HAL
open science

Consistency of the regression estimator with functional data under long memory conditions

Karim Benhenni, Sonia Hedli-Griche, Mustapha Rachdi, Philippe Vieu

To cite this version:

Karim Benhenni, Sonia Hedli-Griche, Mustapha Rachdi, Philippe Vieu. Consistency of the regression estimator with functional data under long memory conditions. Statistics and Probability Letters, 2008, 78 (8), pp.1043-1049. 10.1016/j.spl.2007.11.011 . hal-00622143

HAL Id: hal-00622143

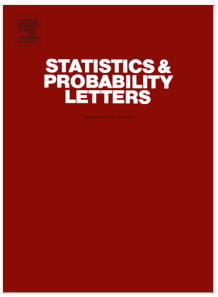
https://hal.science/hal-00622143v1

Submitted on 12 Sep 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Accepted Manuscript



Consistency of the regression estimator with functional data under long memory conditions

K. Benhenni, S. Hedli-Griche, M. Rachdi, P. Vieu

PII: S0167-7152(07)00406-3
DOI: 10.1016/j.spl.2007.11.011
Reference: STAPRO 4838

To appear in: *Statistics and Probability Letters*

Received date: 30 August 2006
Revised date: 13 August 2007
Accepted date: 6 November 2007

Please cite this article as: Benhenni, K., Hedli-Griche, S., Rachdi, M., Vieu, P., Consistency of the regression estimator with functional data under long memory conditions. *Statistics and Probability Letters* (2007), doi:10.1016/j.spl.2007.11.011

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

ACCEPTED MANUSCRIPT

Consistency of the regression estimator with functional data under long memory conditions

K. Benhenni [a,1], S. Hedli-Griche[a], M. Rachdi[a] and P. Vieu [b]

[a] *Université de Grenoble 2, Equipe SF&S, LJK UMR CNRS 5224, UFR SHS, BP. 47, F38040 Grenoble Cedex 09, France.*

[b] *Université Paul Sabatier, LSP UMR CNRS 5583, 118, Route de Narbonne, 31062 Toulouse Cedex, France.*

Abstract

We study the nonparametric regression estimation when the explanatory variable takes values in some abstract functional space. We establish some asymptotic results and we give the (pointwise and uniform) convergence of the kernel type estimator constructed from functional data under long memory conditions.

Key words: Nonparametric regression operator, Functional data, Long memory process

1 Introduction

Since the beginning of the sixties, nonparametric statistical methods have been developed intensively where a lot of scientists today collect samples of curves and other functional observations. Functional data have become more and more popular in modern statistics. For an introduction and applications of this field, the books by Ramsay and Silverman (1997, 2002), Bosq (2000) and Ferraty and Vieu (2006) provide some basic methods of analysis along with diverse case studies in several areas including criminology, economics, archeology, and neurophysiology. It should be noticed that the extension of probability theory to random variables taking values in normed spaces (e.g. Banach and Hilbert spaces), including extensions of certain classical theorems, are very useful tools in the recent literature on functional data (Cf Chobanyan et al. (1987), among others).

[1] Corresponding author. E-mail: Karim.Benhenni@upmf-grenoble.fr

ACCEPTED MANUSCRIPT

In this work, we are interested in the problem of estimating the regression operator when the explanatory data are of functional type, under long memory dependent structure. The phenomenon of long memory was known long before the development of the stochastic models. These processes are mainly of interest for modeling dependence structure of processes, for which the usual weak dependence assumptions (such as mixing or association conditions) are not satisfactory. In fact, in many statistical applications, the correlation between observations decreases to zero slower than in the classical models such as the ARMA models. Models of long memory were first introduced by Mondelbrot and Van Ness (1968) for fractional brownian motion. A good introduction of dependent processes can be found in Beran (1994), among others. For a more detailed discussion on long memory time series and their applications, one can refer for example to Hidalgo (1997) for econometric models and Turcotte (1994) for environmetrics or climatic studies.
Frequently, we say that a stationary process $\{Z_t\}_{t\in\mathbb{Z}}$ is a long memory process if for some real $\gamma \in]0,1]$, and some positive constant $\mathcal{C}$, its autocorrelation function satisfies

$$|f_Z(j)| \sim \mathcal{C}\,|j|^{-\gamma} \text{ as } |j| \to +\infty \tag{1}$$

Other equivalent definitions of long memory process involving the spectral density can be found for instance in Cox (1984). In the finite dimensional framework, the problem of nonparametric regression estimation when the error is a long memory process, was extensively studied by many authors, such as Hall and Hart (1990), Csörgö and Mielniczuk (1995), Masry (2001), Estévez and Vieu (2003)...
In the infinite dimensional framework, the problem of functional estimation for nonparametric regression operator under some strong mixing conditions on the functional data was considered by Ferraty and Vieu (2004). They derived the rates of convergence for a functional version of the kernel type estimator. Rachdi and Vieu (2005, 2007) and Benhenni et al. (2007) constructed some curve data-driven criterion for choosing automatically the smoothing parameter for these estimators. More recently, Müller and Stadtmüller (2006) studied general functional linear models.
In this paper, we consider the similar problem of nonparametric estimation of the regression operator when the pairs (X_i, Y_i) for $i \in \mathbb{Z}$ satisfy some long memory conditions. We give two different ways of convergence, pointwise and uniform convergence, for the estimator of the regression operator.
The organization of this paper is as follows. In section 2, we state the functional regression model and we define the nonparametric estimator. In section 3, we set up the different assumptions and we give the convergence results of the estimator through Theorem 3 and Theorem 4. The proofs of all the results are presented in section 4.

ACCEPTED MANUSCRIPT

2 Problem of interest

Let (X, Y) be a random vector valued in $H \times \mathbb{R}$ with $\mathbb{E}|Y| < \infty$, and where (H, d) is a semi-metric space equipped with the semi-metric d. The purpose of this paper is to study the estimation of the regression operator r when we observe identically distributed copies $(X_1, Y_1), \ldots, (X_n, Y_n)$ of (X, Y) in the following model

$$Y_i = r(X_i) + \varepsilon_i \text{ for } i \in \mathbb{Z}$$

The random errors $\varepsilon_1, \ldots, \varepsilon_n$ are assumed to be centered with finite second moments.
Most standard time series models assume that the non-summability covariance sequences $\{f(i)\}_{i\in\mathbb{Z}}$ captures the intuition behind long range dependence (LRD) or *long memory* (see Estévez and Vieu, 2003):

$$\sum_{\tau=-\infty}^{+\infty} |f(\tau)| = +\infty$$

This assumption is not needed here.
The goal of this work is to estimate the unknown operator $r(x) = \mathbb{E}\left(Y|X = x\right)$ for $x \in H$. For this aim, we use the functional version of Nadaraya-Watson kernel estimator (see Ferraty and Vieu, 2004)

$$\widehat{r}_h(x) = \begin{cases} \dfrac{\sum_{i=1}^n Y_i\, K\left(d(x, X_i)/h_n\right)}{\sum_{i=1}^n K\left(d(x, X_i)/h_n\right)} & \text{if } K\left(d(x, X_i)/h_n\right) \neq 0 \\ 0 & \text{if } K\left(d(x, X_i)/h_n\right) = 0 \end{cases}$$

where K is a real valued function defined on $\mathbb{R}_+$ and $h = h_n$ is the bandwidth parameter, such that: $h \in \mathbb{R}_+$ and $\lim\limits_{n\to+\infty} h = 0$. In what follows, we will use the following notations

$$\widehat{r}_h(x) = \frac{\widehat{r}_{h,2}(x)}{\widehat{r}_{h,1}(x)}$$

where

$$\widehat{r}_{h,1}(x) = \frac{1}{n\,\mathbb{E}(\Delta_1(x))} \sum_{i=1}^n \Delta_i(x) \text{ and } \widehat{r}_{h,2}(x) = \frac{1}{n\,\mathbb{E}(\Delta_1(x))} \sum_{i=1}^n Y_i \Delta_i(x)$$

with $\Delta_i(x) = K\left(d(x, X_i)/h\right)$.

The effect of the dependence of the pairs (X_i, Y_i) for $i = 1, \ldots, n$ is controlled by the following quantity:

$$Q(x) = \max(|F(x)|, |G(x)|), \text{ for } x \in H$$

ACCEPTED MANUSCRIPT

where

$$F(x) = \sum_{i \neq j}^{n} f_{(i,j)}(x) \text{ and } G(x) = \sum_{i \neq j}^{n} g_{(i,j)}(x)$$

with

$$f_{(i,j)}(x) = cov(\Delta_i(x), \Delta_j(x)) \text{ and } g_{(i,j)}(x) = cov(\Gamma_i(x), \Gamma_j(x))$$

3 Main results

The following assumptions are needed for the statement of the results concerning the asymptotic performance of the regression operator estimate $\widehat{r}_h$.

About the kernel: we assume that K is strictly decreasing on $(0,1)$ and there exist some positive constants c_1 and c_2 such that

$$c_1 \, \mathbb{1}_{(0,1)}(t) \leq K(t) \leq c_2 \, \mathbb{1}_{(0,1)}(t), \text{ for } t \in \mathbb{R} \tag{2}$$

The kernel K has a first order derivative K' on $(0,1)$ such that for some real constants c_3 and c_4

$$c_3 \leq K'(t) \leq c_4, \text{ for } t \in \mathbb{R} \tag{3}$$

About the concentration of X: we assume that the probability distribution of the functional variable X can be written as

$$c_5 \, \varphi(h) \leq \mathbb{P}\left(X \in \mathcal{B}(x,h)\right) \leq c_6 \, \varphi(h), \text{ for } x \in H \tag{4}$$

where c_5 and c_6 are real constants and $\mathcal{B}(x,t)$ denotes the closed ball of center x and radius t, and $\varphi(t)$ is a positive function such that

$$\lim_{t \to 0} \varphi(t) = 0 \text{ and } \lim_{n \to +\infty} n \, \varphi(h) = +\infty \tag{5}$$

About the regression operator: there exist some constants $0 < c_7 < \infty$ and $\beta > 0$, such that

$$\forall x, y \in H, \; |r(x) - r(y)| \leq c_7 \, d(x,y)^{\beta} \tag{6}$$

About the moments: the response variable Y satisfies

$$\forall x \in H, \; \mathbb{E}(Y^2 | X = x) \leq \sigma(x) < \infty \tag{7}$$

where $\sigma(.)$ is a continuous function at x.

This assumption allows us to deal with unbounded variables (Cf page 63 of Ferraty and Vieu, 2006).

ACCEPTED MANUSCRIPT

About the dependence: we assume that,

$$\lim_{n\to+\infty} n\varphi(h) = \lim_{n\to+\infty} \inf_{x\in S} \frac{n^2\varphi^2(h)}{Q(x)} = +\infty \tag{8}$$
for some compact set S of H

For instance, when f and g satisfy (1) with respectively some γ_1 and γ_2, we have for all $x \in H$:

$$Q(x) \sim c_x \sum_{k=1}^{n} nk^{-\gamma} \sim c_x n^{2-\gamma} \text{ where } \gamma = \gamma_1 \text{ or } \gamma_2$$

for some positive constant c_x depending on the curve x. The condition (8) is true if $\varphi(h) = O(n^{-\gamma/3})$.

The following useful lemmas are needed in the proofs of Theorems 3 and 4.

Lemma 1 *Under condition (6), we have*

$$\mathbb{E}(\widehat{r}_{h,2}(x)) - r(x) = O\left(h^{\beta}\right), \text{ as } n \to +\infty$$

Lemma 2 *Assume that conditions (2), (4), (5), (6), (7) and (8) are satisfied, then we have as $n \to +\infty$*

(i) $\widehat{r}_{h,2}(x) \to \mathbb{E}(\widehat{r}_{h,2}(x))$, *in probability*
(ii) $\widehat{r}_{h,1}(x) \to 1$, *in probability*

The main results are stated in the following theorems.

Theorem 3 *Under conditions of Lemma 2, we have for all $x \in H$*

$$\widehat{r}_h(x) \to r(x), \text{ in probability, as } n \to +\infty$$

The uniform convergence in probability of the estimator $\widehat{r}_h(x)$ is obtained by the following result.

Theorem 4 *If condition (2) is replaced by condition (3) in Theorem 3 and if we assume that*

$$\forall \epsilon_1 > 0, \exists c_8 > 0 \text{ and } \epsilon < \epsilon_1 \text{ such that } \int_0^{\epsilon} \varphi(u)\, du > c_8\, \epsilon\, \varphi(\epsilon), \tag{9}$$

and for some compact set S of H there exist $\alpha > 0$ such that : $S \subset \cup_{l=1}^{l=l_n} \mathcal{B}_l$ where $\mathcal{B}_l, l = 1, \ldots, l_n$ are balls of the functional space H, all having the same radius t_n with $l_n = t_n^{-\alpha}$. (10)

Then we obtain uniformly on the compact set S that

$$\sup_{x\in S} |\widehat{r}_h(x) - r(x)| \to 0, \text{ in probability, as } n \to +\infty \tag{11}$$

ACCEPTED MANUSCRIPT

It should be noticed that these results showed the convergence of the estimators under long range. This result may be considered as an extension of the results given by Ferraty and Vieu (2004) for short range dependence (under strong mixing conditions) where rates of convergence are also obtained. However, the rates of convergence for long range dependence are not obtained by these results and are still an open problem in the infinite dimensional case. On the other hand, it may be possible to obtain rates of convergence by using adapted large deviation inequalities (see Estévez and Vieu, 2003), but nothing guarantees their optimality.

4 Proofs

In what follows, η_i's for $i = 1, 2, 3$ denote some real positive constants.

Proof of Lemma 1. For a sample of identically distributed pairs (X_i, Y_i) for $i = 1, \ldots, n$, we have

$$\begin{aligned} r(x) - I\!E(\widehat{r}_{h,2}(x)) &= r(x) - \frac{1}{n\, I\!E(\Delta_1(x))} \sum_{i=1}^{n} I\!E\left(Y_i\, \Delta_i(x)\right) \\ &= \frac{I\!E((r(x) - r(X_1))\, \Delta_1(x))}{I\!E\left(\Delta_1(x)\right)} \end{aligned}$$

Because the support of the kernel function K is $(0, 1)$, and from the definition of $\Delta_1(x)$, we have

$$|r(X_1) - r(x)|\, \Delta_1(x) \leq \sup_{t \in \mathcal{B}(x,h)} |r(t) - r(x)|\ \Delta_1(x)$$

then it follows from condition (6) that

$$\begin{aligned} I\!E(|r(X_1) - r(x)|\, \Delta_1(x)) &\leq c_7 \sup_{t \in \mathcal{B}(x,h)} d(x,t)^{\beta}\, I\!E(\Delta_1(x)) \\ &\leq c_7\, h^{\beta}\, I\!E(\Delta_1(x)) \end{aligned} \tag{12}$$

and as $\lim_{n \to +\infty} h = 0$, we obtain the result of Lemma 1. □

Proof of Lemma 2. The proof of the result *(i)* is based on the application of Chebychev's inequality. Indeed, because of:

$$\text{for } \epsilon > 0,\ I\!P\left(|\widehat{r}_{h,2}(x) - I\!E(\widehat{r}_{h,2}(x))| > \epsilon\right) \leq \frac{\mathrm{var}(\widehat{r}_{h,2}(x))}{\epsilon^2}$$

ACCEPTED MANUSCRIPT

we need to show that

$$\mathrm{var}(\widehat{r}_{h,2}(x)) \to 0 \text{ as } n \to +\infty$$

For that we have:

$$\begin{aligned}\mathrm{var}\left(\widehat{r}_{h,2}(x)\right) &= \frac{1}{n\,\mathbb{E}^2(\Delta_1(x))}\mathrm{var}\left(Y_1\,\Delta_1(x)\right)\\ &\quad+\frac{1}{n^2\,\mathbb{E}^2(\Delta_1(x))}\sum_{i\neq j}\mathrm{cov}\left(Y_i\,\Delta_i(x), Y_j\,\Delta_j(x)\right)\\ &:= I_{1,n} + I_{2,n}\end{aligned}$$

For the first term, from (12) we have that

$$\begin{aligned}\mathbb{E}(\Delta_1(x)\,Y_1) &= \mathbb{E}\left(\mathbb{E}(Y_1\,\Delta_1(x)|X_1)\right)\\ &= \mathbb{E}((r(X_1)-r(x))\,\Delta_1(x)) + r(x)\mathbb{E}(\Delta_1(x))\\ &= \left(r(x)+O\left(h^\beta\right)\right)\,\mathbb{E}(\Delta_1(x))\end{aligned}$$

then

$$I_{1,n} = \frac{1}{n\,\mathbb{E}^2(\Delta_1(x))}\,\left(\mathbb{E}(\Delta_1^2(x)\,\mathbb{E}(Y_1^2|X_1)) - (r(x)+O(h^\beta))^2\,\mathbb{E}^2(\Delta_1(x))\right)$$

From condition (7), we have

$$I_{1,n} \leq \frac{1}{n}\left(\frac{\mathbb{E}(\Delta_1^2(x))}{\mathbb{E}^2(\Delta_1(x))}(\sup_{t\in B(x,h)}|\sigma(t)-\sigma(x)| + \sup_{t\in B(x,h)}|\sigma(t)|) - (r(x)+O(h^\beta))^2\right)$$

The lower and upper bounds of conditions (2) and (4) give

$$\mathbb{E}(\Delta_1(x)) \geq c_1\,\mathbb{P}\left(X\in\mathcal{B}(x,h)\right) \geq c_1\,c_5\,\varphi(h)$$

and likewise $\mathbb{E}(\Delta_1^2(x)) \leq c_2^2\,c_6\,\varphi(h)$, and using condition (5) we deduce that

$$I_{1,n} \leq \frac{1}{n\,\varphi(h)}\,\left(\eta_1 - \varphi(h)\,\left(r(x)+O\left(h^\beta\right)\right)^2\right) \leq \frac{\eta_1}{n\,\varphi(h)} + o\left(\frac{1}{n\,\varphi(h)}\right)$$

so that

$$I_{1,n} = O\left(\frac{1}{n\,\varphi(h)}\right) \tag{13}$$

The second term $I_{2,n}$, can be directly written as

ACCEPTED MANUSCRIPT

$$I_{2,n} = O\left(\frac{1}{n^2\varphi^2(h)} \sum_{i\neq j} \mathrm{cov}\left(\Gamma_i(x), \Gamma_j(x)\right)\right)$$
$$= O\left(\frac{1}{n^2\varphi^2(h)} G(x)\right)$$

Finally, using the condition (8), we obtain

$$var(\widehat{r}_{h,2}(x)) \to 0 \text{ as } n \to +\infty$$

The result *(ii)* can be derived similarly as *(i)* by taking $Y_i = 1$ and using the fact that $I\!E(\widehat{r}_{h,1}(x)) = 1$. □

Proof of Theorem 3. The result follows by a direct application of the previous Lemmas 1 and 2 together with the following decomposition

$$\widehat{r}_h(x) - r(x) = \frac{1}{\widehat{r}_{h,1}(x)} \left[(\widehat{r}_{h,2}(x) - I\!E(\widehat{r}_{h,2}(x))) - (r(x) - I\!E(\widehat{r}_{h,2}(x))\right]$$
$$- \frac{r(x)}{\widehat{r}_{h,1}(x)} \left(\widehat{r}_{h,1}(x) - 1\right) \tag{14}$$

and, it remains to show that, there exists $\theta > 0$ such that

$$I\!P\left(|\widehat{r}_{h,1}(x)| \leq \theta\right) \to 0. \tag{15}$$

For this aim, since $\widehat{r}_{h,1}(x)$ converges to 1, in probability as $n \to +\infty$, we have

$$\forall \epsilon > 0, I\!P\left(|\widehat{r}_{h,1}(x) - 1| > \epsilon\right) \to 0 \tag{16}$$

and we can notice that

$$I\!P\left[|\widehat{r}_{h,1}(x)| \leq \frac{1}{2}\right] \leq I\!P\left[|\widehat{r}_{h,1}(x) - 1| > \frac{1}{2}\right]$$

By letting $\theta = \epsilon = 1/2$, the proof of (15) is achieved. □

Proof of Theorem 4. If assumptions (3) and (9) are satisfied then, for h small enough (see Lemma 4.4, page 44 in Ferraty and Vieu, 2006)

$$\text{for } x \in H,\ \eta_2\varphi(h) \leq I\!E\left(K\left(\frac{d(x,X)}{h}\right)\right) \leq \eta_3\varphi(h), \tag{17}$$

ACCEPTED MANUSCRIPT

In order to prove (17), we can write

$$\mathbb{E}\left(K\left(\frac{d(x,X_i)}{h}\right)\right) = \int_0^1 K(t)\, d\mathbb{P}^{\frac{d(x,X_i)}{h}}(t)$$

where $d\mathbb{P}^{\frac{d(x,X_i)}{h}}$ denotes the image measure of the real random variable $d(x,X_i)/h$. Now, since the first derivative of K exists, we have $K(t) = K(0) + \int_0^t K'(u)du$, which implies, by applying the Fubini's Theorem, that

$$\begin{aligned}\mathbb{E}\left(K\left(\frac{d(x,X_i)}{h}\right)\right) &= \int_0^1 K(0)\, d\mathbb{P}^{\frac{d(x,X_i)}{h}}(t) + \int_0^1 \left(\int_0^t K'(u)du\right) d\mathbb{P}^{\frac{d(x,X_i)}{h}}(t) \\ &= K(0)\,\varphi(h) + \int_0^1 K'(u)\, \mathbb{P}\left(u \leq \frac{d(x,X_i)}{h} \leq 1\right) du\end{aligned}$$

and the fact that $K(1) = 0$ allows to write

$$\mathbb{E}\left(K\left(\frac{d(x,X_i)}{h}\right)\right) = -\int_0^1 K'(u)\,\varphi(hu)du$$

It suffices to use hypothesis (3) to show that, for $h < \varepsilon_1$ and with $\eta_2 = -c_1\, c_{10}$

$$\mathbb{E}\left(K\left(\frac{d(x,X_i)}{h}\right)\right) \geq \eta_2\,\varphi(h)$$

Concerning the upper bound, it suffices to remark that K is bounded with support $(0,1)$ and uses hypothesis (3) by putting $\eta_3 = \sup_{t\in(0,1)} K(t)$.
In order to complete the proof of this Theorem, we use first the decomposition (14) and then (16). We notice that

$$\begin{aligned}&\sup_{x\in S} |\widehat{r}_h(x) - r(x)| \\ &\leq \sup_{x\in S} |\widehat{r}_h(x) - \mathbb{E}(\widehat{r}_h(x))| + \sup_{x\in S} |\mathbb{E}(\widehat{r}_h(x)) - r(x)| \\ &\leq \sup_{x\in S} |\widehat{r}_{h,2}(x) - r(x)| + \sup_{x\in S} |\widehat{r}_{h,2}(x) - \mathbb{E}(\widehat{r}_{h,2}(x)| + \sup_{x\in S} |\widehat{r}_{h,1}(x) - 1|\end{aligned}$$

According to Lemma 1, we claim immediately that

$$\sup_{x\in S} |\widehat{r}_{h,2}(x) - r(x)| = O\left(h^{\beta}\right) \tag{18}$$

On the other hand, from condition (10), it suffices to look at the uniform convergence on the closest center to x among the balls centers $\{c_1, \ldots, c_{l_n}\}$,

ACCEPTED MANUSCRIPT

$c_{j(x)} = \arg\min_{c_j \in \{c_1,\ldots,c_{l_n}\}} d(c_j, x)$, (Cf pages 75-76 in Ferraty and Vieu (2006)). We have

$$\begin{aligned}
&\mathbb{P}\left(\sup_{x\in S} |\widehat{r}_{h,2}(c_{j(x)}) - \mathbb{E}(\widehat{r}_{h,2}(c_{j(x)}))| > \epsilon\right) \\
&= \mathbb{P}\left(\max_{j=1,\ldots,l_n} |\widehat{r}_{h,2}(c_j) - \mathbb{E}(\widehat{r}_{h,2}(c_j))| > \epsilon\right) \\
&\leq l_n \max_{j=1,\ldots,l_n} \mathbb{P}\left(|\widehat{r}_{h,2}(c_j) - \mathbb{E}(\widehat{r}_{h,2}(c_j))| > \epsilon\right)
\end{aligned}$$

Choosing $l_n = t_n^{-\alpha}$ with $t_n \sim n^{-\zeta}, \zeta > 0$, and using the results of Lemma 2, we obtain in probability

$$\lim_{n\to\infty} \sup_{x\in S} |\widehat{r}_{h,2}(x) - \mathbb{E}(\widehat{r}_{h,2}(x))| = 0 \text{ and } \lim_{n\to\infty} \sup_{x\in S} |\widehat{r}_{h,1}(x) - 1| = 0 \tag{19}$$

From (18) and (19), we claim the final result (11). □

Acknowledgments

The authors wish to express their gratitude to Frédéric Ferraty. His knowledge on nonparametric statistics for functional variables was of great help, in particular, for proofreading and improving the results of this paper.

References

[1] K. Benhenni, F. Ferraty, M. Rachdi and P. Vieu, *Local smoothing regression with functional data*, Computational Statistics, **22** (2007), no. 3, 353–369.

[2] J. Beran, *Statistics for long-memory processes*, Chapman and Hall, New York, 1994.

[3] D. Bosq, *Linear processes in function space: theory and application*, Lecture Notes in Statistics, 149, Springer-Verlag, New-York, 2000.

[4] S. A. Chobanyan, V. I. Tarieladze, and N. N. Vakashnia, *Probability distributions on banach spaces*, D. Reidel Publishing Co, Boston, 1987.

[5] D. R. Cox, *Long-range dependence*, A review. In Statistics: An Appraisal (H. A. David and H. T. David, eds.). Iowa State Univ. Press (1984), 55–73.

[6] S. Csörgö and J. Mielniczuk, *Nonparametric regression under long-range normal errors*, Ann. Stat. **23** (1996), no. 3, 1000–1014.

[7] G. Estévez and P. Vieu, *Nonparametric estimation under long memory dependence*, J. Nonparametric Stat. **15** (2003), no. 4-5, 535–551.

ACCEPTED MANUSCRIPT

[8] F. Ferraty and P. Vieu, *Nonparametric analysis for functional data, with application in regression method*, J. Nonparametric Stat. **16** (2004), no. 1-2, 111–125.

[9] F. Ferraty and P. Vieu, *Nonparametric functional data analysis: Theory and practice*, Springer-Verlag, 2006.

[10] P. Hall and J. D. Hart, *Nonparametric regression with long range dependence*, J. Stoch. Proc. and Their Appli. **36** (1990), 339–351.

[11] J. Hidalgo, *Nonparametric estimation with strongly dependent multivariate tiem series*, J. Time Ser. Anal. **18** (1997), no. 2, 95–122.

[12] B.B. Mandelbrot and J.W. Van Ness , *Fractional Brownian motions, fractional noises and applications*, SIAM Rev. **10** (1968), 422–437.

[13] E. Masry, *Local linear regression estimation under long-range dependence: Strong consistency and rates*, IEEE Trans. on Inform. Theory **47** (2001), no. 7, 2863–2875.

[14] H-G. Müller and U. Stadmüller, *Generalized functional linear models*, Ann. Stat. **33** (2005), 774–805.

[15] M. Rachdi and P. Vieu, *Sélection automatique du paramètre de lissage pour l'estimation non paramétrique de la régression pour des données fonctionnelles*, C. R. Acad. Sci. Paris, Ser. I **341** (2005), 365–368.

[16] M. Rachdi and P. Vieu, *Nonparametric regression estimation for functional data: automatic smoothing parameter selection*, Journal of Statistical Planning and Inference **137** (2007), no. 9, 2784–2801.

[17] J. Ramsay and B. Silverman, *Functional data analysis*, Springer, New York, 1997.

[18] J. Ramsay and B. Silverman, , *Applied functional data analysis: Methods and case studies*, Springer, New York, 2002.

[19] D. L. Turcotte, *Fractal theory and the estimation of extreme floods*, J. Res. Nat. Inst. Standards Tech. **99** (1994), 377–389.

ACCEPTED MANUSCRIPT