MODEL RISK AND DETERMINATION OF SOLVENCY CAPITAL IN THE SOLVENCY 2 FRAMEWORK

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ABSTRACT

This paper investigates the robustness of the Solvency Capital Requirement (SCR) when a log-normal reference model is slightly disturbed by the heaviness of its tail distribution. It is shown that situations with “almost” lognormal data and a rather important variation between the “disturbed” SCR and the reference SCR can be built. The consequences of the estimation errors on the level of the SCR are studied too.

KEYWORDS: Solvency, extreme values.

RESUME

Le présent article s’intéresse à la robustesse du capital de solvabilité (SCR) lorsqu’un modèle de référence log-normal est perturbé légèrement par l’alourdissement de sa queue de distribution. On montre que l’on peut construire des situations avec des données « presque » log-normales et une variation pourtant importante entre le SCR « perturbé » et le SCR de référence. On s’intéresse également aux conséquences des erreurs d’estimation sur le niveau du SCR.

MOTS-CLEFS : Solvabilité, valeurs extrêmes.

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1. INTRODUCTION

The new prudential framework for European insurers (Solvency 2) is based on a risk-based approach. As a matter of fact, the Solvency Capital Requirement (SCR) corresponds to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5 % over a one-year period (see Article 101 of the European Directive).

As a default option, the SCR will be obtained by a common standard formula for all insurers. This standard formula is built according to a modular approach[1] of risks. Nevertheless, another option will consist to use an internal model specified to be be more adapted to the risk effectively supported by insurer. This internal model will be subject to an approval process by the supervisors in order to be used to estimate the SCR. If various approaches are eligible, the purpose is identical: to establish the level of own-funs an insurer needs today to be not in default in one year in 199 cases out of 200.

The retained level of 99.5% implies the requirement to assess suitably a high-order quantile of the interest distribution (generally and in our case, the excess distribution or the distribution of the asset-liability[2] margin). This problematical point is widely built up in the financial literature that is confronted with these questions since the Basel II accords in the banking area. For instance, we can quote ROBERT [1998] or GAUTHIER and PISTRE [2000].

In this new insurance context, the classic asset/liability modeling that accredits a limited attention at the tail distribution modeling can be proved a penalizing point, because they lead at a low-level representation of extreme values. As a consequence, the solvency capital may be underestimate. For instance, this point is illustrated for the modelings of financial assets in BALLOTTA [2004] in case of options and financial guarantees embedded in life insurance contracts and in PLANCHET and THEROND [2005] in the framework of mono-periodic simplified model in non-life insurance for the determination of the SCR and an optimal asset allocation. THÉROND and PLANCHET [2007] draw the intention to the extent of extremes in the determination of Solvency Capital Requirement (SCR).

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[1] The quantitative impact study 5 carried out by the European Commission gives a proficient idea of which will be the standard formula when Solvency 2 guidelines are adapted.

[2] The valuation of assets and liabilities in this framework are specifically designed (see QIS5 technical specifications for example).

In the present paper, we focus on the problematic of an insurer using a partial internal model to compute its SCR. For example, the capital requirement for market and non-life subscription risks are estimated by an internal model designed to compute the Value-at-Risk of the basic own-funds subject to these risks with a confidence level of 99.5 % over a one-year period. And the global SCR is determined using the standard formula in which the capital requirements for these modules have been substituted by these amounts.

To illustrate the possible undervaluation of the capital if an special attention is not given to the extreme values, we develop this point of view in disturbing a simple log-normal reference model in making heavy its tail distribution. As a matter of fact, the log-normal model is often used to fit usual risks such as equity risk or non-life risks[3]. We show that it is possible to obtain some situations in which the basic reference model significantly underestimates the Solvency Capital Requirement, while being not easily discernible statistically with the disturbed model if a detailed attention is not paid to the extreme values : typically, this situation will arise when one try to fit a log-normal distribution to a random series of values generated by a disturbed log-normal distribution. The standards goodness-of-fit tests lead to accept the fitting also this is not the good one. As a consequence, the SCR is under-estimate.

In order to rectify this phenomenon, we suggest an empirical approach in order to decide if modellings of extreme values type must be carry out on the basis of an observed sample. We suggest also to use a "blended" model built by using a Pareto tail with a log-normal distribution with the goal to avoid the undervaluation of the SCR.

2. DESCRIPTION OF THE MODEL

2.1. PRESENTATION

We consider a probability distribution described by its survival function[4] S_0; more precisely we suppose the positive random X (which could be for example the discounted claim amount) is defined by the following survival function:

[3] The standard formula is based on such an approach for the non-life risks for example.

[4] It's more tractable here to use the survival function $S_0(x)=1-F_0(x)$ where F_0 is the corresponding cdf.

$$S_X(x) = \begin{cases} S_0(x) & x \le m \\ \left(\frac{x}{m}\right)^{-\alpha} S_0(m) & x > m \end{cases}.$$

In other words, X is distributed according to the distribution S_0 until the threshold m, and then according to a Pareto distribution with the (unknown) parameters (m,α). In particular, $P(X>m)=S_X(m)=S_0(m)$. In this situation, we will not reconsider motivations which lead to retain the Pareto distribution, but we encourage the reader to consult EMBRECHTS and al. [1997] for theoretical aspects of the question and ZAJDENWEBER [2000] for a practical perspective.

We verify that the above equality defines a decreasing, continuous function if S_0 is continuous, such as $S_X(0)=1$ and $S_X(0)=1$. So, S_X defines a survival function.

The existence of moments of S_X depends on the existence of moments of the same order for the Pareto distribution with parameters (m,α). So the *k*-order moment exists only for $k<\alpha$.

In this present context, we will choose the threshold *m* so that it corresponds to a high quantile of the distribution S_0, for instance such as $S_0(m = 1.5\ \%)$. The "blended" model in this precise case, behaves "almost" like the basis model associated with S_0 (for the portion $1-S_0(m)$ of observations), but differs beyond this threshold. From this definition of S_X, it may be deduced that:

$$P(X>x|X>m) = \frac{P(X>x)}{P(X>m)} = \frac{S_X(x)}{S_X(m)} = \left(\frac{x}{m}\right)^{-\alpha},$$

which means that the distribution of X conditionally to the fact that the threshold m is exceeded, is a Pareto distribution with parameters (m,α). Symmetrically, we find:

$$P\left(X > x \middle| X \leq m\right) = \frac{P\left(x < X \leq m\right)}{P\left(X \leq m\right)} = \frac{S_X\left(x\right) - S_X\left(m\right)}{1 - S_X\left(m\right)} = \frac{S_0\left(x\right) - S_0\left(m\right)}{1 - S_0\left(m\right)}.$$

The quantile function of *X*, for values of *p* lower than $1 - S_0\left(m\right)$ is simply given by:

$$x_p = m \times \left(\frac{1-p}{S_0\left(m\right)}\right)^{-1/\alpha}.$$

This expression is simply obtained with the equality $1 - p = \left(\frac{x}{m}\right)^{-\alpha} S_0\left(m\right)$, valid for $x > m$.

Logically we have: $x_{1-S_0(m)} = m$.

We wish to compare the case where the risk *X* is distributed simply like S_0 and the case where the tail distribution is weighed as above ("blended distribution"). More precisely, we wish to compare the quantile functions in the two situations, for high-order quantiles. From a practical point of view, we desire to compare the Solvency Capital Requirement in the two situations.

In the case where *X* is distributed according to S_0, the quantile function is by definition $x_p = S_0^{-1}\left(1-p\right)$. In this case, we still have of course $x_{1-S_0(m)} = m$.

In the continuation of this work, we consider that the distribution of reference S_0 is lognormal, at the same time because of its simplicity of use and its very major use in the insurance (the log-normal distribution can be considerate as the reference distribution in non-life insurance and is very often used to represent claims amounts) . So, the distribution of *X* is the "blended" distribution built with the log-normal reference distribution S_0 modified with the Pareto tail.

2.2. Specific case of the lognormal distribution

2.2.1. Calculation of the SCR

From now on, we consider first the case where the basis risk X is lognormal, and so, if we denote $F_0 = 1 - S_0$ and ϕ the cdf of a standardized gaussian random variable:

$$x_p^{LN} = VaR_p(X) = S_0^{-1}(1-p) = F_0^{-1}(p) = \mathbf{exp}\left(\mu + \sigma\phi^{-1}(p)\right).$$

We have: $S_0(m) = P\left(Z > \frac{\ln(m)-\mu}{\sigma}\right) = 1 - \phi\left(\frac{\ln(m)-\mu}{\sigma}\right)$ where Z is a standardized gaussian random variable . It may be deduced the explicit expression of the quantile function in the case of the X is now driven by the blended model:

$$x_p = m \times \left(\frac{1-p}{1-\phi\left(\frac{\ln(m)-\mu}{\sigma}\right)}\right)^{-1/\alpha}.$$

In the applications, we fix m while controlling $1 - S_0(m)$ on a rather large level but lower than p; typically in the Solvency 2 context $p = 99.5\%$ and we will choose $S_0(m) = 2\%$ or $S_0(m) = 1\%$. We note $p_0 = 1 - S_0(m)$, the selected level, so that $x_p = S_0^{-1}(1-p_0) \times \left(\frac{1-p}{1-p_0}\right)^{-1/\alpha}$. In the case of lognormal reference distribution S_0, we obtain in consequence for the blended model if $x_p^{MEL} = VaR_p(X)$:

$$x_p^{MEL} = \mathbf{exp}\left(\mu + \sigma\phi^{-1}(p_0)\right) \times \left(\frac{1-p}{1-p_0}\right)^{-1/\alpha},$$

this formula has to be compared with the version obtained from the lognormal direct model:

$$x_p^{LN} = \mathbf{exp}\left(\mu + \sigma\phi^{-1}(p)\right).$$

The ratio of two quantiles gives:

$$r(\alpha)=\frac{x_p^{MEL}}{x_p^{LN}}=\mathbf{exp}\left(\sigma\left(\phi^{-1}(p_0)-\phi^{-1}(p)\right)\right)\times\left(\frac{1-p}{1-p_0}\right)^{-1/\alpha}.$$

By the way, we can notice necessary that this ratio does not depend on the parameter μ. $r(\alpha)$ is a decreasing function of α: when α decreases, the risk associated with the blended distribution increases and as a consequence, if X is a discounted claim amount, the capital requirement to cover it too.

We will be confronted with the situation of model risk in the case where despite a value $r(\alpha)>>1$, a sample derived from the blended model would be difficult to differentiate with a lognormal sample. The lognormal model is very widespread in insurance and in particular, it is on this model that were gauged a part of parameters of the standard formula described in QIS 3. We are going to pay particular attention to examine this situation in the continuation of this paper.

2.3. ESTIMATION OF THE MODEL PARAMATERS

The estimation of parameters can be performed by the maximum likelihood method. Indeed, because

$$S_X(x)=\begin{cases} S_0(x) & x\le m \\ \left(\frac{x}{m}\right)^{-\alpha}S_0(m) & x>m \end{cases}$$

where $S_0(x)=1-\phi\left(\frac{\mathbf{ln}(x)-\mu}{\sigma}\right)$, the log-likelihood can be written, while noting $\left(x_{(1)},..,x_{(n)}\right)$ the order statistic associated with the sample $(x_1,..,x_n)$ and k the smallest index such as $x_{(k)}\ge m$:

$$l(x_1,..,x_n;\mu,\sigma,\alpha)=\sum_{i=1}^{k-1}\mathbf{ln}\left(\frac{1}{\sigma x_{(i)}\sqrt{2\pi}}\mathbf{exp}\left[-\frac{1}{2}\left(\frac{\mathbf{ln}\left(x_{(i)}\right)-\mu}{\sigma}\right)\right]\right)+\sum_{i=k}^{n}\mathbf{ln}\left(\alpha m^{\alpha}S_0(m)x_{(i)}^{-\alpha-1}\right)$$

which leads after calculation to:

$$l\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right)=c-\left(k-1\right)\ln\left(\sigma\right)-\frac{1}{2}\sum_{i=1}^{k-1}\left(\frac{\ln\left(x_{(i)}\right)-\mu}{\sigma}\right)^2$$
$$+\left(n-k+1\right)\ln\left(\alpha\right)+\alpha\left(n-k+1\right)\ln\left(m\right)-\alpha\sum_{i=k}^{n}\ln\left(x_{(i)}\right)+\left(n-k+1\right)S_0\left(m\right)$$

with a constant *c*. Because of the presence of $k=\min\left\{i;x_{(i)}\geq m\right\}$, the expression of log-likelihood is not easily usable in this form. Nevertheless, we can break up the problem of maximization by noticing that:

$$\max_{(\mu,\sigma,m,\alpha)} l\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right)=\max_{m}\max_{(\mu,\sigma,\alpha)} l\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right).$$

We just need to compute $\left(\hat{\mu},\hat{\sigma},\hat{\alpha}\right)$ which solve $\max_{(\mu,\sigma,\alpha)} l\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right)$ with a given value of *m*. So we must solve $\frac{\partial l}{\partial\mu}\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right)=0$, $\frac{\partial l}{\partial\sigma}\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right)=0$ and $\frac{\partial l}{\partial\alpha}\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right)=0$. We notice in which time *m* is fixed, the expressions of partial derivatives of the log-likelihood are the classic expressions of two subjacent distributions, on the ranges of data with regard to them. The estimators of μ and σ are thus the classic empirical estimators for the gaussian sample $\left(\ln x_{(i)};i=1,..,k-1\right)$:

$$\hat{\mu}=\frac{1}{k-1}\sum_{i=1}^{k-1}\ln x_{(i)} \text{ et } \hat{\sigma}=\sqrt{\frac{1}{k-1}\sum_{i=1}^{k-1}\left(\ln x_{(i)}-\hat{\mu}\right)}.$$

The estimator of tail parameter α is given by the following expression:

$$\hat{\alpha}=\frac{n-k+1}{\sum_{i=k}^{n}\ln\left(\frac{x_{(i)}}{m}\right)}.$$

It remains to eliminate *m*, unknown, in the above equation. In practice we can proceed in the following way:

- we fix k (while starting for example by $k = 95\% \times n$, where n denote the sample size);
- we calculate $\hat{\mu}$ and $\hat{\sigma}$;
- we calculate $\hat{m} = \mathbf{exp}\left(\hat{\mu} + \hat{\sigma}\phi^{-1}\left(k/n\right)\right)$;
- the estimator (pseudo maximum likelihood) of tail parameter α is given by the expression: $\hat{\alpha} = \dfrac{n-k+1}{\sum_{i=k}^{n} \mathbf{ln}\left(\dfrac{x_{(i)}}{\hat{m}}\right)}$

We obtain a value $l(k)$ of log-likelihood; we restart with $k' > k$ and we retain the estimation of parameters associated with the maximal value of the sequence $l(k)$ thus obtained.

In principle, we will notice that the above estimators are skewed (even if as estimators of the maximum likelihood they are asymptotically without skew).

2.4. ISSUE ON THE LEVEL OF THE CAPITAL OF THE PARAMETER ESTIMATION

BOYLE and WINDCLIFF [2004] underline the importance of the phase of parameters estimation, because of the loss of information on this level, in the relevance of the results provided by an theoretical model. As in this case, we have closed formulas for the quantile function in each model, the level of Solvency Capital Requirement will be simply estimate, in the blended model by:

$$\hat{x}_p^{MEL} = \mathbf{exp}\left(\hat{\mu} + \hat{\sigma}\phi^{-1}\left(p_0\right)\right) \times \left(\frac{1-p}{1-p_0}\right)^{-1/\hat{\alpha}},$$

and in the lognormal model, by:

$$\hat{x}_p^{LN} = \mathbf{exp}\left(\hat{\mu} + \hat{\sigma}\phi^{-1}\left(p\right)\right).$$

2.4.1. Case of the lognormal model

We verify easily that the function $f_a(x,y) = \mathbf{exp}(x+ay)$ is convex and we deduce with the Jensen's inequality (DACUNHA-CASTELLE and DUFLO [1982]) that:

$$E\left(\hat{x}_p^{LN}\right) = E\,\mathbf{exp}\left(\hat{\mu} + \hat{\sigma}\phi^{-1}(p)\right) \geq \mathbf{exp}\left(E(\hat{\mu}) + E(\hat{\sigma})\phi^{-1}(p)\right).$$

As in the lognormal model the parameter μ is estimated without skew, and that is possible to substitute $\hat{\sigma}$ by its corrected version of skew $\tilde{\sigma} = \sqrt{\frac{n}{n-1}}\hat{\sigma}$, we conclude that:

$$E\left(\hat{x}_p^{LN}\right) \geq x_p^{LN} = \mathbf{exp}\left(\mu + \sigma\phi^{-1}(p)\right).$$

In other words, the estimation procedure of the Solvency Capital Requirement in lognormal model leads to overestimate it on average. Of course, this is true only if the real distribution of *X* is lognormal, so that there is no error of specification of the underlying model.

2.4.2. Case of the blended model

We assume now that the underlying distribution of *X* is the blended one. In this case, we must examine the behavior of $f_{a,b}(x,y,z) = \mathbf{exp}\left(x + ay + \frac{b}{z}\right)$ with $b = \mathbf{ln}\left(\frac{1-p_0}{1-p}\right) > 0$. A simple matrix calculation makes it possible to verify the positivity of associated Hessian matrix and equally the convex nature of $f_{a,b}$. Unfortunately, it is not easy to deduce the meaning of the skew on the SCR estimation, because of the parameters is not anymore without skew.

The numerical simulations tend to highlight a negative skew, i.e. a underestimation of the SCR, which constitutes a penalizing point in practice (see below). At this stage we can summarize the possible situations as follow :

- the underlying risk if lognormal and the 99,5% quantile is estimated with the assumption that the underlying risk is lognormal ;

- the underlying risk if distributed following the blended distribution and the 99,5% quantile is estimated with the assumption that the underlying risk is distributed following the blended distribution ;

- the underlying risk if lognormal and the 99,5% quantile is estimated with the assumption that the underlying risk is distributed following the blended distribution ;

- the underlying risk if distributed following the blended distribution and the 99,5% quantile is estimated with the assumption that the underlying risk is lognormal ;

We will show that the fourth situation is the most penalizing one and that the bias of underestimation of the SCR can be minimized by choosing the blended distribution to estimate the model parameters.

2.5. NUMERICAL APPLICATION

From a practical point of view, the estimation of the SCR is not executed on observed data but on simulated values resulting from a model (the "internal model"); for instance we can consult THEROND and PLANCHET [2007]. The constraints of calculation make that it is not possible to dispose of an arbitrarily large number of achievements of the simulated asset-liability margin and that the estimation of the SCR will have to be effected on a modest size sample. Indeed, an internal model is complex and a run is very costly in terms of time. So it is only possible to generate, by simulation, a relatively small sample of the variable denoted X at the beginning of this paper, let's say between 1000 and 5000 realizations.

So, the modeling of the asset-liability margin is crucial about the determination of the level of capital.

2.5.1. <u>Simulation of the blended distribution</u>

The simulation of a sample resulting from the blended distribution can be obtained simply in the following way:

- drawing of a value u uniformly distributed on $[0,1]$;
- if $u > p_0$, drawing of x in the Pareto distribution with parameters (m,α);

- if $u < p_0$, drawing of x in distribution $S(x) = \frac{S_0(x) - S_0(m)}{1 - S_0(m)}$.

In this last case, the simulation can be carried out with a rejection method: we make a drawing in the lognormal distribution, and we refuse it if the obtained value is higher than *m*. Indeed, like:

$$P(X > x | X \le m) = \frac{S_0(x) - S_0(m)}{1 - S_0(m)},$$

This leads exactly to the conditional distribution we need.

2.5.2. Results

For the numerical application, we retain:

Threshold distribution (p_0)	98.50%
SCR threshold (p)	99.50%
m (threshold distribution)	353.554

lognormal		**Pareto**	
μ =	**5**	γ =	**353.554**
σ =	**0.4**	α =	**3.9**

With these assumptions, the theoretical value of SCR in the blended model and the lognormal model reference is equal to 113%. In others words, to use the lognormal model leads to underestimating the capital requirement of more than 10 % if the model, from which the data result, is the blended model.

So we generate 2 samples of 1000 achievements of each 2 models and we study the adequacy of the sample resulting from the “blended” distribution with a lognormal distribution. The following fitted distribution is obtained:

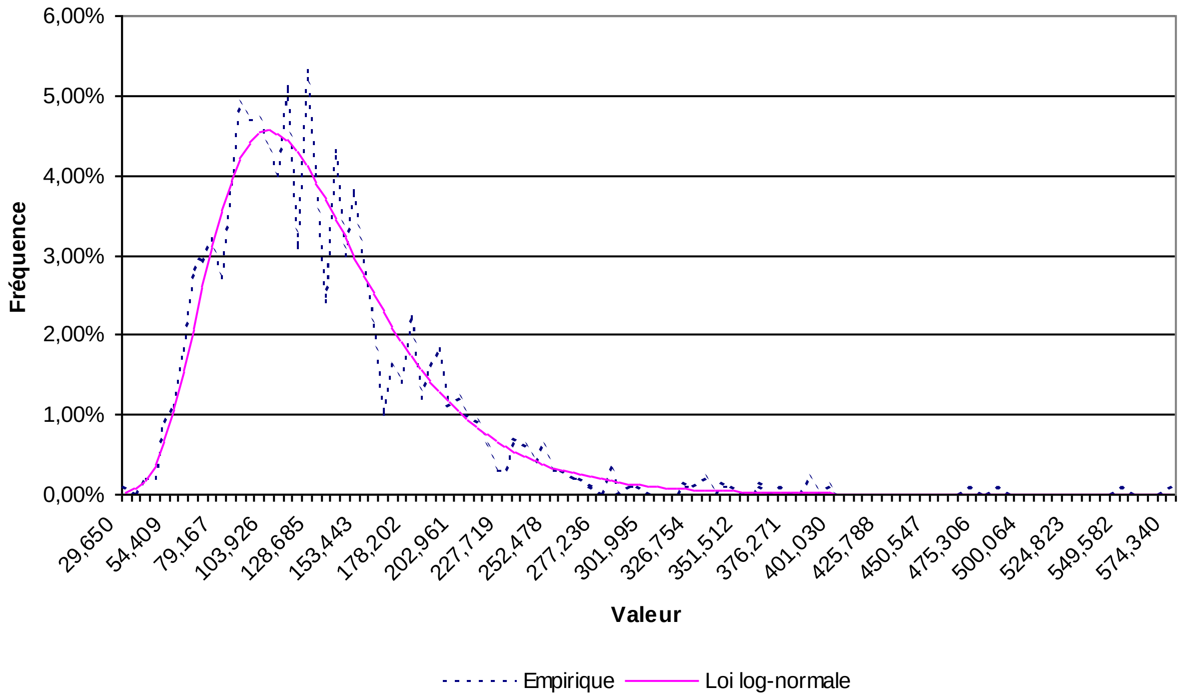

Fig. 1 : *Adjustment of the lognormal distribution on a blended sample*

The adjustment is widely accepted by a chi-square test. A too prompt analysis would lead to accept an inadequate fitting with the reality of the data. It is necessary to examine the behavior of the tail distribution to avoid this fitting error.

2.5.3. Identification of the extreme values

We notice that if we fix a probability $p > p_0$, then the probability that the *p*-order quantile of the lognormal distribution is exceeded in the blended distribution is:

$$\pi(p) = 1 - \left(\frac{\exp\left(\mu + \sigma\phi^{-1}(p)\right)}{m} \right)^{-\alpha} S_0(m)$$

In our example, if $p = 99.8\,\%$ then $\pi(p) = 0.50\,\%$; as a consequence, on a sample of 1000 values, we will get on average two values which exceed $S_0^{-1}(1 - 99.8\,\%)$, whereas there will be 5 values which will exceed this threshold if the subjacent distribution is the blended one. As the number of values N_u exceeding a threshold *u* is approximately normal we obtain:

$$P\left(N_u \geq k\right) \approx 1 - \phi\left(\frac{k - nS(u)}{\sqrt{nS(u)\left(1 - S(u)\right)}}\right).$$

This provides a test to reject the assumption that the subjacent distribution is lognormal by counting the number of excesses of the threshold $S_0^{-1}(1-99,8\%)$ in the sample. For instance, in this application, at the confidence threshold of 10% this rule leads to reject the null assumption that the underlying distribution is the lognormal one as soon as $k \geq 4$. On the sample presented on the above graph we notice thus that 4 points are in this situation:

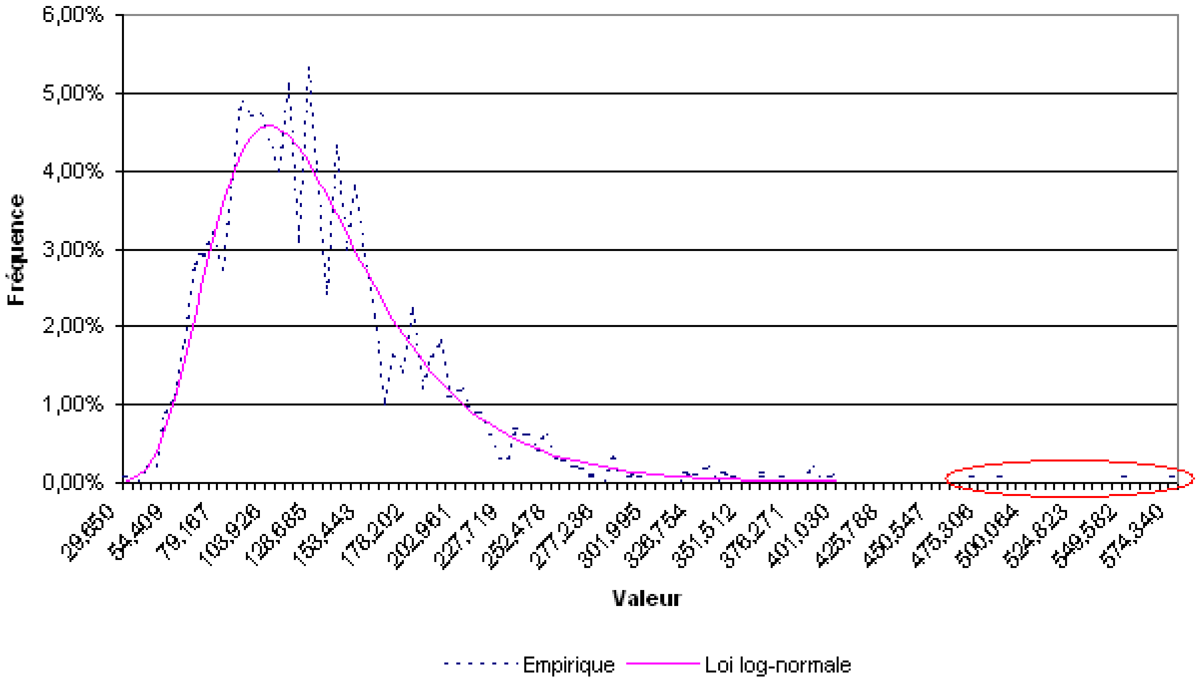

Fig. 2 : *Identification of extreme value*

So we would be led to reject the lognormal adjustment and to use a model taking into consideration the presence of these extreme values. It is important to notice that standards fitting tests as the Chi-square or Kolmogorov-Simrnov do not detect such extreme points.

2.5.4. Adjustment of the blended model

So here we use the blended model to estimate the parameters and derive an estimation of the SCR (defined as the 99.5% quantile of the fitted distribution, *cf*. section 2.1). The adjustment by maximum likelihood of blended model does not present a practical difficulty (*cf*. section

2.4.2). Indeed, the iterative calculation of log-likelihood performed by various values of k reveals a brutal change of slope when $\frac{k}{n} \approx p_0 = 1 - S_0(m)$, as the graph shows it below :

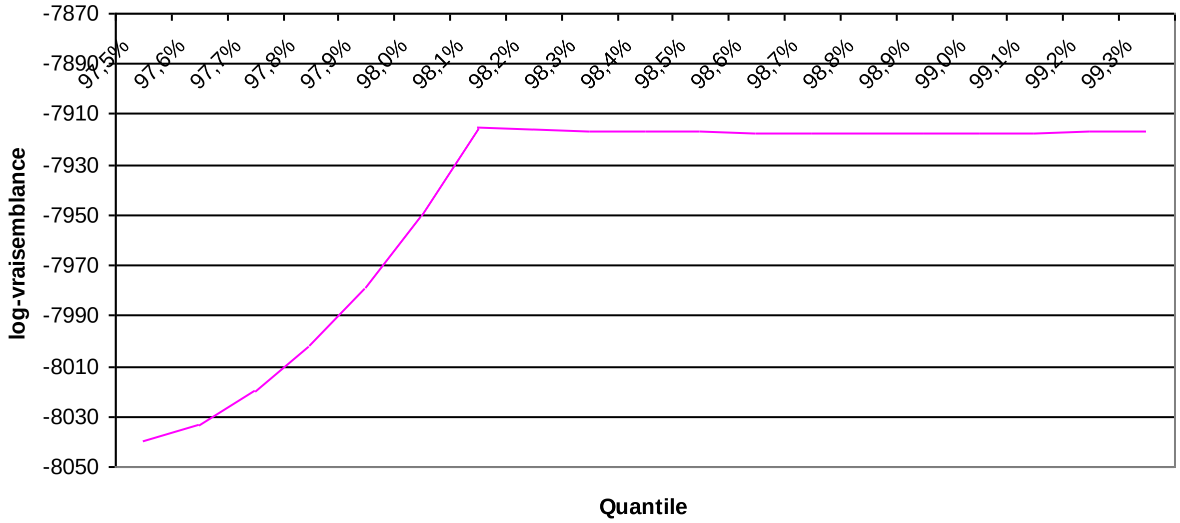

Fig. 3 : *Calculation of maximum likelihood : identification of m*

The values obtained on a "typical" sample arise in the following way:

	Estimation	Theoretical	
μ	4.958	5	
σ	0.386	0.4	
γ	317.097799	353.553971	
α	3.475	3.9	
Estimated ratio =	117%	113%	
Solvency capital requirement			
SCR LN	416.00	415.85	0.0%
SCR mélangé	451.29	468.59	-3.7%

The estimation of SCR in lognormal sample is relatively robust in the case of a sample of size 1000. However, we observe an slight underestimation of the capital in the case of the blended model. But, in the end, we can retain if the data result from the blended model, the fact of considering that they are really issued from a lognormal sample leads to an important underestimation of the capital requirement. Moreover within the framework of the wellspecified model, the estimation still leads to a light underestimation, but this estimation is of course better than the one with the lognormal hypothesis.

This example underlines the importance of an appropriate tail distribution modeling to avoid an important underestimation of an (relatively) high-level quantile.

3. CONCLUSION

The results presented here, within a very simplified framework, underline once again the lack of robustness that is inherent in the criterion of fixing of the Solvency Capital Requirement in the Solvency 2 prudential framework. This is the consequence that one needs to estimate the 99.5% quantile of the net asset-liabilities distribution and, because this distribution can only be approximate by Monte-Carlo methods with a relatively small numbers of points. In this context, using a empirical estimator for this quantile is not possible and a parametric model must be choosen and fitted. The aim of this paper is to focus on the necessity to choose a specification that avoid the underestimation of the probability of extreme values. Frim thie point of view, it is quite natural to use a Pareto distribution for the tail of the distribution, because the extreme values theory tells us that, for very high quantile, it is the asymptotic situation. We just suggest here to force the asymptotic distribution at lower quantiles.

So it seems essential to us that the implementation methods of the ruin probability criterion are clarified in the long term and notably that the constraints on the modelling of the tail distribution are specified within the framework of an internal model. These constraints must be expressed on three levels: for the asset modelling, for the liability modelling, and finally within the framework of the exploitation of the empirical distribution of a asset-liability margin simulated from "way out" of the model.

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