Grain Buiding Ordering

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Abstract. Given a set E, the partitions of E are usually ordered by merging of classes. In segmentation procedures, this ordering often generates small parasite classes. A new ordering, called "grain building ordering", or GBO, is proposed. It requires a connection over E and states that $A \preccurlyeq B$, with $A, B \subseteq E$, when each connected component of B contains a connected component of A. TheGBO applies to sets, partitions, and numerical functions. Thickenings ψ with respect to the GBO are introduced as extensive idempotent operators that do not create connected components. The composition product $\psi\gamma$ of a connected opening by a thickening is still a thickening. Moreover, when $\{\gamma_i, i \in I\}$ is a granulometric family, then the two sequences $\{\psi\gamma_i, i \in I\}$ and $\{\gamma_i\psi, i \in I\}$ generate hierarchies, from which semi-groups can be derived. In addition, the approach allows us to combine any set of partitions or of tessellations into a synthetic one.

1 Introduction

In image processing, the segmentation techniques, which aim to partition the space of definition of the image under study, often generate a few correct classes. They are large and representative, but surrounded by a multitude of small parasitic other ones. Figure 1, depicts a typical example of the phenomenon. Several authors, such as Ph. Salembier et Al. [9] or J. Crespo et Al. [3], among others, propose solutions by merging of flat zones that satisfy convenient criteria, in association with some constraints (e.g. not to subdivide the small zones). In [14], P. Soille and J. Grazzini try to stamp out the parasites by imposing the presence of one extremum at least of the image inside each segmented class. In [1], the small regions are removed by erosions of partitions.

Small classes turn out to be inherent in the segmenting techniques. In case of connective segmentation, for example, they satisfy the chosen criterion, in the same way as the large classes. If they are reduced by intersection of constraints, they become singletons [11], or they are absorbed by the background [7]. The trouble is shifted, but not solved: what to do, then, with this background, or with these unclassified singletons? The solution adopted in [11] (figures 11 and 12) consists in building the influence zones of the large classes, which absorb the small ones. Independently, the above authors often did the same, but surreptitiously, for not catching attention on a procedure that resembles to cooking, rather than to nice theorems.

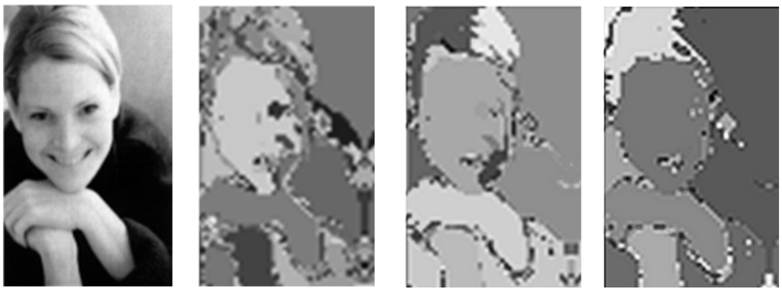

Fig. 1. Segmentations quasi-flat zones of increasing slopes λ ; as λ increases, the details of the face progressively vanish, though the parasite small grains remain (by courtesy of Noyel et Al. [6]).

The elimination of the small classes refers to a wider question: what does the usual ordering on partitions stand for? According to this so called "refinement ordering", one goes from a smaller partition to a larger one uniquely by merging classes, hence by only removing frontiers and not by moving them, or by creating new ones. This rapidly leads to ambiguous situations whose paradigm is depicted in figure 2. In case of the figure, must we introduce some choice in a segmenting approach which is basically deterministic?

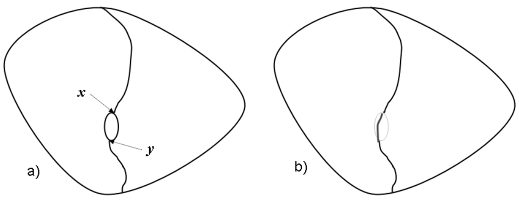

Fig. 2. The usual ordering on partitions can suppress the small class in a) only by merging it with one of the two large ones. Could another ordering divide up the small central class among the two others, as in b)?

When passengers are fully packed in the metro, and that one person leaves the carriage, does one of his neighbors suddenly swell and monopolize the whole free space for himself? At the end of a war, do both winners and losers decide, as an absolute rule, not to move any frontier? In figure 2 as well, we would like to refuse the ukase of the refinement ordering, and to share the small class among the two large ones, by joining x to y by a simple arc. If we proceed this way, then every class of the resulting partition contains at least one class of the initial one. It is exactly this property that we will now raise to an axiom [1].

2 Grain building ordering (GBO)

Given set E, we consider the lattice $\mathcal{P}(E)$ of all its subsets, and provide it with an arbitrary connection $\mathcal{C}^*$, said to be standard. Unlike inclusion, where $A \subseteq B$ means that any *point* of A belongs to B, the ordering relation introduced below holds on the *$\mathcal{C}^*$-components de* $\mathcal{P}(E)$, hence its generic appellation of connected ordering. We will indicate a few notation. The image of $\mathcal{P}(E)$ under operation ψ is written by $\mathcal{P}_\psi$

$$\mathcal{P}_\psi = \{\psi(X), X \in \mathcal{P}(E)\}.$$

In order to avoid confusion between the various openings that intervene, we denote by $\dot{A}_x$, the $\mathcal{C}^*$-component of A at point x (instead of $\gamma_x(A)$). When the labelling of the $\mathcal{C}^*$-components of A is not necessary, one just writes $\dot{A}$ (with $\dot{A} \subseteq A$). When the context is not ambiguous the expression "C^*- connected component" is replaced by "component", or by "grain", and " C^*-connected opening" by "connected opening". On the other hand, we keep the same symbol $\subseteq$ to designate the setwise inclusion as well as the ordering relation it induces between operators (i.e. $\gamma \subseteq \gamma'$ iff $\gamma(A) \subseteq \gamma'(A)$ for all $A \subseteq E$).

Let $A \in \mathcal{P}(E)$, of connected components $\dot{A}_i$. An anti-extensive grain operator is an operation on A that suppresses some $\dot{A}_i$ and leave unchanged the others, according to an increasing binary criterion that holds on each grain separately. When the grain operator γ is idempotent, we speak of connected opening. It may be the matter of an area threshold, or of the radius of the inscribed disc, or of any external attribute.

An extensive and idempotent operation $\psi : \mathcal{P}(E) \rightarrow \mathcal{P}(E)$, is traditionally called thickening, or sometimes idempotent thickening [10]. In addition we assume here that the thickening ψ does not create connected components. Given connection C^*, neither the grain operators and their connected openings, nor thickenings, involve the lattice structure of $\mathcal{P}(E)$, but its inclusion ordering only. Therefore, these three notions apply for any partial ordering on $\mathcal{P}(E)$.

Proposition 1. *When $\mathcal{P}(E)$ is equipped with connection $\mathcal{C}^*$, then the relation*

$$A \preccurlyeq B \text{ iff each } \dot{B} \subseteq B \text{ contains at least one } \dot{A} \tag{1}$$

defines over $\mathcal{P}(E)$ a $\mathcal{C}^*$-connected ordering, called grain building ordering *(in short: GBO).*

[1] A first version of this work, with all proofs and supplementary results, was presented in a worksop at ESIEE, in April 2010 [13]. This initial work inspired Ch.Ronse with several other orderings on partial partitions, and with the resulting optimisations. They can be found in these proceedings [8].

For example, a $\subseteq$-thickening ψ that does not create connected components is not only $\subseteq$-extensive, but also $\preccurlyeq$-extensive, and as its idempotence is independent of the ordering, it turns out to be a $\preccurlyeq$-thickening as well. It is the same for any connected opening γ. We have indeed that $\gamma(A) \succeq A$, since every $\mathcal{C}^*$-component of $\gamma(A)$ contains a $\mathcal{C}^*$-component of A, namely itself. Note that the $\preccurlyeq$-order of $\gamma(A)$ and A is the opposite of that of inclusion. Remark also that the axiom of the conditional union, which characterizes the connection, does not intervene in Proposition 1.

Every set ordering relation extends to partitions via their classes. In the present case, we can state the following:

Corollary 2. *The set property (1) generates an ordering on the space $\mathcal{D}$ of the partitions of E with $\mathcal{C}^*$-connected classes, where $D_1 \preccurlyeq D_1$, $D_1, D_2 \in \mathcal{D}$, when each class of D_2 contains one class of D_1 at least.*

Clearly, the set GBO does not preserve inclusion, neither connection $\mathcal{C}^*$, since when $A \preccurlyeq B$ some $\mathcal{C}^*$-components of the smaller set, A, may lie partly or even completely outside of B, as shown in figure 3. In case of sets, GBO is thus non comparable to inclusion, though, for partitions, it is more restrictive than the usual refinement:

$$D_1 \leq D_2 \quad \Rightarrow \quad D_1 \preccurlyeq D_2, \quad D_1, D_2 \in \mathcal{D}, \tag{2}$$

an implication which is no longer true for partial partitions. For extending the setwise GBO to functions, one proceeds by comparing each section $X_f(t) = \{x : x \in E, f(x) \geq t\}$ to the analogue $X_g(t)$, by putting

$$f \preccurlyeq g \quad \Leftrightarrow \quad X_f(t) \preccurlyeq X_g(t), \quad \forall t \in \bar{\mathbb{R}} \text{ (or } \forall t \in \bar{\mathbb{Z}}) \ .$$

which defines a numerical ordering, which is illustrated by Figure 4.

The set GBO does not induce any lattice: if $\dot{A}_1$ and $\dot{A}_2$ are two $\preccurlyeq -$components of A, each of both sets $\dot{A}_1 = A_1$ and $\dot{A}_2 = A_2$ is an upper bound of A, but there is no upper bound of A smaller than A_1 and A_2. Therefore one can still introduce increasing mappings.

By duality under complementation, Relation (1) induces the following one:

$$A^\star \preccurlyeq B^\star \text{ if every } (\dot{B^c}) \subseteq B^c \text{ contains at least one} (\dot{A^c}) \subseteq A^c, \tag{3}$$

which is still an ordering relation. The two relations (1) and (3) are not equivalent, neither incompatible, and their logical intersection defines a third ordering, of type homotopic type, in that it makes symmetrical the roles of grains and pores. Other orderings associated with connections and partitions can be defined [13] [8]. In particular, one can take the logical intersection between GBO and inclusion, which eliminates all outside small objects. This yields the restriction of the GBO to the partial partitions with same support. The $\preccurlyeq$-thickenings below satisfy this double ordering.

This partial GBO governs the variations of some physical phenomena, such as changes of metallic grains under fatigue. It appears also in political changes (e.g. the dismemberment or the Ottoman Empire at the end of 19th century).

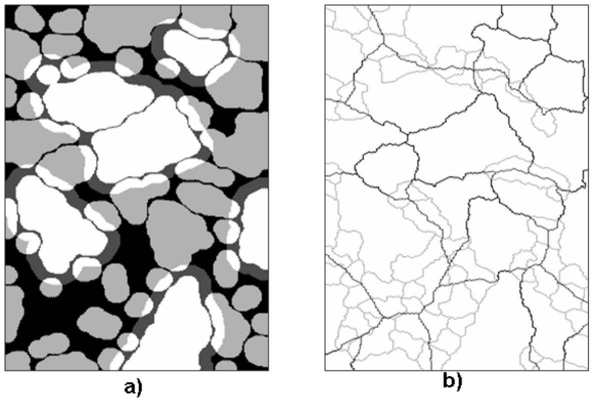

Fig. 3. GBO: a) for sets: the dilates of a few grains (in white) generate a set larger than the initial one ; b) for partitions: in dark grey, the larger partition.

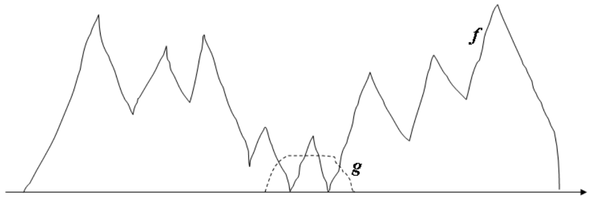

Fig. 4. GBO for numerical functions. Function g, in dotted lines, is the greater.

3 Thickening and $\preccurlyeq$ ordering

3.1 $\preccurlyeq$-thickening from connected opening

We will now construct operations that simplify sets and partitions, by sorting out certain main regions which then expand and cover the whole space. Their choice is governed by an opening, and their expansion by a thickening. The simplest, but the most worked out case, occurs when the opening is $\mathcal{C}^*$-connected:*

Proposition 3. *Given a connection $\mathcal{C}^*$ on $\mathcal{P}(E)$, let $\gamma : \mathcal{P}(E) \rightarrow \mathcal{P}(E)$ be a $\subseteq$-anti-extensive grain operator, and $\psi : \mathcal{P}(E) \rightarrow \mathcal{P}(E)$ be a $\subseteq$-thickening that does not create connected components. The composition product $\psi\gamma$ is a thickening for the GBO, and we have*

$$I \preccurlyeq \psi\gamma = \gamma\psi\gamma = (\psi\gamma)^2. \tag{4}$$

The product $\gamma\psi$ also behaves as a thickening, up to factor ψ:

$$I \preccurlyeq \psi\gamma\psi = \gamma\psi\gamma\psi = (\gamma\psi)^n \qquad n > 1. \tag{5}$$

Below, the grain operators of the proposition are always connected openings. Note that the proposition does not inform us on the distance between connected components in $\psi\gamma$ and $\gamma\psi$.

Extension to partitions Proposition 3 can be stated in terms of partitions of E into connected classes. Let $\mathcal{D}$ be the set of these partitions, $D \in \mathcal{D}$ and D_x the class of D at point x. The set connected opening γ induces on $\mathcal{D}$ the following operation $\gamma_{\mathcal{D}}$

$$D_x[\gamma_{\mathcal{D}}(D)] = \gamma(D_x) = D_x \quad \text{if} \quad x \in \gamma(D_x)$$
$$D_x[\gamma_{\mathcal{D}}(D)] = \{x\} \quad \text{if not}$$

Proposition 3 extends to partitions by replacing $\gamma : \mathcal{P}(E) \rightarrow \mathcal{P}(E)$ by $\gamma_{\mathcal{D}} : \mathcal{D}(E) \rightarrow \mathcal{D}(E)$, and by using a thickening $\psi : \mathcal{D}(E) \rightarrow \mathcal{D}(E)$.

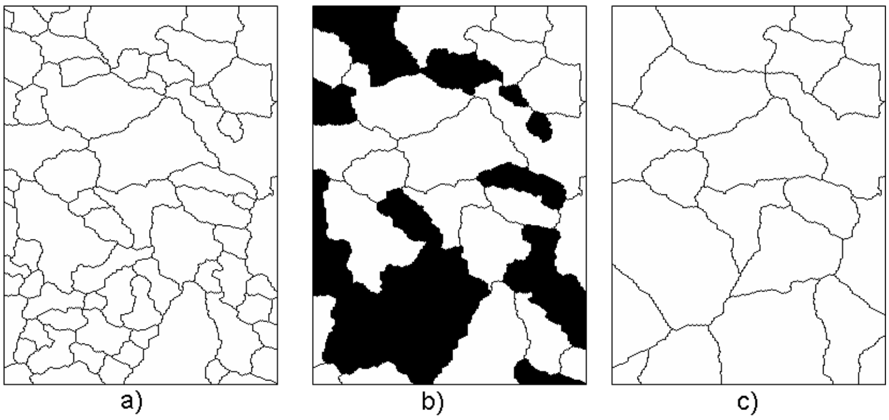

Fig. 5. a) Initial tessellation A, b) opening $\gamma(A)$ that suppresses grains according to their inscribed disc (here for radius ≤ 15); c) Voronoi thickening $\psi\gamma(A)$ of $\gamma(A)$, which is identical to its opening $\gamma\psi\gamma(A)$.

$\preccurlyeq$-thickening from non-connected opening When opening γ is not connected, then Proposition 3 is no longer valid, and is replaced by a more specific result.

Proposition 4. *Let γ be an opening on $\mathcal{P}(E)$ that acts independently on connected components, and let $\psi : \mathcal{P}(E) \rightarrow \mathcal{P}(E)$ be a $\subseteq$-extensive operator that does not create $\mathcal{C}^*$- components. Denote by $(\widetilde{\gamma\psi})\gamma(A)$ the union of those $\mathcal{C}^*$-components of $\gamma\psi\gamma(A)$ that contain a $\mathcal{C}^*$-component of $\gamma(A)$. The composition* *product* $(\widetilde{\gamma\psi})$ *is then* $\preccurlyeq$*-extensive on* $\mathcal{P}_\gamma = \gamma[\mathcal{P}(E)]$*:*

$$\gamma(A) \preccurlyeq (\widetilde{\gamma\psi})^j \gamma(A) \preccurlyeq (\widetilde{\gamma\psi})^{j+1} \gamma(A). \tag{6}$$

The idempotence of ψ is not necessary, and the condition, on γ, of individual processing is satisfied by the usual openings by convex structuring elements. For finite sets of $E = \mathbb{Z}^2$ the limit $\mu = (\widetilde{\gamma\psi})^n \gamma = (\widetilde{\gamma\psi})^{n+1} \gamma$ is reached after n steps, $n < \infty$ (see Figure 6).

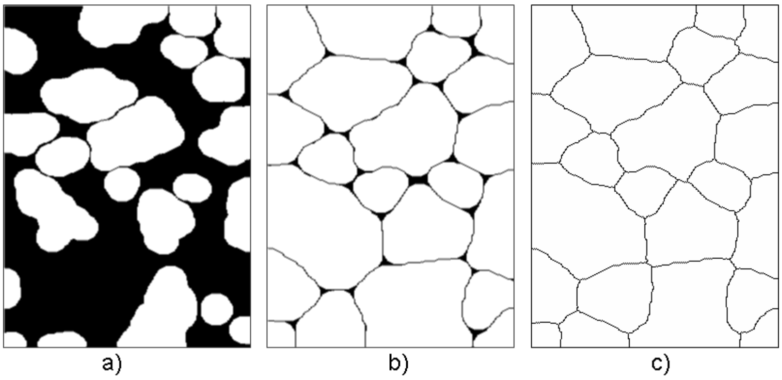

Fig. 6. The initial tesselation is that of figure 7; a) opening of the classes by a dodecagon of size 15 ; b) limit opening μ , c) limit Voronoi thickening $\psi\mu$ (and $\gamma\psi\mu = \mu$).

Tessellations, partitions and Voronoi thickenings

Tesselations In $\mathbb{R}^2$, it is convenient to distinguish between a partition and the opening of its classes. Following R. Miles, we shall call "tesselation" any set of $\mathbb{R}^2$ whose all $\mathcal{C}^*$-components but one are topologically open, the last one being a locally finite union of simple arcs. These contours are called "cleavages", and the open classes "tassels" [13]. When the cleavages class is connected, then the tassels are simply connected. The practical interest of a tessellation is that its open classes can always be handled as subsets of $\mathcal{P}(\mathbb{R}^2)$.

Voronoi thickening in $\mathbb{R}^2$ Start from the family $\mathcal{G}_0$ of all locally finite unions of disjoints open sets. Let $A = \cup \dot{A}_k \in \mathcal{G}_0 \subseteq \mathcal{P}(\mathbb{R}^2)$. The *zone of influence of* $\dot{A}_k$ is the set of all points closer to $\dot{A}_k$ than to any other $\dot{A}_p \in A, p \neq k$, and the *Voronoi thickening* of A is the union $\psi(A)$ of all zones of influence. The complement set $[\psi(A)]^c$ is a locally finite union of simple arcs [4], called skeleton by zones of influence. Therefore, the operator ψ is a $\subseteq$-thickening on $\mathcal{G}_0$ that does not create connected components, hence a $\preccurlyeq$-thickening. Consider now a grain opening $\gamma : \mathcal{G}_0 \to \mathcal{G}_0$, then Rel. (4) implies that the composition product $\psi\gamma : \mathcal{G}_0 \to \mathcal{G}_0$ is still a $\preccurlyeq$-thickening.

Although ψ is not $\preccurlyeq$-increasing in general, it becomes $\preccurlyeq$-increasing for those pairs A and $A' \in \mathcal{G}_0$, such that $A' = A \cup B$, $B \in \mathcal{G}_0$, the $\mathcal{C}^*$-components of B being disjoint from those of A.

Proposition 5. *Let $A, A', B \in \mathcal{G}_0$, with $A' = A \cup B$, and $B \cap A = \varnothing$. Then the Voronoi thickening ψ is $\preccurlyeq$-increasing, i.e.*

$$\{A' = A \cup B, \quad B \cap A = \varnothing\} \quad \Rightarrow \quad \{A' \preccurlyeq A \quad \Rightarrow \quad \psi(A') \preccurlyeq \psi(A)\}. \qquad (7)$$

Voronoi thickening in $\mathbb{Z}^2$ One cannot transpose the above approach directly to $\mathbb{Z}^2$, because the involved digital distances do not ensure that the connectivity of the seeds is preserved under growing. We must proceed by sequences of elementary operations which do maintain homotopy at each step (chap. XI-E in [10]) such as G. Bertrand's topological watersheds [2], in a binary and complemented version. Freedom is left for the succession of the elementary thickenings, so that one can well approximate the final equidistant cleavages of the Euclidean homologues. Moreover, Proposition 5 extends to $\mathcal{P}(\mathbb{Z}^2)$ when ψ is the opposite of a topological watershed.

3.2 Hierarchies of thickenings based on connected opening

Consider, in $\mathbb{R}^2$ or in $\mathbb{Z}^2$, a family $\{\gamma_j,\ j \in J\}$ of connected openings that depend on the integers $j \in J$, and the Voronoi thickening ψ. We now construct hierarchies of connected thickenings $\psi\gamma_j$. Remark firstly that $\subseteq$-decreasingness of the γ_i is equivalent to their $\preccurlyeq$-increasingness (the γ_i are connected openings, and each connected component of $\gamma_j(A)$ is also a connected component of $\gamma_i(A)$)

$$\{j \geq i \quad \Rightarrow \quad \gamma_j \subseteq \gamma_i\} \Leftrightarrow \{j \geq i \quad \Rightarrow \quad \gamma_j \succeq \gamma_i\}.$$

Hierarchies can be obtained in two ways, according as we focus on the increasingness of $j \to \gamma_j\psi$, or as we look for semi-groups. The second approach generates a more powerful structure, but requires sequences of operations.

Hierarchies of ordered operators

Proposition 6. *Let ψ be a thickening by zones of influence, and let $\{\gamma_j,\ j \in J\}$ be a $\preccurlyeq$-increasing family of connected openings, both in $\mathbb{R}^2$ or in $\mathbb{Z}^2$. Then the two thickenings $\{\psi\gamma_j,\ j \in J\}$ and $\{\gamma_j\psi,\ j \in J\}$ form two chains for the GBO:*

$$j \geq i \quad \Rightarrow \quad \psi\gamma_j \succcurlyeq \psi\gamma_i \quad \text{and} \quad \gamma_j\psi \succcurlyeq \gamma_i\psi \qquad i, j \in J.$$

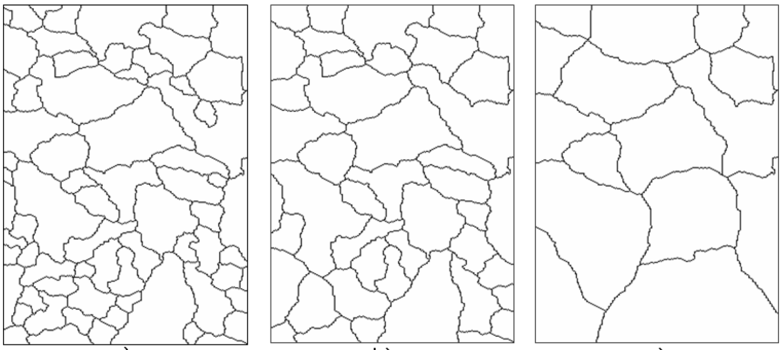

Fig. 7. a) Initial tessellation ψ, b) and c) thickening $\psi\gamma_{10}\psi\gamma_0$, and then $\psi\gamma_{20}\psi\gamma_{10}\psi\gamma_0$. The results are ordered according to both $\preccurlyeq$, and to the semi-group.

Hierarchies by semi-groups Consider the $\preccurlyeq$-connected thickening $\psi\gamma$ and let $\gamma = \gamma_j$ decrease according to $j \in J$,

$$j \geq i \quad \Rightarrow \gamma_j \leq \gamma_i, \qquad i, j \in J \ .$$

The connected components $\psi\gamma_j$ are unchanged under γ_i, and by idempotence of ψ, we obtain

$$j \geq i \quad \Rightarrow \ (\psi\gamma_i)(\psi\gamma_j) = (\psi\gamma_j).$$

Consequently, the $\{\psi\gamma_j\}$ generate, by sequential composition, the Matheron semi-group M_j:

$$M_i M_j = M_j M_i = M_j = (\psi\gamma_j)...(\psi\gamma_2)(\psi\gamma_1), \tag{8}$$

where the M_j are increasing for the GBO, since

$$j > i \quad \Rightarrow M_j = (\psi\gamma_j)...(\psi\gamma_{i+1})M_i \succ M_i.$$

Figure 7 illustrates such a progression.

Saliency and Hierarchy Unlike the hierarchies based on the refinement ordering, those on the GBO involve two saliencies for each edge, at least in the case of Voronoi reconstructions that we study here. A new edge appears either at the lowest level, or when a new grain is generated at level i. Then it does not change as long as the two grains it separates are still present in the hierarchy, and disappears permanently when one of these grains vanishes, at level $j > i$.

An example of this double saliency is depicted in Figure 8. A hierarchy has been produced by applying the semi group of operators (8) to the tessellation of Figure 7a). The pyramid is represented in a synthetic way by the two numerical

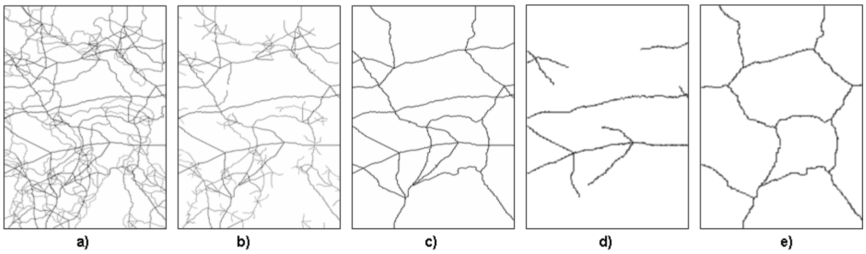

Fig. 8. a) and b), the two saliencies of a $\preceq$-hierarchy; c) and d), the respective cross sections of these saliencies at level 42; e) set diifference between c) and d), which results in partition 42 of the $\preceq$-hierarchy.

functions of Figure 8a) and b). By selecting all frontiers darker than 42 in Figure 8a) we obtain the family of those frontiers that appear before step 42 (Fig. 8c)). Similarly, the threshold of 8b) at level 42 provides all frontiers that disappeared before step 42 (Fig. 8d)). The set difference between the two sections results in the partition at level 42 in the hierarchy (Fig. 8e)).

Mixing two hierarchies Segmentation processing often leads to hierarchies where a sequence of partitions is ordered by refinement (symbol $\leq$). This occurs, for example, in maps of watersheds when one weights the edges between adjacent basins according to their flooding level. Let $\{D_i, i \in I\}$ be such a sequence of partitions, with

$$i \leq j \quad \Rightarrow \quad D_i \leq D_j \quad \Rightarrow \quad D_i \preccurlyeq D_j$$

Consider a thickening $\psi\gamma$ that $\preccurlyeq$-enlarges D_i i.e. $D_i \preccurlyeq \psi\gamma(D_i)$. As $\psi\gamma$ is not $\preccurlyeq$-

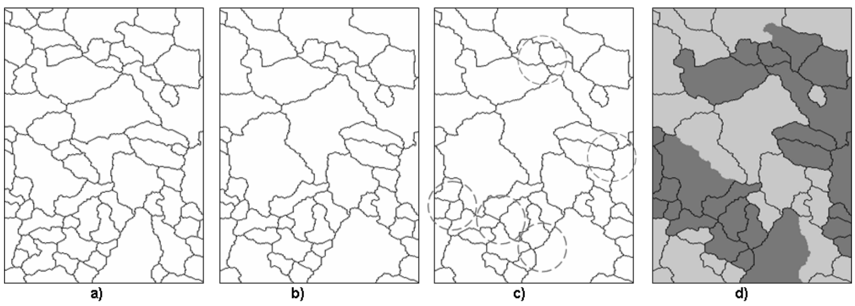

Fig. 9. a) partition D_i; b) partition $D_j \geq D_i$; c) $\psi\gamma(D_i)$, where the dotted rings indicate the places of changes; d) partition $D_j^* = D_j \sqcup \psi\gamma(D_i)$.

increasing, we cannot write $\psi\gamma(D_i) \preccurlyeq \psi\gamma(D_j)$; the hierarchical structure seems to be lost. However, the partition $\psi\gamma(D_i)$ is composed of the partial partition D'_i of all classes of D_i left unchanged under $\psi\gamma$, and of the partial partition D''_i of all the other classes of $\psi\gamma(D_i)$. Let S' and S'' be the two corresponding supports, with $S' \cup S'' = E$. Take the restriction of D_j to set S' and that of $\psi\gamma(D_i)$ to set S'', and define by D^*_j the partition of E which is obtained by the concatenation $\sqcup$ of these two partial partitions:

$$D^*_j = (D_j)_{in\ S'} \ \sqcup (\psi\gamma(D_i))_{in\ S''}.$$

The partition D^*_j is equal to $\psi\gamma(D_i)$ in S'', and $\preccurlyeq$-larger than $\psi\gamma(D_i)$ elsewhere, hence $D^*_j \succeq \psi\gamma(D_i)$. Moreover, D^*_j is invariant under $\psi\gamma$, since all its classes are invariant under γ. We can write

$$D_i \preccurlyeq \psi\gamma(D_i) \preccurlyeq D^*_j = \psi\gamma(D^*_j).$$

Suppose now that the γ's are themselves ordered, i.e. that they form the granulometry $\{\gamma_p, p \in P\}$. Then, for $p \leq q$ we can write

$$p \leq q \text{ and } \ i \leq j \quad \Rightarrow \quad D_i \preccurlyeq \psi\gamma_p(D_i) \preccurlyeq \psi\gamma_p(D^*_j) \preccurlyeq \psi\gamma_q\psi\gamma_p(D^*_j).$$

We find again the semi-goup (8).

$\preccurlyeq$-thickening a low level The GBO can also serve as a tool for filtering. Consider a level D_{i_0} in the hierarchy $\{D_i, i \in I\}$ that we want to $\preccurlyeq$-amend for reducing its small particles (e.g. Figure 10b)). One can perform some $\preccurlyeq$-thickening $\psi\gamma$, which produces the new partition $\psi\gamma(D_{i_0})$ of Figure 10c), and apply to $\psi\gamma(D_{i_0})$ the criterion which already allowed us to suppress edges in the initial hierarchy $\{D_i, i \in I\}$. Indeed, one can check by comparing Figures 10 a) and c) that most of the long previous edges are still in place, in the case of this example at least.a) Initial tessellation; b) additional noisy small classes; c) thickening $\psi\mu$ of b), by a dodecagonal opening γ of size 5.

4 Conclusion

The grain building ordering presented here, as well as the other orderings studied [8] model how partitions of the space are reorganized, and enlarged, in some physical processes. It does it in a more realistic way than the usual refinement ordering, but in compensation, it leads to less simple properties (e.g. two saliencies instead of one). In practice, it allows to eliminate small parasite classes in partitions, and also to "average" different partitions closed enough to each other (this last point, not presented above, was already developed in [12]).

Acknowledgement

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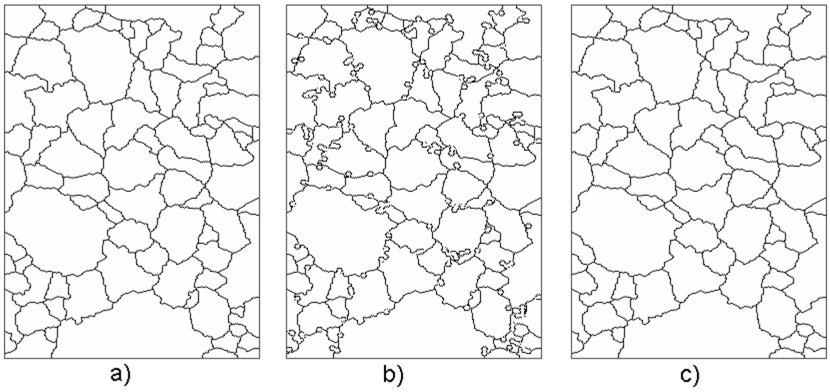

Fig. 10. a) Initial tessellation; b) additional noisy small classes; c) thickening $\psi\mu$ of b), by a dodecagonal opening γ of size 5.

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