

# A dynamic approach for the vehicle routing problem with stochastic demands

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# Agenda

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- The Vehicle Routing Problem with Stochastic Demands (VRPSD)
- The Multiple Scenario Approach (MSA)
- Computational results
- Conclusions and future work

# The VRPSD

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- Extension of the Vehicle Routing Problem
  - Set of customers
  - Fleet of vehicles
  - Design a set of routes minimizing distance
  - Stochastic Demands
    - Known distribution law
- Route failure
  - Demand realization exceeds remaining capacity
  - Split delivery

# The VRPSD

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- Some applications
  - Milk recollection
  - Garbage recollection
  - Heating oil delivery

# Motivation

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- Traditional approaches
  - Two-stage approach
    - Robust a-priori routing
    - Recourse actions in case of failure
  - Assumes vehicles cannot be rerouted
- Why a dynamic approach?
  - Increasing availability of low-cost positioning systems
  - Real-time communication with vehicles
  - Allows further optimization

# State of the art

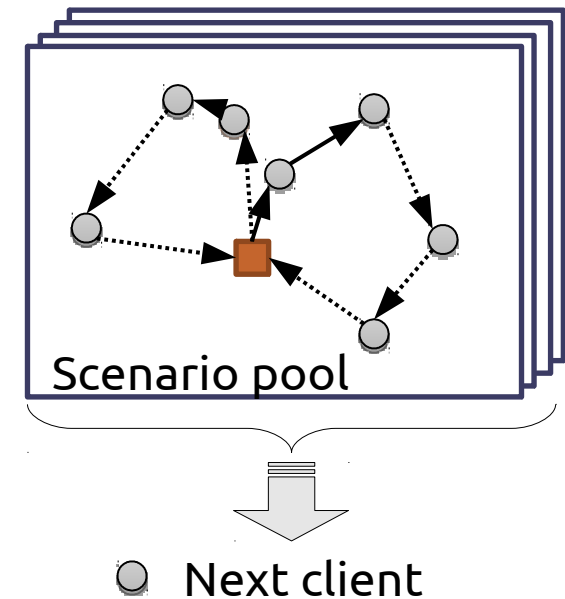
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- Reoptimization/dynamic approach
  - Nova & Storer (2009), Secomandi (2001), Secomandi & Margot (2009)
  - No a-priori routes
  - Optimization performed at each decision time
  - Discrete uniform demands
- Assumptions
  - Single vehicle
  - Failure
    - Split delivery
    - Replenishment trip

# Multiple Scenario Approach (MSA)

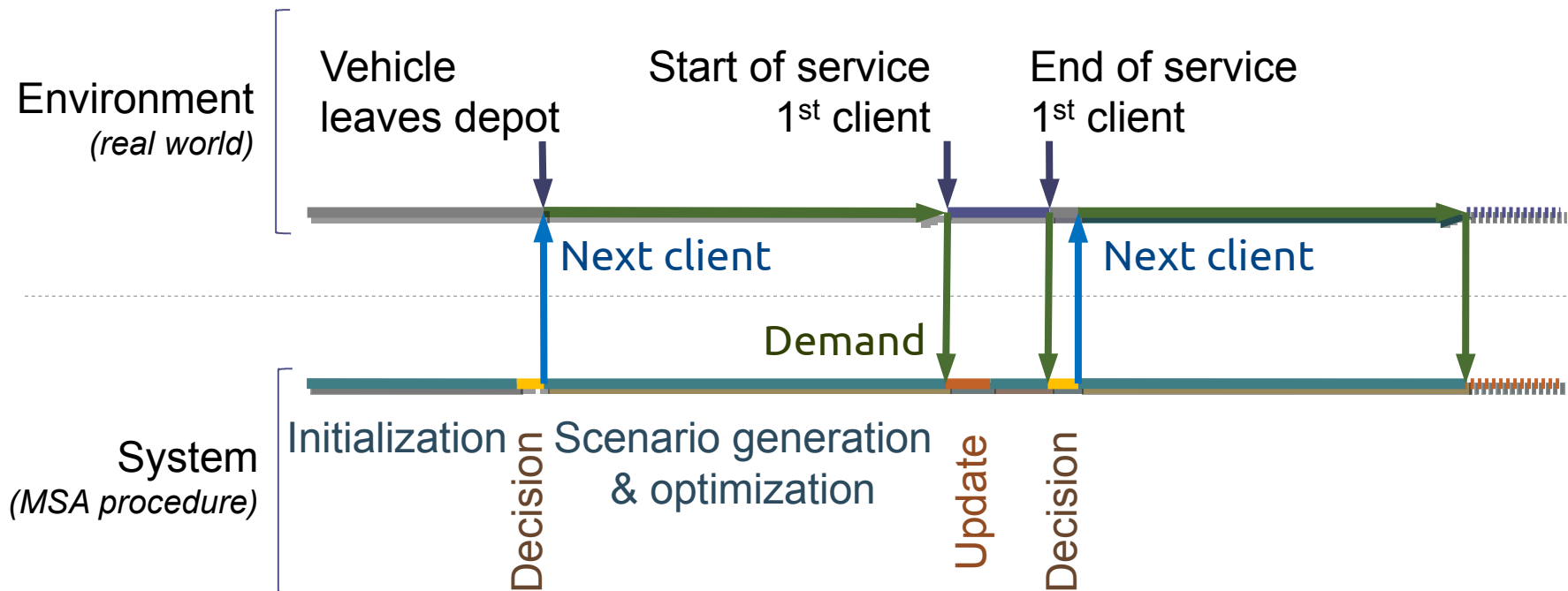
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- Optimization framework for online optimization
  - Van Hentenryck and Bent (2006)
- Runs throughout the day
- Scenario pool
  - Realizations of the random variables
- Decision process
  - Which client next?
  - Aggregate scenario knowledge
- Java implementation (jMSA)



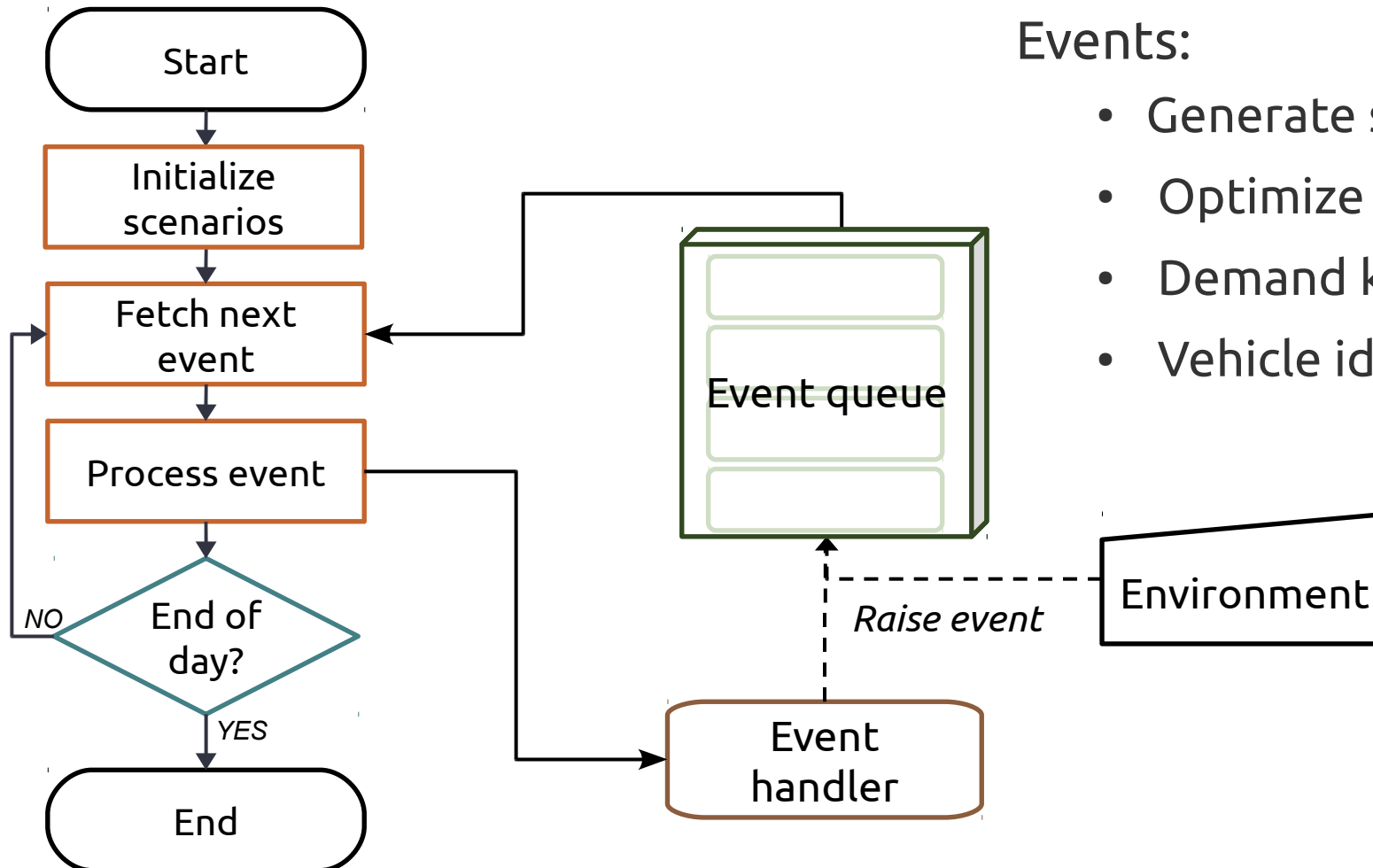
# MSA - Events

- Time line of events





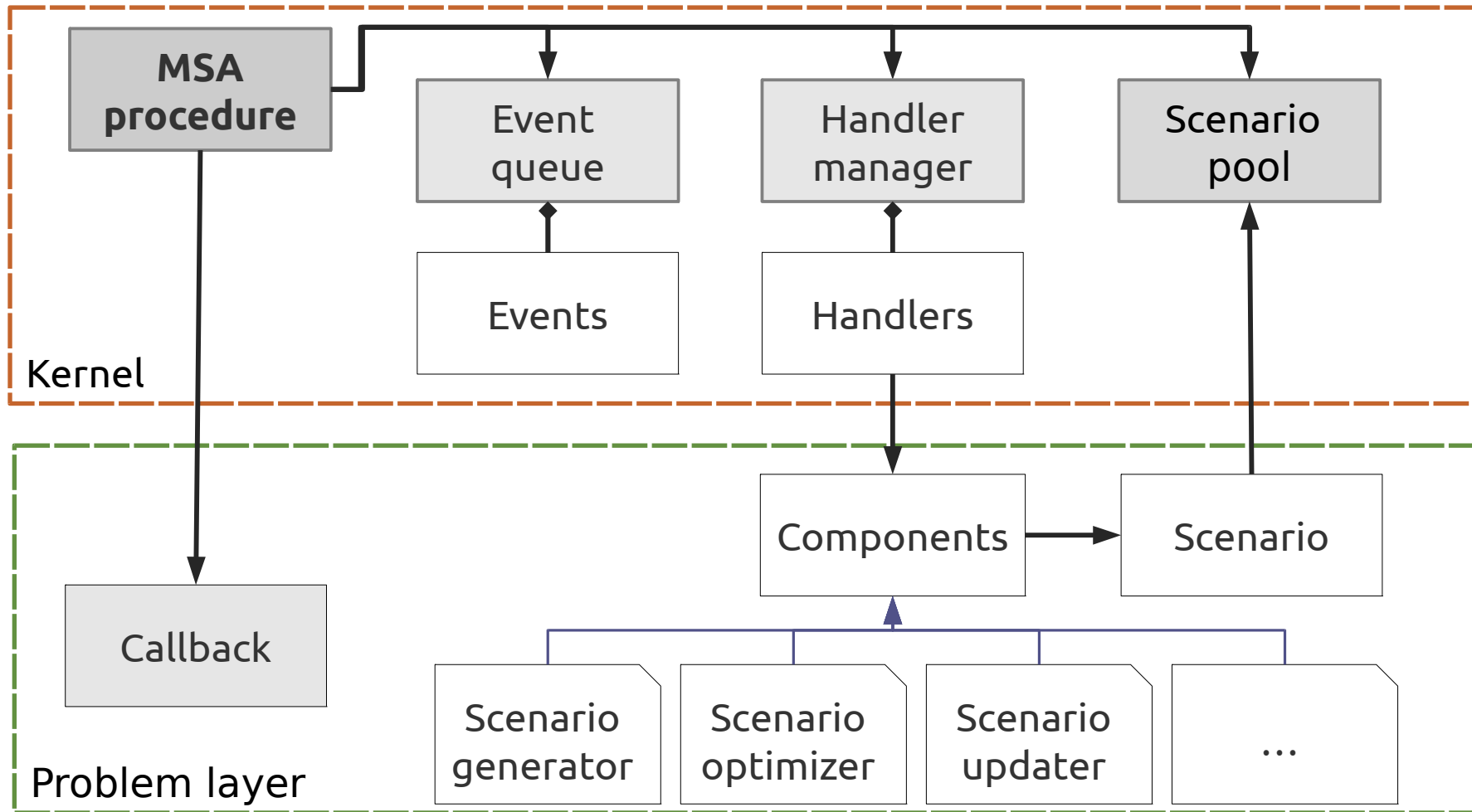
# MSA - Algorithm



## Events:

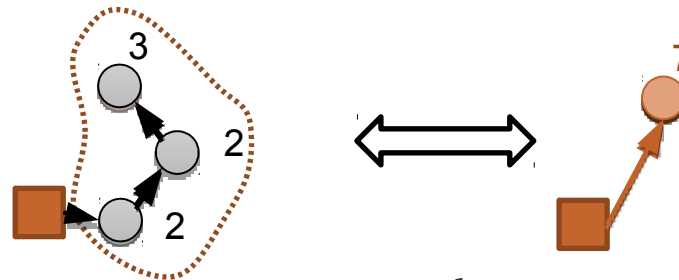
- Generate scenarios
- Optimize scenarios
- Demand known
- Vehicle idle

# jMSA - Design

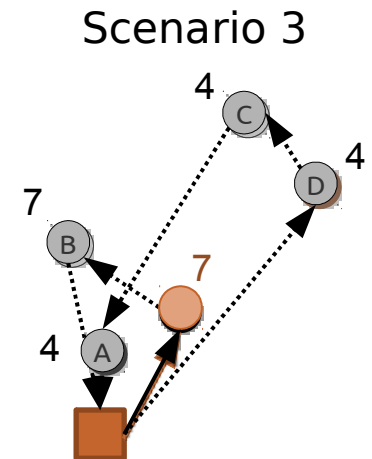
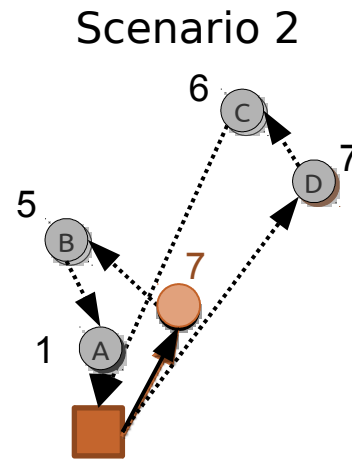
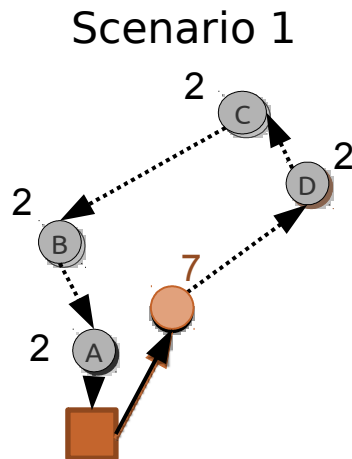


# jMSA – Scenario pool

- Shrinking of executed route



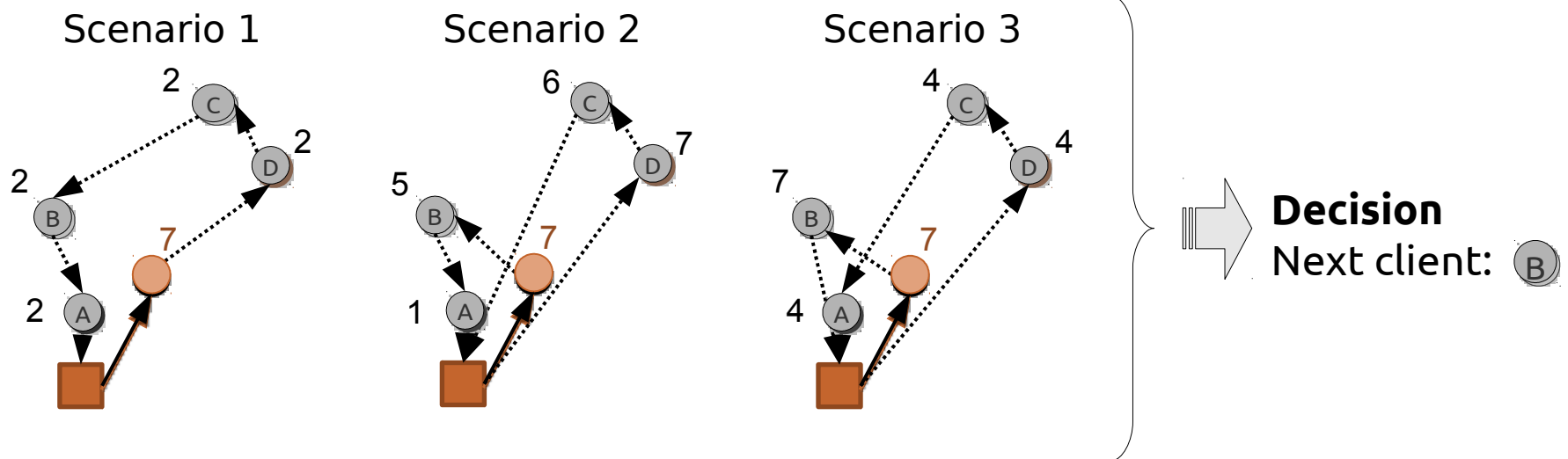
- Sampling of the customer demands



Vehicle capacity :  
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# jMSA - Decision

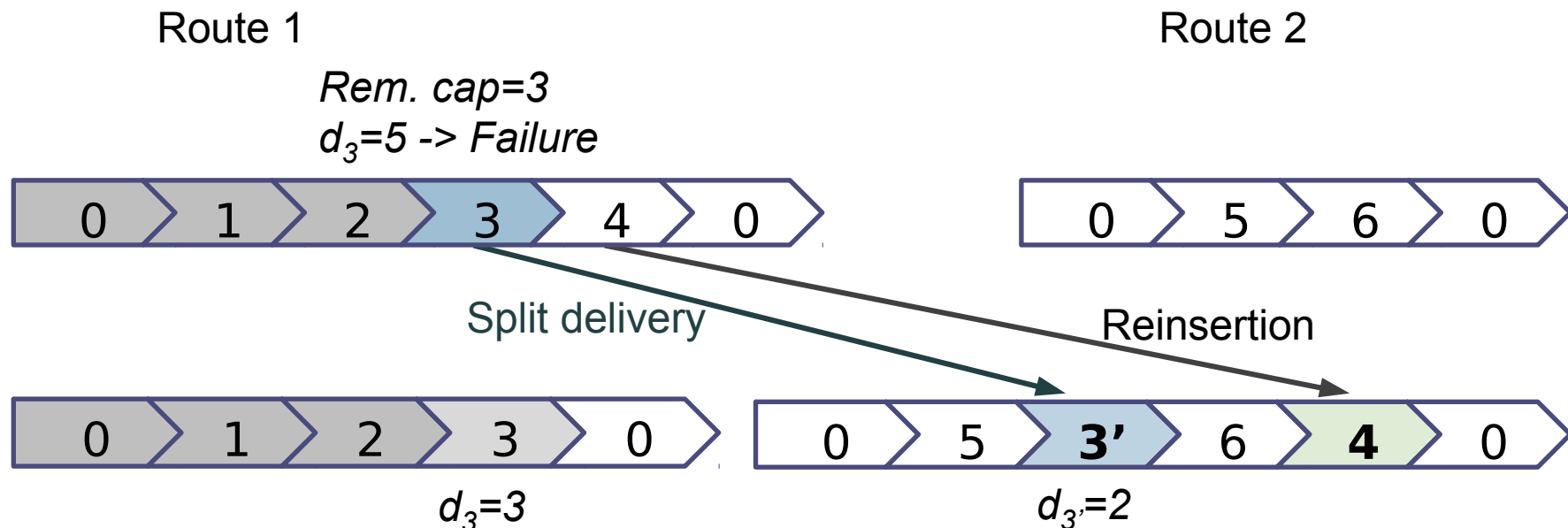
- Consensus
  - Select the customer that appears first with the highest frequency



- Others
  - Regret, detour regret ... dominated

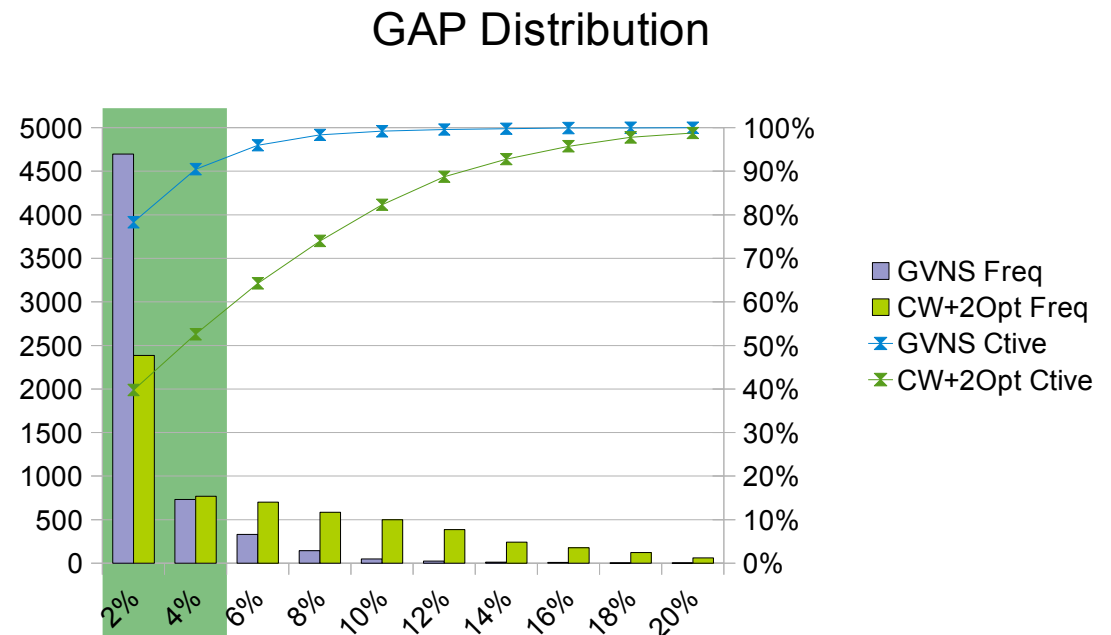
# jMSA – Route failures

- Occurs when the customer demand exceeds vehicle capacity
  - Split delivery
  - Reinsertion of following customers



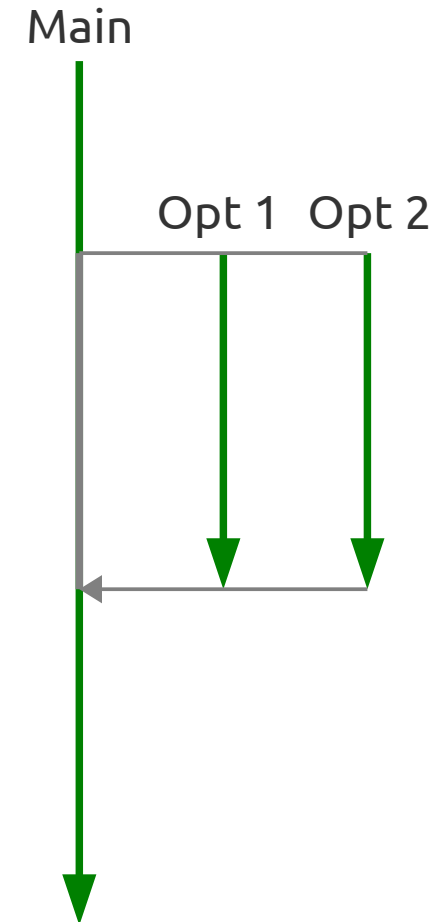
# jMSA - Optimization

- Adaptive Variable Neighborhood Search (AVNS)
  - Swap, 2-opt, Or-opt, string-exchange
  - Adaptive neighborhood selection
    - Based on performance
    - Selection wheel
- Performance
  - Fast (~200ms)
  - Average gap: 1.4%



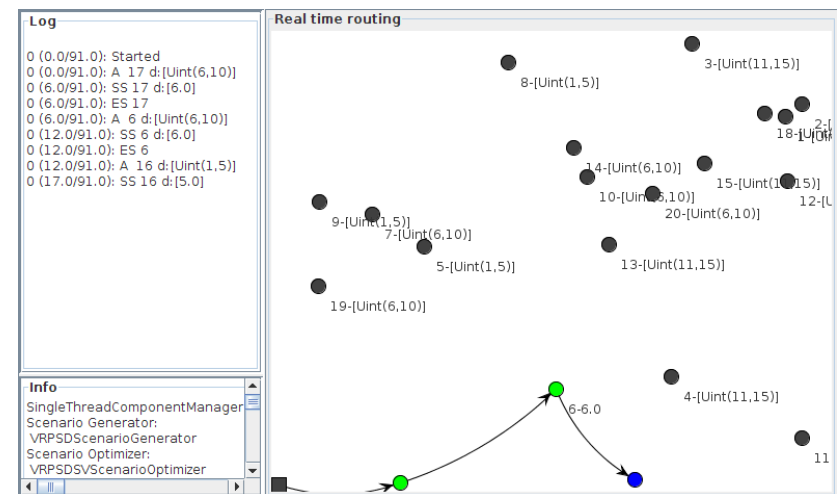
# jMSA - Parallelization

- Parallelization of
  - Scenario generation
  - Scenario optimization
- Benefits
  - Increased reactivity
  - More optimization in the same time



# Computational experiments

- Instances from Novoa (2005)
  - 30, 40 and 60 customers
    - Randomly distributed in 1x1 square
    - **Uniform discrete** demand distribution
  - 2 vehicle capacities
- Simulation
  - MSA used as a *black-box*
  - *100 runs per instance*
  - *3-5 min per run*
  - *3000 runs in total (10 days)*





# Computational experiments

- Average value of information
  - Gap to perfect information (in %)

		(size, capacity)						Average
Algorithm		(30,137)	(30,87)	(40,183)	(40,116)	(60,274)	(60,175)	
<i>Secomandi 2001</i>	<b>1s_n2_r</b>	12.25	11.82	11.15	12.87	13.88	19.63	12.25
	<b>1s_n2reopt_r</b>	16.02	12.32	12.48	12.97	14.02	19.75	16.02
<i>Nova &amp; Storer 2009</i>	<b>1s_stostat_r</b>	4.67	5.06	3.66	5.27	3.52	12.35	4.67
	<b>2s_stostat_r</b>	3.45	<b>3.62</b>	<b>2.95</b>	<b>5.38</b>	<b>2.78</b>	10.70	3.45
<b>MSA</b>		<b>0.91</b>	4.07	3.55	6.26	2.94	<b>2.00</b>	<b>3.16</b>

- Faster decisions (ms vs min)
- Flexibility
  - Other distributions

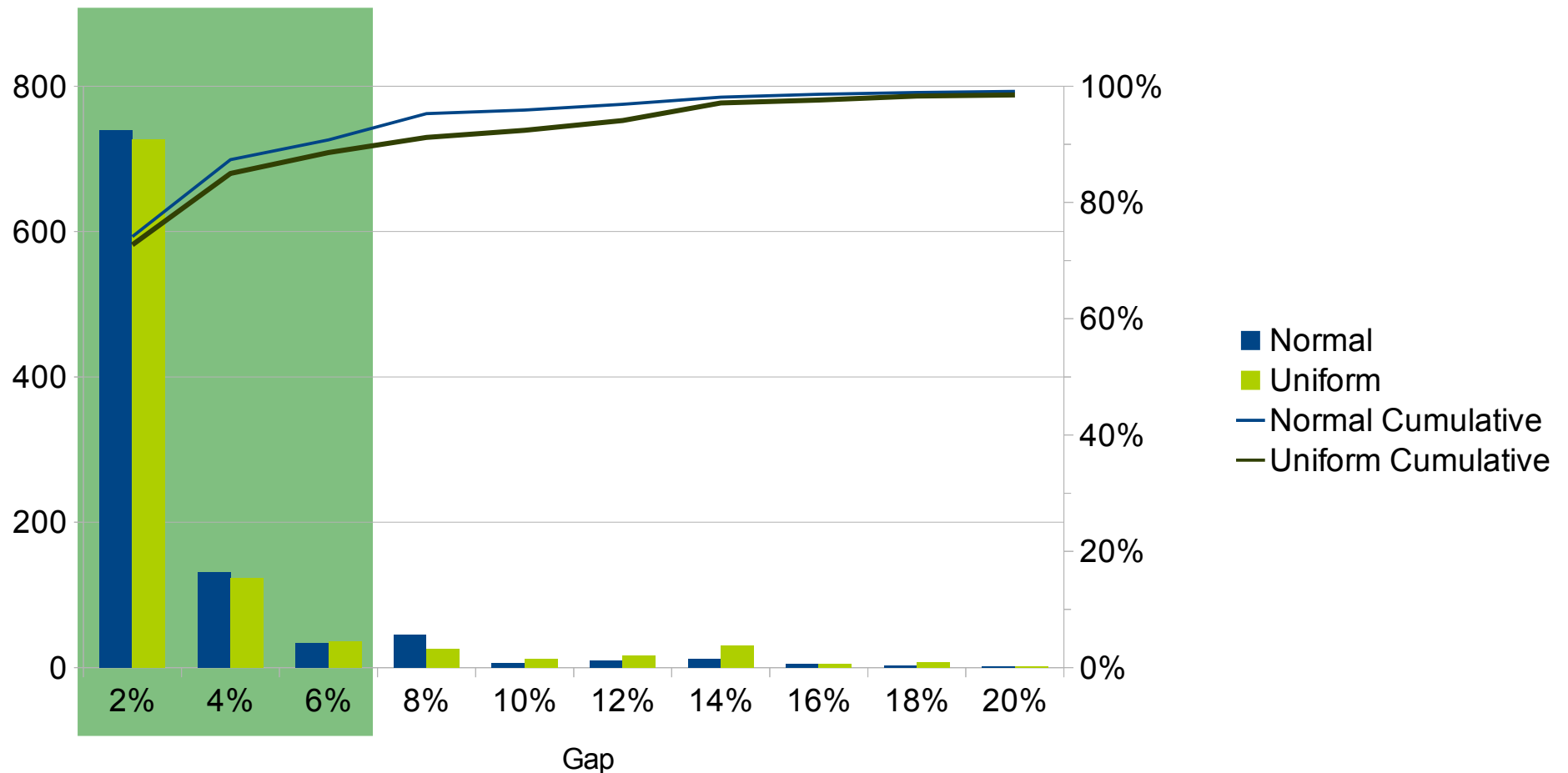
# Computational experiments

- Normal distribution
  - Based on Novoa (2005) instances
  - e.g.:  $U_{int}(11,15) \rightarrow N(13,1)$

Algorithm		(size, capacity)						Average
		(30,137)	(30,87)	(40,183)	(40,116)	(60,274)	(60,175)	
MSA	Uniform	0.91	4.07	3.55	6.26	2.94	2.00	<b>3.29</b>
MSA	Normal	0.66	3.61	3.39	6.21	2.23	1.92	<b>3.01</b>

# Computational experiments

- GAP distribution for size 60 instances



# Conclusions

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- New approach for the VPRSD
  - Dynamic
    - Increased reactivity
  - Generic
    - No assumption on demand distributions
    - Additional constraints
- Future work
  - Extend to multiple vehicles
  - Design specific decision process
    - Use stochastic modeling

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# Questions & Answers

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Thank you for your attention

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