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# STEADY-STATE RESPONSE OF A ROTOR EXCITED BY COMBINED ROTATIONAL AND TRANSLATIONAL BASE MOTIONS

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**Abstract** In the transportation domain, the on-board rotor in bending is subjected not only to rotating mass unbalance but also to support movements. The equations of motion in bending of the rotating rotor take into account the geometric asymmetry of disks and/or shaft and consider six types of deterministic support motions. The application of Lagrange's equations using the finite element method based on the theory of Timoshenko leads to the equations of motion which highlight periodic parametric terms due to the asymmetry of the rotor and time-varying parametric terms due to the rotational base excitations. When the rotor base is subjected to combined rotation and sinusoidal translation, analytical solutions are derived and analyzed by means of Campbell diagrams and steady-state responses.

**Key words:** Dynamics of rotors, on-board rotor dynamics, base excitation, analytical solution.

## 1 Introduction

Rotordynamics with a fixed support has been extensively predicted [1-3]. Other works concentrate on the behavior of a rotor in the seismic excitation cases [4-6]. The effect of the flexible foundation on the rotor dynamics has been studied using a finite element modeling [7] and an experimental investigation of a rotor [8]. In [9], the nonlinear bearing coefficients are considered to be piecewise constant using the El Centro earthquake data. Certain study focuses on the experimental behavior of a rotor under shock base excitation [10]. When the rotor support is induced by a deterministic harmonic motion, the response to this excitation [11], the stability chart [12], the comparisons between experimental and numerical orbits [13] as well as the active vibration control of the rotor [14] are studied and presented. In this paper, we present a symmetrical rotating rotor model whose base is subjected to a constant angular velocity and a periodic translation. Campbell diagrams, mass unbalance responses and responses to translational base excitations

are plotted as a function of the base rotations which can drastically change the behavior of the rotor.

## 2 Dynamical behavior modeling

The rotor support is supposed to be sufficiently rigid compared to the flexible shaft of the rotor. Its motion is described by three rotations and three translations. The investigated rotor and the coordinate system attached to the support are presented in Fig. 1. The derivation of the equations of motion has been established in [2,10,12] and is briefly described as follows:

- The disc is characterized by its kinetic energy because it is considered rigid.
- The elementary kinetic energy of an elementary shaft volume is considered as a generalization of the disc case. Moreover, the strain energy of a shaft finite element is calculated using the Timoshenko beam theory.
- The mass unbalance located on the disc is characterized by its kinetic energy.

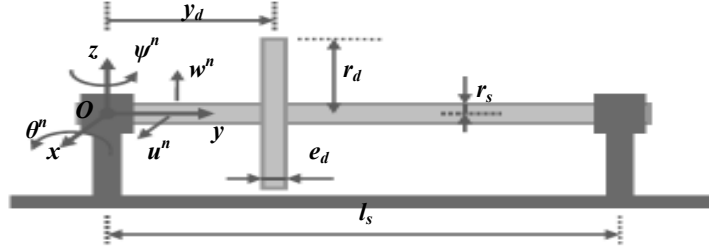


Fig. 1. Description of the investigated rotor.

The nodal displacement vector is defined by  $\mathbf{q}^n = \langle u^n, w^n, \theta^n, \psi^n \rangle^T$ . The equations of motion of the finite element rotor model are obtained after applying the Lagrange's equations to the energies for the disc, the shaft finite element as well as the mass unbalance and assembling the corresponding matrices:

$$\mathbf{M}_r(t)\ddot{\mathbf{q}} + \mathbf{C}_r(t)\dot{\mathbf{q}} + \mathbf{K}_r(t)\mathbf{q} = \mathbf{F}_r(t) \quad (1)$$

where  $\mathbf{M}_r(t)$ ,  $\mathbf{C}_r(t)$  and  $\mathbf{K}_r(t)$  are the parametric matrices of mass, damping and stiffness of an asymmetrical rotating rotor on moving base.  $\mathbf{F}_r(t)$  is the total force vector. Now, the shaft and the disc are assumed to be symmetric and the base excitations are assumed to consist of a constant angular velocity  $\omega^x$  (or  $\omega^y$ ) around the  $Ox$  axis (or the  $Oy$  axis) and a periodic translation along the  $Oz$  axis given by  $z_O = Z_O \sin(\Omega^2 t)$ . The equations of motion (1) in this case include constant terms and may be rewritten in one of the two following forms (Eq. (2) corresponds to  $\omega^x$  and Eq. (3) to  $\omega^y$ ):

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}^g\Omega\dot{\mathbf{q}} + \left(\mathbf{K} + \left(\mathbf{K}_{b1}^{re} + \mathbf{K}_b^{gse}\right)\omega^{x2}\right)\mathbf{q} = & \left(\mathbf{V}_u^c\Omega^2 + \mathbf{V}_{u,b1}^c\omega^{x2}\right)\cos(\Omega t) \\ & + \left(\mathbf{V}_u^s\Omega^2 + \mathbf{V}_{u,b1}^s\omega^{x2}\right)\sin(\Omega t) + \mathbf{V}_b^s Z_O \left(\Omega^{z2} + \omega^{x2}\right)\sin(\Omega^z t) \end{aligned} \quad (2)$$

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \left(\mathbf{C}^g\Omega + \mathbf{C}_b\omega^y\right)\dot{\mathbf{q}} + \left(\mathbf{K} + \mathbf{K}_{b2}^{re}\Omega\omega^y + \mathbf{K}_{b3}^{re}\omega^{y2}\right)\mathbf{q} = & \left(\mathbf{V}_u^c\Omega^2 \right. \\ & + \left.\mathbf{V}_{u,b2}^c\left(\Omega\omega^y + \omega^{y2}\right)\right)\cos(\Omega t) + \left(\mathbf{V}_u^s\Omega^2 + \mathbf{V}_{u,b2}^s\left(\Omega\omega^y + \omega^{y2}\right)\right)\sin(\Omega t) \\ & - 2\mathbf{V}_b^c Z_O\Omega^z\omega^y\cos(\Omega^z t) + \mathbf{V}_b^s Z_O \left(\Omega^{z2} + \omega^{y2}\right)\sin(\Omega^z t) \end{aligned} \quad (3)$$

where  $\Omega$  is the speed of rotation of the rotor.  $\mathbf{V}_u$ ,  $\mathbf{V}_b$  and  $\mathbf{V}_{u,b}$  are load vectors associated respectively with the mass unbalance, the force inertia due to base motions and that due to coupling between both phenomena. The superscript “g” refers to the gyroscopic effect, “re” is the rotational effects due to the base rotation and “gse” is the geometric stiffening effects corresponding to the centrifugal stress due to the base rotation. In our case, the system of Eq. (1) contains constant matrices (according to Eqs. (2) and (3)) and is transformed into a first-order system:

$$\begin{Bmatrix} \dot{\mathbf{q}} \\ \ddot{\mathbf{q}} \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_r^{-1}\mathbf{K}_r & -\mathbf{M}_r^{-1}\mathbf{C}_r \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{M}_r^{-1}\mathbf{F}_r(t) \end{Bmatrix} \quad (4)$$

The eigenvalues of the above matrix is calculated for finding the natural frequencies  $f$  of the rotor. Here only the steady-state responses are calculated. The general solution of the equations of motion (4) is not considered and its particular solution can be expressed by:

$$\mathbf{q} = \mathbf{q}_u^c \cos(\Omega t) + \mathbf{q}_u^s \sin(\Omega t) + \mathbf{q}_b^c \cos(\Omega^z t) + \mathbf{q}_b^s \sin(\Omega^z t) \quad (5)$$

where  $\mathbf{q}_u^c$ ,  $\mathbf{q}_u^s$ ,  $\mathbf{q}_b^c$  and  $\mathbf{q}_b^s$  are unknown displacement amplitude vectors. Use  $\mathbf{q}$  to obtain the velocity and acceleration vectors. Substituting them in Eq. (4) gives:

$$\begin{aligned} \begin{bmatrix} \mathbf{K}_r - \mathbf{M}_r\Omega^2 & \mathbf{C}_r\Omega \\ -\mathbf{C}_r\Omega & \mathbf{K}_r - \mathbf{M}_r\Omega^2 \end{bmatrix} \begin{Bmatrix} \mathbf{q}_u^c \\ \mathbf{q}_u^s \end{Bmatrix} = & \begin{Bmatrix} \mathbf{F}_u^c + \mathbf{F}_{u,b}^c \\ \mathbf{F}_u^s + \mathbf{F}_{u,b}^s \end{Bmatrix} \\ \begin{bmatrix} \mathbf{K}_r - \mathbf{M}_r\Omega^{z2} & \mathbf{C}_r\Omega^z \\ -\mathbf{C}_r\Omega^z & \mathbf{K}_r - \mathbf{M}_r\Omega^{z2} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_b^c \\ \mathbf{q}_b^s \end{Bmatrix} = & \begin{Bmatrix} \mathbf{F}_b^c \\ \mathbf{F}_b^s \end{Bmatrix} \end{aligned} \quad (6)$$

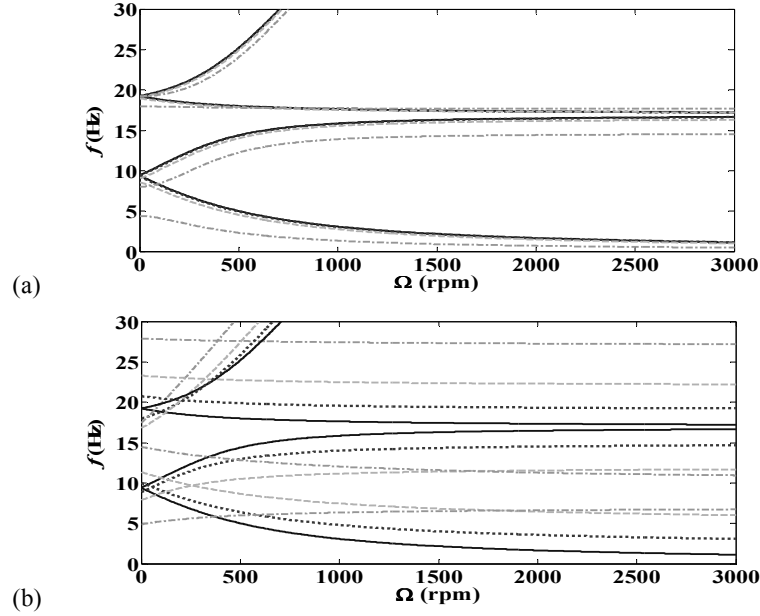
### 3 Results and Discussion

The material properties and the geometry of the rotor are given in Table 1.

**Table 1.** Main characteristics of the investigated rotor.

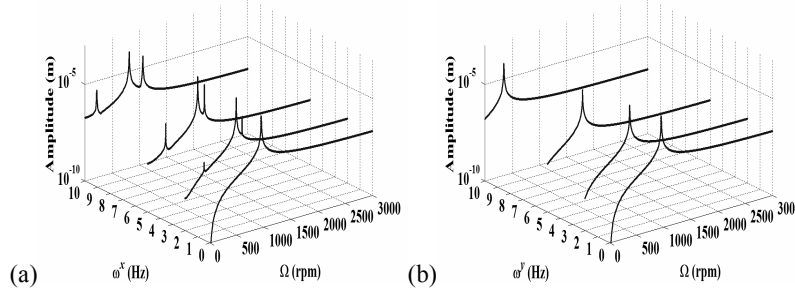
Density of both disc and shaft material	7800 kg/m <sup>3</sup>
Radius, thickness and location of the disc	0.4 m, 0.05 m, 0.4/3 m
shaft material (E, $\nu$ )	$2 \times 10^{11}$ N/m <sup>2</sup> , 0.3
Radius and length of the shaft	0.01 m, 0.4 m
Mass unbalance	40 g mm, 0°

Figure 2 shows the Campbell diagrams of the rotor subject either to  $\omega^x$  or  $\omega^y$  angular velocity. The base rotation  $\omega^x$  breaks the symmetry of the rotor behavior; see its Campbell diagram presented in Fig. 2(a). The rotational effects due to  $\omega^x$  decrease the natural frequencies while the centrifugal stiffening relative to  $\omega^x$  increases them especially at the higher speeds of rotation of the rotor. Unlike the previous case, the solicitation due to the base rotation  $\omega^y$  does not bring any asymmetry to the rotor, but shifts the natural frequencies; see its Campbell diagram presented in Fig. 2(b).



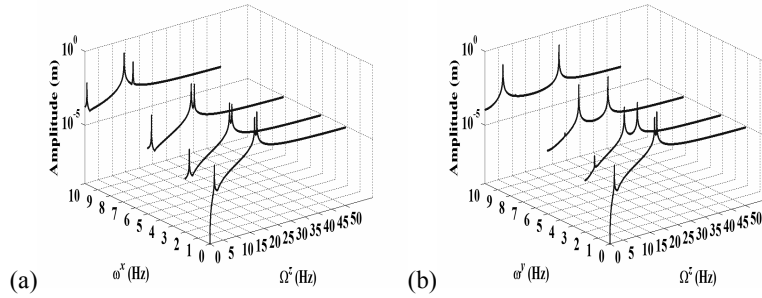
**Fig. 2.** Campbell diagrams for the first four natural frequencies of the rotor during the support rotation: (a) with  $\omega^x=0$  (solid line),  $4\pi$  rad/s (2 Hz, dotted line),  $10\pi$  rad/s (5 Hz, dashed line),  $20\pi$  rad/s (10 Hz, dashed-dotted line); or (b) with  $\omega^y=0$ ,  $4\pi$ ,  $10\pi$ ,  $20\pi$  rad/s.

Figure 3 represents the mass unbalance responses at the middle of the shaft subject either to  $\omega^x$  or  $\omega^y$  angular velocity. The rotor has only one critical speed in the presence of  $\omega^y$  (see Fig. 3(b)), but three in the presence of  $\omega^x$  (see Fig. 3(a)). These responses confirm that the rotor keeps a symmetrical behavior in the case of the base rotation around the  $Oy$  axis, but has a non-symmetrical behavior in the case of the base rotation around the  $Ox$  axis. We note that the position of the critical speed is shifted towards the lower values of the speed of rotation of the rotor when the angular velocity  $\omega^y$  increases.



**Fig. 3.** Mass unbalance responses during the support rotation around: (a) the  $Ox$  axis; or (b) the  $Oy$  axis.

Figure 4 presents the evolution of the frequency responses at the middle of the rotor running at a speed of rotation of 2000 rpm and subject to a harmonic base translation of amplitude  $2 \times 10^{-4}$  m in the  $z$  direction. In the presence of  $\omega^x$  or  $\omega^y$  angular velocity, the responses exhibit always three critical frequencies (three peaks) corresponding to the rotor natural frequencies at 2000 rpm. In the case of  $\omega^y$ , the harmonic translation along the  $Oz$  axis produces a very small peak compared to the other two peaks (see Fig. 4(b)). This is because Eq. (3) presents a negative cosine term containing the frequency  $\Omega^z$  of the base translation.



**Fig. 4.** Dynamical responses to the periodic base translation at 2000 rpm speed of rotation of the rotor during the base rotation around: (a) the  $Ox$  axis; or (b) the  $Oy$  axis.

## 4 Conclusions

A finite element model is presented for analyzing the dynamical behavior of a symmetrical on-board rotor whose support is subjected to combined constant rotation and sinusoidal translation. The rotational effects and the geometric stiffening effects relative to the centrifugal stressing due to the support rotation are taken into account. It is shown that support rotation not only changes the natural frequencies of the rotor but can also break the symmetry of the dynamical behavior in the case of the rotation around a transverse axis. The sinusoidal support translation does not change the rotor natural frequencies. It has only an influence on the rotor responses and the shape of the orbits.

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