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Erratum to "Constrained estimators of treatment parameters in semiparametric models" [Statist. Probab. Lett. 77 (2007) 914-919]

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# Erratum <br> Constrained Estimators of Treatment Parameters in Semiparametric Models <br> [Statist. Probab. Lett 77 (2007) 914-919] 

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#### Abstract

In [Przystalski and Krajewski, 2007. Constrained estimators of treatment parameters in semiparametric models, Statist. Probab. Lett. 77, 914-919] we presented a method of estimation of treatment effects in a semiparametric model with one smoothing term under additional conditions on their linear functions and its application to hypothesis testing. In this note we will present corrected version of the proof of theorem 3.1.


Key words: cubic smoothing splines, estimation, hypothesis testing, loess.

This note is aimed at correcting a minor mistake in the proof of Theorem 3.1 in Przystalski and Krajewski (2007). The notation is the same as that in Przystalski and Krajewski (2007). Here is corrected version.
Theorem 3.1. If the smoother matrix $\mathbf{S}$ is symmetric then the estimate of $\boldsymbol{\beta}$ in model (1) obtained by using the penalized least squares criterion (4), satisfying condition (6), is equal to

$$
\boldsymbol{b}=\widehat{\boldsymbol{\beta}}+\mathbf{D}^{-1} \mathbf{R}^{\prime}\left\{\mathbf{R D}^{-1} \mathbf{R}^{\prime}\right\}^{-1}(\mathbf{r}-\mathbf{R} \widehat{\boldsymbol{\beta}})
$$

and its covariance matrix to $\operatorname{Var}(\boldsymbol{b})=\sigma^{2} \mathbf{C C}^{\prime}$, where $\mathbf{D}=\mathbf{X}^{\prime}(\mathbf{I}-\mathbf{S}) \mathbf{X}$ and $\mathbf{C}=\left(\mathbf{I}-\mathbf{D}^{-1} \mathbf{R}^{\prime}\left\{\mathbf{R D}^{-1} \mathbf{R}^{\prime}\right\}^{-1} \mathbf{R}\right) \mathbf{D}^{-1} \mathbf{X}^{\prime}(\mathbf{I}-\mathbf{S})$.
Proof: Let us assume that the smoother matrix $\mathbf{S}$ is symmetric and we want to find an estimate of $\boldsymbol{\beta}$ in model (1) satisfying condition (6) using the criterion (4). For this purpose we construct the function

$$
F(\boldsymbol{\beta}, \boldsymbol{f}, \boldsymbol{\lambda})=\|\boldsymbol{y}-\mathbf{X} \boldsymbol{\beta}-\boldsymbol{f}\|^{2}+\boldsymbol{f}^{\prime}\left(\mathbf{S}^{-1}-\mathbf{I}\right) \boldsymbol{f}-2 \boldsymbol{\lambda}^{\prime}(\mathbf{R} \boldsymbol{\beta}-\mathbf{r}) .
$$

By differentiating function $F$ with respect to $\boldsymbol{\beta}, \boldsymbol{f}$ and $\boldsymbol{\lambda}$ we obtain the following system
of vector equations

$$
\begin{aligned}
& \frac{\partial F(\boldsymbol{\beta}, \boldsymbol{f}, \boldsymbol{\lambda})}{\partial \boldsymbol{\beta}}=-2 \mathbf{X}^{\prime}(\boldsymbol{y}-\mathbf{X} \boldsymbol{\beta}-\boldsymbol{f})-2 \mathbf{R}^{\prime} \boldsymbol{\lambda}=\mathbf{0} \\
& \frac{\partial F(\boldsymbol{\beta}, \boldsymbol{f}, \boldsymbol{\lambda})}{\partial \mathbf{f}}=-2(\boldsymbol{y}-\mathbf{X} \boldsymbol{\beta}-\boldsymbol{f})+2\left(\mathbf{S}^{-1}-\mathbf{I}\right) \boldsymbol{f}=\mathbf{0} \\
& \frac{\partial F(\boldsymbol{\beta}, \boldsymbol{f}, \boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}}=\mathbf{R} \boldsymbol{\beta}-\mathbf{r}=\mathbf{0}
\end{aligned}
$$

If we get $\boldsymbol{f}$ from the second equation as

$$
\boldsymbol{f}=\mathbf{S}(\boldsymbol{y}-\mathbf{X} \boldsymbol{\beta})
$$

and we put it into the first equation, we obtain

$$
\mathbf{X}^{\prime}[\boldsymbol{y}-\mathbf{X} \boldsymbol{\beta}-\mathbf{S}(\boldsymbol{y}-\mathbf{X} \boldsymbol{\beta})]+\mathbf{R}^{\prime} \boldsymbol{\lambda}=\mathbf{0}
$$

Solving this equation with respect to $\boldsymbol{\beta}$ we get that

$$
\boldsymbol{\beta}=\left\{\mathbf{X}^{\prime}(\mathbf{I}-\mathbf{S}) \mathbf{X}\right\}^{-1}\left(\mathbf{X}^{\prime}(\mathbf{I}-\mathbf{S}) \boldsymbol{y}+\mathbf{R}^{\prime} \boldsymbol{\lambda}\right) .
$$

Such $\boldsymbol{\beta}$ we put into the third equation and we have

$$
\mathbf{R}\left[\left\{\mathbf{X}^{\prime}(\mathbf{I}-\mathbf{S}) \mathbf{X}\right\}^{-1}\left(\mathbf{X}^{\prime}(\mathbf{I}-\mathbf{S}) \boldsymbol{y}+\mathbf{R}^{\prime} \boldsymbol{\lambda}\right)\right]=\mathbf{r}
$$

If we put $\mathbf{D}=\mathbf{X}^{\prime}(\mathbf{I}-\mathbf{S}) \mathbf{X}$, and notice that $\left\{\mathbf{X}^{\prime}(\mathbf{I}-\mathbf{S}) \mathbf{X}\right\}^{-1} \mathbf{X}^{\prime}(\mathbf{I}-\mathbf{S}) \boldsymbol{y}$ is equal to $\widehat{\boldsymbol{\beta}}$ in (2), we get the following equality

$$
\mathbf{R} \widehat{\boldsymbol{\beta}}+\mathbf{R D}^{-1} \mathbf{R}^{\prime} \boldsymbol{\lambda}=\mathbf{r}
$$

Because $\mathbf{R D}^{-1} \mathbf{R}^{\prime}>0$, we may write the optimal $\boldsymbol{\lambda}$ as

$$
\widehat{\boldsymbol{\lambda}}=\left\{\mathbf{R D}^{-1} \mathbf{R}^{\prime}\right\}^{-1}(\mathbf{r}-\mathbf{R} \widehat{\boldsymbol{\beta}})
$$

Finally, the estimate of $\boldsymbol{\beta}$ may be written as

$$
\boldsymbol{b}=\widehat{\boldsymbol{\beta}}+\mathbf{D}^{-1} \mathbf{R}^{\prime}\left\{\mathbf{R D}^{-1} \mathbf{R}^{\prime}\right\}^{-1}(\mathbf{r}-\mathbf{R} \widehat{\boldsymbol{\beta}})
$$

## References

Przystalski M., Krajewski P. (2007), Constrained estimators of treatment parameters in semiparametric models, Statist. Probab. Lett 77, 914-919.

