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# SUBPROBLEM METHOD WITH DUAL FINITE ELEMENT FORMULATIONS FOR ACCURATE THIN SHELL MODELS

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<u>Abstract</u> – A subproblem method with dual finite element magnetostatic and magnetodynamic formulations is developed to correct the inaccuracies near edges and corners coming from thin shell models, that replace thin volume regions by surfaces. The surface-to-volume correction problem is defined as one of the multiple subproblems applied to a complete problem, considering successive additions of inductors and magnetic or conducting regions, some of these being thin regions. Each subproblem is independently solved on its own domain and mesh, which facilitates meshing and solving while controlling the importance and usefulness of each correction. Parameterized analyses of thin regions are efficiently performed.

#### Introduction

The finite element (FE) subproblem method (SPM) provides advantages in repetitive analyses and helps in improving the solution accuracy [1-2]. Each SP considers the solution of previous SPs through surface sources (SSs) and volume sources (VSs) instead of starting a new complete FE solution for any variation of geometrical or physical data. Each SP is defined on its particular geometry and mesh.

The thin shell (TS) FE representation of thin regions in magnetic problems is herein placed at the hearth of the SPM, to define both parameterized and correction schemes. The TS model is used to avoid meshing the thin regions, and consequently lighten the mesh of their surrounds [3-4]. For that, the volume thin regions are reduced to surfaces with interface conditions (ICs) linked to 1-D analytical distributions (throughout the shell thickness) that however generally neglect end and curvature effects. The SPM is herein applied to correct the inaccuracies of the field distribution and losses proper to the TS FE representation of thin regions, mainly in the vicinity of their edges and corners. Prior to such corrections, the SPM naturally allows parameterized analyses of the thin region characteristics: permeability, conductivity and thickness.

The developments are performed for both magnetic vector potential and magnetic field FE magnetostatic and magnetodynamic formulations, with attention to the proper discretization of the constraints involved in each SP. The method is illustrated and validated on a practical test problem.

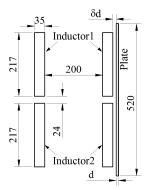
### **Series of Coupled Subproblems**

A complete problem is split into a series of SPs that define a sequence of changes, with the complete solution replaced the sum of the SP solutions. Each SP p is defined in its particular domain. It is governed by magnetostatic or magnetodynamic equations and constrained with VSs and SSs [1-2], of which some components originate from previous problems q. VSs express changes, from SP q to SP p, of permeability and conductivity of volume regions, while SSs express changes of ICs through surfaces. Mesh-to-mesh projections are required to express these sources in each new SP [2].

The surface-to-volume correction SP consists in suppressing the TS model and simultaneously replacing it by a FE volume model in a domain reduced to the thin region and its surrounds. This is defined via SSs that are the opposite of the so-known TS ICs [1-2], and simultaneously via VSs expressing the so-considered volume of the thin region that replaces the air region. The SSs have complementary strong and weak natures depending on the b- or h-formulation used, that necessitate adequate discrete tools to be accurately defined [2]. From a TS solution with given permeability, conductivity and thickness, a next SP can consider changes of these parameters directly via SSs that suppress the previous model simultaneously with new TS ICs. All these changes generally lead to local modifications of the solution, which thus allows to reduce the calculation domain and its mesh in the surrounds of the thin regions.

#### **Application Example**

The test problem is based on TEAM problem 21, with two inductors and a thin plate (Fig. 1, *left*). An example of magnetodynamic SP scheme first considers the inductors and a TS model of the plate, followed by the surface-to-volume correction SP with the actual volume plate alone. The TS model error is pointed out through the relative correction of the Joule losses density (Fig. 1, *right*) and the longitudinal magnetic flux (Fig. 2, *left*), for different plate parameters and both **b**- and **h**-formulations. Significant errors (up to 70% for the loss density and 30% for the magnetic flux) occur near the plate ends. The corrected solution is checked to be very close to the complete volume FE solution (Fig. 2, *right*). The relative importance of each correction is a useful indicator of the quality of the TS model or of the solution sensitivity to some parameters. All the steps of the method will be detailed, illustrated and validated in the extended paper. A general SP scheme with inductors alone, TS regions added, TS parameter changes and corresponding surface-to-volume corrections will be applied and validated for both 2-D and 3-D geometries, allowing to point out its efficiency and accuracy in parameterized analyses.



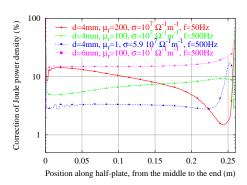
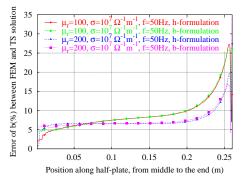


Fig. 1. Geometry of TEAM problem 21 (*left*); relative correction of the Joule power density along the plate for different plate parameters: thickness d, permeability  $\mu_r$ , conductivity  $\sigma$  and frequency f (*right*)



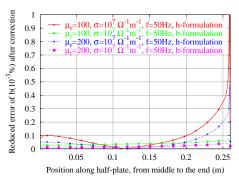


Fig. 2. Relative correction of the longitudinal magnetic flux along the plate (*left*) and comparison of the corrected solution with a classical FE volume model, with effects of  $\mu_{\tau}$ ,  $\sigma$ , f and d = 4mm

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