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Drag and lift forces on clean spherical air bubbles in a rotating flow

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Abstract

We study here the motion of a spherical gas bubble in a solid body rotating flow of clean silicon oil, to avoid contamination of interface. From the measurement of its equilibrium position, lift and drag forces are determined. Five different silicon oils have been used, providing five different viscosities and Morton numbers. For spherical bubbles the drag coefficient is at first order the same as the one of clean spherical bubbles in a uniform flow. For clean spherical bubbles, the measured lift coefficient proves to be in good agreement with the expression of Legendre and Magnaudet based on their numerical simulations. We propose however a correlation which improves significantly the prediction of the lift coefficient for small Reynolds numbers. In contrast to the case of contaminated bubbles, we observed no dependence of the lift coefficient on the Rossby number.

Introduction

The lift force exerted by a non-uniform flow on a gas bubble is of the utmost importance in the transverse distribution of phases. Several studies have considered the case of the solid body rotation around an horizontal axis (SBRH). Naciri (1992) worked with a rotating tank full of mixtures of water and glycerin. For Re numbers ranging from around 10 to around 100 and slightly deformed contaminated bubbles, he obtained a lift coefficient of the order of 0.2 to 0.3. Magnaudet and Legendre (1998) simulated numerically a fixed clean bubble in a SBRH. The influence of the shear on the drag coefficient was noticed and a law for the lift coefficient has been derived for $Re > 50$. Van Nierop et al. (2007) revisited Naciri experiment. They also worked with mixtures of glycerin and water, hereby with contaminated bubbles. One of the main result of the paper is that for low Re configurations ($Re < 5$) the lift coefficient become negative. Rastello et al. (2007, 2009) worked with the same kind of experiment but with water. The low-Re case was not investigated in this paper and the existence of negative lift at small Re could not be investigated. It has been shown however that in this Reynolds number range, contamination of the interface of the bubbles makes them spinning on themselves, inducing an additional lift component due to Magnus-like effects. The present study also deals with single bubbles in a SBRH. The difference is that silicon oil is used as a fluid, which makes it possible to have clean bubbles, i.e. no contamination by surface active molecules as it is the case for polar liquids such as water. We focus here on the lift and drag coefficients for spherical bubbles, the ellipsoidal case being under investigation. The

parameters that control the bubble behavior in the spherical range are the Reynolds number, the Rossby number and the Morton number. Hence, different sizes of bubbles have been used, different rotation speed for the tank, as well as five different Newtonian oils to vary the viscosity, ranging from 100 to 0,65 centiStokes.

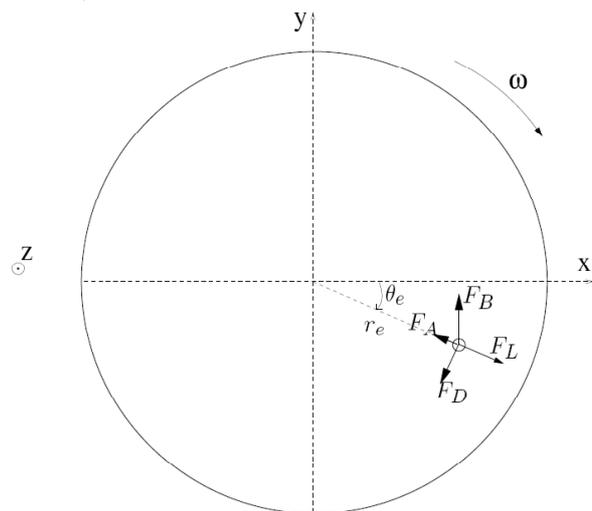


Figure 1 : The forces applied to a bubble at equilibrium at moderate to high Reynolds numbers. F_B is the buoyancy force, F_D , the drag force, F_L the lift force. F_A takes into account the added mass force and pressure gradient effects. In this sketch, the angular velocity ω and equilibrium angle θ_e are negative.

For a spherical bubble, moving relatively to the fluid at a

moderate to large Reynolds number, the dynamical equation commonly adopted to describe bubble motion is (Magnaudet and Eames 2000):

$$\rho_l V_b C_A \frac{d\mathbf{v}}{dt} = -\rho_l V_b \mathbf{g} + \rho_l V_b (C_A + 1) \frac{D\mathbf{U}}{Dt} + \rho_l V_b C_L (\mathbf{U} - \mathbf{v}) \times (\nabla \times \mathbf{U}) + \frac{1}{2} \rho_l C_D A_b |\mathbf{U} - \mathbf{v}| (\mathbf{U} - \mathbf{v}) \quad (2.1)$$

where ρ_l is the liquid density, V_b the volume of the bubble, A_b its projected area, \mathbf{v} bubble's velocity, \mathbf{g} gravity acceleration and \mathbf{U} the velocity of the undisturbed ambient flow taken at the centre of the bubble. The forces considered in this equation are respectively buoyancy \mathbf{F}_B , added mass and pressure gradient \mathbf{F}_A , lift \mathbf{F}_L and drag \mathbf{F}_D . C_A is the added mass coefficient. Its value is 0.5 for spherical bubbles (Magnaudet and Eames 2000). C_L is the lift coefficient and C_D the drag coefficient. In the present experiment the undisturbed flow field \mathbf{U} is a solid body rotating flow at constant angular velocity ω around a horizontal axis (SBRH). In that case all forces balance when the flow is stationary and the bubble comes to an equilibrium position (see figure 1) where $\mathbf{v} = 0$. With $\mathbf{U} = \omega r \mathbf{e}_\theta$, equation (2.1) becomes:

$$C_L = \frac{1}{2} \left(1 + C_A - \frac{g \sin \theta_e}{r_e \omega^2} \right) \quad (2.2)$$

$$C_D = \frac{4}{3} \frac{g d \cos \theta_e}{r_e^2 \omega^2}$$

where (r_e, θ_e) are the polar coordinates (r, θ) of the centre of the bubble at equilibrium and d the diameter of the bubble. Thus, if one measures r_e and θ_e , d and ω , one can calculate the values of C_L and C_D . At low Reynolds numbers the decomposition of equation (2.1) does not hold anymore. The lift force cannot be parametrized as in (2.1) (Saffman 1965, 1968; Legendre and Magnaudet 1997). Also, history terms are becoming important as Reynolds number decreases (Candelier et al. 2004; Wakaba and Balachandar 2005). Subsequently, the use of equations (2.2) to measure the force coefficients is not justified. Moreover, it is not obvious that the bubble still reaches an equilibrium position in that case.

As described in Rastello et al. (2009) three non-dimensional numbers are necessary to describe the flow. The three following numbers are chosen here: $Re = (v-U)d/v = \omega r_e d/v$ the Reynolds number based on bubble's diameter and on the relative velocity; $Ro = (v-U)/\omega d = r_e/d$ the Rossby number that takes into account the effects of rotation; $Mo = g v^4 \rho^3 / \sigma^3$ the Morton number to account for the effects of fluids properties.

Experimental Facility

The device is a cylindrical tank ($\varnothing=10$ cm, $L=10$ cm) rotating around an horizontal axis. The tank is made of Plexiglas (see figure 2). A motor is coupled to the tank, using a toothed belt. For this set of experiments, the range of rotation rates for the tank is $[4 \text{ rad.s}^{-1}, 27 \text{ rad.s}^{-1}]$. A complete description of the flow with and without bubble

can be found in Rastello et al. (2007, 2009). The flow in the absence of bubble is a solid body rotation. The experimental procedure is the following : the temperature (T) of the fluid contained in the tank is first measured using a thermometer. A bubble is then injected in the tank. After that, the tank is set in rotation. The time necessary for the fluid to reach constant angular velocity and for the bubble to be stabilized at its equilibrium position is waited. Then images are taken with a Phantom V4.3 high speed camera. The 2000 images recorded for each run are processed using Matlab.



Figure 2: Picture of the experimental device showing the tank and the high-speed camera.

Results and Discussion

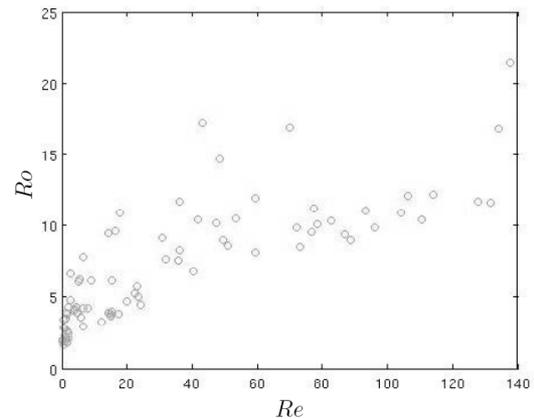


FIGURE 3. Re - Ro working diagram. \circ : Rhodorsil® 47V100, \square : Rhodorsil® 200® 1cst, \triangle : Dow Corning 200® 0.

As usual (Clift et al. 1978), bubbles are considered to be spherical if the minor to major axis ratio lies within 10% of unity. Different points have been tackled to be sure of the accuracy of the foregoing measurements. The spectrum of the equilibrium positions of every bubbles have been checked. No frequencies (eg no wake feedback on the bubble) have been noticed. Indeed, low Ro numbers ($Ro < 6$) also correspond to low Re number (see figure 3). The corresponding bubbles have very small wakes that dissipate rapidly and do not influence them back. Data for bubbles in silicon oil exhibit much less noise than for bubbles in water. The small movements of the bubble (order of magnitude: tenth of a millimeter) around its equilibrium position are only linked to an oscillation with frequency ω . To obtain the equilibrium value of the parameters (C_L , C_D , Re ...) an average over the 2000 points is taken.

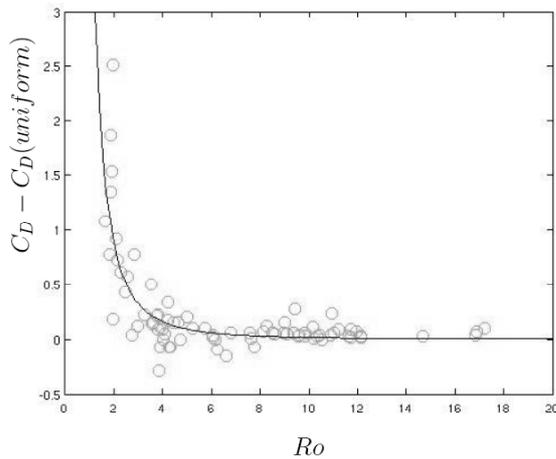


Figure 4 : Drag coefficient versus Rossby number.
Full line : $5/Ro^{5/2}$.

The drag coefficient is first plotted as a function of the Re number in figure 4. The present experimental points fall at first order on the curve governing the drag coefficient for clean bubble in a uniform flow. This contrasts with the previous study performed with similar spherical bubbles, but in water (Rastello et al. 2009). In that case, the experimental points were falling on the curve that describes solid spheres in uniform flow, indicating that bubbles were contaminated. Here, the bubbles appear to have a clean interface. This confirms that, even with small bubbles, the use of silicon oil insure the possibility to keep a sliding condition at bubbles' interfaces. As the flow is not uniform, it is of interest to look at the influence of the Ro number on the drag. For a linear shear flow, Legendre and Magnaudet (1998) find that the drag is increasing with the local shear $S=Ro^{-1}$. The value in uniform flow $C_{D(uniform)}$ must be then corrected to take into account this increase. The simple correlation proposed by these authors is written as

$$C_D = C_{D(uniform)} \times \left(1 + \frac{0.55}{Ro^2}\right)$$

Although the flow is different, our experimental data are compared to this drag law in figure 5. The data exhibit no particular trend when divided by $C_{D(uniform)}$ and clearly, no agreement with the above Ro dependence law is found. This incited us to reconsider the question (tackle the problem) from another point of view. It has been assumed that the drag coefficient could be decomposed at first approximation, as the sum of the drag coefficient of Mei et al. (1994) plus a correction term, function of Re and Ro. For the sake of simplicity, the correction term has been taken equal to a product of two functions, which yields for C_D the following expression

$$C_D(Re, Ro) = C_{D(uniform)} + f(Re)g(Ro)$$

The Ro dependence is then looked for by plotting $C_D(Re, Ro) - C_{D(uniform)}$ as a function of Ro (figure 6). It can be seen that $C_D/C_{D(uniform)}$ in average well follows a $Ro^{5/2}$ law. The deviations from this average law are linked to the Re variations and to some probable dispersion in the data. Given the precision of our measurements, it has not been possible to determine the Re dependence, so the drag coefficient can just be written as

$$C_D(Re, Ro) = C_{D(uniform)} + 5 \times \frac{f_1(Re)}{Ro^{5/2}}$$

with $f_1(Re)$ of the order 1of for the working plan (Re, Ro) investigated: $(Re < 140, Ro < 20)$. Except for low Re numbers ($Ro^{-1} > 0.2$), the influence of Ro on the drag is more important by a factor of 10 that the one obtained for linear shear flows Magnaudet and Legendre (1998). As discussed in Rastello et al. (2009), this result is not really surprising. In a linear shear flow, the increase of Ro^{-1} is only related to an increase of shear, while in a SBRH both the shear and the curvature of the streamlines go up. So, we can expect a dissymmetry of the wake which is more accentuated when Ro^{-1} increases, hence a stronger effect on the drag. Also, it is worthwhile noting that the C_D variation with Ro observed here for the moderate Re numbers is of the same order of magnitude as the one we found for contaminated bubbles in water (Rastello et al. 2009).

The lift coefficient is plotted versus Re number in figure 7. Our data are compared with the results from the numerical simulations of Magnaudet and Legendre (1998). Both fall onto the same curve, unlike in Rastello et al. (2009), where the measured lift coefficients were systematically higher than those calculated. The good agreement between the present data and the simulations is consistent with the previous results on the drag. It confirms that the interfaces are clean, so that the boundary condition is of the "shear-free" type as in the calculation. It brings an additional evidence that the higher values obtained in water by Rastello et al. (2009) with the same experimental procedure are not imputable to some errors in the measurements, but result from the contamination of the interface which modifies the boundary conditions. As shown by visualizations, the higher values are caused by the existence of a thin layer of fluid that spins around the contaminated bubble interface because of the quasi non-slip condition and thus creates a "Magnus" like extra lift component, which superposes to the rotation- induced lift.

Both experimental and numerical points in the figure appear to be well fitted by Magnaudet and Legendre (1998)'s correlation for $Re > 50$:

$$C_L = 0.5 + 1.2 \times Re^{-\frac{1}{3}} - 6.5 \times Re^{-1}$$

However this correlation does not fit the experimental and numerical data below. So, we propose another expression that works well both for large and low Re numbers covered in this study:

$$C_L = 0.5 + 4 \times \left(1 - \frac{6}{5Re^{\frac{1}{6}}}\right) \times \exp(-Re^{\frac{1}{6}})$$

It is noted that this equation asymptotically behaves as equation the former for $Re > 150$. Magnaudet and Legendre (1998) observed no Ro dependence for the lift coefficient. Here also we are not able to exhibit any Ro dependence for the lift coefficient. If there is one, it should be so small that it will fall within the dispersion of our points. Moreover, if existing, the Ro dependence would be really smaller than the one that has been observed for contaminated bubbles. In this last case, C_L was found to increase by a factor of the order of 1.2-1.4 with Ro, when Re is greater than 70.

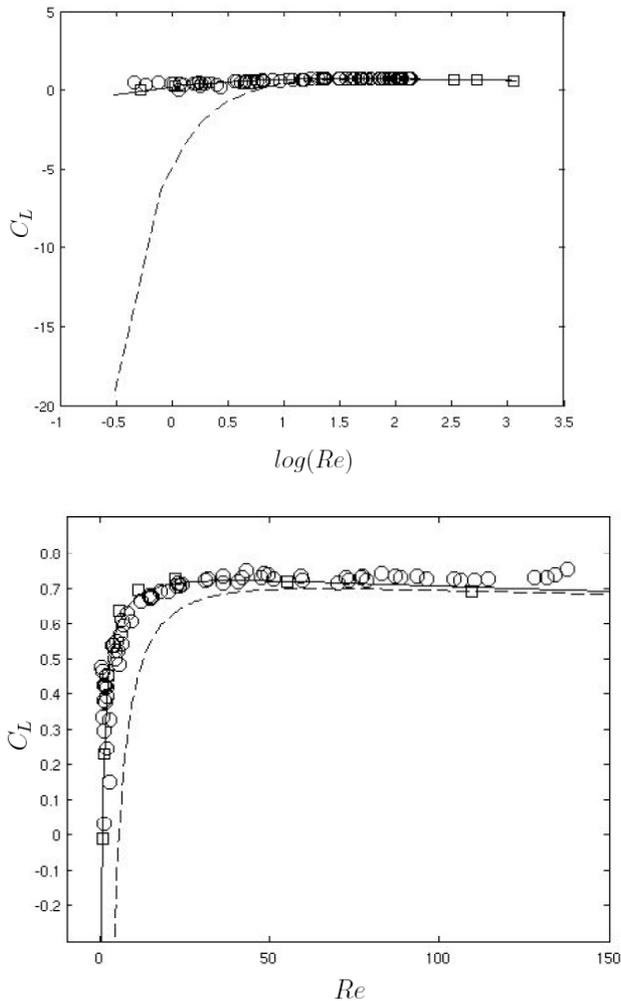


Figure 5 : Lift coefficient plotted in function of Reynolds number.

\circ : present experiments; \square : Magnaudet and Legendre
full line: $0.5 + 4 \times (1 - 6/(5Re^{1/5})) \times \exp(-Re^{1/5})$

dashed line: $0.5 + 1.2 \times Re^{-1/3} - 6.5 \times Re^{-1}$

Because $Re = \omega r_e d / \nu$, different conditions are required to achieve an experiment with a low Reynolds number. A fluid with a high viscosity is obviously better suited, together with a small diameter for the bubble. Also, the fluid velocity at the stabilization point r_e must be small. Unfortunately this velocity is controlled by the two previous parameters (ν , d), hence, ω and r_e cannot be varied independently. This is shown by eliminating the angle θ_e between equations 2.2 and 2.3 (Bluemink et al. 2010). If the drag coefficient is estimated by $C_D = 16Re$, the equilibrium position r_e for very high viscosity and very low viscosity fluids can respectively be approximated by

$$\frac{r_e}{d} = \frac{gd}{3\nu\omega}$$

$$\frac{r_e}{d} = \frac{g}{d(C_L - 1 - C_A)\omega^2}$$

Therefore, increasing (resp. decreasing) ω will decrease (resp. increase) r_e . For the rotation be stable, the tank needs to turn at least faster than 4 rad.s⁻¹. Owing to this lower limit, the only possibility to obtain data at low Reynolds number, even for the highest fluid viscosity, is to decrease d . Hence, as we decrease in Re , r_e decreases. Such a

decrease is problematic for the measurements. As mentioned in Rastello et al. (2009), bubbles always move slightly around their equilibrium position. The motions are typically of the order of the tenth of a millimetre and have generally a negligible influence on the calculation of the mean quantities. This is no more true at very low Reynolds number where C_L steeply tends to zero. In that case, the relative importance of these slight motion rises, due to the concomitant decrease of y_e and of r_e that results from the particular position of the equilibrium point (near the center and the horizontal x-axis). The instantaneous values of Re , C_D and C_L then begin to exhibit significant oscillations. Their amplitudes remain reasonable (less than 30% of the mean) for Re and C_D , but become higher and higher for C_L as Re decreases. For $Re < 3-5$, the amplitude of the oscillations of C_L is such that we can wonder whether the mean coefficient calculated from the equilibrium position is still meaningful. This more especially concerns the negative values of the lift coefficient. In the same way as van Nierop et al. (2007) observed a change of sign of the lift for Re around 5, we measure here negative values of C_L (in average) when Re is about 1. This reversal of the lift for $Re=1$ seems consistent with the trends obtained when extrapolating the numerical data of Magnaudet and Legendre (1998) and the value provided by the fit above. However for the reason earlier mentioned, we may have some doubt about the relevancy of these negative lift coefficients. This is all the more true, that the validity of equation 2.1 is not justified over this range of Re , which makes the use of equation 2.2 to determine the force coefficients questionable. Hence, it is difficult to conclude whether a lift reversal does actually take place in our experiment or the lift monotonously decreases up to zero with decreasing Reynolds number.

Conclusions

This article is an extension of our previous work on contaminated bubbles Rastello et al. (2009). The purpose was to measure the drag and lift coefficient on clean bubbles in a SBRH flow. To ensure the bubbles' cleanliness, experiments have been achieved with silicon oils.. For spherical bubbles the following results have been obtained. The drag coefficient versus the Re number has been shown to be follow, at first order, the known-curve for clean bubbles in uniform flows. The influence of the Ro number has been quantified, showing, in the (Re , Ro) range in which we work an increase of the drag coefficient when the Ro number decreases. The lift coefficients measured coincide with the one obtained numerically by Magnaudet and Legendre (1998) for a SBRH. A fit of both their and ours results has been proposed. It covers the four decades of all results. No influence of the Ro number has been pointed out. Special attention was paid to the low Re numbers. It has been exhibited that for very low values of the Re number a measurement of the lift coefficient is not realistic with a SBRH experiment.

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