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# Multivariate MA( $\infty$ ) processes with heavy tails and random coefficients 

SHUYAN LIU ${ }^{1}$ and JOHAN SEGERS ${ }^{2}$


#### Abstract

Many interesting processes share the property of multivariate regular variation. This property is equivalent to existence of the tail process introduced by B. Basrak and J. Segers [1] to describe the asymptotic behavior for the extreme values of a regularly varying time series. We apply this theory to multivariate MA( $\infty$ ) processes with random coefficients.


Key words : extremes, heavy tails, regular variation, tail process.

## 1 Introduction

We interested in multivariate infinite-order moving average process with random coefficient matrices, defined for $t \in \mathbb{Z}$ by

$$
\begin{equation*}
X_{t}=\sum_{i=0}^{\infty} C_{i}(t) \xi_{t-i} \tag{1.1}
\end{equation*}
$$

where $\left(\xi_{t}\right)_{t \in \mathbb{Z}}$ is a sequence of i.i.d. regularly varying random vector in $\mathbb{R}^{q}$ and $\left\{C_{i}(t), i \geq\right.$ $0, t \in \mathbb{Z}\}$ is an array of random $d \times q$ matrices. The tail behavior of such process is usually controlled by a moment condition on the matrix $C_{i}(t)$, this has been shown by Hult and Samorodnitsky 4. We adapt the conditions as follows.

[^0]Condition 1.1 Suppose that $\mathbf{P}\left(\bigcap_{i \geq 0}\left\{\left\|C_{i}(0)\right\|=0\right\}\right)=0$, and there is some $\varepsilon \in(0, \alpha)$ such that

$$
\begin{gather*}
\sum_{i} \mathbf{E}\left\|C_{i}(0)\right\|^{\alpha-\varepsilon}<\infty \text { and } \sum_{i} \mathbf{E}\left\|C_{i}(0)\right\|^{\alpha+\varepsilon}<\infty, \quad \text { if } \alpha \in(0,1) \cup(1,2) ;  \tag{1.2}\\
\mathbf{E}\left(\sum_{i}\left\|C_{i}(0)\right\|^{\alpha-\varepsilon}\right)^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}}<\infty, \quad \text { if } \alpha \in\{1,2\} ;  \tag{1.3}\\
\mathbf{E}\left(\sum_{i}\left\|C_{i}(0)\right\|^{2}\right)^{\frac{\alpha+\varepsilon}{2}}<\infty, \quad \text { if } \alpha \in(2, \infty) ; \tag{1.4}
\end{gather*}
$$

To apply the results in Hult and Samorodnitsky [4, a 'predictability' assumption is required.

Condition 1.2 Suppose there is a filtration $\left(\mathcal{F}_{j}, j \in \mathbb{Z}\right)$ such that for all $t \in \mathbb{Z}$ and $i \in \mathbb{N} \cup\{0\}$

$$
\begin{gather*}
C_{i}(t) \in \mathcal{F}_{i-t}, \quad \xi_{t-i} \in \mathcal{F}_{i-t+1}, \quad\left(\text { or } \xi_{j} \in \mathcal{F}_{1-j}\right),  \tag{1.5}\\
\mathcal{F}_{j} \text { is independent of } \sigma\left(\xi_{-j}, \xi_{-j-1}, \ldots\right) . \tag{1.6}
\end{gather*}
$$

If the sequence $\left(C_{i}(t)\right)$ is independent of the sequence $\left(\xi_{i}\right)$, we can set $\mathcal{F}_{j}=\sigma\left(\left(C_{k}(t)\right)_{k \geq 0, t \in \mathbb{Z}},\left(\xi_{k}\right)_{k \geq 1-j}\right)$.

Throughout the paper we will use the following notations, $\mathbb{E}^{q}=[-\infty, \infty]^{q} \backslash\{0\}$, $\mathbb{E}_{u}^{q}=\left\{x \in \mathbb{E}^{q} \mid\|x\|>u\right\}$ and $S^{q-1}=\left\{x \in \mathbb{R}^{d} \mid\|x\|=1\right\}$. A function $f$ is called regularly varying at infinity with index $\tau \in \mathbb{R}$, if $\lim _{x \rightarrow \infty} f(u x) / f(x)=u^{\tau}$ for all $u>0$.

Definition 1.1. A random vector $X$ in $\mathbb{R}^{d}$ is regularly varying with index $\alpha>0$ if there exists a non-null Radon measure $\mu$ on $\mathbb{E}^{d}$ and a regularly varying function $V$ of index $-\alpha$ such that, as $x \rightarrow \infty$,

$$
\begin{equation*}
\frac{\mathbf{P}\left\{x^{-1} X \in \cdot\right\}}{V(x)} \xrightarrow{v} \mu(\cdot), \tag{1.7}
\end{equation*}
$$

where $\xrightarrow{v}$ denote the vague convergence of measures.
Relation 1.7) is equivalent to

$$
\frac{\mathbf{P}\left\{\|X\|>u x, x^{-1} X \in \cdot\right\}}{\mathbf{P}\{\|X\|>x\}} \xrightarrow{v} u^{-\alpha} \mathbf{P}\{\Theta \in \cdot\},
$$

as $x \rightarrow \infty$. The law of $\Theta$ is called spectral measure.
The random sequence $\left(X_{t}\right)$ given by 1.1 is built from two components which satisfy the following basic assumptions.

A1 The $q$-dimensional random vector $\xi_{t}$ is regularly varying of index $\alpha \in(0, \infty)$ and with spectral measure $\mathcal{L}(\Theta)$ on $S^{q-1}$, i.e. there exists a non-null Radon measure $\mu$ on $\mathbb{E}^{q}$ such that, as $x \rightarrow \infty$,

$$
\frac{\mathbf{P}\left\{x^{-1} \xi_{0} \in \cdot\right\}}{\mathbf{P}\left\{\left\|\xi_{0}\right\|>x\right\}} \xrightarrow{v} \mu(\cdot),
$$

where the measure $\mu$ has the following representation, for all $u>0$,

$$
\mu\left\{x \mid\|x\|>u, \frac{x}{\|x\|} \in \cdot\right\}=u^{-\alpha} \mathbf{P}\{\Theta \in \cdot\} .
$$

A2 The array of random matrices $\left\{C_{i}(t), i \geq 0, t \geq \mathbb{Z}\right\}$ of dimension $d \times q$ is independent of the sequence $\left(\xi_{t}\right)$ and stationary when index over $t \in \mathbb{Z}$.

## 2 Joint regular variation

The following theorem is adapted version of Theorem 3.1 in [4].
Theorem 2.1 [4] Suppose that Condition 1.1 and 1.2 hold, then the series $X_{t}$ given by (1.1) converges a.s. and

$$
\begin{equation*}
\frac{\mathbf{P}\left\{x^{-1} X_{0} \in \cdot\right\}}{\mathbf{P}\left\{\left\|\xi_{0}\right\|>x\right\}} \xrightarrow{v} \sum_{i=0}^{\infty} \mathbf{E}\left[\mu \circ C_{i}(0)^{-1}(\cdot)\right], \tag{2.8}
\end{equation*}
$$

as $x \rightarrow \infty$ on $\mathbb{E}^{q}$.
Observe that by assumption A1, the function $u \mapsto \mathbf{P}\left\{\left\|\xi_{0}\right\|>u\right\}$ is regularly varying with index $-\alpha<0$. Therefore if the limit on the right-hand side in (2.8) is a non-null Radon measure, then vector $X_{0}$ is regularly varying with index $\alpha$. As we shall see slightly stronger condition is needed to ensure regular variation of vector $X_{0}$. Consider for $u>0$

$$
\begin{aligned}
\mathbf{E}\left[\mu \circ C_{i}(0)^{-1}\left(\mathbb{E}_{u}^{d}\right)\right] & =\mathbf{E}\left[\mu\left\{x \mid\left\|C_{i}(0) x\right\|>u\right\}\right] \\
& =\mathbf{E} \iint \mathbb{I}_{\left\{\left\|C_{i}(0) r \theta\right\|>u\right\}} d\left(-r^{-\alpha}\right) \mathbf{P}_{\Theta}(d \theta) \\
& =u^{-\alpha} \mathbf{E}\left\|C_{i}(0) \Theta\right\|^{\alpha} .
\end{aligned}
$$

Thus if

$$
\begin{equation*}
0<\sum_{i=0}^{\infty} \mathbf{E}\left\|C_{i}(0) \Theta\right\|^{\alpha}<\infty \tag{2.9}
\end{equation*}
$$

the random vector $X_{0}$ is regularly varying.
Let us define a random vector of length $(t-s+1) d$ for $s, t \in \mathbb{Z}$ with $s \leq t$

$$
\tilde{X}(s, t)=\left(X_{s}, \ldots, X_{t}\right)^{\prime} .
$$

By the definition of the series $X_{t}$ (1.1), we have

$$
\begin{equation*}
\tilde{X}(s, t)=\sum_{i=0}^{\infty} \tilde{C}_{i}(s, t) \xi_{t-i}, \tag{2.10}
\end{equation*}
$$

where $\left\{\tilde{C}_{i}(s, t)\right\}$ is an array of random matrices of dimension $(t-s+1) d \times q$ defined by

$$
\begin{equation*}
\tilde{C}_{i}(s, t)=\left(C_{i-t+s}(s), C_{i-t+s+1}(s+1), \ldots, C_{i-1}(t-1), C_{i}(t)\right)^{\prime}, \quad i \geq 0 \tag{2.11}
\end{equation*}
$$

where $C_{i}(\cdot)=0$, if $i<0$. By condition 1.5 and the definition of the sequence $\left\{\tilde{C}_{i}(s, t)\right\}$ (2.11), we have

$$
\tilde{C}_{i}(s, t) \in \mathcal{F}_{i-t}, \text { for all } s, t \in \mathbb{Z}, i \in \mathbb{N} \cup\{0\}
$$

Hence Condition 1.2 holds for the sequence $\left\{\tilde{C}_{i}(s, t)\right\}$ and $\left(\xi_{t}\right)$. In order to obtain the convergence of type 2.8 ) for the series $\tilde{X}(s, t)$ defined by 2.10 , it is sufficient to show that the array $\left\{\tilde{C}_{i}(s, t)\right\}$ satisfy Condition 1.1 . For this, the following lemma is needed.

Lemma 2.2 Let $A=\left(A_{1}, \ldots, A_{n}\right)^{\prime}$ be a block matrix of dimension $n d \times q$ with $n$ blocks, $A_{i}$ be a $d \times q$ matrix with entries in $\mathbb{R}, i=1, \ldots, n$. Then

$$
\begin{equation*}
\|A\| \leq \sum_{i=1}^{n}\left\|A_{i}\right\|, \tag{2.12}
\end{equation*}
$$

where $\|\cdot\|$ denote the matrix norm.
Proof. Without loss of generality, we consider the vector norm $\left\|\left(x_{1}, \ldots, x_{d}\right)^{\prime}\right\|_{d}=$ $\max _{1 \leq i \leq d}\left\{\left\|x_{i}\right\|\right\}$ on $\mathbb{R}^{d}$. We may decompose the matrix $A$ as

$$
\begin{equation*}
A=\sum_{i=1}^{n} B_{i} \tag{2.13}
\end{equation*}
$$

with $B_{i}$ is a $n d \times q$ matrix defined by

$$
B_{i}=\left(0, \ldots, A_{i}, \ldots, 0\right)^{\prime}
$$

where $A_{i}$ is in the $i$ th position. We have, for all $x \in \mathbb{R}^{d}$

$$
\left\|B_{i} x\right\|_{n d}=\left\|\left(0, \ldots, A_{i} x, \ldots, 0\right)^{\prime}\right\|_{n d}=\left\|A_{i} x\right\|_{d}, \quad i=1, \ldots, n
$$

Therefore

$$
\begin{equation*}
\left\|B_{i}\right\|=\sup _{\|x\|_{q} \leq 1}\left\{\left\|B_{i} x\right\|_{n d}\right\}=\sup _{\|x\|_{q} \leq 1}\left\{\left\|A_{i} x\right\|_{d}\right\}=\left\|A_{i}\right\| . \tag{2.14}
\end{equation*}
$$

Inequality (2.12) follows from (2.13) and (2.14).
By Lemma 2.2 and the definition of the sequence $\left\{\tilde{C}_{i}(s, t)\right\}$ 2.11, we obtain

$$
\begin{equation*}
\left\|\tilde{C}_{i}(s, t)\right\| \leq \sum_{j=(i-t+s) \vee 0}^{i}\left\|C_{j}(t-i+j)\right\| . \tag{2.15}
\end{equation*}
$$

Using (2.15) and the following inequality, for $a_{i}>0, i=1, \ldots, n$,

$$
\begin{equation*}
\left(\sum_{i=1}^{n} a_{i}\right)^{p} \leq c \sum_{i=1}^{n} a_{i}^{p}, \tag{2.16}
\end{equation*}
$$

where $c=1$ if $p \leq 1, c=n^{p-1}$ if $p>1$, we obtain, if $\alpha \in(0,1) \cup(1,2)$, for some $0<\varepsilon<\alpha$,

$$
\begin{align*}
\sum_{i=0}^{\infty} \mathbf{E}\left\|\tilde{C}_{i}(s, t)\right\|^{\alpha-\varepsilon} & \leq \sum_{i=0}^{\infty} \mathbf{E}\left(\sum_{j=(i-t+s) \vee 0}^{i}\left\|C_{j}(t-i+j)\right\|\right)^{\alpha-\varepsilon} \\
& \leq c_{1} \sum_{i=0}^{\infty} \sum_{j=(i-t+s) \vee 0}^{i} \mathbf{E}\left\|C_{j}(t-i+j)\right\|^{\alpha-\varepsilon} \\
& =c_{1} \sum_{j=0}^{t-s} \sum_{i=0}^{\infty} \mathbf{E}\left\|C_{i}(t-j)\right\|^{\alpha-\varepsilon} \tag{2.17}
\end{align*}
$$

where $c_{1}$ is a constant depending on $\alpha-\varepsilon$ and $t-s$. The latter inequality in combination with the condition (1.2) and the stationarity of the sequence $\left\{C_{i}(t)\right\}$ implies

$$
\sum_{i=0}^{\infty} \mathbf{E}\left\|\tilde{C}_{i}(s, t)\right\|^{\alpha-\varepsilon}<\infty
$$

By the similar method for 2.17, we obtain, for some $0<\varepsilon<\alpha$,

$$
\begin{gathered}
\sum_{i=0}^{\infty} \mathbf{E}\left\|\tilde{C}_{i}(s, t)\right\|^{\alpha+\varepsilon} \leq c_{2} \sum_{j=0}^{t-s} \sum_{i=0}^{\infty} \mathbf{E}\left\|C_{i}(t-j)\right\|^{\alpha+\varepsilon}, \text { if } \alpha \in(0,1) \cup(1,2), \\
\mathbf{E}\left(\sum_{i=0}^{\infty}\left\|\tilde{C}_{i}(s, t)\right\|^{\alpha-\varepsilon}\right)^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}} \leq c_{3} \sum_{j=0}^{t-s} \mathbf{E}\left(\sum_{i=0}^{\infty}\left\|C_{i}(t-j)\right\|^{\alpha-\varepsilon}\right)^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}}, \text { if } \alpha \in\{1,2\}
\end{gathered}
$$

and

$$
\mathbf{E}\left(\sum_{i=0}^{\infty}\left\|\tilde{C}_{i}(s, t)\right\|^{2}\right)^{\frac{\alpha+\varepsilon}{2}} \leq c_{4} \sum_{j=0}^{t-s} \mathbf{E}\left(\sum_{i=0}^{\infty}\left\|C_{i}(t-j)\right\|^{2}\right)^{\frac{\alpha+\varepsilon}{2}}, \text { if } \alpha \in(2, \infty),
$$

where $c_{2}, c_{3}$ and $c_{4}$ are the constants depending on $\alpha, \varepsilon$ and $t-s$. As a consequence, under the conditions of Theorem 2.1, it is not only the marginal distribution of $\left(X_{t}\right)$ that is regularly varying, the same holds for all finite dimensional distributions.

Corollary 2.3 Under the conditions of Theorem 2.1, the stationary process $\left(X_{t}\right)$ given by (1.1) satisfies, for $s, t \in \mathbb{Z}$ with $s \leq t$

$$
\begin{equation*}
\frac{\mathbf{P}\left\{x^{-1}\left(X_{s}, \ldots, X_{t}\right)^{\prime} \in \cdot\right\}}{\mathbf{P}\left\{\left\|\xi_{0}\right\|>x\right\}} \xrightarrow{v} \sum_{i=0}^{\infty} \mathbf{E}\left[\mu \circ \tilde{C}_{i}(s, t)^{-1}(\cdot)\right]=: \nu_{s, t}(\cdot), \tag{2.18}
\end{equation*}
$$

as $x \rightarrow \infty$ on $\mathbb{E}^{(t-s+1) d}$, where the sequence of random matrices $\left\{\tilde{C}_{i}(s, t)\right\}$ is defined by 2.11. In particular, if $X_{0}$ is regularly varying, then the process $\left(X_{t}\right)$ is jointly regularly varying, i.e. for all $s, t \in \mathbb{Z}$ withs $\leq t$, random vector $\left(X_{s}, \ldots, X_{t}\right)^{\prime}$ are regularly varying.

## 3 Tail process

The joint regular variation of the sequence $\left(X_{t}\right)$ in particular means that there exists a process $\left(Y_{t}\right)_{t \in \mathbb{Z}}$ in $\mathbb{R}^{d}$ with $\mathbf{P}\left\{\left\|Y_{0}\right\|>y\right\}=y^{-\alpha}$ for $y \geq 1$ such that for all $s, t \in \mathbb{Z}$ with $s \leq t$ and as $x \rightarrow \infty$

$$
\mathcal{L}\left(x^{-1}\left(X_{s}, \ldots, X_{t}\right) \mid\left\|X_{0}\right\|>x\right) \rightsquigarrow \mathcal{L}\left(Y_{s}, \ldots, Y_{t}\right),
$$

see Theorem 2.1 in [1]. The process $\left(Y_{t}\right)$ is called the tail process of $\left(X_{t}\right)$. Moreover the so-called spectral process $\left(\Theta_{t}=\frac{Y_{t}}{\left\|Y_{0}\right\|}\right)_{t \in \mathbb{Z}}$ is independent of $\left\|Y_{0}\right\|$, see Theorem 3.1 in [1].

To understand the tail process in this case, it is helpful for instance to consider a set $B$ in $\mathbb{E}^{(t-s+1) d}$ of the form $B=\mathbb{E}_{b_{s}}^{d} \times \cdots \times \mathbb{E}_{b_{t}}^{d}$ for arbitrary real constant $b_{i}>0$, $i=s, \ldots, t$. The limit of the probabilities, as $x \rightarrow \infty$,

$$
\frac{\mathbf{P}\left\{x^{-1}\left(X_{s}, \ldots, X_{t}\right)^{\prime} \in B\right\}}{\mathbf{P}\left\{\left\|\xi_{0}\right\|>x\right\}}
$$

is the sum

$$
\begin{aligned}
& \sum_{i=0}^{\infty} \mathbf{E}\left[\mu \circ \tilde{C}_{i}(s, t)^{-1}(B)\right] \\
= & \sum_{i=0}^{\infty} \mathbf{E}\left[\mu\left\{x \mid \tilde{C}_{i}(s, t) x \in B\right\}\right] \\
= & \sum_{i=t-s}^{\infty} \mathbf{E}\left[\mu\left\{x \mid\left\|C_{i-t+s}(s) x\right\|>b_{s}, \ldots,\left\|C_{i}(t) x\right\|>b_{t}\right\}\right] \\
= & \sum_{i=t-s}^{\infty} \mathbf{E}\left[\int \mathbf{P}_{\Theta}(d \theta) \int \mathbb{I}_{\left\{\left\|C_{i-t+s}(s) r \theta\right\|>b_{s}\right\}} \cdots \mathbb{1}_{\left\{\left\|C_{i}(t) r \theta\right\|>b_{t}\right\}} d\left(-r^{-\alpha}\right)\right] \\
= & \sum_{i=t-s}^{\infty} \mathbf{E}\left[\int \mathbf{P}_{\Theta}(d \theta)\left(\max \left\{\frac{b_{s}}{\left\|C_{i-t+s}(s) \theta\right\|}, \ldots, \frac{b_{t}}{\left\|C_{i}(t) \theta\right\|}\right\}\right)^{-\alpha}\right] \\
= & \sum_{i=t-s}^{\infty} \mathbf{E}\left[\min \left\{\frac{\left\|C_{i-t+s}(s) \Theta\right\|^{\alpha}}{b_{s}^{\alpha}}, \ldots, \frac{\left\|C_{i}(t) \Theta\right\|^{\alpha}}{b_{t}^{\alpha}}\right\}\right] .
\end{aligned}
$$

In particular, if $s=t$ and $b_{s}=1$, one can apply this to obtain the following limit, as $x \rightarrow \infty$,

$$
\begin{equation*}
\frac{\mathbf{P}\left\{\left\|X_{0}\right\|>x\right\}}{\mathbf{P}\left\{\left\|\xi_{0}\right\|>x\right\}} \rightarrow \sum_{i=0}^{\infty} \mathbf{E}\left\|C_{i}(0) \Theta\right\|^{\alpha} . \tag{3.19}
\end{equation*}
$$

The following result is the multivariate version of Breiman's lemma [3] which appears as Proposition A. 1 in [2].

Lemma 3.1 Let $Z$ be a $q$-dimensional random vector and let $A$ be a $d \times q$ random matrix, independent of $Z$. Assume that $Z$ is multivariate regularly varying of index $\alpha \in(0, \infty)$, i.e. there exists a non-null Radon measure $\mu$ on $\mathbb{E}^{q}$ such that, as $x \rightarrow \infty$,

$$
\frac{\mathbf{P}\left\{x^{-1} Z \in \cdot\right\}}{\mathbf{P}\{\|Z\|>x\}} \xrightarrow{v} \mu(\cdot) .
$$

If $\mathbf{E}\|A\|^{\beta}<\infty$ for some $\beta>\alpha$, then in $\mathbb{E}^{d}$, as $x \rightarrow \infty$,

$$
\frac{\mathbf{P}\left\{x^{-1} A Z \in \cdot\right\}}{\mathbf{P}\{\|Z\|>x\}} \xrightarrow{v} \mathbf{E}\left[\mu \circ A^{-1}(\cdot)\right] .
$$

Theorem 3.2 Let $\left(X_{t}\right)_{t \in \mathbb{Z}}$ be a stationary process given by (1.1). Suppose that Condition 1.1 and 1.2 hold. If $\mathbf{P}\left(\cap_{i \geq 0}\left\{\left\|C_{i}(0) \Theta\right\|=0\right\}\right)=0$, then for $s, t \in \mathbb{Z}$ with $s \leq 0 \leq t$
and bounded and continuous function $f:\left(\mathbb{R}^{d}\right)^{t-s+1} \rightarrow \mathbb{R}$,

$$
\begin{align*}
& \mathbf{E}\left[\left.f\left(\frac{X_{s}}{\left\|X_{0}\right\|}, \ldots, \frac{X_{t}}{\left\|X_{0}\right\|}\right) \right\rvert\,\left\|X_{0}\right\|>x\right]  \tag{3.20}\\
& \rightarrow \frac{1}{\sum_{i=0}^{\infty} \mathbf{E}\left\|C_{i}(0) \Theta\right\|^{\alpha}} \sum_{i=0}^{\infty} \mathbf{E}\left[f\left(\frac{C_{i+s}(s) \Theta}{\left\|C_{i}(0) \Theta\right\|}, \ldots, \frac{C_{i+t}(t) \Theta}{\left\|C_{i}(0) \Theta\right\|}\right)\left\|C_{i}(0) \Theta\right\|^{\alpha}\right]
\end{align*}
$$

as $x \rightarrow \infty$, where $C_{i}(\cdot)=0$ if $i<0$.
Proof. Let $h:\left(\mathbb{R}^{d}\right)^{t-s+1} \rightarrow \mathbb{R}$ be bounded and continuous. In view of convergence (3.19) and Corollary 2.3, as $x \rightarrow \infty$,

$$
\begin{align*}
& \mathbf{E}\left[\left.f\left(\frac{X_{s}}{x}, \ldots, \frac{X_{t}}{x}\right) \right\rvert\,\left\|X_{0}\right\|>x\right]  \tag{3.21}\\
& \rightarrow \frac{1}{\sum_{i=0}^{\infty} \mathbf{E}\left\|C_{i}(0) \Theta\right\|^{\alpha}} \int h\left(y_{s}, \ldots, y_{t}\right) \mathbb{I}_{\left\{\left\|y_{0}\right\|>1\right\}} \nu_{s, t}(d \mathbf{y}) .
\end{align*}
$$

Note

$$
\tilde{X}(s, t)=\left(X_{s}, \ldots, X_{t}\right)^{\prime}=\sum_{i=0}^{\infty} \tilde{C}_{i}(s, t) \xi_{t-i},
$$

where the sequence $\left\{\tilde{C}_{i}(s, t)\right\}$ is defined by 2.11 . By $2.15,2.2$, Condition 1.1 and the stationarity of sequence $\left\{C_{i}(t)\right\}$, we have for some $\beta>\alpha$,

$$
\mathbf{E}\left\|\tilde{C}_{i}(s, t)\right\|^{\beta} \leq c \sum_{j=(i-t+s) \mathrm{V} 0}^{i} \mathbf{E}\left\|C_{j}(t-i+j)\right\|^{\beta}=c \sum_{j=(i-t+s) \vee 0}^{i} \mathbf{E}\left\|C_{j}(0)\right\|^{\beta}<\infty,
$$

where $c$ is a constant depending on $\beta$ and $t-s$. Using Lemma 3.1, we obtain, for each $i \geq 0$

$$
\begin{equation*}
\frac{\mathbf{P}\left\{x^{-1} \tilde{C}_{i}(s, t) \xi_{t-i} \in \cdot\right\}}{\mathbf{P}\left\{\left\|\xi_{0}\right\|>x\right\}} \xrightarrow{v} \mathbf{E}\left[\mu \circ \tilde{C}_{i}(s, t)^{-1}(\cdot)\right]=: \nu_{s, t}^{(i)}(\cdot), \tag{3.22}
\end{equation*}
$$

as $x \rightarrow \infty$, in $\mathbb{E}^{(t-s+1) d}$. By the definition of the measure $\nu_{s, t}$ in 2.18),

$$
\begin{equation*}
\nu_{s, t}(\cdot)=\sum_{i=0}^{\infty} \nu_{s, t}^{(i)}(\cdot) . \tag{3.23}
\end{equation*}
$$

We denote

$$
H(x)=\mathbf{E}\left[h\left(x^{-1} \tilde{C}_{i}(s, t) \xi_{t-i}\right) \mathbb{I}_{\left\{\left\|C_{i-t}(0) \xi_{t-i}\right\|>x\right\}}\right] / \mathbf{P}\left\{\left\|\xi_{0}\right\|>x\right\} .
$$

On the one hand, it follows from (3.22), as $x \rightarrow \infty$,

$$
\begin{align*}
H(x) & =\int h\left(y_{s}, \ldots, y_{t}\right) \mathbb{I}_{\left\{\left\|y_{0}\right\|>1\right\}} \mathbf{P}_{x^{-1} \tilde{C}_{i}(s, t) \xi_{t-i}}(d \mathbf{y}) / \mathbf{P}\left\{\left\|\xi_{0}\right\|>x\right\} \\
& \rightarrow \int h\left(y_{s}, \ldots, y_{t}\right) \mathbb{I}_{\left\{\left\|y_{0}\right\|>1\right\}} \nu_{s, t}^{(i)}(d \mathbf{y}) . \tag{3.24}
\end{align*}
$$

On the other hand, by independence between $\tilde{C}_{i}(s, t)$ and $\xi_{t-i}$ and assumption A1, as $x \rightarrow \infty$,

$$
\begin{align*}
H(x) & =\int \mathbf{E}\left[h\left(\tilde{C}_{i}(s, t) \mathbf{y}\right) \mathbb{I}_{\left\{\left\|C_{i-t}(0) \mathbf{y}\right\|>1\right\}}\right] \mathbf{P}_{x^{-1} \xi_{t-i}}(d \mathbf{y}) / \mathbf{P}\left\{\left\|\xi_{0}\right\|>x\right\} \\
& \rightarrow \int_{0}^{\infty} \mathbf{E}\left[h\left(\tilde{C}_{i}(s, t) \Theta r\right) \mathbb{I}_{\left\{\left\|C_{i-t}(0) \Theta\right\| r>1\right\}}\right] d\left(-r^{-\alpha}\right) \tag{3.25}
\end{align*}
$$

Since for $i<t, \nu_{s, t}^{(i)}\left\{\left(y_{s}, \ldots, y_{t}\right) \mid y_{s}=\cdots=y_{0}=0\right\}=1$,

$$
\begin{equation*}
\int h\left(y_{s}, \ldots, y_{t}\right) \mathbb{I}_{\left\{\left\|y_{0}\right\|>1\right\}} \nu_{s, t}^{(i)}(d \mathbf{y})=0, \quad i=0, \ldots, t-1 \tag{3.26}
\end{equation*}
$$

In combination with (3.23), (3.24), (3.25) and (3.26), it follows that

$$
\begin{align*}
& \int h\left(y_{s}, \ldots, y_{t}\right) \mathbb{I}_{\left\{\left\|y_{0}\right\|>1\right\}} \nu_{s, t}(d \mathbf{y}) \\
= & \sum_{i=t}^{\infty} \int h\left(y_{s}, \ldots, y_{t}\right) \mathbb{I}_{\left\{\left\|y_{0}\right\|>1\right\}} \nu_{s, t}^{(i)}(d \mathbf{y}) \\
= & \sum_{i=t}^{\infty} \int_{0}^{\infty} \mathbf{E}\left[h\left(\tilde{C}_{i}(s, t) \Theta r\right) \mathbb{I}_{\left\{\left\|C_{i-t}(0) \Theta\right\| r>1\right\}}\right] d\left(-r^{-\alpha}\right) . \tag{3.27}
\end{align*}
$$

Considering (3.21) and (3.27), we have, as $x \rightarrow \infty$,

$$
\mathbf{E}\left[\left.f\left(\frac{X_{s}}{x}, \ldots, \frac{X_{t}}{x}\right) \right\rvert\,\left\|X_{0}\right\|>x\right] \rightarrow I
$$

where
$I=\frac{1}{\sum_{i=0}^{\infty} \mathbf{E}\left\|C_{i}(0) \Theta\right\|^{\alpha}} \sum_{i=t}^{\infty} \int_{0}^{\infty} \mathbf{E}\left[h\left(C_{i-t+s}(s) \Theta r, \ldots, C_{i}(t) \Theta r\right) \mathbb{I}_{\left\{\left\|C_{i-t}(0) \Theta\right\| r>1\right\}}\right] d\left(-r^{-\alpha}\right)$.
Applying this relation to the function $h\left(x_{s}, \ldots, x_{t}\right)=f\left(\frac{x_{s}}{\left\|x_{0}\right\|}, \ldots, \frac{x_{t}}{\left\|x_{0}\right\|}\right)$ to see that, as $x \rightarrow \infty$, the left-hand side of (3.20) converges to
$\frac{1}{\sum_{i=0}^{\infty} \mathbf{E}\left\|C_{i}(0) \Theta\right\|^{\alpha}} \sum_{i=t}^{\infty} \int_{0}^{\infty} \mathbf{E}\left[f\left(\frac{C_{i-t+s}(s) \Theta}{\left\|C_{i-t}(0) \Theta\right\|}, \ldots, \frac{C_{i}(t) \Theta}{\left\|C_{i-t}(0) \Theta\right\|}\right) \mathbb{I}_{\left\{\left\|C_{i-t}(0) \Theta\right\| r>1\right\}}\right] d\left(-r^{-\alpha}\right)$.
By Fubini's theorem, this is equal to the right-hand side of 3.20 .

## 4 Applied models

### 4.1 Heavy tailed multivariate ARMA(1,1) process

Assume that $\left(\left(A_{t}, B_{t}, \xi_{t}\right)\right)_{t}$ is an i.i.d. sequence of random vectors in $\mathbb{R}^{d \times d} \times \mathbb{R}^{d \times d} \times \mathbb{R}^{d}$, for some $d \geq 1$. Throughout we assume that $\xi_{t}$ 's are regularly varying random vectors. Suppose that a stationary sequence $\left(X_{t}\right)_{t \in \mathbb{Z}}$ with value in $\mathbb{R}^{d}$ satisfies a multivariate random coefficient $\operatorname{ARMA}(1,1)$ equation of the following form

$$
\begin{equation*}
X_{t}=A_{t} X_{t-1}+B_{t} \xi_{t-1}+\xi_{t} \tag{4.28}
\end{equation*}
$$

Iterating this equation backwards we arrive at the following $\mathrm{MA}(\infty)$ representation of this process

$$
\begin{equation*}
X_{t}=\xi_{t}+\sum_{i \geq 1}\left(A_{t-i+1}+B_{t-i+1}\right) \prod_{j=0}^{i-2} A_{t-j} \xi_{t-i} \tag{4.29}
\end{equation*}
$$

namely, the stationary solution can be represented as (1.1) with

$$
C_{i}(t)= \begin{cases}I d, & i=0  \tag{4.30}\\ \left(A_{t-i+1}+B_{t-i+1}\right) \prod_{j=0}^{i-2} A_{t-j}, & i \geq 1\end{cases}
$$

From Theorem 3.1 in Hult and Samorodnitsky [4] we obtain the following result.
Corollary 4.1 Suppose that $\xi \in R V(\alpha, \mu)$ and there is some $0<\varepsilon<\alpha$ such that

$$
\begin{equation*}
\mathbf{E}\|A\|^{\alpha+\varepsilon}<1, \mathbf{E}\|B\|^{\alpha+\varepsilon}<\infty \tag{4.31}
\end{equation*}
$$

Then the series (4.29) converges a.s. and

$$
\begin{equation*}
\frac{\mathbf{P}\left\{u^{-1} X_{0} \in \cdot\right\}}{\mathbf{P}\left\{\left\|\xi_{0}\right\|>u\right\}} \xrightarrow{v} \mathbf{E}\left[\sum \mu \circ C_{j}(0)^{-1}(\cdot)\right] \tag{4.32}
\end{equation*}
$$

as $u \rightarrow \infty$ on $\overline{\mathbb{R}}^{d} \backslash\{0\}$.
Proof. Since $C_{0}(0)=I d$, we have $\mathbf{P}\left(\bigcap_{j \geq 0}\left\{\left\|C_{j}(0)\right\|=0\right\}\right)=0$.
For $\alpha \in(0,1) \cup(1,2)$, we have

$$
\begin{align*}
\sum \mathbf{E}\left\|C_{i}(0)\right\|^{\alpha-\varepsilon} & =1+\sum_{i \geq 1} \mathbf{E}\left\|\left(A_{-i+1}+B_{-i+1}\right) \prod_{j=0}^{i-2} A_{-j}\right\|^{\alpha-\varepsilon} \\
& \leq 1+\sum_{i \geq 1} \mathbf{E}\|A+B\|^{\alpha-\varepsilon} \mathbf{E} \prod_{j=0}^{i-2}\left\|A_{-j}\right\|^{\alpha-\varepsilon} \\
& =1+\mathbf{E}\|A+B\|^{\alpha-\varepsilon} \sum_{i \geq 1}\left(\mathbf{E}\|A\|^{\alpha-\varepsilon}\right)^{i-1} \\
& =1+\mathbf{E}\|A+B\|^{\alpha-\varepsilon} \sum_{i \geq 0}\left(\mathbf{E}\|A\|^{\alpha-\varepsilon}\right)^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon} i} \frac{\alpha-\varepsilon}{\alpha+\varepsilon} \tag{4.33}
\end{align*}
$$

By Jensen's inequality, the last term is bounded by

$$
\begin{equation*}
1+\mathbf{E}\|A+B\|^{\alpha-\varepsilon} \sum_{i \geq 0}\left(\mathbf{E}\|A\|^{\alpha+\varepsilon}\right)^{i \frac{\alpha-\varepsilon}{\alpha+\varepsilon}} \tag{4.34}
\end{equation*}
$$

In the case of $\alpha \in(0,1)$,

$$
\begin{equation*}
\mathbf{E}\|A+B\|^{\alpha-\varepsilon}<\mathbf{E}\|A\|^{\alpha-\varepsilon}+\mathbf{E}\|B\|^{\alpha-\varepsilon} \tag{4.35}
\end{equation*}
$$

In the case of $\alpha \in(1,2)$

$$
\begin{equation*}
\mathbf{E}\|A+B\|^{\alpha-\varepsilon} \leq\left(\mathbf{E}\|A+B\|^{\alpha+\varepsilon}\right)^{\frac{\alpha-\varepsilon}{\alpha+\varepsilon}} \leq\left[\left(\mathbf{E}\|A\|^{\alpha+\varepsilon}\right)^{\frac{1}{\alpha+\varepsilon}}+\left(\mathbf{E}\|B\|^{\alpha+\varepsilon}\right)^{\frac{1}{\alpha+\varepsilon}}\right]^{\alpha-\varepsilon} \tag{4.36}
\end{equation*}
$$

Combining (4.33), 4.34, 4.35 and 4.36 proves

$$
\sum \mathbf{E}\left\|C_{i}(0)\right\|^{\alpha-\varepsilon}<\infty
$$

Similarly as in 4.33, we have

$$
\begin{equation*}
\sum \mathbf{E}\left\|C_{i}(0)\right\|^{\alpha+\varepsilon} \leq 1+\mathbf{E}\|A+B\|^{\alpha+\varepsilon} \sum_{i \geq 0}\left(\mathbf{E}\|A\|^{\alpha+\varepsilon}\right)^{i} \tag{4.37}
\end{equation*}
$$

If $\alpha+\varepsilon<1$, then

$$
\begin{equation*}
\mathbf{E}\|A+B\|^{\alpha+\varepsilon}<\mathbf{E}\|A\|^{\alpha+\varepsilon}+\mathbf{E}\|B\|^{\alpha+\varepsilon} \tag{4.38}
\end{equation*}
$$

else

$$
\begin{equation*}
\mathbf{E}\|A+B\|^{\alpha+\varepsilon} \leq\left[\left(\mathbf{E}\|A\|^{\alpha+\varepsilon}\right)^{\frac{1}{\alpha+\varepsilon}}+\left(\mathbf{E}\|B\|^{\alpha+\varepsilon}\right)^{\frac{1}{\alpha+\varepsilon}}\right]^{\alpha+\varepsilon} \tag{4.39}
\end{equation*}
$$

Combining 4.37, 4.38 and 4.39 proves

$$
\sum \mathbf{E}\left\|C_{i}(0)\right\|^{\alpha+\varepsilon}<\infty
$$

For $\alpha \in\{1,2\}$, using Lemma 3.2.1 in Kwapien and Woyczynski [5] it follows that

$$
\begin{align*}
& \mathbf{E}\left(\sum\left\|C_{i}(0)\right\|^{\alpha-\varepsilon}\right)^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}} \\
\leq & {\left[\sum\left(\mathbf{E}\left\|C_{i}(0)\right\|^{\alpha+\varepsilon}\right)^{\frac{\alpha-\varepsilon}{\alpha+\varepsilon}}\right]^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}} } \\
= & {\left[1+\sum_{i \geq 1}\left\{\mathbf{E}\left\|\left(A_{-i+1}+B_{-i+1}\right) \prod_{j=0}^{i-2} A_{-j}\right\|^{\alpha+\varepsilon}\right\}^{\frac{\alpha-\varepsilon}{\alpha+\varepsilon}}\right]^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}} } \\
\leq & {\left[1+\sum_{i \geq 1}\left\{\mathbf{E}\|A+B\|^{\alpha+\varepsilon} \mathbf{E} \prod_{j=0}^{i-2}\left\|A_{-j}\right\|^{\alpha+\varepsilon}\right\}^{\frac{\alpha-\varepsilon}{\alpha+\varepsilon}}\right]^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}} } \\
= & {\left[1+\left(\mathbf{E}\|A+B\|^{\alpha+\varepsilon}\right)^{\frac{\alpha-\varepsilon}{\alpha+\varepsilon}} \sum_{i \geq 0}\left(\mathbf{E}\|A\|^{\alpha+\varepsilon}\right)^{i \frac{\alpha-\varepsilon}{\alpha+\varepsilon}}\right]^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}} . } \tag{4.40}
\end{align*}
$$

Considering 4.39 and 4.40 we get

$$
\mathbf{E}\left(\sum\left\|C_{i}(0)\right\|^{\alpha-\varepsilon}\right)^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}}<\infty
$$

For $\alpha \in(2, \infty)$, using Lemma 3.2.1 in Kwapien and Woyczynski [5] it follows that

$$
\begin{aligned}
\mathbf{E}\left(\sum\left\|C_{i}(0)\right\|^{2}\right)^{\frac{\alpha+\varepsilon}{2}} & \leq\left[\sum\left(\mathbf{E}\left\|C_{i}(0)\right\|^{\alpha+\varepsilon}\right)^{\frac{2}{\alpha+\varepsilon}}\right]^{\frac{\alpha+\varepsilon}{2}} \\
& \leq\left[1+\left(\mathbf{E}\|A+B\|^{\alpha+\varepsilon}\right)^{\frac{2}{\alpha+\varepsilon}} \sum_{i \geq 0}\left(\mathbf{E}\|A\|^{\alpha+\varepsilon}\right)^{i \frac{2}{\alpha+\varepsilon}}\right]^{\frac{\alpha+\varepsilon}{2}}
\end{aligned}
$$

Considering (4.39) and 4.41 we get

$$
\mathbf{E}\left(\sum\left\|C_{i}(0)\right\|^{2}\right)^{\frac{\alpha+\varepsilon}{2}}<\infty
$$

Here is another proof of Corollary 4.1
Proof. We denote

$$
\begin{gather*}
\tilde{X}_{t}=\left(X_{t}, \xi_{t}, \xi_{t+1}\right)^{\prime}  \tag{4.42}\\
\tilde{A}_{t}=\left(\begin{array}{ccc}
A_{t} & B_{t} & I d \\
0 & 0 & I d \\
0 & 0 & 0
\end{array}\right)
\end{gather*}
$$

and

$$
Z_{t}=\left(0,0, \xi_{t+1}\right)^{\prime}
$$

Then the relation 4.28 can be wrote as

$$
\begin{equation*}
\tilde{X}_{t}=\tilde{A}_{t} \tilde{X}_{t-1}+Z_{t} \tag{4.43}
\end{equation*}
$$

Iterating this equation we get

$$
\tilde{X}_{t}=\sum_{i=0}^{\infty} C_{i}(t) Z_{t-i}
$$

where $C_{0}(t)=I d, C_{1}(t)=\tilde{A}_{t}$ and

$$
C_{i}(t)=\left(\begin{array}{ccc}
\prod_{j=0}^{i-1} A_{t-j} & B_{t-i+1} \prod_{j=0}^{i-2} A_{t-j} & \left(A_{t-i+2}+B_{t-i+2}\right) \prod_{j=0}^{i-3} A_{t-j} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right), i \geq 2
$$

It is equivalent to prove the convergence 4.32 of vector $\tilde{X}_{t}$ defined by 4.42 .
By Lemma 2.2 and the properties of matrix norm, for $i \geq 1$

$$
\begin{equation*}
\left\|C_{i}(t)\right\| \leq \prod_{j=0}^{i-1}\left\|A_{t-j}\right\|+\left\|B_{t-i+1}\right\| \prod_{j=0}^{i-2}\left\|A_{t-j}\right\|+\left\|A_{t-i+2}+B_{t-i+2}\right\| \prod_{j=0}^{i-3}\left\|A_{t-j}\right\| \tag{4.44}
\end{equation*}
$$

where $A_{i}=B_{i}=I d$ if $i>t$. It follows from 2.16), if $\alpha \in(0,1) \cup(2,1)$

$$
\begin{align*}
& \sum_{i \geq 0} \mathbf{E}\left\|C_{i}(0)\right\|^{\alpha-\varepsilon}  \tag{4.45}\\
\leq & 1+c \sum_{i \geq 1}\left[\mathbf{E} \prod_{j=0}^{i-1}\left\|A_{t-j}\right\|^{\alpha-\varepsilon}+\mathbf{E}\left(\left\|B_{t-i+1}\right\| \prod_{j=0}^{i-2}\left\|A_{t-j}\right\|\right)^{\alpha-\varepsilon}\right. \\
& \left.+\mathbf{E}\left(\left\|A_{t-i+2}+B_{t-i+2}\right\| \prod_{j=0}^{i-3}\left\|A_{t-j}\right\|\right)^{\alpha-\varepsilon}\right] \\
= & 1+c\left(\sum_{i \geq 1}\left(\mathbf{E}\|A\|^{\alpha-\varepsilon}\right)^{i}+\mathbf{E}\|B\|^{\alpha-\varepsilon} \sum_{i \geq 0}\left(\mathbf{E}\|A\|^{\alpha-\varepsilon}\right)^{i}\right. \\
& \left.+\mathbf{E}\|A+B\|^{\alpha-\varepsilon} \sum_{i \geq 0}\left(\mathbf{E}\|A\|^{\alpha-\varepsilon}\right)^{i}+2^{\alpha-\varepsilon}\right)
\end{align*}
$$

where $c$ is a constant depending on $\alpha-\varepsilon$. By Jensen's inequality,

$$
\begin{equation*}
\mathbf{E}\|A\|^{\alpha-\varepsilon} \leq\left(\mathbf{E}\|A\|^{\alpha+\varepsilon}\right)^{\frac{\alpha-\varepsilon}{\alpha+\varepsilon}} \tag{4.46}
\end{equation*}
$$

Combining 4.45, 4.46, 4.38 and 4.39 proves

$$
\sum_{i \geq 0} \mathbf{E}\left\|C_{i}(0)\right\|^{\alpha-\varepsilon}<\infty
$$

Similarly we can prove

$$
\sum_{i \geq 0} \mathbf{E}\left\|C_{i}(0)\right\|^{\alpha+\varepsilon}<\infty
$$

For $\alpha \in\{1,2\}$, using Lemma 3.2.1 in Kwapien and Woyczynski [5] it follows that

$$
\begin{equation*}
\mathbf{E}\left(\sum\left\|C_{i}(0)\right\|^{\alpha-\varepsilon}\right)^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}} \leq\left[\sum\left(\mathbf{E}\left\|C_{i}(0)\right\|^{\alpha+\varepsilon}\right)^{\frac{\alpha-\varepsilon}{\alpha+\varepsilon}}\right]^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}} \tag{4.47}
\end{equation*}
$$

By 2.16 and 4.44, $\sum\left(\mathbf{E}\left\|C_{i}(0)\right\|^{\alpha+\varepsilon}\right)^{\frac{\alpha-\varepsilon}{\alpha+\varepsilon}}$ is bounded by the right-hand side of 4.45 which is bounded. Hence

$$
\begin{equation*}
\mathbf{E}\left(\sum\left\|C_{i}(0)\right\|^{\alpha-\varepsilon}\right)^{\frac{\alpha+\varepsilon}{\alpha-\varepsilon}}<\infty \tag{4.48}
\end{equation*}
$$

By similar method of 4.48), we get

$$
\mathbf{E}\left(\sum\left\|C_{i}(0)\right\|^{2}\right)^{\frac{\alpha+\varepsilon}{2}}<\infty .
$$

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