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ERRATA TO: ON KAZHDAN-LUSZTIG CELLS IN TYPE B

CÉDRIC BONNAFÉ

ABSTRACT. Based on a combinatorial result about the domino insertion algorithm [6], which appears to be false, the proof of our main result in [2] is not complete. We explain here what can be saved from [2]: more precisely, we explain what is the scope of validity of [2, Theorem 1.5].

In [2], we have found, using brute force computations, some (not all) Kazhdan-Lusztig relations (let us call them the *elementary relations*) between very particular elements of a Weyl group of type B . This shows in particular that the equivalence classes generated by the elementary relations are contained in Kazhdan-Lusztig cells.

It was announced in [6, Theorems 1.2 and 1.3] that the elementary relations generate the equivalence classes defined by the domino insertion algorithm (let us call them the *combinatorial cells*). As a consequence, we “deduced” that the combinatorial cells are contained in the Kazhdan-Lusztig cells [2, Theorem 1.5], thus confirming conjectures of Geck, Iancu, Lam and the author [3, Conjectures A and B]. However, as it was explained in a revised version of [6] (see [7]), the equivalence classes generated by the elementary relations are in general strictly contained in the combinatorial cells. This has no consequence on most of the intermediate results in [2], but changes the scope of validity of [2, Theorem 1.5]. Indeed, for some special cases of the parameters, T. Pietraho [5] has found that the elementary relations generate the combinatorial cells. So part of [2, Theorem 1.5] can be saved: the aim of this note is to explain precisely what is proved and what remains to be proved.

REMARK - The fact that [6, Theorems 1.2 and 1.3] is false does not imply that the result stated in [2, Theorem 1.5] is also false: it just means that its proof is

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not complete and we still expect the statement to be correct (as conjectured in [3, Conjectures A and B]).

1. PROVED AND UNPROVED RESULTS FROM [2]

Unproved results. We keep the notation of [2]. First of all, the proof of the Theorem stated in the introduction of [2], so its statement remains a conjecture (and similarly for the Corollary stated at the end of this introduction). Also, [2, Theorem 1.5(a)] is still a conjecture. However, [2, Theorem 1.5(b)] is still correct: its proof must only be adapted, using Pietraho's results [5].

Theorem 1. *Let $r \geq 0$ and assume that $b = ra > 0$. Let $? \in \{L, R, LR\}$ and $x, y \in W_n$ be such that $x \approx_?^r y$. Then $x \sim_? y$.*

The proof of Theorem 1 will be given in the next section. It must also be noted that [2, Theorem 1.5] is also valid if $b > (n-1)a$ (see [4, Theorem 7.7] and [1, Corollaries 3.6 and 5.2]).

Proved results. Apart from the above mentioned results, all other intermediate results (about computations of Kazhdan-Lusztig polynomials, structure constants, elementary relations) are correct.

2. PROOF OF THEOREM 1

In [2, §7.1], we have introduced, following [6], three elementary relations \smile_1 , \smile_2^r and \smile_3^r : for adapting our argument to the setting of [5], we shall need to introduce another relation, which is slightly stronger than \smile_3^r .

Definition 2. *If w and w' are two elements of W_n , we shall write $w \frown_3^r w'$ whenever $w' = tw$ and $|w(1)| > |w(2)| > \dots > |w(r+2)|$. If $r \geq n-1$, then, by convention, the relation \frown_3^r never occurs.*

Using this definition, Pietraho's Theorem [5, Theorem 3.11] can be stated as follows:

Pietraho's Theorem. *The relation \approx_R^r is the equivalence relation generated by \smile_1 , \smile_2^r and \frown_3^{r-1} .*

It is easy to check that, if $w \frown_3^r w'$, then $w \smile_3^r w'$. Therefore, Theorem 1 follows from [2, Lemmas 7.1, 7.2 and 7.3] and the argument in [2, §7.2].

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