Decision support with a markovian approach for maintenance context activities

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ABSTRACT: Today, maintenance strategies and their analyses remain a worrying problem for companies. Our study, presented in this paper, shows that our indicator can provide help for expert decision (maintenance manager) in the context of condition-based maintenance. The paper deals with the proposition of using Hidden Markov Models to track and estimate the degradation of a system, according to observations (maintenance activities registered in a database). In a first time, the degradation level of process was established by a "classical" degradation laws (statistical laws). In a second step, this level was established by Hidden Markov Model (probabilistic laws).Tests conducted on the synthesis model, for which degradation levels were known, allowed us to implement the method.

1 INTRODUCTION

Industrial processes need to be maintained to prevent breakdown. Some years ago, maintenance activities were only deployed to repair process after the problem occurs. Nowadays, in an international market context, companies need to improve their productivity. In this context, maintenance strategies are included in reliability engineering (Moubray 1997). In Phimister (Phimister et al. 2003), authors split maintenance into two kinds of activities: technical activities and management activities. More details can be found in (Wireman 2004). Figure 1 shows different kinds of maintenance policies. A review of different maintenance strategies can be found in Bérenguer (Bérenguer et al. 2004) and Cotaina (Cotaina et al. 2000). In some specific cases, maintenance policy could be imposed, like it is the case for nuclear plant in France.

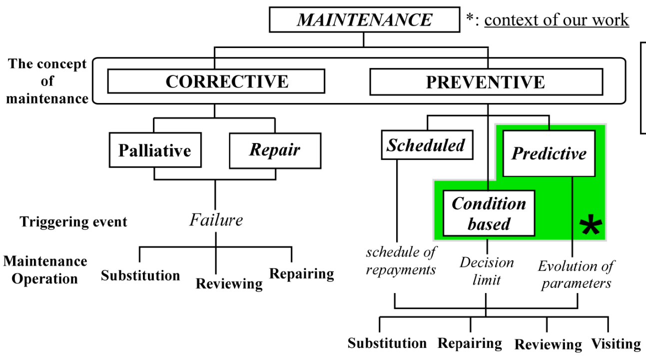

Figure 1. Maintenance policies

This field includes technological aspects, personal organization, logistic… Different kinds of maintenance policy can be applied: preventive or corrective according to manager strategies.

In case of preventive maintenance, different strategies should be used like planned preventive maintenance or condition-based maintenance. Planned actions would be program using feedback from experience which used statistical frequencies of defaults. Condition-based maintenance could use specific features extracted from the process like vibration sensors, oil analyzers… Then according to these indicators, maintenance actions can be performed. In many cases, defaults are preceded by specific series of events. Black smoke behind a car informs that engine could stop. Dark clouds indicate it will rain… Events which precede default could inform about imminence of it. Valdez-Florez (Valdez-Florez and Feldman 1989) survey researches on model optimization for repair, replacement, and inspection of systems subject to stochastic deterioration. Simeu-Abazi (Simeu-Abazi and Sassine 1999) adopted a modular modelling approach, based on a cellular decomposition of the system. They use stochastic Petri nets and Markov chains to implement various maintenance strategies in complex production workshops. A parametric decision framework (multi-threshold policy) is proposed to choose sequentially the best maintenance actions and to schedule future inspections, using on-line monitoring information on the system deterioration level (Castanier et al. 2003), (Dieulle et al. 2003). Deterioration of technical systems can often be classified into discrete states, and transitions between these states can be modelled using a Markov process. Instead of using an exponential distribution, it may be more realistic to assume that a general probability distribution describes staying time in one of these states (Welte 2008). Soro (Soro et al. 2010) proposed a model for evaluating availability, production rate and reliability function of multi-state degraded systems subject to minimal repairs and imperfect preventive maintenance. System status is considered to degrade with its use. These degradations may decrease system efficiency. It is assumed that the system can consecutively degrade into several discrete states, which are characterized by different performance rates, ranging from

perfect functioning towards complete failure. Nevertheless, global performances are difficult to be controlled because system environment changes. Operating modes are dependent on product flows, and ageing of components modifies continuously system characteristics. Thus today, most maintenance strategies are not well adapted to these requirements because purely reactive (fixing or replacing equipment after it fails) or time-scheduled (Wang 2002).
Ben Salem (Ben Salem et al. 2006) propose a model of the degradation of an n-component system. This modelling can be performed in two steps:

- modelling degradation of the different system components,
- from these models, establish an overall degradation model taking into account the functional dependencies between components.

As in these studies, we show that a degradation level of a process can be proposed to the expert, from series of "field" events. In this study, we try to learn, without "a priori", this default signature. The originality of our work, is to use maintenance activities as an indicator (Figure 2). Works, presented in this paper, take part of condition monitoring systems. Using observations provided on the process, we try to generate an availability indicator which can be used by maintenance manager to plan actions dynamically (Figure 1 and Figure 2). According to system availability, preventive maintenance could be scheduled to prevent uncontrolled stops of system.

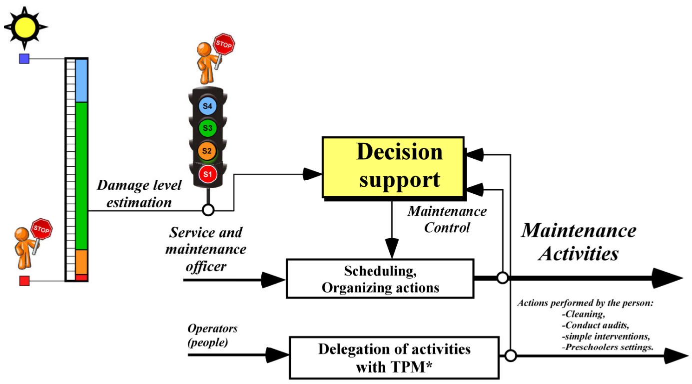

Figure 2. Works goals

The replacement of components for which failure is thought to be imminent, can be performed when the component is strongly damaged according to different use criteria, or when it has reached a critical condition. The success of this approach depends on the ability to predict remaining life of the component and when to perform the replacement (Bérenguer 2008), (Bouvard et al. 2008).
Hidden Markov Models (HMM) have been used, with success, to model sequences of events like, for example, in speech recognition. To improve results of these methods, model parameters should be adjusted to match event characteristics (states, topology…). In this study, we use the same strategy to learn events which can be observed on an industrial process. Model topology is configured to provide an availability explanation to our model. When system is started, model will indicate a high level of availability. When system is stopped by defaults, model will be in the "off" state (red state: it is too late to prevent default). Our new estimator is compared with "classical" degradation laws. These degradation laws are used as references.
In the next part, we introduce maintenance strategies and our works are located in this context. In part 2, we recall some "classical" reliabilities laws. We give more details for Kaplan-Meier law and Cox model, which have been implemented. In part 3, our strategy to use HMM for availability indicator implementation is presented. In the last part, we compare results of "classical" degradation laws with our HMM availability indicator on a synthesis model.

2 RELIABILITY STUDY OF MAINTAIN PROCESS

Reliability is often used by maintenance expert. In this section, some usual lifetime laws are presented. These laws can be used as well for medical studies as for industrial context. Main properties of these laws (probability density, reliability functions, failure rate) are defined and applied in reliability applications (Ebeling 1997), (Birolini 1994). Commonly, parametric models and nonparametric models based on proportional risk are used (Bertholon et al. 2006). Different classical laws of degradations exist: exponential law, normal law (Laplace-Gauss), Lognormal law (or Galton law), Weibull. We do not present these laws in this paper.
Efficient maintenance is related to a pertinent estimation of components lifetime. This estimation is based on experience feedback. Lifetime study of each system can be split into two options which need a great analysis of this experience feedback:

- *System reliability is its ability to perform what it has to do in its usual conditions during a given time (N.F.E.N 13306X60-319 2001), (C.E.I 1985). Reliability expert tries to plan new maintenance strategy using reliability evolution.*
- *Durability is the ability for a system to perform its goals, given using and maintaining conditions, until a limit state is reached (N.F.E.N 13-306 2001). Durability expert tasks should consist in estimation of remaining lifetime of working systems. This kind of study needs to take into account using conditions, parts replacement, to estimate system lifetime.*

2.1 *Kaplan-Meier law*

The Kaplan-Meier method, used in cross-disciplinary field ((Brookmeyer and Crowley 1985), (Cheuk-Kit and Eng Wie 2007)), provides an estimation of survival functions, with not necessarily regular time intervals, instead of actuarial tables[1]. Survival curves can be used to analyse evolution of populations over the time. These techniques (also called product limit estimators) are used for analysis of survival data, whether for persons (e.g. cancer) or products (wear tools resistance).

S(*t*) is the survival function. According to data ($y_1,\dots y_n$), we can provide unbiased empirical Kaplan-Meier estimator:

$$S_n(t) = \prod_{i\in\{1,\dots,n\}, y_i \le t} \left(\frac{n-i}{n-i+1}\right)^{\delta_i} \quad n \ne i \tag{1}$$

n: number of risk episodes (stop)
Where $\delta_i = 1$ if y_i is an uncensored data (0 if censored data). Lo (Lo et al. 1989) and Bitouzé (Bitouzé et al. 1999) propose a definition of this estimator for a measure of concentration of according S(*t*) to real distribution, in a non-asymptotic context.

2.2 *Cox model*

Cox regression model is an useful method to study impact of variables on survival time of a process (Kalbfleisch and Prentice 2002), of patients (Breslow 1973), (medical study)... It is applied to survival data, i.e., time variables, censored variables and explanatory variables. This model is based on a maximum likelihood estimation, developed by (Cox 1972). Principle of Cox model is to link event happening to explanatory variables. For example, in medical field, we try to assess impact of a pretreatment on the healing time of a patient. Cox model can be compared with classical regression models: events (modelled by date) should be linked with explanatory variables. Specificity of this approach is its ability to assess relationship between hazard and explanatory variables without assumptions on the shape of baseline hazard function. It contains real parameters and unknown functions (hence appearance of semi-parametric methods that take into account this double aspect). It is based on the proportional hazards assumption (instantaneous risk of event happening can be written as the product of a function that depends on time and a function that depends only on specimen features). It can be applied to any situation where event duration is studied. Cox is based on the assumption of proportional hazards. In proportional hazard models, the instantaneous risk is written:

$$\begin{aligned} h(t \mid z,\theta) &= \lim_{dt\to 0} \frac{1}{\Delta t} P(t < T \le t+\Delta t \mid T > t, Z = z) \\ h(t \mid z,\theta) &= \alpha_0(t) f_\beta(z) \end{aligned} \tag{2}$$

With: $Z = (Z_1,\dots Z_p)^T$: a vector of covariates, α_0 : basis risk, unknown, independent of Z, β : the regression parameter, unknown, $f_\beta(z)$: relative risk, independent of time, *T*: random variable that characterizes the time or the process is stopped. For example: risk ratio for two individuals is independent of time. Regressors $Z_1,\dots Z_p$, quantitative or qualitative are called prognostic factors (age, sex, treatment, ...). In the Cox model, the instantaneous risk for an individual *i* is written:

$$\begin{aligned} h(t \mid Z_i) &= \alpha_0(t)\exp(\beta_1 Z_{i,1} + \dots, \beta_p Z_{i,p}) \\ h(t \mid Z_i) &= \alpha_0(t)\exp(\beta^T Z_i) \end{aligned} \tag{3}$$

$\alpha_0(t)$ is any function which depends only on time (basis risk is unknown and independent of Z_i), and β_1, $\beta_2,\dots,\beta_p$ are constants with: $\alpha_0(t)$, $\beta = (\beta_1, \dots, \beta_p)^T$ is the unknown regression parameters. With: $Z_i = (Z_{i,1},\dots Z_{i,p})$: a components vector. S_0 is the basic survival function associated with α_0. In this case, we have the following relationship: $S(t \mid Z_i) = \left[S_0(t)\right]\exp(\beta^T Z_i)$. This provides an estimate of S knowing β the estimation vector. To estimate the components of the vector β from an ordered sample ($y_{(1)},\dots y_{(n)}$), we calculate the partial likelihood function of Cox (if no censored data):

$$L(y_{(1)},\dots y_{(n)};\beta) = \prod_{i=1}^{n} \frac{\exp(\beta^T Z_i)}{\sum_{k\in R(y_{(i)})} \exp(\beta^T Z_k)} \tag{4}$$

Note:

- $\exp(\beta_i)$ relative rate of subjects for which X_j=1 compared to those for which X_j = 0,
- $\exp(\beta_i)$ >1: harmful effect ; $\exp(\beta_i)$ =1: no effect ; $\exp(\beta_i)$ <1: positive effect.

3 DECISION SUPPORT BASED ON HIDDEN MARKOV MODELS

Stochastic models are representations of dynamic systems based on probabilities. Stochastic processes were firstly developed in the early 20th century by a Russian mathematician, Andrei Andreyevich Markov. His statistical study of language has led to the markovian hypothesis, which can be summarized as follows: "Future evolution of a system only depends on its present state". This hypothesis implies that current state of a system contains all information provided by its past. Therefore, it is a very important assumption. In practice, this condition is rarely satisfied. However, approximation by Markovian models could provide good modelling results (Hopp (Hopp and Wu 1998) proposes a maintenance model under Markovian deterioration is developed

[1] These methods combine observations by random or predefined intervals. It enables to estimate and to obtain a hazard rate representation.

in which maintenance and replacement actions are permitted and states are completely observable). Meier-Hirmer (Meier-Hirmer et al. 2009) proposes a model used for the maintenance of railway tracks. In this paper, semi-regeneration properties at the inspection times and associated Markov renewal techniques are used in order to compute the long-term mean costs.

Risk analysis of dynamical systems by classical markovian approaches considers only two states (On / Failure (Stop)). Between perfect working condition and complete failure state, industrial systems generally have a large set of degraded states in which system continues to provide service, even if it does not produce fully. "*These degraded statements need to be taken into account to properly assess service level of industrial systems and this is especially true with regard to production systems (Innal et al. 2008)"*.

Hypothesis: events preceding a crash are often recurrent. Specific series of events should inform about the next failure. Some examples can illustrate this hypothesis.

- In mechanical systems, noises, vibrations precede failure. Loss of performances reflects failures or technical defaults,
- In computers, suspect pointer movements, loss of performances, application malfunctions like web browser may reflect virus presence on computer…

Our approach tries to understand "this signature" using HMM. Hidden process will match system state (or subsystem state) and observations will be observable part of processes (Figure 3). Our works (Vrignat et al. 2010) show that it is possible to model degradation levels of a "continuous" process. Hidden process will fit to system or subsystem states (Run, Degradation level 1, ..., Degradation level N, failure) and observations will be information which can be collected on the system (Figure 3).

Topology of the models used is shown in Figure 6.

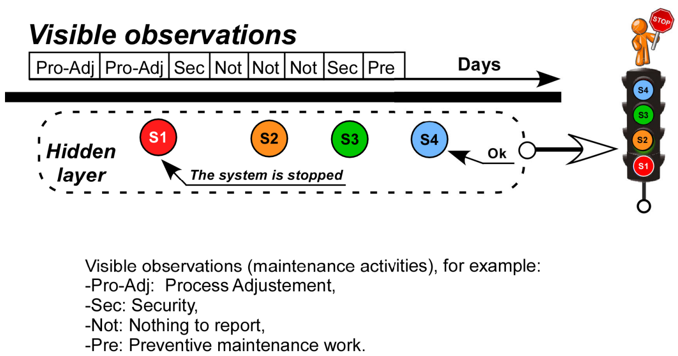

Figure 3. Visible and hidden layers (system states)

In another transposable example, Figure 4 gives more details. In this example, the surfer and the old man can not see itself. After model training, the old man is able to report what the surfer makes, if the surfer sends a message.

Section 4.1 provides more details on the model.

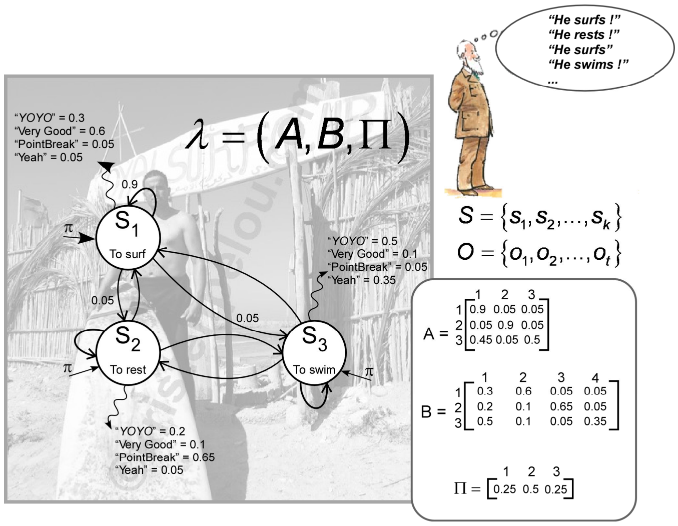

Figure 4. The "language" of surfer

3.1 *HMM approach*

The aim of this paper is not to present exhaustively Hidden Markov Model. For readers interested in more details, we recommend to read papers Rabiner (Rabiner 1989) and Aupetit (Aupetit et al. 2008) which presents HMM general problems. In this paper, we use the same notation for models. A model $\lambda = (A, B, \Pi)$ is described by three matrices:

$$A = \{a_{ij} = P(S_j | S_i)\}; \sum_{j=1}^{N} a_{ij} = 1 \tag{5}$$

Corresponding to transition probabilities between hidden states.

$$B = \{b_i(o_T) = P(o_T | S_i)\}; \sum_{j=1}^{N} b_i(o_j) = 1 \tag{6}$$

Corresponding to probabilities of observations considering states.

$$\Pi = \{\pi_i = P(S_i)\}; \sum_{i=1}^{N} \pi_i = 1 \tag{7}$$

Corresponding to initial state probabilities.

Learning Hidden Markov Model consist in estimating the parameter vector λ on the basis of a set of observation sequences. The learning algorithm most commonly used is the Baum-Welch algorithm (Baum 1972). This algorithm is derived from the EM algorithm (Expectation-Maximization). The Baum-Welch algorithm solves the problem of learning with the criterion of Maximum Likelihood. For a sequence of observations o, this criterion is to find the HMM λ^* which has the highest probability of generating the sequence o that is to say, maximizing $P(O{=}o|\lambda)$. The Baum-Welch algorithm is a proce-

dure which iteratively re-estimates matrices A, B and Π from an initial HMM. Baum-Welch algorithm provides a local optimum of the likelihood function. By applying this learning with different initial models, it is possible to obtain either a global optimum or a near optimal model for the considered criterion. Among all the criteria used for learning HMM, the criterion of segmental K-means is different from others. For this criterion, we seek to maximize the probability $P(O,S=Q^*|\lambda)$, with Q^* corresponding to the sequence of hidden states that most likely generates the sequence as calculated by the Viterbi algorithm (Viterbi 1967). The segmental k-means algorithm can adjust the model parameters iteratively from an initial model. Just as Baum-Welch algorithm, this algorithm could provide a local optimum result.
The two previous modes of learning have properties to preserve initial structure of models. When initial model probability is zero then the corresponding probability is zero in the learnt model. It is however important to note that a non-zero probability in the initial model may become zero in the learnt model. This phenomenon often occurs when certain symbols are not observed in learning sequences. A single occurrence of a "missing" symbol in a new sequence will cause non-recognition by the HMM: probability will be zero. To handle this problem, we introduce a smoothing step after the learning step. For each probability, not forced to zero by the model structure, an epsilon is added. Constraints of stochastic matrices are obtained by normalizing sums to 1. This smoothing introduces a distortion of optimal learnt model. We therefore distinguish, in the following, non-smoothed and smoothed learning.
Once the model is characterized either by the Baum-Welch algorithm or the segmental k-means algorithm, with or without smoothing, we seek to estimate, the most likely sequence of statements on new observations sequences.
For the learning's with the Baum-Welch algorithm, we estimate most probable states at a given time using Forward variables (Rabiner 1989). Let $\alpha_t(j)$ be the probability of generating the observation sequence $O = \{o_1, o_2,\dots,o_t\}$ and being in state q_t at time t, that is to say: $\alpha_t(j) = P(o_1,o_2...o_t,Q_t = s_j|\lambda)$.
The most probable state at time t is defined by $argmax_{j=1..k}\alpha_t(j)$.
For learning's with the segmental k-means, we consider the most probable state at a given time using the latest state of the optimal path given by Viterbi algorithm (Viterbi 1967). Considering the previous notation, the most probable state is defined by: $\delta_t(j) = max_{q_1,\dots,q_{t-1}} P(Q_1 = q_1,\dots,Q_{t-1} = q_{t-1}, Q_t = S_j, o_1,\dots,o_t / \lambda)$.
Figure 5 summarizes methods adopted for different tests (HMM).

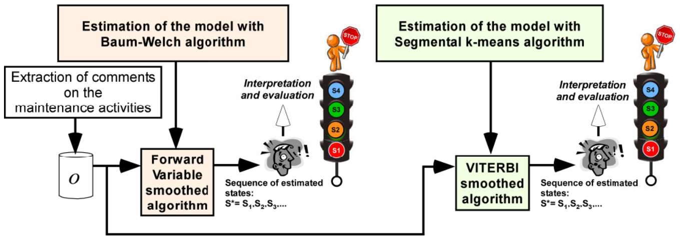

Figure 5. Methods adopted for different tests

3.2 *Validation of a Hidden Markov Model with a synthesis model*

To describe algorithms used and HMM among three chosen models (Figure 6), we first conducted several tests on a synthesis model (Figure 7).

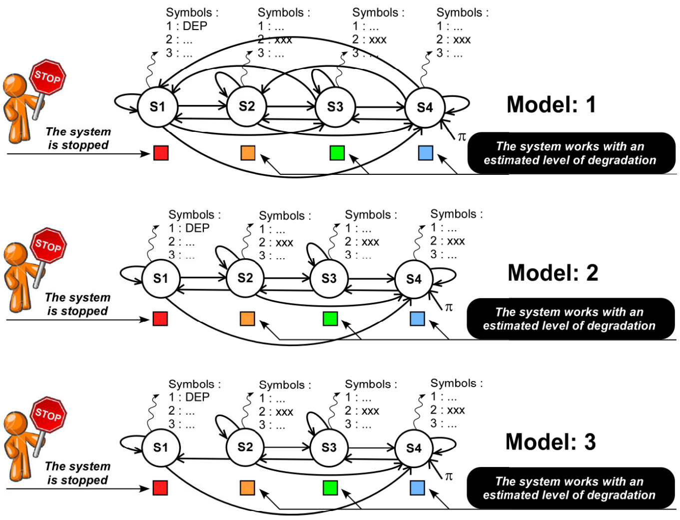

Figure 6. Three topologies with four states

The chosen models corresponds to a system, that can be degraded or can be repaired with 4 states (Figure 7).
Model 1 describes a topology without constraints (free topology). For example, state estimation can move from state S4 to S1 brutally. In this case, there is no possible warning for expert, before the stop situation. Model 2 describes a topology with constraints. Before ending in S1, there will be an obligatory passage in S3 and S2. In Model 3, we removed one degree of freedom (probability of transition S1 to S2). This model is more "absorbent" to S1 as the Model 2.
Synthesis model that was chosen with Model 2 (Figure 6). All numerical values of μ and λ as well as emissions of symbols are perfectly mastered (Figure 7: definition of benchmark for tests). We chose two different laws (physical/measurement aspects) for generation of symbols (observations) by the states. Uniform law and normal law are a good approximation of the real case. Then we used the 8000 observations of the synthesis model for the qualification and ability to recognize a "priori" model with the HMM algorithms.

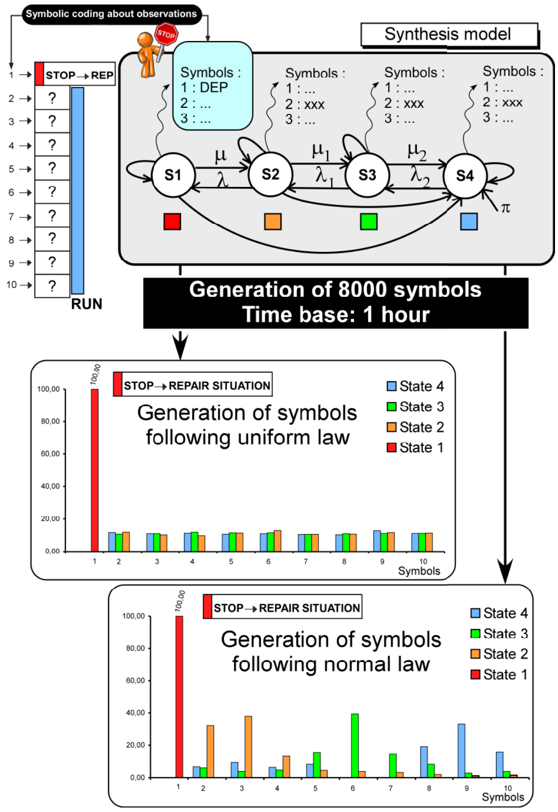

Figure 7. Synthesis model

We have established the distribution of various stop (State 1 corresponded: Repair situation), different survival functions about our synthesis system (Figure 8 (a) and (b)).

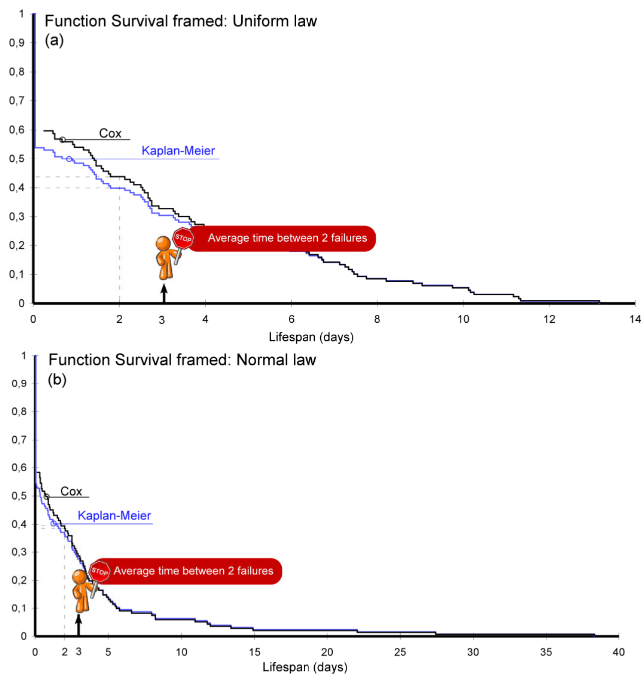

Figure 8: Survival functions by Kaplan-Meier and Cox

First indicators are statistical reference informations. These informations can be used by the expert but the question remains:
How to put the decision threshold for maintenance intervention (Figure 9)?

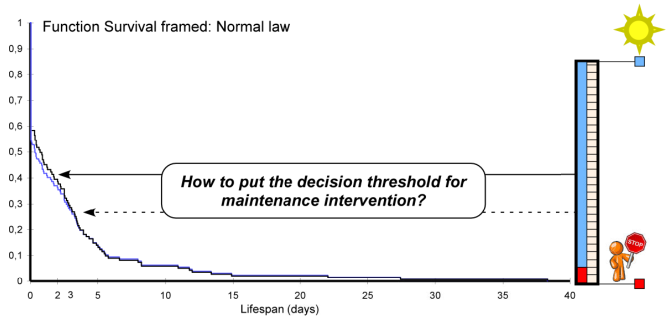

Figure 9: How to put the decision ?

To answer this question, we perform different tests on these HMMs algorithms used according to the three selected models (Figure 6). The results presented in **Erreur ! Source du renvoi introuvable.** correspond to a succession of tests. These tests are summarized in Figure 10.

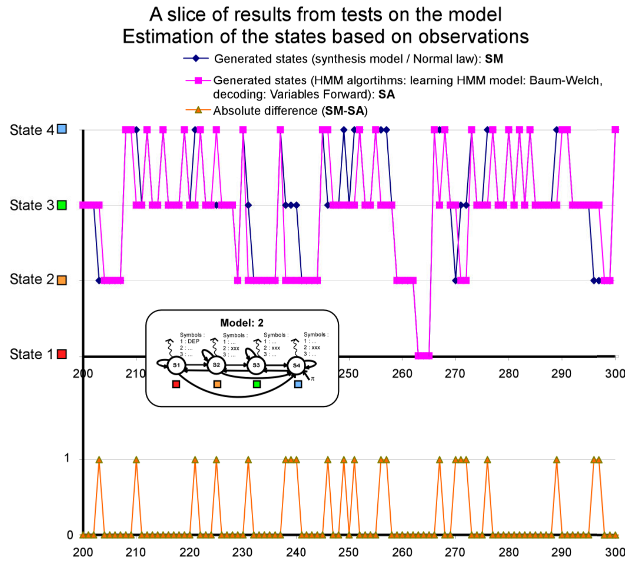

Figure 10: A slice of results from tests on the model

Different tests show that model 2, is the better to assimilate synthesis datas (**Erreur ! Source du renvoi introuvable.** (B)). Best results are: Generated states (Model 2: Uniform Law) / Learning: Baum-Welch / Decoding: Forward variables. Correlation rate between SM-SA is good (0,708). In other cases, errors percentage between SM and SA (%) is important.

Table 1. Results between the 3 models

Generated states (Model 1: Uniform Law) / Learning: Baum-Welch / Decoding: Forward variables		**Generated states (Model 1: Uniform Law) / Learning: Segmental K-means / Decoding: Viterbi**	
Errors percentage between SM and SA (%)	70,4	Errors percentage between SM and SA (%)	59,7
Maximum value of the absolute value (SM-SA)	2	Maximum value of the absolute value (SM-SA)	2
Correlation rate between SM-SA	0,045	Correlation rate between SM-SA	0,312
Generated states (Model 1: Normal Law) / Learning: Baum-Welch / Decoding: Forward variables		**Generated states (Model 1: Normal Law) / Learning: Segmental K-means / Decoding: Viterbi**	
Errors percentage between SM and SA (%)	69,2	Errors percentage between SM and SA (%)	60,4
Maximum value of the absolute value (SM-SA)	2	Maximum value of the absolute value (SM-SA)	2
Correlation rate between SM-SA	0,281	Correlation rate between SM-SA	0,351

Generated states (Model 2: Uniform Law) / Learning: Baum-Welch / Decoding: Forward variables	
Errors percentage between SM and SA (%)	56,5
Maximum value of the absolute value (SM-SA)	2
Correlation rate between SM-SA	0,304

Generated states (Model 2: Uniform Law) / Learning: Segmental K-means / Decoding: Viterbi	
Errors percentage between SM and SA (%)	58,1
Maximum value of the absolute value (SM-SA)	2
Correlation rate between SM-SA	0,253

Generated states (Model 2: Normal Law) / Learning: Baum-Welch / Decoding: Forward variables	
Errors percentage between SM and SA (%)	28,6
Maximum value of the absolute value (SM-SA)	1
Correlation rate between SM-SA	0,708

Generated states (Model 2: Normal Law) / Learning: Segmental K-means / Decoding: Viterbi	
Errors percentage between SM and SA (%)	31,1
Maximum value of the absolute value (SM-SA)	2
Correlation rate between SM-SA	0,618

Generated states (Model 3: Uniform Law) / Learning: Baum-Welch / Decoding: Forward variables	
Errors percentage between SM and SA (%)	61,3
Maximum value of the absolute value (SM-SA)	2
Correlation rate between SM-SA	0,073

Generated states (Model 3: Uniform Law) / Learning: Segmental K-means / Decoding: Viterbi	
Errors percentage between SM and SA (%)	61,1
Maximum value of the absolute value (SM-SA)	2
Correlation rate between SM-SA	0,073

Generated states (Model 3: Normal Law) / Learning: Baum-Welch / Decoding: Forward variables	
Errors percentage between SM and SA (%)	58,2
Maximum value of the absolute value (SM-SA)	2
Correlation rate between SM-SA	0,209

Generated states (Model 3: Normal Law) / Learning: Segmental K-means / Decoding: Viterbi	
Errors percentage between SM and SA (%)	54,5
Maximum value of the absolute value (SM-SA)	2
Correlation rate between SM-SA	0,347

Our goal is to provide a S2 state which has a "high sense" for preventive maintenance (do not detect default, neither too late nor too early which could provide unnecessary or earlier interventions). S2 state is "on" only 1 hour before default occurs (Figure 11). Table 1 summarizes results of tests with both algorithms used (learning HMM: Baum-Welch, states estimated: Variables Forward). In case (Figure 11), after 1 hour, failure is detected in 50% of cases (Table 1).

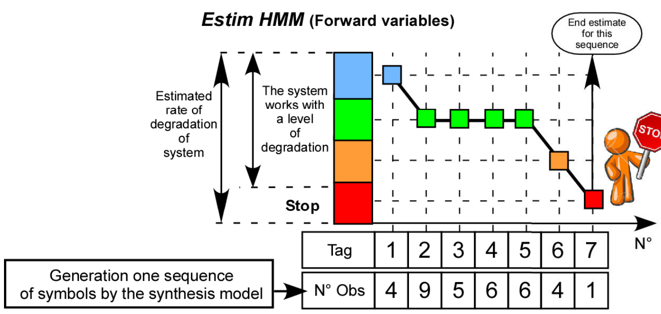

Figure 11: Test on a sequence sample

Table 1. Failure probability provided by S2 state "on" – synthesis model

Prediction Forward variables Model 2	at + 1 hour	at + 2 hours	at + 3 hours	at + 4 hours
	50%	67%	67%	100%

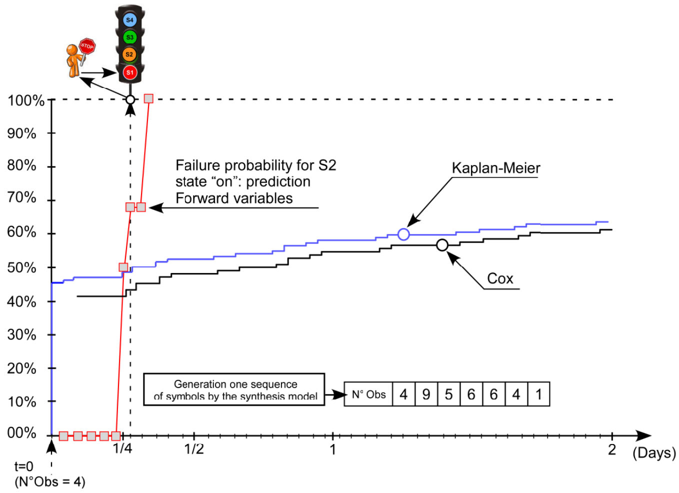

Figure 12: Triggering of "S2 state"

Figure 12 shows additional information that is available to the expert. We also show that our approach is better than using degradation laws (Kaplan-Meier and Cox). Once the S2 state is detected, the expert should react.

4 CONCLUSION

In this paper, we propose a new method to evaluate availability level of a system based on observed events on this system or on maintenance activities applied on this system. This new indicator is based on HMM.
To improve efficacy of this indicator, we use a synthesis model, for which degradation levels were perfectly known.
This model was chosen to provide progressive degradation of a process (oriented model).
With tests which have been made, we show that our model is able to follow "real" degradation level with enough accuracy. Tests were carried out offline, so we cannot assess the effects of scheduling dynamically maintenance activities. Before considering implementation in real-time situation, we will explore possibilities of process simulation for which we could apply dynamic scheduling of maintenance.

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