Bedload Transport. Part 1: Two-Phase Model and 3D Numerical Implementation

Julien Chauchat[1], Mickael Pailha[2] , Pascale Aussillous[3],
Marc Médale[3] and Elisabeth Guazzelli[3]

[1] LEGI CNRS UMR 5519, Université Joseph Fourier, INPG, Grenoble, France
[2] Manchester Center for Nonlinear Dynamics and School of Mathematics, University of Manchester, United Kingdom

[3] IUSTI CNRS UMR 6595 - Aix-Marseille Université (U1), France
julien.chauchat@polytech.univ-mrs.fr and elisabeth.guazzelli@polytech.univ-mrs.fr

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We have developed a three dimensional numerical model (Chauchat and Médale, 2010) based on the two-phase modeling having a Newtonian rheology for the fluid phase and Coulomb-type friction for the particulate phase which has been developed by Ouriemi et al. (2009a) to study bedload transport in pipe flows. The governing equations are discretized by a finite element scheme and a penalisation method is introduced to cope with the incompressibility constraint. A regularisation technique is used to deal with the visco-plastic behaviour of the granular phase. We have performed three-dimensional computations for bedload transport in rectangular cross-section duct when the bed interface remains fixed. This numerical model which captures the complex coupling between the granular media and the fluid should enable a better understanding of the sediment transport mechanisms in these duct flows (Ouriemi et al., 2009b). The geometry corresponds to the one used in Pailha et al. (2011) where the authors investigate experimentally the behaviour of the flowing granular layer. The presented model and numerical results will be used for a detailed comparison in a near future.

I Introduction

The transport of sediment or more generally the transport of particles by a fluid flow is a problem of major importance in geophysical flows such as coastal or river morphodynamic or in industrial flows with the hydrate or sand issues in oil production and granular transport in food or pharmaceutical industries. This problem has been extensively studied in the literature since the middle of the twentieth century but poorly understood actually (Einstein, 1942; Meyer-Peter and Muller, 1948; Bagnold, 1956; Yalin, 1963).

Recently, Ouriemi et al. (2009a) have proposed a two-phase model describing the bed-load transport in laminar flows that allows to incorporate more physics than in previous modelling based on particle flux or erosion deposition approaches. This two-phase model is based on a Newtonian rheology for the fluid phase and a frictional rheology for the particulate phase Forterre and Pouliquen (2008) while the fluid-particle interaction is assumed to follow a Darcy law. This approach allows to predict the threshold of motion for the particle phase and give a description of the flow inside the mobile granular layer. Away from the threshold of motion, a simple analytical model for the particle flux is obtained which gives a quite satisfactory description of experimental observations of bed-load transport in pipe flows (Ouriemi et al., 2009a; Pailha et al., 2011).

Based on this theoretical model we have developed a 3D Finite Element numerical model that allows to simulate bed-load transport in 2D or 3D configurations (Chauchat and Médale, 2010). It is restricted to the cases where the granular bed does not change its shape in the course of time. The viscoplastic behaviour of the granular phase characterized by the existence of a yield stress is implemented using a regularisation technique. This numerical model is used here to investigate the influence of the granular rheology, Coulomb or $\mu(I)$ (Forterre and Pouliquen, 2008), on the mobile granular layer characteristics (velocity profiles, thickness of the layer and particle flux) for a wide range of fluid flow rate.

II Two-phase model and Numerical Implementation

The present model is based on Jackson (2000) averaged equations using the closures developed by Ouriemi et al. (2009a). These equations are summarized hereafter in dimensionless form using the following scaling: the length are scaled by H, the channel height, and the stresses are scaled by $\Delta\rho g H$, and therefore the time is scaled by $\eta/\Delta\rho g H$ where $\Delta\rho = \rho_p - \rho_f$. The general problem is expressed in terms of the solid volume fraction ϕ, the mixture velocity $\overrightarrow{u^m}$ and the particulate velocity $\overrightarrow{u^p}$. In this paper we only present the mixed-fluid model equation (1) which is the sum of the fluid and particulate phases momentum equations assuming no-slip velocity between the fluid and the particles ($\overrightarrow{u_p} = \overrightarrow{u_m}$) (Chauchat and Médale, 2010).

$$\begin{aligned} \nabla . \left(\overrightarrow{u^m}\right) &= 0 \\ Ga\frac{H^3}{d^3}(1+R_\rho)\frac{D\overrightarrow{u^m}}{Dt} &= -\nabla p^f - \nabla p^p + \frac{\rho_m \overrightarrow{g}}{\Delta\rho \parallel \overrightarrow{g} \parallel} + \nabla . \left(\frac{\eta_e + \eta_p}{\eta}\overline{\overline{\dot{\gamma}^m}}\right) \end{aligned} \tag{1}$$

In these equations, $\overline{\overline{\dot{\gamma}^m}} = \nabla\overrightarrow{u^m} + (\nabla\overrightarrow{u^m})^T$, $R_\rho = \rho_f/\rho_p$ represents the density ratio and $Ga = d^3\rho_f\Delta\rho g/\eta^2$ is the Galileo number where d is the particle diameter. The Galileo number is a Reynolds number based on the settling velocity of particles. p^f and p^p represent the fluid and particulate pressure respectively. The latter is assumed to be hydrostatic and then its vertical gradient is equal to the apparent weight of the particles. To close these equations we need to prescribe the effective fluid viscosity $\eta_e = \eta(1 + 5/2\phi)$ (Einstein, 1906). Following (Jop et al., 2006), the frictional stress can be written as $\tau^p = \eta_p\overline{\overline{\dot{\gamma}^p}}$, with $\eta_p = \mu(I)p^p/ \parallel \overline{\overline{\dot{\gamma}^p}} \parallel$ where the friction coefficient is expressed as $\mu(I) = \mu_s + (\mu_2 - \mu_s) / (I_0/I + 1)$ as a function of the inertial number $I = \parallel \overline{\overline{\dot{\gamma}^p}} \parallel \eta_f/(\alpha\, p^p)$ (Cassar et al., 2005; Forterre and Pouliquen, 2008). The parameter α is the linked to the permeability K which is defined following Kozeny-Carman $K = \frac{(1-\phi)^3}{k_{CK}\phi^2}\, d^2 = \alpha\, d^2$ with $k_{CK} \approx 180$ (Happel and Brenner, 1973). The coefficient μ_s corresponds to the static friction and μ_2 to the dynamical friction whereas I_0 is an empirical parameter that have been estimated to $I_0 = 1$ by Cassar et al. (2005). This formulation of the frictional viscosity is based on the $\mu(I)$ rheology but we can assume a simpler rheology by considering a constant friction coefficient μ_s corresponding to a Coulomb rheology therefore $\eta_p = \mu_s p^p/ \parallel \overline{\overline{\dot{\gamma}^p}} \parallel$.

The main issue for the numerical solution of these equations comes from the divergence of the particulate viscosity as the particulate shear rate tends toward zero (*i.e.* in the static zone). The basic idea to overcome this issue consists in regularizing the viscosity by adding a small quantity (λ) to the denominator of the particulate viscosity $\eta_p = \mu_s p^p/(\parallel \overline{\overline{\dot{\gamma}^p}} \parallel + \lambda)$ then the divergence is controlled by this parameter and the viscosity is kept finite. In other words, the static zone in the frictional rheology is replaced by a very viscous one. We have analysed the influence of the parameter λ on the model solution using a Coulomb rheology in Chauchat and Médale (2010), the reader is referred to this article for more details. It is possible to regularize the $\mu(I)$ rheology using the same technique:

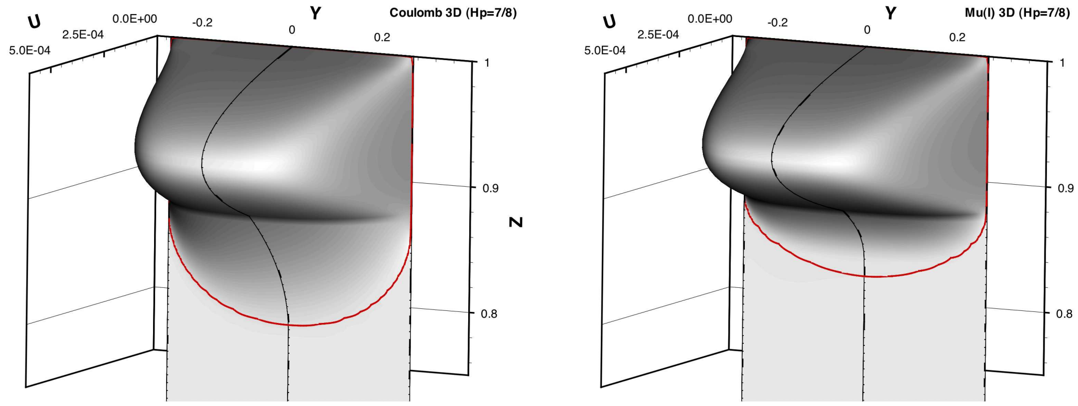
$$\eta_p = \frac{\mu_s p^p}{\parallel \overline{\overline{\dot{\gamma}^p}} \parallel + \lambda} + \frac{(\mu_2 - \mu_s)p^p}{\lambda + \parallel \overline{\overline{\dot{\gamma}^p}} \parallel + I_0\, \alpha\, p^p/\eta_f}$$

The present numerical model results from the discretisation by the standard Finite Element Method of the variational formulation associated with equation (1). The non-linearities in the governing equations are solved by a Newton-Raphson algorithm by taking the first variation of the variational formulation. In our implementation, we use piecewise quadratic polynomial approximation for the velocity and piecewise linear discontinuous approximation for the pressure. In the computations, we have employed a 27-nodes hexahedra element (H27) for the velocities.

The incompressibility constraint is solved by a penalisation method (penalty parameter set to 10^9).

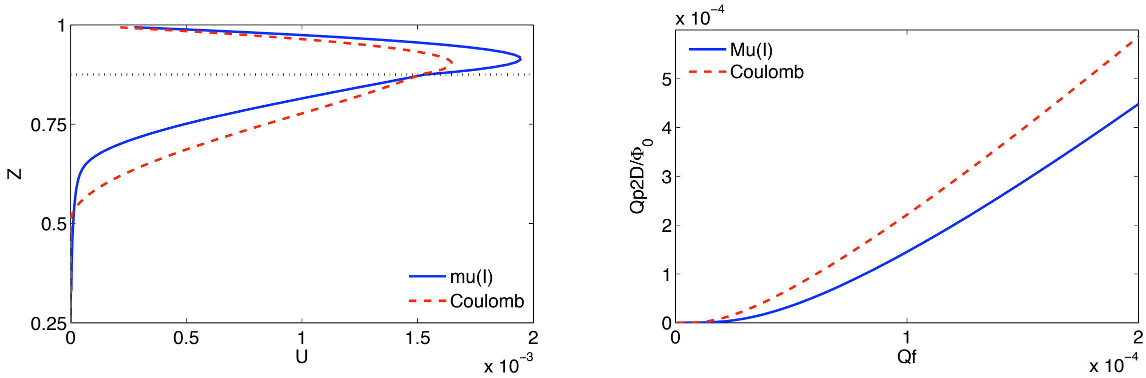
III Results and discussion

We present the results of the previous two-phase numerical model applied to the flow of a Newtonian fluid over a granular bed. The geometry used for the simulations is the same as in Pailha et al. (2011). This is a rectangular duct of aspect ratio $W/H = 0.538$ and fulfilled with a fluid-particles mixture at $\phi = 0.55$ in the lower part ($H_p = 7/8$) and with pure fluid ($\phi = 0$) in the upper part of the duct. The regularisation parameter is set to $\lambda = 10^{-6}\ s^{-1}$ and we solved by FEM the mixed-fluid formulation of the two-phase flow model for a 3x40x80 mesh with a requested absolute residual lower than 10^{-11} per degree of freedom. Since the vertical plane at the center of the duct ($y = 0$) is a symmetry plane of the problem we only solve for one half of the domain with appropriate symmetry conditions on the plane ($y = 0$).



(a) Coulomb friction (b) $\mu(I)$ friction

Figure 1: Velocity profile obtained by numerical simulations for a fluid flow rate $Q_f = 1.79\ 10^{-5}\ m^3/s$ with a) the Coulomb friction rheology ($dp/dx = -0.163$) and b) the $\mu(I)$ rheology ($dp/dx = -0.2$).



(a) Velocity profile (b) Particle flux

Figure 2: Comparison of the numerical results for the experimental 3D configuration using Coulomb friction and $\mu(I)$ friction rheology for a) the velocity profiles for both rheology at a flow rate $Q_f = 1e-4$ and b) the particle flux Q_p^{2D}/ϕ_0 versus the fluid flow rate.

Figure 1 shows the mixture velocity profile (u^m) in a cross section of the duct. The red thick solid line represents the position of the "static granular bed". The fluid and the mixture are sheared in both z and y directions inducing an increase in the friction compared with the two-dimensional case. Due to this complex shear the use of a three-dimensional model is needed. This figure also shows the influence of the granular rheology (Coulomb or $\mu(I)$) on the shape of the mobile granular layer for a given fluid flow rate (Q_f) . In particular, we observe that a greater longitudinal pressure gradient is needed to reach the same flow rate for the $\mu(I)$ rheology than for the Coulomb one. The thickness of the flowing granular layer is thinner with the $\mu(I)$ rheology compared to the one predicted with a Coulomb rheology. This is due to the increased frictional viscosity in the $\mu(I)$ rheology therefore the driving fluid shear stress at the bed interface is dissipated on a thinner layer in this latter case.

We have conducted a parametric study to analyse the influence of the driving pressure drop on the particulate flow rate thanks to an Arc Length Continuation algorithm. We have validated this algorithm by comparison with the analytical solution of Ouriemi et al. (2009a) for the thickness of the mobile granular layer $H_p - H_c$ and the particle flux Q_p in the 2D case (See Chauchat et al. (2010b) for details). Figure (2a) shows the velocity profiles for a given flow rate ($Q_f = 1e-4$). In the $\mu(I)$ case the vertical velocity gradient is very small in a rather thick layer above the fixed bed compared with the Coulomb case. As a consequence, for a given flow rate the particle flux predicted using a Coulomb rheology is greater than the one predicted using the $\mu(I)$ rheology. Figure 2b illustrates the influence of the granular rheology on the particle flux Q_p^{2D}/ϕ_0. At a non-dimensionalised flow rate of $Q_f = 10^{-4}$ a decrease of approximately 65% of the the 2D particle flux is observed with the $\mu(I)$ rheology. This sensitivity of the particle flux to the rheology is one of the important question that we would like to answer in the joint theoretical, numerical, and experimental investigation presented in Pailha et al. (2011).

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