\mathbf{k},\downarrow}$. Let's notice that $[\hat{S}_{+,\mathbf{q}}, c_{\mathbf{k}-\mathbf{q}',\uparrow}^+ c_{\mathbf{k},\downarrow}] = 0$, such that collective and single particle modes are not intrinsically coupled. In the following we will use exact commutation rules to write equation of motions for these normal modes of the SP2DEG. We will note $\left(\frac{d\hat{A}}{dt} \right)_{2DEG} = [\hat{A}, \hat{H}_{SP2DEG}] / i\hbar$ the time derivative of $\hat{A}$ related to $\hat{H}_{SP2DEG}$ only. Further, we will make use of linear response theory to derive quantities like :

$$\left\langle \left\langle \hat{A}; \hat{B} \right\rangle \right\rangle_{\omega} = -\frac{i}{\hbar} \lim_{\varepsilon \rightarrow 0^+} \int_0^{+\infty} \left\langle [\hat{A}(t), \hat{B}] \right\rangle_0 e^{-i\omega t - \varepsilon t} dt \quad (8)$$

which gives the linear response of an observable $\hat{A}$ to a perturbation coupled linearly to $\hat{B}$ in the considered Hamiltonian²².

$\langle\langle\hat{A};\hat{B}\rangle\rangle_\omega$ has the following equations of motion :

$$\langle\langle\hat{A};\hat{B}\rangle\rangle_\omega = -\frac{i}{\omega} \langle\langle\dot{\hat{A}};\hat{B}\rangle\rangle_\omega - \frac{1}{\hbar\omega} \langle[\hat{A},\hat{B}]\rangle_0 \quad (9a)$$

$$= \frac{i}{\omega} \langle\langle\hat{A};\dot{\hat{B}}\rangle\rangle_\omega - \frac{1}{\hbar\omega} \langle[\hat{A},\dot{\hat{B}}]\rangle_0 \quad (9b)$$

where $\dot{\hat{A}} = [\hat{A}, \hat{H}] / i\hbar$ is the time derivative of $\hat{A}$ related to the considered Hamiltonian. We will note $\langle\langle\hat{A};\hat{B}\rangle\rangle_\omega^{2DEG}$ when $\dot{\hat{A}}$ is replaced by $(d\hat{A}/dt)_{2DEG}$.

1. Single particle modes dynamics

The kinetic Hamiltonian $\hat{H}_{Kin} = \sum_{\mathbf{k},\sigma} E_{\mathbf{k}} c_{\mathbf{k},\sigma}^\dagger c_{\mathbf{k},\sigma}$ and the mean-field Zeeman Hamiltonian conserve the single-particle modes :

$$\begin{aligned} & \left[c_{\mathbf{k}-\mathbf{q},\uparrow}^\dagger c_{\mathbf{k},\downarrow}, \hat{H}_{Kin} + Z_e(B) \hat{S}_{z,\mathbf{q}=\mathbf{0}} \right] = \\ & (E_{\mathbf{k}} - E_{\mathbf{k}-\mathbf{q}} - Z_e) c_{\mathbf{k}-\mathbf{q},\uparrow}^\dagger c_{\mathbf{k},\downarrow} \end{aligned}$$

But the Coulomb Hamiltonian $\hat{H}_{Coulomb} = \frac{1}{2} \sum_{\mathbf{q} \neq 0, \mathbf{k}', \sigma', \mathbf{k}, \sigma} V_{\mathbf{q}} c_{\mathbf{k}+\mathbf{q},\sigma}^\dagger c_{\mathbf{k}'-\mathbf{q},\sigma'}^\dagger c_{\mathbf{k}',\sigma'} c_{\mathbf{k},\sigma}$ couples a single particle mode to multi-pair modes having a spin +1 :

$$\begin{aligned} & \left[c_{\mathbf{k}-\mathbf{q},\uparrow}^\dagger c_{\mathbf{k},\downarrow}, \hat{H}_{Coulomb} \right] = \\ & + \sum_{\mathbf{q}' \neq 0, \mathbf{k}', \sigma} V_{\mathbf{q}'} \left(c_{\mathbf{k}-\mathbf{q}+\mathbf{q}',\uparrow}^\dagger c_{\mathbf{k}',\sigma}^\dagger c_{\mathbf{k}'-\mathbf{q}',\sigma} c_{\mathbf{k},\downarrow} \right) \\ & - \sum_{\mathbf{q}' \neq 0, \mathbf{k}', \sigma} V_{\mathbf{q}'} \left(c_{\mathbf{k}-\mathbf{q},\uparrow}^\dagger c_{\mathbf{k}',\sigma}^\dagger c_{\mathbf{k}'-\mathbf{q}',\sigma} c_{\mathbf{k}-\mathbf{q}',\downarrow} \right) \end{aligned} \quad (10)$$

where $V_{\mathbf{q}'}$ is the space Fourier transform of the bare Coulomb interaction. It follows from Eq. (10) that $\hat{H}_{Coulomb}$ does not conserve the SPE motion, but introduces an infinite hierarchy where a single electron-hole pair of the Fermi sea (a SPE) couples to multiple pairs having the same global spin. Approximations can be made : in the rhs terms of Eq. (10), some conserve the SPE motion and renormalize it, others introduce a scattering effect, the so-called spin-Coulomb drag²³, and can be described by an electron-electron scattering time²⁴ τ_{e-e} . The former consists in making the random phase approximation (RPA) on single

mode dynamics²⁵, *i.e.*, keeping in Eq. (10) only terms which can be written as a product of a SF-SPE with an occupation number $\hat{n}_{\mathbf{k},\sigma}$, and replacing $\hat{n}_{\mathbf{k},\sigma}$ by its average value $\langle \hat{n}_{\mathbf{k},\sigma} \rangle_0$. Then the Coulomb factor $V_{\mathbf{q}'}$ has to be replaced by a local field factor G_{xc} which accounts for the effective dynamical exchange-field produced by other electrons²⁶ (a part of what has been suppressed in making the RPA). Adding a damping rate $\eta = \hbar/\tau_{e-e}$ due to the scattering leads to the SF-SPE equation of motion that we will use in the following :

$$i\hbar \left(\frac{d}{dt} c_{\mathbf{k}-\mathbf{q},\uparrow}^+ c_{\mathbf{k},\downarrow} \right)_{2DEG} = \left(E_{\mathbf{k}} - E_{\mathbf{k}-\mathbf{q}} - Z_e + 2G_{xc} \langle \hat{S}_{z,\mathbf{q}=0} \rangle_0 \right) c_{\mathbf{k}-\mathbf{q},\uparrow}^+ c_{\mathbf{k},\downarrow} + G_{xc} (\langle \hat{n}_{\mathbf{k},\downarrow} \rangle_0 - \langle \hat{n}_{\mathbf{k}-\mathbf{q},\uparrow} \rangle_0) \hat{S}_{+,\mathbf{q}} - i\eta c_{\mathbf{k}-\mathbf{q},\uparrow}^+ c_{\mathbf{k},\downarrow} \quad (11)$$

Eq. (11) evidences the renormalized Zeeman energy, *i.e.*, the spin-flip energy of single electrons :

$$Z^* = Z_e - 2G_{xc} \langle \hat{S}_{z,\mathbf{q}=0} \rangle_0 \quad (12)$$

Compared to the bare Zeeman energy Z_e , Z^* is enhanced by Coulomb-exchange between spin-polarized electrons, a phenomenon linked to the spin-susceptibility enhancement²⁷. Each SF-SPE is characterized by two wavevectors $\mathbf{k}$ and $\mathbf{q}$. At $\mathbf{q} = 0$, SF-SPE are degenerate to Z^* . When $\mathbf{q} \neq 0$ the degeneracy is lifted by the kinetic spread of velocities which depend on the initial momentum $\mathbf{k}$.

2. Collective modes dynamics

Along the SF-SPE, the above spin polarized SP2DEG develops collective modes, the so-called spin-flip waves (SFW). SFW dynamics are described by the $\hat{S}_{+,\mathbf{q}}$ operators dynamics. As $\hat{H}_{Coulomb}$ conserves the macroscopic spin, it follows :

$$\left[\hat{S}_{+,\mathbf{q}}, \hat{H}_{Coulomb} + Z_e(B) \hat{S}_{z,\mathbf{q}=0} \right] = -Z_e(B) \hat{S}_{+,\mathbf{q}} \quad (13)$$

But, the kinetic Hamiltonian couples collective states to the spin current $\hat{\mathbf{J}}_{+,\mathbf{q}} = \frac{\hbar}{m^*} \sum_{\mathbf{k}} (\mathbf{k} - \frac{\mathbf{q}}{2}) c_{\mathbf{k}-\mathbf{q},\uparrow}^+ c_{\mathbf{k},\downarrow}$ carried by single particle states :

$$\left[\hat{S}_{+,\mathbf{q}}, \hat{H}_{Kin} \right] = \hbar \mathbf{q} \cdot \hat{\mathbf{J}}_{+,\mathbf{q}} \quad (14)$$

Finally the equation of motion of collective SFW modes writes :

$$\left(\frac{d}{dt} \hat{S}_{+, \mathbf{q}} \right)_{2DEG} = i\omega_e \hat{S}_{+, \mathbf{q}} - i\mathbf{q} \cdot \hat{\mathbf{J}}_{+, \mathbf{q}} \quad (15)$$

where $\omega_e = Z_e(B)/\hbar$ is the frequency of the Larmor's electron mode.

One is left with evaluating the spin-current dynamics to find the 2DEG electron spin waves. The spin current evolution is dominated by single particle states dynamics as $\hat{H}_{Coulomb}$ does not conserve $\hat{\mathbf{J}}_{+, \mathbf{q}}$ and destroys the coherence between the single particle objects composing $\hat{\mathbf{J}}_{+, \mathbf{q}}$. As seen from Eq. (11), the exchange field produced by the spin fluctuation $\hat{S}_{+, \mathbf{q}}$ drives the spin current. The interplay between the spin-current dynamics and the $\hat{S}_{+, \mathbf{q}}$ dynamics then determines both the SFW dispersion and its damping. The relevant spin-current response is the transverse spin-conductivity²⁰ $\tilde{\sigma}_+$, which links the spin current to the gradient of the exciting exchange field :

$$\langle \hat{\mathbf{J}}_{+, \mathbf{q}} \rangle_{\omega} = \mathbf{q} \tilde{\sigma}_+(\mathbf{q}, \omega) G_{xc} \langle \hat{S}_{+, \mathbf{q}} \rangle_{\omega} \quad (16)$$

where $\langle \rangle_{\omega}$ is the expectation value at frequency ω . The spin-conductivity has an imaginary part originating from the damping of SF-SPE, intrinsically due to $\hat{H}_{Coulomb}$ or any source of disorder acting on transverse spin degrees of freedom. Consequently, the real part of the spin conductivity determines the SFW dispersion, while the imaginary part determines its damping. It is worth noting, that the spin wave damping originates from the kinetic motion of the conduction electrons and from the topology of the conduction band, a 2D parabolla. In a Luttinger liquid²⁸, the conduction band is linear in $\mathbf{k}$ and 1D, thus Eq. (14) conserves the macroscopic spin. It breaks the coupling between $\hat{S}_{+, \mathbf{q}}$ and SF-SPEs, which are coupled to charge degrees of freedom by $\hat{H}_{Coulomb}$. This property is at the origin of the well known spin-charge separation²⁹ occurring in Luttinger liquids. Injecting Eq. (16) into Eq. (15) and solving it in the frequency domain for long wavelength ($q \ll k_F$) leads to :

$$\frac{d}{dt} \hat{S}_{+, \mathbf{q}} = i\tilde{\omega}_q \hat{S}_{+, \mathbf{q}} \quad (17)$$

with $\tilde{\omega}_q$ a complex pulsation:

$$\text{Re } \tilde{\omega}_q = \omega_e - q^2 G_{xc} \lim_{q \rightarrow 0, \omega \rightarrow 0} \text{Re } \tilde{\sigma}_+ \quad (18a)$$

$$\text{Im } \tilde{\omega}_q = q^2 G_{xc} \lim_{q \rightarrow 0, \omega \rightarrow 0} \text{Im } \tilde{\sigma}_+ \quad (18b)$$

In the following we note $\sigma_+ = \lim_{q \rightarrow 0, \omega \rightarrow 0} \text{Im } \tilde{\sigma}_+$, the imaginary spin-conductivity. It was calculated in Ref.²⁰ and some corrections were added in Ref.¹⁹ which gave also an experimental evidence of the kinetic damping law found in Eq. (18b). We highlight that these q^2 laws are valid in the longwavelength limit when the SFW propagates far from the SF-SPE continuum (see Ref.¹⁸). When close to this continuum, the stronger coupling with SF-SPEs introduces corrections to the above laws and one should better replace the dispersion law with the pole appearing in the transverse spin susceptibility (see Ref.¹⁸) which will be derived in the next paragraph.

3. Electron spin-susceptibility

The transverse spin-susceptibility, defined by the ratio of the expectation value $\langle \hat{S}_{+, \mathbf{q}} \rangle_\omega$ to the perturbing potential $g_e \mu_B b_{+, \mathbf{q} \omega}$, where $b_{+, \mathbf{q} \omega}$ is the amplitude at the same pulsation ω and wavevector $\mathbf{q}$ of the exciting magnetic field, is given by :

$$\chi_+(\mathbf{q}, \omega) = \left\langle \left\langle \hat{S}_{+, \mathbf{q}}; \hat{S}_{-, -\mathbf{q}} \right\rangle \right\rangle_\omega^{2DEG} \quad (19)$$

Straightforward calculations using the equation of motion Eq. (11) lead to :

$$\chi_+(\mathbf{q}, \omega) = \left\langle \left\langle \hat{S}_{+, \mathbf{q}}; \hat{S}_{-, -\mathbf{q}} \right\rangle \right\rangle_\omega^{2DEG} = \frac{\Pi_{\downarrow\uparrow}(\mathbf{q}, \omega)}{1 + G_{xc} \Pi_{\downarrow\uparrow}(\mathbf{q}, \omega)} \quad (20)$$

where we have introduced the transverse Lindhardt-type response¹⁸ :

$$\Pi_{\downarrow\uparrow}(\mathbf{q}, \omega) = \sum_{\mathbf{k}} \frac{\langle \hat{n}_{\mathbf{k}-\mathbf{q}, \uparrow} \rangle_0 - \langle \hat{n}_{\mathbf{k}, \downarrow} \rangle_0}{Z^* + E_{\mathbf{k}-\mathbf{q}} - E_{\mathbf{k}} - \hbar\omega - \eta} \quad (21)$$

A comparison between the above spin-susceptibility expression and the one given by local spin-density approximation¹⁸, gives the expression of the local field factor G_{xc} :

$$G_{xc} = -\frac{2}{n_{2D}^2 L^2} \frac{1}{\zeta} \frac{\partial E_{xc}}{\partial \zeta} \quad (22)$$

where E_{xc} is the exchange-correlation part of the ground state energy³⁰.

SFW appear as poles of $\chi_+(\mathbf{q}, \omega)$, one finds in the long wavelength limit, another expression for $\text{Re } \tilde{\omega}_q$:

$$\text{Re } \tilde{\omega}_q = \omega_e - \frac{1}{|\zeta|} \frac{Z_e}{Z^* - Z_e} \frac{\hbar}{2m^*} q^2 \quad (23)$$

Alternatively, if one uses the approximated equation of motion Eq. (15), one finds the spin susceptibility in the long wavelength limit :

$$\chi_+(\mathbf{q}, \omega) = -\frac{2 \langle \hat{S}_{z, \mathbf{q}=\mathbf{0}} \rangle_0}{\hbar\omega - \hbar\tilde{\omega}_q} \quad (24)$$

B. Transverse spin dynamics equations with s-d dynamical coupling

Now, we keep lines (4b) to (4d) in the s-d Hamiltonian, and we reconsider collective transverse spin dynamics. In the following, the derivative $d\hat{A}/dt = [\hat{A}, \hat{H}]/i\hbar$ takes into account the coherent coupled dynamics due to lines (4b) and (4d) in the s-d Hamiltonian, but reduced to first order terms : higher order correlation terms like $\sum_{\mathbf{q}'} \delta \hat{S}_{z, \mathbf{q}+\mathbf{q}'} \cdot \hat{M}_{+, -\mathbf{q}'}^{(1)}$ have been dropped.

1. Electron dynamics

We find :

$$\frac{d}{dt} \hat{S}_{+, \mathbf{q}} = \dot{S}_{+, \mathbf{q}} - \frac{i}{\hbar} K \hat{M}_{+, \mathbf{q}}^{(1)} \quad (25)$$

where $\dot{S}_{+, \mathbf{q}} = i\tilde{\omega}_q \hat{S}_{+, \mathbf{q}}$. Compared to the SP2DEG dynamics, the *sd*-dynamical coupling adds the second term of Eq. (25) which is a coherent coupling with Mn transverse degrees of freedom. One key feature is that the collective electron motion naturally couples with $\hat{M}_{+, \mathbf{q}}^{(1)}$ Mn-modes, a Mn precession having a profile, out of the QW plane, following the electron probability distribution. We are left with deriving the equation of motion for these Mn-modes.

2. Manganese dynamics

We obtain the first order equation of motion for Mn spins :

$$\frac{d}{dt}\hat{M}_{+,\mathbf{q}}^{(n)} = \frac{i}{\hbar}g_{Mn}\mu_B B\hat{M}_{+,\mathbf{q}}^{(n)} + \frac{i}{\hbar}K\hat{M}_{+,\mathbf{q}}^{(n+1)} - \frac{i}{\hbar}\Delta_{n+1}\hat{S}_{+,\mathbf{q}} \quad (26)$$

where we have introduced n -profile Overhauser shifts : $\Delta_n = \tilde{\alpha} \left| \left\langle \hat{M}_{z,\mathbf{q}=\mathbf{0}}^{(n)} \right\rangle_0 \right| = \gamma_n/\gamma_1 \Delta$ with $\gamma_n = w^{n-1} \int_0^w \chi^{2n}(y) dy$. The important features are the second and third terms in Eq. (26). The later couples the Mn-precession with collective electron modes. The former couples a n -profile Mn mode to a $(n+1)$ -profile mode, because this coupling is mediated by the 2DEG. Thus, the Mn-dynamics is given by an infinite serie of equations. This is a consequence of the 3D nature of the Mn dynamics. A variable like $\hat{M}_{+,\mathbf{q}}^{(n)}$ describes an oscillation propagating in the plane with a rigid profile in the normal direction, but the out of plane degree of freedom is restored by the possibility for Mn spins to build modes which are combinations of $\hat{M}_{+,\mathbf{q}}^{(n)}$ resulting in different out of plane profiles¹⁰. Obviously, the $\hat{M}_{+,\mathbf{q}}^{(n)}$ are not independent variables because they don't correspond to orthogonal out of plane profiles. Solving the serie of infinite equations requires a projection of $\hat{M}_{+,\mathbf{q}}^{(n)}$ over a set of modes with orthogonal profiles as it was carried out in Ref.¹⁰. Along with modes having a strong mixed nature (electron-Mn modes), we then expect to find a high number of modes having essentially a Mn character, but with orthogonal profiles (Mn modes). The number of Mn modes has to be consistent with the initial number of degrees of freedom present in the system. Ref.¹⁰ found a high number of Mn modes branches which were separated by energies of the order of 0.1Δ . However, in the experimental data of Ref.⁴, only one branch of these Mn modes was apparent. It appears then, that the set of modes chosen in Ref.¹⁰ is not the most appropriate to describe properly all the modes contained in Eqs.(25)-(26), at least in the vicinity of the anticrossing gap (see below). Anyway, this point requires further developments out of the scope of the present study. Indeed, we are particularly interested in discussing mixed electron-Mn modes which are strongly coupled to electrons rather than modes specific to the 3D nature of the Mn dynamics. We can remark that the coupling between $\hat{M}_{+,\mathbf{q}}^{(n)}$ and $\hat{M}_{+,\mathbf{q}}^{(n+1)}$ has a strength given by K , which is very small compared to Δ_n due to the ratio $\left| \left\langle \hat{M}_{z,\mathbf{q}=\mathbf{0}}^{(n)} \right\rangle_0 / \left\langle \hat{S}_{z,\mathbf{q}=\mathbf{0}} \right\rangle_0 \right| \gg 1$. Hence, considering only modes strongly coupled with electron modes is reasonable. As electron modes are naturally coupled to $\hat{M}_{+,\mathbf{q}}^{(1)}$ modes, we will consider the dynamics for these ones only by cutting the infinite serie with

an homothetic approximation :

$$\hat{M}_{+,\mathbf{q}}^{(2)} = (\gamma_2/\gamma_1) \hat{M}_{+,\mathbf{q}}^{(1)} \quad (27)$$

Consequently the set of coupled electron-Mn equations reduces to :

$$\frac{d}{dt} \hat{S}_{+,\mathbf{q}} = i\tilde{\omega}_q \hat{S}_{+,\mathbf{q}} - \frac{i}{\hbar} K \hat{M}_{+,\mathbf{q}}^{(1)} \quad (28a)$$

$$\frac{d}{dt} \hat{M}_{+,\mathbf{q}}^{(1)} = i\omega_{Mn} \hat{M}_{+,\mathbf{q}}^{(1)} - \frac{i}{\hbar} \Delta_2 \hat{S}_{+,\mathbf{q}} \quad (28b)$$

where :

$$\omega_{Mn} = \left(g_{Mn} \mu_B B + K \frac{\gamma_2}{\gamma_1} \right) / \hbar \quad (29)$$

is the natural precession pulsation of the free $\hat{M}_{+,\mathbf{q}}^{(1)}$ mode.

IV. MIXED MN-ELECTRON SPIN WAVES

A. Spin susceptibilities

To find the dynamically coupled modes, we will derive the electron spin susceptibility with help of equations of motion (9a) and (28a)-(28b).

From Eqs.(9a) and (28a), we first get :

$$\left\langle \left\langle \hat{S}_{+,\mathbf{q}}; \hat{S}_{-,-\mathbf{q}} \right\rangle \right\rangle_{\omega} = \frac{\tilde{\omega}_q}{\omega} \left\langle \left\langle \hat{S}_{+,\mathbf{q}}; \hat{S}_{-,-\mathbf{q}} \right\rangle \right\rangle_{\omega} - \frac{K}{\hbar\omega} \left\langle \left\langle \hat{M}_{+,\mathbf{q}}^{(1)}; \hat{S}_{-,-\mathbf{q}} \right\rangle \right\rangle_{\omega} - \frac{2}{\hbar\omega} \left\langle \hat{S}_{z,\mathbf{q}=0} \right\rangle_0$$

then from Eq. (28b), we get :

$$\left\langle \left\langle \hat{M}_{+,\mathbf{q}}^{(1)}; \hat{S}_{-,-\mathbf{q}} \right\rangle \right\rangle_{\omega} = \frac{\omega_{Mn}}{\omega} \left\langle \left\langle \hat{M}_{+,\mathbf{q}}^{(1)}; \hat{S}_{-,-\mathbf{q}} \right\rangle \right\rangle_{\omega} - \frac{\Delta_2}{\hbar\omega} \left\langle \left\langle \hat{S}_{+,\mathbf{q}}; \hat{S}_{-,-\mathbf{q}} \right\rangle \right\rangle_{\omega}$$

hence,

$$\left\langle \left\langle \hat{M}_{+,\mathbf{q}}^{(1)}; \hat{S}_{-,-\mathbf{q}} \right\rangle \right\rangle_{\omega} = -\Delta_2 \frac{\left\langle \left\langle \hat{S}_{+,\mathbf{q}}; \hat{S}_{-,-\mathbf{q}} \right\rangle \right\rangle_{\omega}}{\hbar\omega - \hbar\omega_{Mn}}$$

which finally leads to :

$$\left\langle\left\langle\hat{S}_{+,\mathbf{q}};\hat{S}_{-,-\mathbf{q}}\right\rangle\right\rangle_{\omega}=\frac{(\hbar\omega-\hbar\omega_{Mn})\chi_{+}(\mathbf{q},\omega)}{\hbar\omega-\hbar\omega_{Mn}-\frac{\tilde{\alpha}\Delta_2}{2}\chi_{+}(\mathbf{q},\omega)} \quad (30)$$

and

$$\left\langle\left\langle\hat{M}_{+,\mathbf{q}}^{(1)};\hat{M}_{-,-\mathbf{q}}^{(1)}\right\rangle\right\rangle_{\omega}=\frac{-2\left\langle\hat{M}_{z,\mathbf{q}=\mathbf{0}}^{(1)}\right\rangle}{\hbar\omega-\hbar\omega_{Mn}-\frac{\tilde{\alpha}\Delta_2}{2}\chi_{+}(\mathbf{q},\omega)} \quad (31)$$

Consequently, e-Mn mixed spin excitations appear as poles of the above responses, *i.e.*, are zeros of the propagator :

$$\hbar\omega-\hbar\omega_{Mn}-\frac{\tilde{\alpha}\Delta_2}{2}\chi_{+}(\mathbf{q},\omega) \quad (32)$$

with $\chi_{+}(\mathbf{q},\omega)$ being the spin-susceptibility of the uncoupled SP2DEG described in Section III B.

We can understand the above equation as follows. Consider the Mn point of view ; in the presence of the SP2DEG, the precession frequency of Mn spins is shifted from the normal precession ($g_{Mn}\mu_B B/\hbar$) by two quantities : a blue shift due to static exchange field with spin polarized electrons ($K/\hbar$) and an additional shift due to the dynamic change of the electron spin-polarization. The later is induced by the Mn precession itself. Finally Eq. (32) describes a recursive closed loop where : Mn transverse precession induces electron transverse precession proportional to $\Delta_2\chi_{+}(\mathbf{q},\omega)$, this dynamically changes the electron spin polarization which in turn shifts the Mn precession frequency by an amount $\tilde{\alpha}\Delta_2\chi_{+}(\mathbf{q},\omega)$. Finally, in dropping correlation terms given by Eqs. (4c) and (4d), one finds a collective behavior where electrons and Mn respond adiabatically to the dynamical perturbation from the opposite spin-subsystem.

Similar expressions for the coupled modes propagator have been obtained in previous works. To our knowledge it was first derived in Ref.⁹ for bulk DMS, and more recently using a spin-path integral approach in DMS quantum wells with electrons^{6,10} or bulk DMS with holes⁸. However, none of these works did include the influence of the Coulomb interaction between carriers. Instead of Eq. (32), they resulted in the following propagator :

$$\hbar\omega-\hbar\omega_{Mn}-\frac{\tilde{\alpha}\Delta}{2}\Pi_{\uparrow\downarrow}(\mathbf{q},\omega) \quad (33)$$

where the electron spin-susceptibility was replaced by the non-interacting single-particle response introduced in Eq. (21). Introduction of Coulomb interaction results in strong

qualitative changes in the spectrum which have been partially addressed in Ref.¹⁵ and will be detailed below.

B. Homogeneous modes

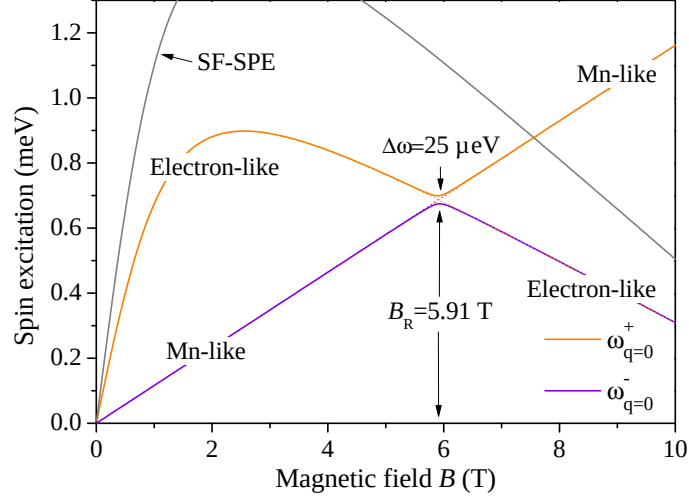


FIG. 1: (Color online). Zone center electron-Mn modes. Dotted lines are the uncoupled electron (upper curve) and Mn (lower curve) modes. Full lines are the solutions $\omega_{q=0}^{\pm}$ of Eq.(34). Out of the resonant field B_R where the modes anticross, branches have electron or Mn character. Sample parameters are, $x_{eff} = 0.23\%$, $T = 2$ K, $w = 150\text{\AA}$ and $n_{2D} = 3.1 \times 10^{11}\text{cm}^{-2}$.

Eq. (33) was used to successfully fit the experiment of Teran et al. (Ref.¹⁷) where homogeneous modes ($q=0$) were probed and shown to experience an anticrossing at a magnetic field B_R such that $\omega_{Mn} = \omega_e$. It is a consequence of the Larmor's theorem that the homogeneous electron mode behaves as if electron were not interacting (Eq. (15) for $q = 0$). Indeed, setting $q = 0$ in Eq. (32), leads to the homogeneous precession modes equation :

$$(\hbar\omega - \hbar\omega_{Mn})(\hbar\omega - \hbar\omega_e) - K\Delta_2 = 0 \quad (34)$$

which solutions are real :

$$\omega_{q=0}^{\pm} = \frac{\omega_e + \omega_{Mn}}{2} \pm \frac{1}{2} \sqrt{(\omega_e - \omega_{Mn})^2 + 4K\Delta_2/\hbar^2} \quad (35)$$

Figure 1 shows the magnetic field dependence of these modes and the gap opening at the resonant field B_R . The upper branch $\omega_{q=0}^+$ (resp $\omega_{q=0}^-$) has an electron character (resp. Mn character) when $B < B_R$ and vice-versa for $B > B_R$. The amplitude of the homogeneous anticrossing gap $\Delta\omega_{q=0} = \sqrt{4K\Delta_2}$ denotes the strength of the dynamical coupling between the two spin subsystems. Detailed discussions on this gap have been given in Ref.^{4,6} and Ref.¹. In particular, Ref.¹ identifies the anticrossing Mn mode as $\hat{M}_{+,q}^{(1)}$ consistent with the homothetic approximation of Eq. (27) used here. The anticrossing gap was found to be $\Delta\omega_{q=0} = \sqrt{4K\Delta_2 - \left(\frac{\hbar}{T_{2e}}\right)^2}$ where $\frac{\hbar}{T_{2e}}$ is the damping rate of the homogeneous uncoupled electron mode, a quantity that we have neglected here in dropping electron-Mn correlation terms contained in Eqs.(4b)-(4d). In Ref.¹, $\frac{\hbar}{T_{2e}}$ was estimated from measurements of the electronic spin wave damping at $q = 0$. A rigorous simultaneous determination of Δ_2 and T_{2e} , lead to the extraction of K from the anticrossing gap and furthermore to the spin-polarization degree ζ of the 2DEG. Data showed that the so-extracted ζ was slightly exceeding the prediction¹⁸ made for ζ in contradiction with other determinations of ζ performed in the same type of samples³¹, which showed that the model used to predict ζ was reliable. A more accurate description of the anticrossing gap taking into account the infinite set of coupled n -profile mode $\hat{M}_{+,q}^{(n)}$ equation of motion might overcome this discrepancy. One should also mention, that the infinite serie of equations must be cut in order to conserve the initial number of degrees of freedom (number of available spins in the system). But finding the right number of Eq. (26)-like to be taken depends on how the total number of degrees of freedom separates into a number of (quasi-) individual modes⁴ and a number of collective electron-Mn modes. Determining this separation is also an important and interesting issue.

C. Spin waves

For $q > 0$, Eqs(32) and (33) give very different qualitative results as illustrated on Fig. 2. Without Coulomb interaction between electrons, uncoupled modes of the electrons are the SF-SPE which are degenerate to Z_e at $q = 0$. The sd dynamical coupling introduces two additional collective mode : the OPW propagating above the SF-SPE domain with a positive dispersion and the IPW propagating below with a negative dispersion. Introducing the

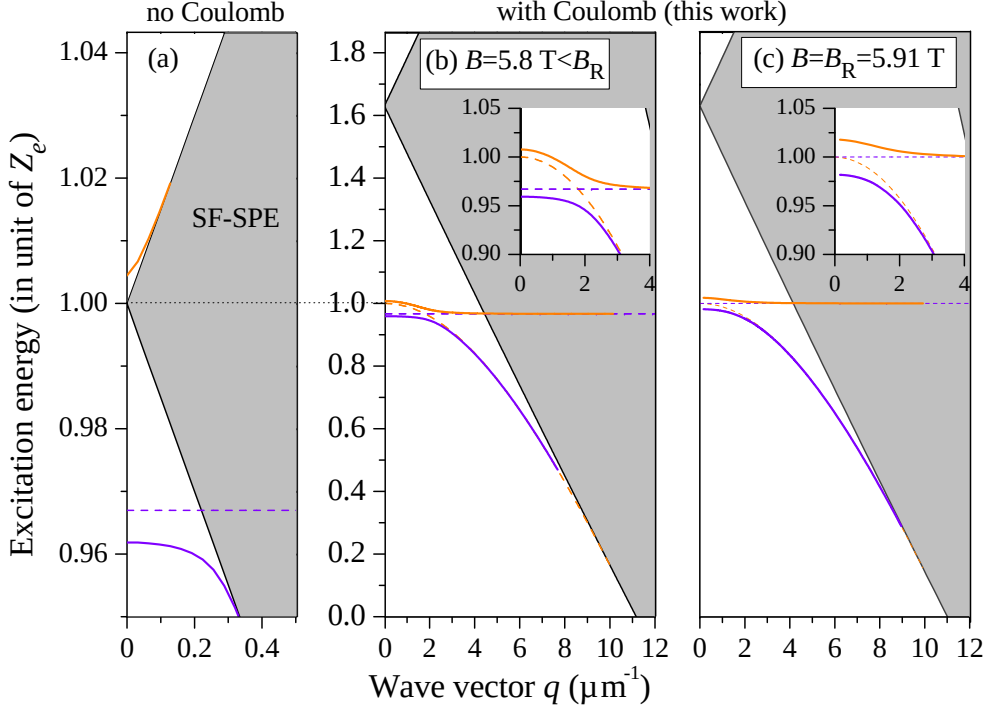


FIG. 2: (Color online). Illustration of the changes introduced by the Coulomb interaction on the mixed modes dispersions for the same sample parameters as in Fig.1. (a) Solutions without Coulomb given by Eq.(33) for $B = 5.8$ T. At $q = 0$ SF-SPE are degenerate to Z_e . The sd dynamical shift introduces two propagating waves : above (resp. below) the SF-SPE domain, the OPW (resp. IPW) propagates with a positive (resp. negative) dispersion. The dashed line is the uncoupled Mn mode (degenerate to $\hbar\omega_{Mn}$). (b) and (c) The Coulomb interaction between electrons is included as in Eq.(32) of this work. Dashed lines are the uncoupled electron and Mn modes. At $q = 0$ the SF-SPE energy is shifted to Z^* . An anticrossing gap opens at the wavevector $q_R(B)$ if $B \leq B_R$. In usual conditions (see Ref.¹⁵) both the OPW and IPW propagate below the SF-SPE continuum with negative dispersions. Dispersions were calculated after setting to zero the kinetic damping rate.

Coulomb interaction between electrons shifts the SF-SPE to higher energies (Z^*) and gives rise to the collective wave SFW propagating below the SF-SPE continuum. The SFW is further coupled to Mn modes through the sd interaction. An evaluation of the coupling between SF-SPE and Mn modes was given in Ref.¹ and found to be negligible. Thus the

Coulomb interaction introduces a shift between the SF-SPE and the SFW energies, and the later is further shifted by the sd dynamical coupling. In realistic conditions, it was shown in Ref.¹⁵ that the Coulomb shifts dominates over the sd dynamical shift. Hence, when Coulomb interaction is taken into account, Eq. (32), except under unrealistic conditions, gives rise to two spin wave modes propagating below the SF-SPE continuum, the IPW and OPW. An anticrossing gap opens at a specific wavevector $q_R(B)$ given by :

$$q_R(B) = \sqrt{|\zeta| (Z^*/Z_e - 1) \frac{2m^*}{\hbar} (\omega_e - \omega_{Mn})} \quad (36)$$

Note that $q_R(B_R) = 0$ and that $\tilde{\omega}_{q_R} = \omega_{Mn} + iq_R^2 G_{xc} \sigma_+$. If $q_R(B) > 0$, compared to the homogenous gap, the anticrossing gap at q_R is dramatically reduced by the kinetic damping of the electron wave and is given by :

$$\omega_{q_R}^+ - \omega_{q_R}^- = \sqrt{4K\Delta_2/\hbar^2 - \eta_{q_R}^2} \quad (37)$$

where $\eta_{q_R} = q_R^2 G_{xc} \sigma_+$. We note that the kinetic damping is the only one considered here. Other sources of damping, as e.g., the ones dropped in Eq. (4d), will of course further reduce the amplitude of the gap.

Fig. 3 illustrates the variation of $q_R(B)$ with the magnetic field. It is always smaller than $q_m = k_{F\downarrow} - k_{F\uparrow}$, the wavevector delimiting the window where the SFW propagates¹⁸. Overlaid in Fig. 3, are the anticrossing gap at q_R and the corresponding damping rate η_{q_R} . In the absence of the kinetic damping, the gap would be given by $\sqrt{4K\Delta_2}$. One sees the dramatic effect of this intrinsic kinetic damping, which kills the gap outside a very narrow range of magnetic fields. As coupling between spin waves of the electron and the Mn spin systems is responsible for the appearance of the carrier induced ferromagnetism⁶, we might conclude that the above disappearance of the gap diminishes the possibilities for ferromagnetic transitions with complex order (out of $q = 0$).

The disappearance of the gap is illustrated in Fig. 4 by comparing the dispersions obtained from the zeros of the propagator in Eq. (32) in the presence or absence of the kinetic damping. In the presence of the damping, the solutions have a non-zero imaginary part for $q > 0$. The corresponding damping rate is plotted in the lower insets of Fig. 4. It is well known that when the frequencies of two coupled oscillators anticross each other, their corresponding damping rates cross themselves. Clearly for $B < 5.7$ T, the mixed modes do

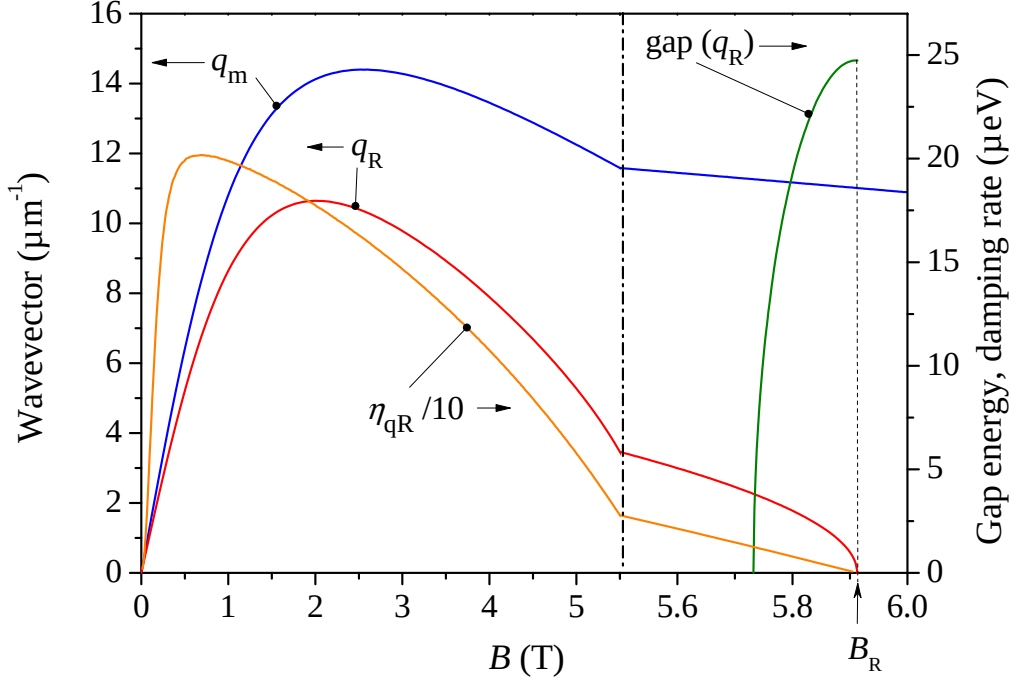


FIG. 3: (Color online). Left axis : variation of the wavevector $q_R(B)$ and the wavevector $q_m = k_{F\downarrow} - k_{F\uparrow}$. Right axis : variation of the anticrossing gap and the kinetic damping rate $\eta_{q_R} = \text{Im} \tilde{\omega}_{q_R}$. The anticrossing gap is killed by the damping rate when the later is of the same order of magnitude as $\sqrt{4K\Delta_2}$. A break in the horizontal axis scale has been introduced to zoom the region close to $B \lesssim B_R$. To calculate the damping rate, we have used a typical SF-SPE scattering time $\tau = 2\text{ps}$ instead of τ_{e-e} ($\sim 150\text{ps}$) to match the experimental conditions of Ref.¹⁹.

not anticross at any q and each branch conserves its former character, Mn-like or electron-like. On the contrary, for $5.7 \text{ T} < B < B_R$, the modes anticross at q_R , and the OPW transfers the kinetic damping (q^2 law) of the SFW to the IPW when $q > q_R$. It is worth to note that this q^2 law for the IPW damping rate was also found in GaMnAs compounds in the ferromagnetic state⁵.

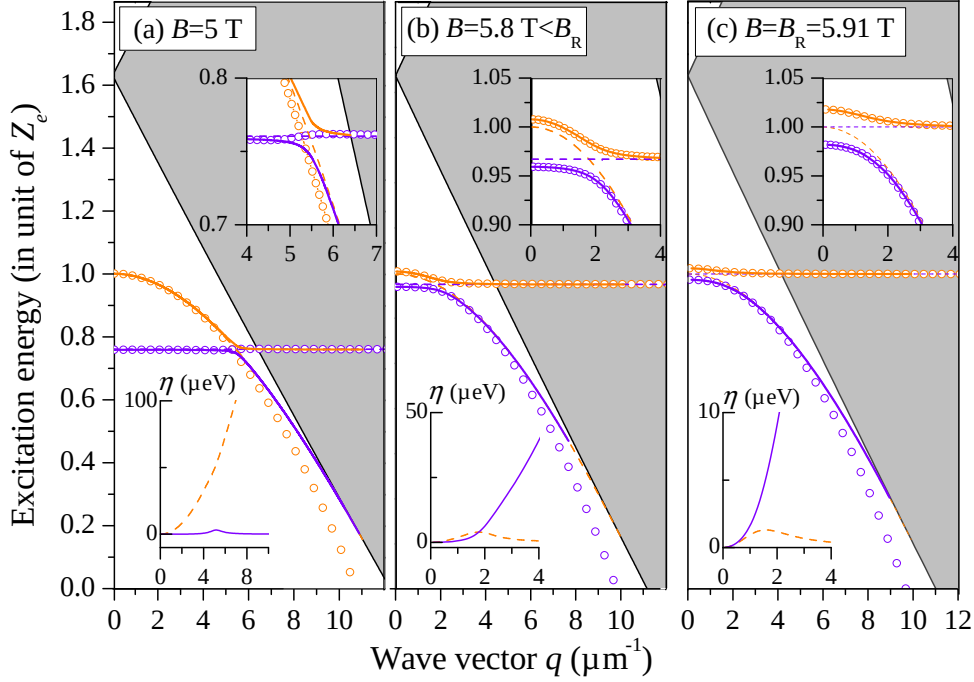


FIG. 4: (Color online). Illustration of the progressive disappearance of the anticrossing gap when B goes away from B_R . Continuous lines (open symbols) are dispersions calculated in absence (presence) of the kinetic damping. Dashed lines are the uncoupled modes. Upper insets : zoom on the anticrossing gap region. Lower insets : variation with q of the damping rate of both the OPW (dashed line) and IPW (straight line).

V. CONCLUSION

In conclusion, we have introduced equations for the spin dynamics in a test-bed diluted magnetic system that allow to take into account the interplay between the Coulomb interaction dynamics and the sd dynamical coupling between the electrons and the localized spins. We have shown how the Coulomb interaction introduces strong qualitative changes : the mixed electron-Mn modes propagate below the SF-SPE continuum and an anticrossing gap is open for a given range of magnetic field. Because of Coulomb interaction, the intrinsic kinetic damping due to the electron motion is always present (the SF-SPE scattering time can not be longer than τ_{e-e}), this damping kills the anticrossing gap outside a very narrow range

of magnetic fields. Our calculations illustrate also how this kinetic damping is transferred to the IPW, a phenomenon found in GaMnAs compounds.

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