A Unified Framework for Reactive Control of Wheeled Mobile Manipulators

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Abstract— **In this paper, we give an overview of the work we have conducted toward a unified modeling framework for wheeled mobile manipulators (WMM). Where most work in the literature often provide models, sometimes simplified, of a given type of WMM, we give an extensive description of the obtention of explicit kinematic and dynamic models of those systems. This modeling framework is particularly well suited for reactive control approaches which, in the case of mobile manipulation missions, are necessary to handle the complexity of the tasks to be fulfilled, the dynamic aspect of the extended workspace and the uncertainties on the knowledge of the environment. We thus also provide a flexible reactive framework allowing the sequencing of operational tasks whose natures are different but also an on-line switching mechanism between constraints that are to be satisfied using the system redundancy.**

I. Introduction

From manipulators executing highly specific and simple tasks in structured environments, robotics missions have evolved towards the service domain where robots are expected to explore (partially) unknown dynamic environments, interact with human beings or manipulate hazardous products. Examples of field or service robotics applications are numerous and they all involve robots whose workspace capacities have to be extended and whose control architecture and strategies must ensure good overall performances in complex missions. The complexity of those missions lies in the fact that an extension of the workspace of robots leads to a partial knowledge of the environment but also in the fact that a complex mission is often an aggregation or sequence of different types of tasks: trajectory tracking, motion in dynamic environment, contact interaction, compliant motion. To fulfil such missions, robotic systems are required to combined both locomotion and manipulation capacities and such systems are called *mobile manipulators*.

The physical aspect of mobile manipulators varies with the type of *mobile manipulation mission* that is considered. The main differences between those systems of a same family consist in the locomotion mean that is used. Apart from very specific application such as underwater applications where submarine-like locomotion is used, we can divide mobile manipulators into two large families: legged systems such as *humanoid robots* and wheeled systems. The need of humanoid-like robot is mostly associated to service missions for which the social aspect of the interaction is of great importance. To this social aspect of the interaction, one must add the need of adaptation to environments primarily designed for human beings. When those two characteristics are not present in the features of the mission to accomplish, the locomotion problem can be partially simplified by the use of wheeled mobile platforms whose design and control is less demanding. Moreover, in several cases, wheeled mobile platforms are more suitable than legged type robots whose locomotion properties are restricted especially in terms of speed and load. This paper focuses on this latter type of mobile manipulators: *wheeled mobile manipulators* (WMM), especially those evolving on flat grounds, *i.e.* whose platform are not characterised by high terrain clearance capabilities. We name WMM any system combining a wheeled mobile platform and one or several manipulators (classically arms).

Those systems have the following characteristics:

- wheeled mobile platforms, and by extension WMM, are nonholonomic systems;
- WMM are often kinematically redundant with respect to the task to be achieved;
- the mass properties, and thus the dynamic properties, of the wheeled platform and those of the manipulator(s) are very different.

To these characteristics, one has to add the constraints associated to the control of robots and robotic mission themselves:

- joint limits avoidance;
- rated input for the actuators;
- singular configurations avoidance;
- obstacle avoidance;
- tip-over of the system and skidding /slipping of wheels.

From this set of constraints and characteristics, different approaches have been developed to control WMM. A first class of approach is inherited from the control schemes that have been developed for manipulators. Those control schemes have been extended to WMM in order to account for their specificities. Among those approaches, we can distinguish the pioneer work of H. Seraji [1] who proposed an extension of kinematic based control laws to the case of a mobile manipulator equipped with a unicycle-like wheeled platform and a manipulator. The innovation of this work was to consider the system without making any explicit difference between the two subsystems and thus to offer an *implicit coordination* control scheme, extending the concept of redundancy to the whole system and not only the manipulator. If for a given problem, one might want to use a trivial explicitly coordinated control scheme of a mobile manipulator, the most generic framework to control mobile manipulators is clearly to consider an implicit coordination of the system. This framework allows to define the control problem without explicitly defining the wheeled platform trajectory thus not

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restricting the set of solutions to a very restricted subset corresponding to a specific platform trajectory. This implicit coordination approach was extended to dynamic model based control schemes by Y. YAMAMOTO and X. YUN in [2] where they showed that the dynamic coupling between the two subsystems does not require to be fully compensated to achieve good tracking performances. Implicitly coordinated approaches to the force control problem of WMM has also been tackled using the dynamic models for some specific type of WMM (see [3] for example). Finally, reactive motion has been studied for WMM in the case of omnidirectional and unicycle wheeled platform using a dynamical model. This was the framework of O. Brock *et al.* in [4]. H. TANNER *et al.* also provide in [5] some results for navigation with obstacles but from the planning point of view and considering multiple WMM coordination.

If those studies provide some major developments for the control of WMM, they do not offer a general framework for the modelling of those systems. A unification of the kinematic modelling of those system is proposed by B. BAYLE *et al.* in [6]. This work is based on the generic classification of wheeled mobile platforms presented in [7] by G. CAMPION *et al.*. M. FRUCHARD *et al.* also provide in [8] a general kinematic control approach based on the *transverse function approach* that was initially developed for wheeled mobile robots ([9]).

Regarding a generic dynamic model for WMM, no previous work has been done to propose a unified expression of this model. Of course, most of the studies of WMM based on a dynamic model provide an expression of this model in the specific case of a given robot. The goal of this paper is thus to provide an explicit formulation of the dynamic model of any kind of WMM that can be used for torque based control for reactive mission execution. We put an emphasis on the reactivity which is, given the set of constraints to satisfy at any time, the complexity of the missions and the uncertainty on the plan execution, a necessary feature of the control of WMM.

This paper is organised in four sections. We recall in the first section the basics of the generic configuration description of WMM. In the second section, we derive a generic dynamic formulation of WMM and its connections to reactive control approaches after having presented the generic kinematic modelling of those systems that was proposed in [10]. In the third section, the reactive control framework of WMM that we implemented both in simulation and experimentally, in the case of robotic missions based on tasks sequencing, is introduced. We finally conclude this study by the perspectives and needs of future work regarding the reactive control of WMM.

II. CONFIGURATION DESCRIPTION OF WHEELED MOBILE MANIPULATORS

Model based control approaches require a clear definition of the configuration parameters of the system whose models are derived. The configuration description of a mechanical system is a well known concept. However, it is of interest to take time to clarify this "simple" notion in the case of the configuration description of WMM.

Configuration of a wheeled mobile manipulator

Considering the four classical types of wheels (fixed, centered, off-centered, Swedish) and using the notations introduced in [7] (cf. figure 1), one can intuitively define the configuration of a wheeled mobile manipulator evolving on a planar surface as the configuration of the holonomic arm mounted on the mobile platform, the location of the wheeled mobile platform in the world reference frame and the configuration of each wheel with respect to the platform. For the sake of conciseness, we here focus on the one arm case but extending the following models to the multiple arms case (*i.e.* multiple operational points case) is straightforward. We then have:

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_m^t & \boldsymbol{\xi}_p^t & \boldsymbol{\varphi}^t & \boldsymbol{\beta}^t \end{bmatrix}^t,$$

where:

- $\mathbf{q}_m$ is the configuration of the manipulator mounted on the platform. Using the modified version of the D.-H. parameters ([11]), the n_m components of $\mathbf{q}_m$ can be naturally chosen as the n_m joint parameters thus leading to a minimal representation of the configuration of the manipulator;
- $\boldsymbol{\xi}_p = \begin{bmatrix} x & y & \vartheta \end{bmatrix}^t$ is the vector of location of the platform (whose size is denoted $m_p = 3$) with respect to the world reference frame $\mathcal{R} = (O, \vec{x}, \vec{y}, \vec{z})$;
- $\boldsymbol{\varphi} = \begin{bmatrix} \boldsymbol{\varphi}_f^t & \boldsymbol{\varphi}_c^t & \boldsymbol{\varphi}_{oc}^t & \boldsymbol{\varphi}_s^t \end{bmatrix}^t$ is the vector of the $N = N_f + N_c + N_{oc} + N_s$ spinning angles of the N_f fixed wheels, N_c centred wheels, N_{oc} off-centred wheels and N_s Swedish wheels respectively;
- $\boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{\beta}_{oc}^t & \boldsymbol{\beta}_c^t \end{bmatrix}^t$ is the vector of the $N_S = N_c + N_{oc}$ steering angles of the N_c centred wheels and N_{oc} off-centred wheels respectively.

The configuration of a WMM is thus defined by a vector $\mathbf{q}$ of size $n = n_m + m_p + N + N_S$. However, apart from classical revolute and prismatic joints, a WMM is subject to rolling without slipping conditions which are joint equations of a specific type. We hereafter describe those conditions.

Rolling without slipping conditions

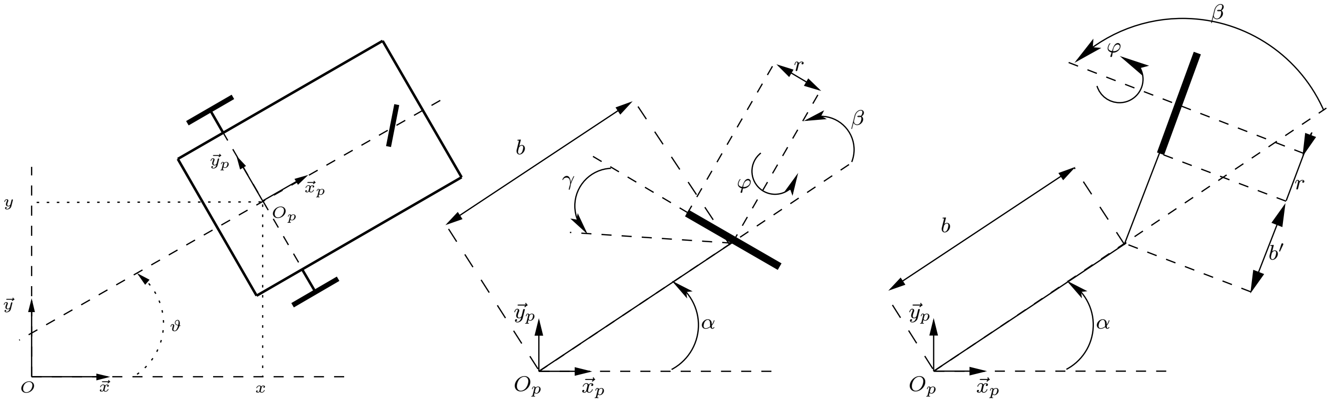
The rolling without slipping conditions of a wheel represent the fact that we consider perfect joint of the wheel with the ground, *i.e.* no slipping of the wheel in its vertical plane and no skidding in its orthogonal plane. Using the configuration description, they can respectively be written, for a given wheel:

$$\begin{bmatrix} -S_{\alpha\beta\gamma} & C_{\alpha\beta\gamma} & bC_{\beta\gamma} \end{bmatrix} R(\vartheta)^T \dot{\boldsymbol{\xi}}_p + rC_\gamma \dot{\varphi} \quad = \quad 0, \tag{1}$$

and:

$$\begin{bmatrix} C_{\alpha\beta} & S_{\alpha\beta} & b' + bS_\beta \end{bmatrix} \; R(\vartheta)^T \, \dot{\boldsymbol{\xi}}_p + b'\dot{\beta} \quad = \quad 0 \;, \tag{2}$$

where (2) does not apply to Swedish wheels, $C_{\alpha\beta}$ and $S_{\alpha\beta}$ are short notations respectively for $\cos(\alpha + \beta)$ and



a) location of a planar platform *b) parameters for fixed, centered and swedish wheels description* *c) parameters for off-centered wheel description*

Fig. 1. Configuration description of wheeled planar platforms.

$\sin(\alpha+\beta)$. $R(\vartheta)$ represents the rotation matrix of angle ϑ around $\vec{z}$.

Those joint equations are most of the time non integrable, *i.e.* nonholonomic. In the specific cases where some combinations of those equations can be integrated, they are often kept as complementary joint equations. This choice can be justified in the practical cases where pure rolling without slipping is never completely respected and thus basing the knowledge of some configuration parameters on this assumption might lead to a drift of the reconstructed (not directly measured by physical sensors) parameters.

III. UNIFIED KINEMATIC AND DYNAMIC MODELLING OF WHEELED MOBILE MANIPULATORS

Given a WMM, the task is expressed in terms of a trajectory to follow, a point to reach or a force to exert for the end-effector of the manipulator. This operational point is associated to a set of operational coordinates whose size is associated to the type of task to fulfil: constrained location, constrained position *etc.* The vector of operational coordinates of the manipulator is written $\boldsymbol{\xi}$ and its size is denoted m. The direct kinematics model of the WMM can then be written:

$$\boldsymbol{\xi} = \mathbf{f}\left(\boldsymbol{\xi}_p, \mathbf{q}_m\right), \qquad (3)$$

where $\mathbf{f}$ is a set of nonlinear functions, independently of the type of platform.

After differentiation of (3), we get:

$$\dot{\boldsymbol{\xi}} = J(\mathbf{q})\,\dot{\mathbf{q}}. \qquad (4)$$

$J(\mathbf{q})$ is the $m \times n$ Jacobian matrix of the WMM.

Equation (4) is not sufficient to fully describe the system's differential kinematics. One needs to also take the rolling without slipping joint equation of the wheels. Those equations can be written in the synthetic form:

$$\begin{bmatrix} J_{1f} \\ J_{1c}(\boldsymbol{\beta}_c) \\ J_{1d}(\boldsymbol{\beta}_d) \\ J_{1s} \end{bmatrix} R(\vartheta)^T \dot{\boldsymbol{\xi}}_p + \begin{bmatrix} J_{2f} \\ J_{2c} \\ J_{2d} \\ J_{2s} \end{bmatrix} \dot{\boldsymbol{\varphi}} = 0, \qquad (5)$$

$$\begin{bmatrix} C_{1f} \\ C_{1c}(\boldsymbol{\beta}_c) \\ C_{1d}(\boldsymbol{\beta}_d) \end{bmatrix} R(\vartheta)^T \dot{\boldsymbol{\xi}}_p + \begin{bmatrix} \mathcal{O}_{N_f \times N_d} \\ \mathcal{O}_{N_c \times N_d} \\ C_{2d} \end{bmatrix} \dot{\boldsymbol{\beta}}_d = 0. \qquad (6)$$

One can easily deduce that:

- $J_1 = \begin{bmatrix} J_{1f}^T & J_{1c}(\boldsymbol{\beta}_c)^T & J_{1oc}(\boldsymbol{\beta}_d)^T & J_{1s}^T \end{bmatrix}^T$ is a $(N \times m_p)$ matrix;
- $J_2 = \begin{bmatrix} J_{2f}^T & J_{2c}^T & J_{2oc}^T & J_{2s}^T \end{bmatrix}^T$ is a diagonal, constant, $(N \times N)$ matrix;
- $C_1 = \begin{bmatrix} C_{1f}^T & C_{1c}(\boldsymbol{\beta}_c)^T & C_{1oc}(\boldsymbol{\beta}_d)^T \end{bmatrix}^T$ is a $((N - N_s) \times m_p)$ matrix;
- and $C_2 = \begin{bmatrix} \mathcal{O} & \mathcal{O} & C_{2oc}^T \end{bmatrix}^T$ is a diagonal, constant, $((N - N_s) \times N_{oc})$ matrix;

(5) and (6) can then be compactly written as:

$$D(\mathbf{q}_p)\,\dot{\mathbf{q}} = 0. \qquad (7)$$

$D(\mathbf{q}_p)$ has $(\bar{h}+h_c)$ rows with $\bar{h}$ the number of nonholonomic constraints and h_c the number of holonomic constraints kept as complimentary joints equations (table I).

In order to be able to control the system without violating the rolling without slipping assumption, it is necessary to find the subset of $\dot{\mathbf{q}}$ such that (7) is always satisfied.

A. Mobility analysis

An analysis of matrix $D(\mathbf{q}_p)$ leads to the conclusion that for a motion of a WMM to be possible, conditions to be respected are only related to the velocity of the wheeled platform. Especially, for a possible motion of the platform to exist, the following constraint, extracted from $D(\mathbf{q}_p)$, has to be met:

$$C_{1fc}(\boldsymbol{\beta}_c)\, R(\vartheta)^T \dot{\boldsymbol{\xi}}_p = 0, \qquad (8)$$

where:

$$C_{1fc}(\boldsymbol{\beta}_c) = \begin{bmatrix} C_{1f} \\ C_{1c}(\boldsymbol{\beta}_c) \end{bmatrix}.$$

$$D(\mathbf{q}_p) = \begin{bmatrix} \mathcal{O}_{m\times m} & \mathcal{O}_{m\times m_p} & \mathcal{O}_{m\times N} & \mathcal{O}_{m\times N_{oc}} & \mathcal{O}_{m\times N_c} \\ \mathcal{O}_{N\times m} & J_1 R(\vartheta)^T & J_2 & \mathcal{O}_{N\times N_{oc}} & \mathcal{O}_{N\times N_c} \\ \mathcal{O}_{N_f\times m} & C_{1f} & \mathcal{O}_{N_f\times N} & \mathcal{O}_{N_f\times N_{oc}} & \mathcal{O}_{N_f\times N_c} \\ \mathcal{O}_{N_c\times m} & C_{1c}(\boldsymbol{\beta}_c) R(\vartheta)^T & \mathcal{O}_{N_c\times N} & \mathcal{O}_{N_c\times N_{oc}} & \mathcal{O}_{N_c\times N_c} \\ \mathcal{O}_{N_{oc}\times m} & C_{1oc}(\boldsymbol{\beta}_{oc}) R(\vartheta)^T & \mathcal{O}_{N_{oc}\times N} & C_{2oc} & \mathcal{O}_{N_{oc}\times N_c} \end{bmatrix}.$$

TABLE I

DETAILED DESCRIPTION OF MATRIX $D(\mathbf{q}_p)$.

This condition on the movement of the platform is such that $\dot{\boldsymbol{\xi}}_p$ is constrained to evolve in a subspace of the space of possible planar velocity. The size of this subspace is denoted $\delta_{mobp} \leq m_p$ and called the *degree of mobility* of the platform. δ_{mobp} is dependent on the platform type and it is shown in [7] that there are actually a limited number of wheeled platform types such that $\delta_{mobp} \neq 0$, *i.e.* such that the platform can move. Considering the set of *feasible wheeled platform type* described in [7] (i.e. wheeled platform such that $\text{rank}(C_{1fc}(\boldsymbol{\beta}_c)) < m_p$), constraint (8) requires to choose $\dot{\boldsymbol{\xi}}_p$ in the range of $\mathcal{N}(C_{1fc}(\boldsymbol{\beta}_c))$, Given, $\Sigma(\boldsymbol{\beta}_c)$ a $(m_p \times \delta_{mobp})$ matrix whose column are a basis of $\mathcal{N}(C_{1fc}(\boldsymbol{\beta}_c))$, any $\dot{\boldsymbol{\xi}}_p$ chosen as:

$$\dot{\boldsymbol{\xi}}_p = R(\vartheta)\Sigma(\boldsymbol{\beta}_c)\mathbf{u}_{mobp}, \quad (9)$$

will satisfy constraint (8). $\mathbf{u}_{mobp}$ is any vector of size δ_{mobp} and is named the mobility command of the wheeled platform. From relation (9), we can then deduce the values of the velocity $\dot{\boldsymbol{\varphi}}$ and $\dot{\boldsymbol{\beta}}_{oc}$ that will lead to the complete satisfaction of constraint (7):

$$\dot{\boldsymbol{\varphi}} = -J_2^{-1} J_1 \Sigma(\boldsymbol{\beta}_c)\mathbf{u}_{mobp}, \quad (10)$$

and:

$$\dot{\boldsymbol{\beta}}_{oc} = -C_{2oc}^{-1} C_{1d} \Sigma(\boldsymbol{\beta}_c)\mathbf{u}_{mobp}. \quad (11)$$

Constraint (7) does not give any guidance for the determination of $\dot{\boldsymbol{\beta}}_c$, but among the set of feasible wheeled platform type, one can demonstrate that the number of components of $\dot{\boldsymbol{\beta}}_c$ that can be steered independently cannot be more that 2. This number is called the degree of steerability of the wheeled platform and is denoted δ_{st}. The δ_{st} independent components of $\dot{\boldsymbol{\beta}}_c$ form the *steerability command* $\mathbf{u}_{st}$ of the wheeled platform. The steering velocity of any other centred wheel can be determined from this steerability command vector (see [12]).

Regarding the manipulator, its mobility command can be simply chosen as:

$$\mathbf{u}_{mobm} = \dot{\mathbf{q}}_m. \quad (12)$$

The sum of $\delta_{mob} + \delta_{st}$ is called the *degree of manoeuvrability* δ_{man} of the WMM. Given the restriction on the type of feasible wheeled platform, it is equal to the degree of freedom n_{dof} of the system [13]:

$$\delta_{man} = n_{dof}.$$

Considering wheeled platform with no more than two steerable wheels (any other case can be deduced from this one), the subset of $\dot{\mathbf{q}}$ satisfying (7) are thus related to the command of the WMM by the relation:

$$\dot{\mathbf{q}} = S(\mathbf{q}_p)\mathbf{u}, \quad (13)$$

where $\mathbf{u} = \begin{bmatrix} \mathbf{u}_{mobm}^t & \mathbf{u}_{mobp}^t & \mathbf{u}_{st}^t \end{bmatrix}^t$ and:

$$S(\mathbf{q}_p) = \begin{bmatrix} \mathcal{I}_m & \mathcal{O}_{m\times\delta_{mobp}} & \mathcal{O}_{m\times\delta_{st}} \\ \mathcal{O}_{m_p\times m} & R(\vartheta)\Sigma(\boldsymbol{\beta}_c) & \mathcal{O}_{m_p\times\delta_{st}} \\ \mathcal{O}_{N\times m} & -J_2^{-1}J_1\Sigma(\boldsymbol{\beta}_c) & \mathcal{O}_{N\times\delta_{st}} \\ \mathcal{O}_{N_{oc}\times m} & -C_{2oc}^{-1}C_{1oc}\Sigma(\boldsymbol{\beta}_c) & \mathcal{O}_{N_{oc}\times\delta_{st}} \\ \mathcal{O}_{\delta_{st}\times m} & \mathcal{O}_{\delta_{st}\times\delta_{mobp}} & \mathcal{I}_{\delta_{st}} \end{bmatrix}.$$

One can easily check that we have $D(\mathbf{q}_p)S(\mathbf{q}_p) = 0$ for any $\mathbf{q}$. An analysis of the number of joint equations that are actually nonholonomic can be determined by studying the span of the vector fields generated by the columns of $S(\mathbf{q}_p)$ (details can be found in [7] and examples in the case of WMM in [13]).

B. Kinematic model

From this subset of generalised velocities, we can now write the kinematic model in a form that can be used to control the system. One first needs to notice that there is no kinematic connection between $\dot{\boldsymbol{\xi}}$ and $\dot{\boldsymbol{\beta}}_c$. Thus, even if we do not deal with the problem of designing the proper steerability command for the WMM in this paper, it is of importance to notice that the full kinematic model description of a WMM is extended to the following relation:

$$\dot{\mathbf{z}} = B(\mathbf{q})\dot{\mathbf{q}}, \quad (14)$$

where:

$$\mathbf{z} = \begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\beta}_c \end{bmatrix},$$

and:

$$B(\mathbf{q}) = \begin{bmatrix} J(\mathbf{q}) & \\ \mathcal{O}_{\delta_{st}\times(n-\delta_{st})} & \mathcal{I}_{\delta_{st}} \end{bmatrix}. \quad (15)$$

Plugging (13) in (14), we get:

$$\dot{\mathbf{z}} = \bar{B}(\mathbf{q})\mathbf{u}, \quad (16)$$

where:

$$\bar{B}(\mathbf{q}) = B(\mathbf{q})S(\mathbf{q}_p), \quad (17)$$

which can also be written:

$$\bar{B}(\mathbf{q}) = \begin{bmatrix} \bar{J}(\mathbf{q}) & \mathcal{O}_{m\times\delta_{st}} \\ \mathcal{O}_{\delta_{st}\times\delta_{mob}} & \mathcal{I}_{\delta_{st}} \end{bmatrix},$$

with:

$$\begin{bmatrix} \bar{J}(\mathbf{q}) & \mathcal{O}_{m\times\delta_{st}} \end{bmatrix} = J(\mathbf{q})S(\mathbf{q}_p).$$

In the kinematic redundant case (*i.e.* $\delta_{mob} > m$), an inverse kinematic model-based controller can be used:

$$\mathbf{u} = \bar{B}(\mathbf{q})^\star \dot{\mathbf{z}} + \left(\mathcal{I}_{n_{dof}} - \bar{B}(\mathbf{q})^\star \bar{B}(\mathbf{q})\right)\mathbf{u}_0, \qquad (18)$$

with $\bar{B}(\mathbf{q})^\star$ a weighted pseudo-inverse of $\bar{B}(\mathbf{q})$, $\dot{\mathbf{z}}$ the desired operational and steering velocity and $\mathbf{u}_0$ any vector. $\bar{B}(\mathbf{q})^\star$ is given by:

$$\bar{B}(\mathbf{q})^\star = \begin{bmatrix} \bar{J}(\mathbf{q})^\star & \mathcal{O}_{\delta_{mob}\times\delta_{st}} \\ \mathcal{O}_{\delta_{st}\times m} & \mathcal{I}_{\delta_{st}} \end{bmatrix}.$$

Access to the system redundancy is given by the term $\left(\mathcal{I}_{n_{dof}} - \bar{B}(\mathbf{q})^\star \bar{B}(\mathbf{q})\right)\mathbf{u}_0$. $\mathbf{u}_0$ can be chosen using gradient descent techniques to locally minimize any *potential function* $\mathcal{P}(\mathbf{q})$ used to represent a constraint that has to be satisfied by the system at a given time. To locally minimize $\mathcal{P}(\mathbf{q})$, we have to ensure that $\dot{\mathcal{P}}(\mathbf{q}) \leq 0$. $\dot{\mathcal{P}}(\mathbf{q})$ can be expressed as:

$$\dot{\mathcal{P}}(\mathbf{q}) = \boldsymbol{\nabla}^t \mathcal{P}(\mathbf{q})\,\dot{\mathbf{q}},$$

where $\boldsymbol{\nabla}\mathcal{P}(\mathbf{q})$ is the gradient of function $\mathcal{P}(\mathbf{q})$. In order not to provide any operational effects and to be compliant with the complementary joint equations, $\dot{\mathbf{q}}$ has to be chosen as:

$$\dot{\mathbf{q}} = S(\mathbf{q}_p)\left(\mathcal{I}_{n_{dof}} - \bar{B}(\mathbf{q})^\star \bar{B}(\mathbf{q})\right)\mathbf{u}_0.$$

We thus have:

$$\dot{\mathcal{P}}(\mathbf{q}) = \boldsymbol{\nabla}^t \mathcal{P}(\mathbf{q})\, S(\mathbf{q}_p)\left(\mathcal{I}_{n_{dof}} - \bar{B}(\mathbf{q})^\star \bar{B}(\mathbf{q})\right)\mathbf{u}_0.$$

Given the idempotency property of matrix projectors such as $\left(\mathcal{I}_{n_{dof}} - \bar{B}(\mathbf{q})^\star \bar{B}(\mathbf{q})\right)$, choosing $\mathbf{u}_0$ as:

$$\mathbf{u}_0 = -K\left(\boldsymbol{\nabla}^t \mathcal{P}(\mathbf{q})\, S(\mathbf{q}_p)\left(\mathcal{I}_{n_{ddl}} - \bar{B}(\mathbf{q})^\sharp \bar{B}(\mathbf{q})\right)\right)^t,$$

where K is a positive definite matrix, leads to $\dot{\mathcal{P}}(\mathbf{q}) \leq 0$. One has to notice that when the operational task to be achieved and the potential function to be optimized are not compatible, the projection of $\mathbf{u}_0$ into $\left(\mathcal{I}_{n_{dof}} - \bar{B}(\mathbf{q})^\star \bar{B}(\mathbf{q})\right)$ will lead to 0. The operational task has a greater priority than the optimisation of the potential function. The priority concept can be extended to multiple potential functions optimized at the same time. The basics of this prioritisation technique can be found in [14]. In the case of WMM, the redundancy degree of the system is not high enough to efficiently optimize multiple potential functions at the same time. However those techniques make a lot of sense with highly redundant robots such as humanoids. In that specific case, the constraints associated to the potential functions can be highly critical and the operational task can be given a priority lower than the priority of the potential functions.

C. Dynamic model

When interaction forces are involved and/or compliant behaviour is desired, the use of a dynamic model of the system is required in order to correctly compensate for the natural dynamic of the system. The implicit and explicit formulation of the dynamics of manipulators is a widely treated subject. Regarding WMM, some papers propose a dynamic modelling in the specific case of a given system (see for example [15] or [3]) and H. TANNER *et al.* in [16] propose a formulation of the dynamic model of the WMM using KANE's approach. However, this work does not give access to a formulation that can be directly used for control purposes. We propose here to derive such an explicit dynamic model based on LAGRANGE formulation. We only partially cover the topic of the dynamic modelling of nonholonomic systems and [17] (among others) can be used as a complete reference book.

The LAGRANGE formulation of the dynamic of a mechanical system can be summarised by the following formula:

$$\frac{d}{dt}\left(\frac{\partial T(\mathbf{q},\dot{\mathbf{q}},t)}{\partial \dot{q}_i}\right) - \left(\frac{\partial T(\mathbf{q},\dot{\mathbf{q}},t)}{\partial q_i}\right) = Q_i, \quad (1 \leq i \leq n), \qquad (19)$$

or:

$$P_i = Q_i, \quad (1 \leq i \leq n),$$

with $P_i = \frac{d}{dt}\left(\frac{\partial T(\mathbf{q},\dot{\mathbf{q}},t)}{\partial \dot{q}_i}\right) - \left(\frac{\partial T(\mathbf{q},\dot{\mathbf{q}},t)}{\partial q_i}\right)$.

This results from the expression of the virtual power principle, expressed in terms of the kinetic energy $T(\mathbf{q},\dot{\mathbf{q}},t)$ of the system. Briefly, we can say that:

- P_i is the force applied to the system which is due to its motion and which is associated to coordinate q_i;
- Q_i is the sum of any other force acting on the system and associated to coordinate q_i.

However relation (19) does not account for the complementary joint equations and any virtual velocity $\dot{\mathbf{q}}^\star$ compatible with the complementary joint equations is such that:

$$D(\mathbf{q})\,\dot{\mathbf{q}}^\star = 0. \qquad (20)$$

The virtual power principle also leads to the following results:

$$(P - Q)^t\,\dot{\mathbf{q}}^\star = 0. \qquad (21)$$

Relations (20) and (21) leads to the conclusion that $\dot{\mathbf{q}}^\star$ has to be orthogonal to the $\bar{h} + h_c$ rows of $D(\mathbf{q})$ and also orthogonal to vector $(P - Q)$. The latter can thus be written as a linear combination of $D(\mathbf{q})$ rows:

$$P_i - Q_i = \sum_{j=1}^{\bar{h}+h_c} \lambda_j D_{ji}, (1 \leq i \leq n),$$

where λ_j are $\bar{h} + h_c$ scalars (historically known as LAGRANGE *multipliers).*

A complete dynamic description of the system is thus given by:

$$P_i = Q_i + \sum_{j=1}^{\bar{h}+h_c} \lambda_j D_{ji}, \quad (1 \leq i \leq n).$$

Computing the kinetic energy of the system and computing the components of P, the dynamic equation of a WMM can always be written in the form:

$$A(\mathbf{q})\,\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} = Q + D(\mathbf{q})^t\,\boldsymbol{\lambda}. \qquad (22)$$

$A(\mathbf{q})$ is the $n \times n$ symmetric, positive definite, inertia matrix of the system. $C(\mathbf{q}, \dot{\mathbf{q}})$ is the $n \times n$ CORIOLIS and centrifugal matrix of the system. Q can be written as the sum of four terms $Q = Q_a + Q_g + Q_d + Q_{ext}$, where:

- $Q_a = M\mathbf{\Gamma}$, the actuation forces acting on the system. We do not provide here any analysis of the actuation of WMM (see [7]) but assuming a non under-actuated system with n_a actuators, $\mathbf{\Gamma}$ is the actual vector of actuation torques (actually force or torque) and M is a $(n \times n_a)$ matrix such that $M_{ik} = 1$ if coordinate q_i is directly actuated by actuator k and $M_{ik} = 0$ otherwise;
- $Q_g = -\mathbf{g}(\mathbf{q})$, the vector of gravity forces acting on the system;
- Q_d, the vector of force disturbances due to friction-like effects in the mechanism;
- Q_{ext}, the vector of external interaction forces.

Regarding Q_{ext}, the location $\boldsymbol{\xi}_{c_l}$ of the interaction/contact on the WMM can be related to the configuration by a geometrical relation:

$$\boldsymbol{\xi}_{c_l} = \mathbf{g}_{c_l}(\mathbf{q}),$$

that can be differentiated leading to a kinematic relation of the type:

$$\dot{\boldsymbol{\xi}}_{c_l} = J_{c_l}(\mathbf{q})\,\dot{\mathbf{q}}.$$

The kineto-static principle of equivalence allows to compute Q_{ext}, the equivalent generalised force associated to this contact force at $\boldsymbol{\xi}_{c_l}$, as a function of the contact forces $\mathbf{f}_{c_l}$. We thus have:

$$Q_{ext} = -J_{c_l}(\mathbf{q})^t\,\mathbf{f}_{c_l}.$$

We do not make here any assumption about the compatibility between the contact force applied at $\boldsymbol{\xi}_c$ and the motion of the controlled system. We rather focus on the specific case where this contact interaction occurs at the end-effector of the WMM but also assume some resistive torque for the steering axes of the steerable wheels. This latter assumption is realistic when the contact point of the wheels with the ground is actually more a line than a point thus leading to some amplified resistive friction effects of the ground on the wheels with respect to the steering axis. In that specific case, we have $J_c(\mathbf{q}) = B(\mathbf{q})$ and thus:

$$Q_{ext} = -B(\mathbf{q})^t\,\mathbf{f}_c.$$

From here, the dynamic equation of motion can be written:

$$A(\mathbf{q})\,\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\,\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + B(\mathbf{q})^t\,\mathbf{f}_c = M\mathbf{\Gamma} + D(\mathbf{q})^t\,\boldsymbol{\lambda}. \tag{23}$$

$D(\mathbf{q})^t\,\boldsymbol{\lambda}$ is actually the projection of the reaction forces at each joint whose equation has been chosen as a complementary joint equation. Assuming that only the rolling without slipping joint equation have been kept as complementary joint equations, $\boldsymbol{\lambda}$ precisely represents the vector of the tangential reaction forces (collinear with plane $(O, \vec{x}, \vec{y})$) of the wheels on the ground. If the non slipping assumption is not realistic (that can be the case in outdoor, non-structured ground applications) and using a friction model, one can lead a study of the slipping/skidding conditions for the system based on (23). In some applications, this assumption is valid and one may want to restrict the study to those motions that do not violate the constraints. To do so, one can eliminate the term $D(\mathbf{q})^t\,\boldsymbol{\lambda}$ in (23) by multiplying both sides by $S(\mathbf{q})^t$. This leads to:

$$\begin{gathered} S(\mathbf{q})^t A(\mathbf{q})\,\ddot{\mathbf{q}} + S(\mathbf{q})^t C(\mathbf{q}, \dot{\mathbf{q}})\,\dot{\mathbf{q}} + \\ S(\mathbf{q})^t\,\mathbf{g}(\mathbf{q}) + S(\mathbf{q})^t B(\mathbf{q})^t\,\mathbf{f}_c = S(\mathbf{q})^t M\mathbf{\Gamma}. \end{gathered} \tag{24}$$

The rolling without slipping conditions are supposed to be respected and thus $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are restricted to:

$$\dot{\mathbf{q}} = S(\mathbf{q})\,\mathbf{u} \text{ and } \ddot{\mathbf{q}} = \dot{S}(\mathbf{q})\,\mathbf{u} + S(\mathbf{q})\,\dot{\mathbf{u}}.$$

Plugging into (24), we get the *reduced dynamic model* of the WMM or *constraints consistent dynamic model*:

$$\bar{A}(\mathbf{q})\,\dot{\mathbf{u}} + \bar{C}(\mathbf{q}, \dot{\mathbf{q}})\,\mathbf{u} + \bar{\mathbf{g}}(\mathbf{q}) + \bar{B}(\mathbf{q})^t\,\mathbf{f}_c = \bar{\mathbf{\Gamma}}, \tag{25}$$

where:

- $\bar{A}(\mathbf{q})$ is the $n_{dof} \times n_{dof}$, symmetric, positive definite, constraint consistent inertia matrix of the system. It can be decomposed as:
$$\bar{A}(\mathbf{q}) = \begin{bmatrix} A_m(\mathbf{q}_m) & \bar{A}_{p/m}(\mathbf{q}) \\ \bar{A}_{p/m}(\mathbf{q})^t & \bar{A}_p(\mathbf{q}) + \bar{A}_{mp}(\mathbf{q}) \end{bmatrix},$$
with:
  - $A_m(\mathbf{q}_m)$ the $n_m \times n_m$ inertia matrix of the manipulator;
  - $\bar{A}_p(\mathbf{q})$ the $(n_{dof} - n_m) \times (n_{dof} - n_m)$ constraint consistent inertia matrix of the wheeled platform without accounting for the presence of the manipulator;
  - $\bar{A}_{mp}(\mathbf{q})$ the $(n_{dof} - n_m) \times (n_{dof} - n_m)$ constraint consistent inertia matrix term representing the influence of the presence of the manipulator on the wheeled platform;
  - $\bar{A}_{p/m}(\mathbf{q})$ the $n_m \times (n_{dof} - n_m)$ constraint consistent inertia matrix representing the influence of the platform acceleration on the manipulator inertial properties.
- $\bar{C}(\mathbf{q}, \dot{\mathbf{q}})$ is the $n_{dof} \times n_{dof}$ constraint consistent matrix of the Coriolis and centrifugal effects of the system which can also be decomposed in terms of manipulator, platform and mutual effects;
- $\bar{\mathbf{\Gamma}} = \bar{M}\mathbf{\Gamma}$, with $\bar{M} = S(\mathbf{q})^t M$.

Given any $(\mathbf{u}, \dot{\mathbf{u}})$ computed from (18) and any consistent $\mathbf{f}_c$ (consistent with $\dot{\boldsymbol{\xi}}$ in terms of controlled operational direction), one can compute the needed torque $\mathbf{\Gamma}_{op}$ to fully compensate for the dynamic effects of the desired motion of the system:

$$\bar{\mathbf{\Gamma}}_{op} = \bar{A}(\mathbf{q})\,\dot{\mathbf{u}} + \bar{C}(\mathbf{q}, \dot{\mathbf{q}})\,\mathbf{u} + \bar{\mathbf{g}}(\mathbf{q}_b) + \bar{B}(\mathbf{q})^t\,\mathbf{f}_c,$$

and:

$$\mathbf{\Gamma}_{op} = \bar{M}^{\star}\left(\bar{A}(\mathbf{q})\,\dot{\mathbf{u}} + \bar{C}(\mathbf{q}, \dot{\mathbf{q}})\,\mathbf{u} + \bar{\mathbf{g}}(\mathbf{q}_b) + \bar{B}(\mathbf{q})^t\,\mathbf{f}_c\right). \tag{26}$$

Assuming that the system is not under-actuated, (26) leads to an actuation torque that perfectly compensate for the dynamic effects of the system providing that the weighting matrix for the weighted pseudo-inverse of matrix $\bar{B}(\mathbf{q})$ is chosen as the inertia matrix $\bar{A}$ of the system (see [18] and [13] for a proof in the case of any type of WMM). In the case of force-based potential functions to locally optimize, access to the redundancy is also given in the torque space:

$$\bar{\mathbf{\Gamma}} = \bar{\mathbf{\Gamma}}_{op} + \left(\mathcal{I}_{n_{dof}} - \bar{B}(\mathbf{q})^t \, \bar{B}(\mathbf{q})^{t[1,2]}\right) \bar{\mathbf{\Gamma}}_0.$$

$\bar{B}(\mathbf{q})^{t[1,2]}$ (see [19] for a complete definition of $[1,2]-$generalised inverses) actually has to be chosen as an inertia matrix weighted pseudo-inverse in order to fully compensate for the system dynamic.

This framework can be extended to the operational space formulation of O. KHATIB that was primarily developed for manipulators. Details of this extension are given in [13].

IV. Reactive control for wheeled mobile manipulators

Using model-based actuation torque or velocity computation as presented in section III, we have developed a framework allowing a reactive execution of complex mobile manipulation missions based on the sequencing of operational space tasks for the WMM. This was validated in simulation using a *Matlab/Simulink* simulator developed to simulate the dynamics of the WMM, test different control laws and implement our framework for reactive control. We also implemented this framework using the $G^{en}om$ control architecture of the LAAS Laboratory in Toulouse (France) on a real robot: H_2Bis + GT6a (see[20],[21]).

A. Framework description

We consider missions based on the sequencing of operational tasks whose nature can be different: free-space motion, constrained motion, force interaction. A mission can then be represented as a state machine where each state or mode represents a specific operational task. The possible transitions between states are described and transition conditions are mainly based on sensors information. This allows to describe a large number of missions.

Regarding the operational trajectories, they can either be pre-planned or generated on-line. The actuators' inputs are then computed based either on inverse kinematics or on both inverse kinematics and dynamics. In both cases, the computed input can be written:

$$\mathbf{c} = \underbrace{\mathbf{c}_{op}}_{\text{operational task}} + \underbrace{\left(\mathcal{I}_{n_{dof}} - E(\mathbf{q})^\star E(\mathbf{q})\right)\mathbf{c}_0}_{\mathbf{c}_{red}\,:\,\text{redundancy}},$$

where, in the kinematic case, $\mathbf{c} = \mathbf{u}$, $\mathbf{c}_{op} = \bar{B}(\mathbf{q})^\star \dot{\mathbf{z}}$, $E(\mathbf{q}) = \bar{B}(\mathbf{q})$ and $\mathbf{c}_0 = \mathbf{u}_0$. In the torque formulation (dynamic or even static when neglecting inertial effects), we have $\mathbf{c} = \bar{\mathbf{\Gamma}}$, $\mathbf{c}_{op} = \bar{\mathbf{\Gamma}}_{op}$, $E(\mathbf{q}) = \bar{B}(\mathbf{q})^{t\star}$ and $\mathbf{c}_0 = \bar{\mathbf{\Gamma}}_0$.

B. Actuator input

Considering the case where redundancy is formulated in the kinematic framework, rated input actuators (in terms of velocity and acceleration) have to be respected. To do so, we first have to ensure that the operational task term is such that:

$$|\mathbf{c}_{op}| \leq \mathbf{c}_{max}, \tag{27}$$

where $\mathbf{c}_{max}$ is the vector of maximum velocities for the actuators of the system. This constraint can be easily met when designing the input trajectories. We then have to find the largest positive value of a real α_s such that we have:

$$|\mathbf{c}_{op} + \alpha_s \mathbf{c}_{red}| \leq \mathbf{c}_{max}.$$

This allows to scale down (or eventually up) the redundancy term in order the constraint on maximum velocities to be respected. It also keeps the direction of descent of the gradient method used to compute $\mathbf{c}_0$, when it is chosen as a potential function to locally optimize. α_s can be computed as:

$$min\left(\left(\frac{|c_{max,i}.sign(c_{red,i}) - c_{op,i}|}{|c_{red,i}|}\right)_{i=1\ldots n_{dof}}, \alpha_{s_{max}}\right).$$

This lead to a value of α_s between 0 and $\alpha_{s_{max}}$. $\alpha_{s_{max}}$ is an upper limit used not to over amplify the redundancy term. It can for example be chosen using *Armijo*'s rule. A similar reasoning process can be led to respect the constraints on maximum acceleration. However in that case, it is also important to choose $\mathcal{C}^2$-class potential functions so that $\boldsymbol{\nabla}\mathcal{P}_i(\mathbf{q})$ and $\boldsymbol{\nabla}\dot{\mathcal{P}}_i(\mathbf{q})$ are continuous functions.

C. Active constraint

The criticalities of the constraints listed in Introduction can be different given the type of task to be executed. Moreover, those constraints are not always active. For example, the obstacle avoidance constraint has to be respected at any time. However, it is not be necessary to take it into account when far from obstacles. The same reasoning can be applied to any type of constraint and we thus designed a deterministic arbitration mechanism that at each time evaluates the criticality, with respect to the current executed task, of each potential function associated to the constraints and chooses which constraint is of greater importance at a given time. To avoid fast switching between two antagonist constraints, activation and deactivation thresholds for a given constraint can be chosen differently. Reactive and smooth (in terms of continuity of the control signals) switches between the constraints and the associated function to locally optimize using redundancy are then accordingly operated. A simple way to obtain smooth transition is to design a switching function $\beta_s(t)$ linearly varying between 0 and 1, 0 being the value of the function when the transition starts and 1 its value when the transition ends. The transition potential function is thus respectively given by:

$$\mathcal{P}(t) = \begin{cases} \mathcal{P}_{old}(t) \\ \mathcal{P}_{new}(t)\beta(t)^2 + \mathcal{P}_{old}(t)\left(1-\beta(t)^2\right) \\ \mathcal{P}_{new}(t) \end{cases} .$$

In order to ensure an efficient optimisation of the potential function associated to a given constraint, one has to make sure that the ratio $\frac{\mathbf{c}_{red}}{\mathbf{c}_0}$ is not too small and that α_s is not to small. The first case corresponds to a situation where the operational task to be achieved and the potential function to be optimized are not compatible. In that case, O. BROCK *et al.* propose in [4] to release some operational constraints momentarily in order to respect the constraints associated to the potential function. The second case occurs when the operational trajectories are very demanding in terms of actuation in which case the scaling of the redundancy term can lead to a drastic reduction of the efficiency of the optimized function. We use a threshold based mechanism (threshold on α_s value) leading to the modification of the operational trajectories in terms of speed of execution. This allows to redistribute the actuation capacities of the system. In case of pre-planned trajectories, no complete re-planning is necessary and a simple adaptation of the original plan is sufficient. When trajectories are generated on-line, they are filtered using a low pass filter that is designed with varying parameters so that the constraints on the actuators inputs are met and so that the actuation power of the system can be redistributed. The direction of the operational movement is kept identical, only the speed of execution is reduced.

V. CONCLUSION

In this paper we have presented a unified framework for the reactive control of wheeled mobile manipulators dedicated to the execution of complex robotic missions composed of different types of operational tasks that are dynamically sequenced.

This framework is mainly based on model-based control strategies and relies on the generic kinematic and dynamic modeling of WMM. These models give access to the redundancy of the system. This redundancy allows, using local optimisation techniques, the satisfaction of the internal and external constraints associated to the control of such systems. Arbitration and switching between critical constraints, scaling of the actuators' inputs and operational trajectories offer the required flexibility in the realisation of a complex robotic mission.

Further developments will be conducted especially to fully integrate and automate the design of the steering wheels' control within the proposed framework. In the case of outdoor applications, the rolling without slipping assumption may be too strong and we will also try to extend this framework to the case where a friction model of the couple wheel/ground might be useful in order to correctly model the dynamic behaviour of the WMM and thus correctly compensate for it.

Web resources: Movies of the simulation and experimental results can be found at http://www.stanford.edu/~vpadois/

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