

New Efficient Method to Generate Optimal 2^n -PSK STTCs with a Large Number of Transmit Antennas

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Abstract

In this paper, we propose a new method called coset partitioning to generate easily and efficiently the best space-time trellis codes (STTCs), thereby reducing considerably the usual important search time. This method, used in the case of 2^n -PSK STTCs, is an extension of the set partitioning proposed by Ungerboeck based on the coset and lattice Calderbank's approach. The main guideline of the coset partitioning is to divide the set of MIMO symbols into cosets, not into simple sets as the set partitioning. The case of a large number of transmit antennas is considered because the time to obtain the best STTCs with an exhaustive search is very important when the number of transmit antennas increases. Using the coset partitioning, new 4-PSK STTCs with 5 to 6 transmit antennas are generated. Simulation results show that the new codes slightly outperform the best corresponding known codes. Thanks to the effectiveness of the coset partitioning, for the first time, 4-PSK STTCs with 7 and 8 transmit antennas and 8-PSK STTCs with 5 to 6 transmit antennas are obtained. Their performance is also analyzed by simulation.

1 Introduction

In [1], Tarokh *et al.* have introduced the concept of space-time trellis codes (STTCs) to take benefit from the detrimental effect of multipath fading on wireless communications. In order to improve the reliability and/or data rate of the wireless communications, STTCs take advantage of both the spatial diversity provided by the multiple input and output (MIMO) antennas and the coding gain provided by the trellis encoder. In [1], the rank and determinant criteria have been proposed to analyze the performance of STTCs in the case of slow Rayleigh fading channels. In [2], Chen *et al.* have presented the trace criterion for slow and fast Rayleigh fading channels. Based on the Euclidean distance, this criterion governs the performance of codes in the case of a great product between the number of transmit and receive antennas. Thanks to these criteria, after a systematic search, many codes have been proposed in [1, 2, 3, 4], but only for MIMO systems with up to 4 transmit antennas. In [5, 6], Abdool-Rassool *et al.* have presented 4-PSK STTCs with 5 and 6 transmit antennas obtained via a systematic search. The main problem of these previous published code searches is the computational time, particularly when the number of transmit antennas, the number of states and the modulation indices increase. For example, there are several billions of 4 states 4-PSK STTCs with 4 transmit antennas which must be analyzed to find the best STTCs.

In order to reduce drastically the search time, a new method called coset partitioning is presented to generate 2^n -PSK STTCs with a large number of transmit antennas without the usual time-consuming exhaustive/systematic search. In [7], the coset partitioning has been used to design optimal 2^n -PSK STTCs, but only for a small number of transmit antennas. This method is an extension for MIMO systems of the set partitioning proposed by Ungerboeck to design trellis coded modulations (TCMs) [8, 9, 10], based on the lattice and coset Calderbank's approach [11].

In the coset partitioning, the MIMO symbols are regrouped into cosets, not into simple sets, as in the set partitioning. Thus, the number of codes generated by the coset partitioning is reduced compared to the number of codes obtained with the set partitioning. Therefore, the time to find optimal STTCs via the coset partitioning is considerably shortened, especially when the number

of transmit antennas and/or the number of states increase.

The paper is organized as follows. Section II describes the representation of STTCs and the existing design criteria. In section III, the new method is presented with several design examples. Section IV gives new 4-PSK and 8-PSK STTCs with 5 to 8 and with 5 to 6 transmit antennas respectively. In section V, the performance of these new codes is compared to that of the best known codes.

2 System model and design criteria

2.1 Representation of a space-time trellis encoder

We consider a 2^n -PSK space-time trellis encoder with n_T transmit antennas and n_R receive antennas. For $n = 2$, the encoder is shown in Fig. 1.

The encoder is composed of one input block of n bits and ν memory blocks of n bits. A state is defined by the binary values of the ν memory blocks. At each time $t \in \mathbb{Z}$, all the bits of a block are replaced by the n bits of the previous block. For each block i with $1 \leq i \leq \nu + 1$, the j^{th} bit with $1 \leq j \leq n$ is associated to n_T coefficients $g_{j,i}^k \in \mathbb{Z}_{2^n}$, $1 \leq k \leq n_T$. With these $n_T \times n(\nu + 1)$ coefficients, a generator matrix \mathbf{G} with n_T lines and $\nu + 1$ blocks [12] is obtained :

$$\mathbf{G} = [G_1^1 \cdots G_n^1 | \cdots | G_1^{\nu+1} \cdots G_n^{\nu+1}] \quad (1)$$

with $G_j^i = [g_{j,i}^1 \cdots g_{j,i}^{n_T}]^T \in \mathbb{Z}_{2^n}^{n_T}$. The matrix $[\cdots]^T$ is the transpose of the matrix $[\cdots]$.

At each time t , the encoder output $\mathbf{y}_{x^t} = [y_1^t y_2^t \cdots y_{n_T}^t]^T \in \mathbb{Z}_{2^n}^{n_T}$ is given by

$$\mathbf{y}_{x^t} = \mathbf{G} \mathbf{x}^t \quad (2)$$

where $\mathbf{x}^t = [x_1^t \cdots x_n^t \cdots x_1^{t-\nu} \cdots x_n^{t-\nu}]^T$ is the extended-state at time t of the $L = n(\nu + 1)$ length shift register realized by the input block followed by the ν memory blocks.

Each encoder output y_k^t is mapped onto a 2^n -PSK signal $s_k^t = \exp(j \frac{\pi}{2^{n-1}} y_k^t)$. Each output signal s_k^t is sent to the k^{th} transmit antenna. At each time t , the symbols transmitted simultaneously over the fading MIMO channel are given by $\mathbf{s}^t = [s_1^t s_2^t \cdots s_{n_T}^t]^T$.

An encoder can also be represented by a trellis, as shown in Fig. 2 for a 4 states 4-PSK STTC.

In the trellis, the states are described by the points and the transitions between the states by the lines. Each transition corresponds to an extended-state. The vector $\mathbf{y}_i \in \mathbb{Z}_4^{n_T}$ represents the MIMO symbol associated to an extended-state. The index i is computed as the decimal value of this extended-state, with $x_{1,1}^t$ the least significant bit. In this example, the trellis representation corresponds to the generator matrix $\mathbf{G} = [y_1 y_2 | y_4 y_8]$.

In the general case, for a 2^n -PSK STTC, there are 2^n transitions originating from a same state or merging into a same state. The MIMO symbols belong to $\mathbb{Z}_{2^n}^{n_T}$.

2.2 Design criteria

The main design criteria have been established in [1, 2] in order to decrease the bit and frame error rates. In this paper, only the case of slow fading channels is considered, i.e. the channel fading coefficients are constant for each frame of L_f symbols. Besides, we assume that a maximum likelihood algorithm is used to estimate the transmitted symbols.

The main goal of this design is to reduce the pairwise error probability (PEP) which is the probability that the decoder selects an erroneous frame. It is possible to represent a coded frame of L_f MIMO symbols starting at $t = 0$ by a $n_T \times L_f$ matrix $\mathbf{S} = [\mathbf{s}^0 \mathbf{s}^1 \dots \mathbf{s}^{L_f-1}]$, where \mathbf{s}^t is the t^{th} MIMO symbol. An error occurs if the decoder decides that another frame $\mathbf{E} = [\mathbf{e}^0 \mathbf{e}^1 \dots \mathbf{e}^{L_f-1}]$ was transmitted. Let us define the $n_T \times L_f$ difference matrix $\mathbf{B} = \mathbf{E} - \mathbf{S}$:

$$\mathbf{B} = \begin{bmatrix} e_1^0 - s_1^0 & \dots & e_1^{L_f-1} - s_1^{L_f-1} \\ \vdots & \ddots & \vdots \\ e_{n_T}^0 - s_{n_T}^0 & \dots & e_{n_T}^{L_f-1} - s_{n_T}^{L_f-1} \end{bmatrix} \quad (3)$$

The $n_T \times n_T$ product matrix $\mathbf{A} = \mathbf{B}\mathbf{B}^*$ is introduced, where \mathbf{B}^* denotes the hermitian of \mathbf{B} . We define $r = \min(\text{rank}(\mathbf{A}))$, where \mathbf{A} is computed for all pairs of different coded frames ($\mathbf{E} \neq \mathbf{S}$). The design criteria depend on the value of the product rn_R .

First case: $rn_R \leq 3$:

In this case, for a slow Rayleigh fading channel, two criteria have been proposed [1, 13] to reduce

the PEP:

- \mathbf{A} has to be a full rank matrix for any pair (\mathbf{E}, \mathbf{S}) with $\mathbf{E} \neq \mathbf{S}$.
- The coding gain is related to the inverse of $\eta = \sum_d N(d)d^{-n_R}$, where $N(d)$ is defined as the average number of error events with determinant $d = \det(\mathbf{A})$. The best codes must have the minimum value of η .

Second case: $rn_R > 3$:

In [2], it is shown that for a large value of rn_R , which corresponds to a large number of independent SISO channels, the PEP is minimized if the sum of all the eigenvalues of the matrix \mathbf{A} is maximized.

Since \mathbf{A} is a square matrix, the sum of all the eigenvalues is equal to the trace of the matrix \mathbf{A} :

$$\text{tr}(\mathbf{A}) = \sum_{k=1}^{n_T} \lambda_k = \sum_{k=1}^{n_T} \left(\sum_{t=0}^{L_f-1} |e_t^k - s_t^k|^2 \right) \quad (4)$$

For each pair of codewords, $\text{tr}(\mathbf{A})$ is computed. The minimization of the PEP amounts to using a code which has the maximum value of the minimum trace computed for all different pairs of coded frames (\mathbf{E}, \mathbf{S}) . In [13], it is also stated that to minimize the frame error rate (FER), the number of error events with minimum trace has to be minimized.

In this paper, we consider the case $rn_R > 3$ which is encountered when the minimum rank of \mathbf{A} is greater than 1 and there are at least 2 receive antennas.

3 Coset partitioning

3.1 Preliminary

Each MIMO symbol belongs to the additive group $\mathbb{Z}_2^{n_T}$. The set

$$C_0 = 2^{n-1} \mathbb{Z}_2^{n_T} \quad (5)$$

is a normal subgroup of $\mathbb{Z}_{2^n}^{nT}$ such as $v = -v, \forall v \in C_0$. It is possible to partition the group $\mathbb{Z}_{2^n}^{nT}$ into $2^{nT(n-1)}$ cosets as:

$$\mathbb{Z}_{2^n}^{nT} = \bigcup_{p \in \mathbb{Z}_{2^{n-1}}^{nT}} (p + C_0) = \bigcup_{p \in \mathbb{Z}_{2^{n-1}}^{nT}} C_p \quad (6)$$

where $C_p = p + C_0, \forall p \in \mathbb{Z}_{2^{n-1}}^{nT}$.

Using the cosets C_p , it is possible to make a new partition of $\mathbb{Z}_{2^n}^{nT}$:

$$\mathbb{Z}_{2^n}^{nT} = \bigcup_{k=0}^{n-1} E_k \quad (7)$$

where $E_0 = C_0$. For $1 \leq k \leq n-1$, the other sets E_k are defined by :

$$E_k = \bigcup_{p_k} (p_k + C_0) \quad (8)$$

where $p_k \in 2^{n-k-1}\mathbb{Z}_{2^k}^{nT} \setminus 2^{n-k}\mathbb{Z}_{2^{k-1}}^{nT}$. The set \mathbb{Z}_1^{nT} contains only the nul element of $\mathbb{Z}_{2^n}^{nT}$. For $1 \leq k \leq n-1$, each coset $p_k + C_0 \subset E_k$ is called 'relative to' $q_k = 2p_k \in E_{k-1}$. For example, Table 1 shows the partition of the group \mathbb{Z}_2^4 .

3.2 Presentation of the coset partitioning

In [8, 9, 10], Ungerboeck has proposed the set partitioning to design the TCMs in the case of single input and single output (SISO) systems. The set partitioning can be stated by the following rules:

Rule 1 *Each point of the constellation has the same number of occurrences.*

Rule 2 *In the trellis, transitions originating from a same state or merging into a same state should be assigned subsets which contain signal points separated by the largest Euclidean distances.*

Rule 3 *Parallel paths should be assigned signal points separated by the largest Euclidean distances. Since the trellis of STTCs has no parallel paths, this rule is not relevant for STTCs design.*

Calderbank *et al.* give an alternative to set partitioning [11]: the constellation must be a subgroup of an abelian group. The subgroup is divided into cosets. At each time t , the encoder selects one coset, then one element of this coset.

The proposed method called coset partitioning can be stated by the following properties.

Property 1 *The used MIMO symbols are equally probable.*

Property 2 *The MIMO symbols originating from or merging to a same state belong to the same coset.*

Property 3 *The elements of each coset must be separated by the largest Euclidean distances.*

For a $2^{n\nu}$ states 2^n -PSK STTC, the generator matrix has $\nu + 1$ blocks of n columns of n_T lines. A characteristic of the STTCs designed with the coset partitioning is that the n columns of each block $[G_1^i \cdots G_n^i]$ of \mathbf{G} , with $1 \leq i \leq \nu + 1$ generate a subgroup Λ_i of $\mathbb{Z}_{2^n}^{n_T}$:

$$\Lambda_i = \left\{ \sum_{j=1}^n x_j G_j^i \bmod 2^n / x_j \in \{0, 1\} \right\} \quad (9)$$

with $\text{card}(\Lambda_i) = 2^n$.

In order to obtain a subgroup, the n columns must be selected as follows. The first column G_1^i must belong to C_0^* . If $j - 1$ first columns have been previous selected and create a subgroup $\Lambda_{i,j-1}$, the columns G_j^i with $2 \leq j \leq n$ and $1 \leq i \leq \nu + 1$ must belong to C_0^* or to the cosets relative to an element of $\Lambda_{i,j-1}$ and must not belong to $\Lambda_{i,j-1}$.

Thus, the n columns of each block generate a subgroup. For each subgroup, it is possible to compute the minimal Euclidean distance between its elements. The sets of n columns generating the subgroups with the largest value of the minimal Euclidean distance are used to design the generator matrix \mathbf{G} . After these choices, it is possible to permute the columns and the lines within each block in order to obtain other codes which fulfill the properties of the coset partitioning. Further on, the codes with the best trace are searched.

Rule 1 of set partitioning is fulfilled by the STTCs designed via the coset partitioning because these STTCs are balanced codes [14, 15]. A STTC is balanced if and only if the generated MIMO symbols \mathbf{y} have the same number of occurrences $n(\mathbf{y}) = n_0 \geq 1$ when the input data are sent by a binary memoryless source with equally probable symbols. If the columns of the generator matrix \mathbf{G} generate a subgroup of $\mathbb{Z}_{2^n}^{n_T}$, then the corresponding STTC is balanced [15].

For a STTC designed with coset partitioning, the set

$$\Lambda = \sum_{i=1}^{\nu+1} \Lambda_i \quad (10)$$

of generated MIMO symbols is a subgroup of $\mathbb{Z}_{2^n}^{n_T}$, where Λ_i is the subgroup generated by the n columns of the block i of \mathbf{G} . Thus, for these codes, rule 1 of set partitioning is respected.

If the code is designing with the coset partitioning, rule 2 of the set partitioning is also followed. In fact, in the case of set partitioning, the symbols originating from or merging to a same state belong to a **subset** which contains symbols separated by the largest Euclidean distances. In the case of coset partitioning, the MIMO symbols originating from or merging to a same state belong to a **coset** which contains MIMO symbols separated by the largest Euclidean distances.

However, in the case of coset partitioning, the selection of 'optimal' cosets is more restrictive than the selection of 'optimal' subsets used by set partitioning. Therefore, the number of STTCs obtained via the coset partitioning is significantly reduced to the best ones.

3.3 Design examples for 4-PSK 4 states STTCs with n_T transmit antennas

The MIMO symbols belong to $\mathbb{Z}_4^{n_T}$. This group can be divided into 2 subsets: $E_0 = C_0$ and $E_1 = \bigcup(g + C_0)$, with $g \in \mathbb{Z}_2^{n_T} \setminus [0 \cdots 0]^T$. The generator matrix \mathbf{G} has 2 blocks of 2 columns: $B_i = [G_1^i, G_2^i]$, where $G_j^i \in \mathbb{Z}_4^{n_T}$ for $1 \leq i \leq 2$ and $1 \leq j \leq 2$.

To design optimal STTCs, the columns of each block must generate a subgroup of $\mathbb{Z}_2^{n_T}$. The subgroup $\Lambda_1 = \left\{ \sum_{j=1}^2 x_j G_j^1 \bmod 4/x_j \in \{0, 1\} \right\}$ generated by the first block is also denoted Λ_1^F because it is used to generate the MIMO symbols originating From a same state, as will be shown later. The subgroup $\Lambda_2 = \left\{ \sum_{j=1}^2 x_j G_j^2 \bmod 4/x_j \in \{0, 1\} \right\}$ generated by the second block is also denoted Λ_1^M because it is used to generate the MIMO symbols Merging to a same state, as will be shown later. The set of generated MIMO symbols is given by $\Lambda = \left\{ \sum_{i=1}^2 \sum_{j=1}^2 x_j^i G_j^i \bmod 4/x_j^i \in \{0, 1\} \right\}$. Thus, each coset of $\Lambda/\Lambda_1^F = \{g + \Lambda_1^F/g \in \Lambda_1^M\}$ contains the MIMO symbols originating from a same state. In the same way, each coset of $\Lambda/\Lambda_1^M = \{g + \Lambda_1^M/g \in \Lambda_1^F\}$ contains the MIMO symbols merging to a same state. The trellis is shown in

Fig. 3.

To respect the coset partitioning, the Euclidean distances between the different elements of each coset of Λ/Λ_1^F and Λ/Λ_1^M must have the largest values. To obtain these optimal cosets, it is sufficient to design the optimal subgroups. In fact, the distance spectrum of each coset of Λ/Λ_1^F and Λ/Λ_1^M , i.e. the repartition of Euclidean distances between the elements of each coset of Λ/Λ_1^F and Λ/Λ_1^M , is identical to the distance spectrum of Λ_1^F and Λ_1^M respectively. Besides, for a 2^n -PSK modulation, $d_E^2(0, v) = d_E^2(0, -v)$ where $d_E^2(0, v)$ is the squared Euclidean distance between the MIMO signals corresponding to the MIMO symbols 0 and v . As the opposite of each MIMO symbol of one block is generated, the number of constraints used to find the optimal block is reduced compared to the set partitioning.

A exhaustive search has been performed to find the best 4-PSK blocks for 5 to 8 transmit antennas whose columns generate a subgroup of $\mathbb{Z}_2^{n_T}$. The optimal blocks are based on the permutation of the lines and/or the columns of the blocks presented in Table 2. In this table, the notation "1/3" must be read "1 or 3".

Let us consider the distance spectrum of one block of \mathbf{G} . The distance spectra generated by the proposed blocks are optimal and given in Fig. 4. The best codes are constituted by 2 optimal blocks.

3.4 Design example for 16 states 4-PSK STTCs with n_T transmit antennas

The MIMO symbols belong to $\mathbb{Z}_4^{n_T}$. This group can be divided into 2 sets of cosets: $E_0 = C_0$ and $E_1 = \bigcup(g + C_0)$ with $g \in \mathbb{Z}_2^{n_T} \setminus [0 \dots 0]^T$. The matrix \mathbf{G} has 3 blocks of 2 columns: $B_1 = [G_1^1, G_2^1]$, $B_2 = [G_1^2, G_2^2]$ and $B_3 = [G_1^3, G_2^3]$, where $G_j^i \in \mathbb{Z}_4^{n_T}$ for $1 \leq i \leq 3$ and $1 \leq j \leq 2$. Hence, the generator matrix is $\mathbf{G} = [G_1^1 G_2^1 | G_1^2 G_2^2 | G_1^3 G_2^3]$.

In the case of a 16 states 4-PSK STTC designed via the coset partitioning, the trellis can be represented as shown by Fig. 5. On the left side of the trellis, there are the MIMO symbols \mathbf{y}_i , with $0 \leq i \leq 63$, originating from a same state. On the right side of the trellis, there are the

MIMO symbols merging into a same state.

In this case, the generator matrix is $\mathbf{G} = [\mathbf{y}_1\mathbf{y}_2|\mathbf{y}_4\mathbf{y}_8|\mathbf{y}_{16}\mathbf{y}_{32}]$. To design \mathbf{G} , there are two steps

The first step is to select the columns of the B_1 and B_3 . The first block and the last block of the generator matrix generate respectively the subgroups $\Lambda_1^F = \{\mathbf{y}_0, \mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3 = \mathbf{y}_1 + \mathbf{y}_2 \bmod 4\}$ and $\Lambda_1^M = \{\mathbf{y}_0, \mathbf{y}_{16}, \mathbf{y}_{32}, \mathbf{y}_{48} = \mathbf{y}_{16} + \mathbf{y}_{32} \bmod 4\}$. Each coset of Λ/Λ_1^F and of Λ/Λ_1^M corresponds to the transitions originating from a same state and the transitions merging into a same state respectively. The selection of B_1 and B_3 is identical to the previous section. Thus, B_1 and B_3 correspond to one of the blocks proposed in the previous section after an eventual permutation of lines and/or columns.

The second step is the selection of the columns G_1^2 and G_2^2 of the second block. These columns must be selected via the previous stated properties to obtain a subgroup.

Finally, the columns of each block are permuted to obtain the codes with the best trace.

3.5 Design examples for 8-PSK 4 states STTCs with n_T transmit antennas

In this case, the MIMO symbols belong to $\mathbb{Z}_8^{n_T} = E_0 \cup E_1 \cup E_2$, where $E_0 = C_0 = 4\mathbb{Z}_2^{n_T}$, $E_1 = \bigcup(g_1 + C_0)$ with $g_1 \in 2\mathbb{Z}_2^{n_T} \setminus [0 \cdots 0]^T$ and $E_2 = \bigcup(g_2 + C_0)$ with $g_2 \in \mathbb{Z}_4^{n_T} \setminus 2\mathbb{Z}_2^{n_T}$. The generator matrix \mathbf{G} is constituted by 2 blocks of 3 columns $B_1 = [G_1^1 G_2^1 G_3^1]$ and $B_2 = [G_1^2 G_2^2 G_3^2]$, where $G_j^i \in \mathbb{Z}_8^{n_T}$ for $1 \leq i \leq 2$ and $1 \leq j \leq 3$.

Each block must generate a subgroup of $\mathbb{Z}_8^{n_T}$ with 8 elements. In this case, the subgroup of MIMO symbols generated by the encoder is $\Lambda = \left\{ \sum_{i=1}^2 \sum_{j=1}^3 x_j^i G_j^i \bmod 8/x_j^i \in \{0, 1\} \right\}$. Besides, the subgroups $\Lambda_1 = \Lambda_1^F = \left\{ \sum_{j=1}^3 x_j G_j^1 \bmod 8/x_j \in \{0, 1\} \right\}$ and $\Lambda_2 = \Lambda_1^M = \left\{ \sum_{j=1}^3 x_j G_j^2 \bmod 8/x_j \in \{0, 1\} \right\}$ are generated by B_1 and B_2 respectively. The 8 MIMO symbols originating from a same state or merging into a same state are the elements of one of the 8 cosets of Λ/Λ_1^F and Λ/Λ_1^M respectively.

For 8-PSK STTCs, the optimal blocks are formed by one column of E_0 , one column of E_1 and

one column of E_2 i.e. $G_1^i \in C_0$, $G_2^i \in (p + C_0)$, with $2p = G_1^i$ and $G_3^i \in (q + C_0)$, with $2q = G_2^i$.

Remark: Some codes may have the first j columns of the $(\nu + 1)^{th}$ block with $1 \leq j \leq n - 1$, equal to the null vector. In this case, the number of states is $2^{n\nu-j}$. The columns which generate the subgroup Λ_1^M of the MIMO symbols which merge into a same state, are the first j columns of ν^{th} block and the last $(n - j)$ columns of the $(\nu + 1)^{th}$ block.

3.6 Example of search time reduction

To show the usefulness of the coset partitioning, the case of 4 states 4-PSK STTCs with 3 transmit antennas is considered. To find the best codes, the minimum trace of codes must be computed. With an exhaustive search, the number of generator matrices is equal to $4^{12} = 16\ 777\ 216$.

Chen *et al.* have proposed a suboptimal method in [3]. Using this method, 65 792 generator matrices are designed. This number corresponds to 0.39% of the possible generator matrices. Rassool *et al.* have proposed in [6] a method which ensures to obtain the optimal STTCs. The number of generator matrices is 1 398 101, i.e. 8.33% of the possible generator matrices.

With the proposed method, each block must generate a subgroup. There are $2 \times 7 \times (7+8) = 210$ possibilities to select 2 no null columns (including the permutations between the columns). As there are two blocks, the number of generator matrices is $210^2 = 44\ 100$. These matrices are generated in just one second with a simple computing program. So, it is sufficient to search the best codes among these 0.26% of the possible codes. The proposed method reduces the search time and leads to better STTCs than the corresponding Chen's codes.

When the number of transmit antennas and/or the number of states increase, the percentage of codes generated by the coset partitioning decreases. For example, we consider a 4 states 4-PSK STTC with 5 transmit antennas. The number of generator matrices is $4^{5 \times 4} = 4^{20}$ and the number of possibilities to select 2 no null columns to generate a subgroup is $2 \times 31 \times (31 + 32) = 3\ 906$ (including the permutation between the columns). The generator matrices contain 2 blocks. Thus, the number of codes generated by the coset partitioning is $3\ 906^2$, corresponding to 0.0014% of all the possible 4 states 4-PSK STTCs with 5 transmit antennas.

4 New codes

New codes have been generated via the coset partitioning. Tables 3 and 4 show new 4/16/32 states 4-PSK STTCs for 5 transmit antennas and 6 transmit antennas respectively and the corresponding Rassool's STTCs [5, 6].

The trace of the proposed codes is equal to the trace of the corresponding Rassool's codes. However, for the new codes, the Euclidean distances between the MIMO symbols originating from or merging to a same state are greater than the corresponding Euclidean distances of the Rassool's codes.

Table 5 shows the best 4 states 4-PSK STTCs with 7 and 8 transmit antennas. In the previous publications, no STTC has been proposed with more than 6 transmit antennas. Table 6 shows the best 8 states 8-PSK STTCs with 5 and 6 transmit antennas. In the previous publications, no 8-PSK STTC has been proposed with more than 4 transmit antennas.

5 Code Performance

The performance of each code is evaluated by simulation over slow Rayleigh fading channels. The channel fading coefficients are independent samples of a complex Gaussian process with zero mean and variance 0.5 per dimension. These channel coefficients are assumed to be known by the decoder. Each frame consists of 130 4-PSK or 8-PSK MIMO symbols. For the simulation, there are 2 receive antennas. The decoding is performed by the Viterbi's algorithm.

Figs. 6 and 7 show the performance of the STTCs for 5 and 6 transmit antennas presented in Tables 3 and 4. Each new code outperforms slightly the best corresponding Rassool's code. The performance of the codes with 7 and 8 transmit antennas is presented in Fig. 8. The performance of the new 8-PSK codes proposed in Table 6 is shown in Fig. 9.

6 Conclusion

In order to decrease significantly the time to search the best STTCs, a new and simple method called coset partitioning has been presented in this paper. This method is based on a coset approach of the set partitioning used for MIMO systems. The general rule of this new method is to regroup the MIMO symbols originating from or merging to the same state into cosets, not into simple sets as the set partitioning. These cosets are obtained via a division of the generator matrix into blocks. The columns of each block must generate a subgroup of $\mathbb{Z}_{2^n}^{n_T}$ and the MIMO symbols generated by the first block and those generated by the last block must be separated by the largest Euclidean distance respectively. Thus, the number of codes which must be analyzed is considerably reduced. For example, it is sufficient to analyze 0.0014% of all the possible 4 states 4-PSK STTCs with 5 transmit antennas to find the optimal codes. To emphasize the important search time reduction, the case of STTCs with a great number of transmit antennas has been considered. New 4-PSK codes with 5 to 8 transmit antennas and new 8-PSK codes with 5 to 6 transmit antennas, obtained with the coset partitioning, have been proposed. When similar codes are available, simulation results show that the new codes slightly outperform the best previous published codes.

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Table captions:

Table 1: Partition of the group \mathbb{Z}_4^2

Table 2: Optimal 4-PSK blocks

Table 3: 4-PSK STTCs based on the Euclidean distance criterion with 5 transmit antennas

Table 4: 4-PSK STTCs based on the Euclidean distance criterion with 6 transmit antennas

Table 5: New 4-PSK STTCs based on the Euclidean distance criterion with 7 and 8 transmit antennas

Table 6: New 8-PSK STTCs based on the Euclidean distance criterion with 5 and 6 transmit antennas

Figure captions:

Figure 1: Space-time trellis encoder for 4-PSK and n_T transmit antennas

Figure 2: Example of a trellis obtained for a 4 states 4-PSK STTC

Figure 3: Example of a coset representation for a 4 states 4-PSK STTC

Figure 4: Distance spectra of the optimal blocks of 2 columns in the case of 4-PSK modulation

Figure 5: Example of a coset representation for a 16 states 4-PSK STTC

Figure 6: Performance of 4/16/32 states 4-PSK STTCs with 5 transmit antennas and 2 receive antennas

Figure 7: Performance of 4/16/32 states 4-PSK STTCs with 6 transmit antennas and 2 receive antennas

Figure 8: Performance of 16 states 4-PSK STTCs with 7 & 8 transmit antennas and 2 receive antennas

Figure 9: Performance of 8 states 8-PSK STTCs with 5 & 6 transmit antennas and 2 receive antennas

Table 1:

C_0	$\begin{bmatrix} 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \end{bmatrix}$	E_0
$C_0 + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 2 & 2 \\ 1 & 3 & 1 & 3 \end{bmatrix}$	E_1
$C_0 + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 3 & 3 \\ 0 & 2 & 0 & 2 \end{bmatrix}$	
$C_0 + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 3 & 1 & 3 \end{bmatrix}$	

Table 2:

n_T	Optimal blocks
5	$\begin{bmatrix} 0 & 0 & 2 & 2 & 2 \\ 2 & 2 & 1/3 & 1/3 & 1/3 \end{bmatrix}^T$
6	$\begin{bmatrix} 0 & 0 & 2 & 2 & 2 & 0 \\ 2 & 2 & 1/3 & 1/3 & 1/3 & 1/3 \end{bmatrix}^T$
7	$\begin{bmatrix} 0 & 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 1/3 & 1/3 & 1/3 & 1/3 & 1/3 \end{bmatrix}^T$
8	$\begin{bmatrix} 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 1/3 & 1/3 & 1/3 & 1/3 & 1/3 & 1/3 \end{bmatrix}^T$

Table 3:

States	Code	\mathbf{G}	Trace
4	Rassool <i>et al.</i>	$\begin{bmatrix} 2 & 3 & 2 & 0 \\ 0 & 2 & 3 & 2 \\ 3 & 2 & 2 & 3 \\ 2 & 3 & 2 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix}$	26
	New 1	$\begin{bmatrix} 2 & 3 & 1 & 2 \\ 0 & 2 & 3 & 2 \\ 0 & 2 & 2 & 0 \\ 2 & 3 & 2 & 0 \\ 2 & 3 & 1 & 2 \end{bmatrix}$	26
16	Rassool <i>et al.</i>	$\begin{bmatrix} 2 & 0 & 1 & 2 & 2 & 0 \\ 1 & 2 & 3 & 2 & 1 & 2 \\ 3 & 2 & 2 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 & 2 & 3 \\ 1 & 2 & 2 & 0 & 3 & 2 \end{bmatrix}$	40
	New 2	$\begin{bmatrix} 0 & 2 & 2 & 3 & 0 & 2 \\ 0 & 2 & 0 & 2 & 2 & 3 \\ 2 & 1 & 2 & 3 & 0 & 2 \\ 2 & 1 & 2 & 3 & 2 & 3 \\ 2 & 1 & 2 & 1 & 2 & 1 \end{bmatrix}$	40
32	Rassool <i>et al.</i>	$\begin{bmatrix} 2 & 1 & 2 & 3 & 0 & 3 & 0 & 1 \\ 2 & 3 & 2 & 0 & 1 & 0 & 0 & 2 \\ 2 & 2 & 1 & 2 & 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 2 & 2 & 3 & 0 & 0 \\ 0 & 2 & 2 & 3 & 3 & 2 & 0 & 2 \end{bmatrix}$	44
	New 3	$\begin{bmatrix} 0 & 2 & 2 & 1 & 2 & 1 & 0 & 1 \\ 0 & 2 & 2 & 3 & 0 & 0 & 0 & 2 \\ 2 & 1 & 2 & 3 & 2 & 1 & 0 & 3 \\ 2 & 1 & 2 & 1 & 2 & 1 & 0 & 3 \\ 2 & 3 & 0 & 2 & 2 & 3 & 0 & 1 \end{bmatrix}$	44

Table 4:

States	Code	\mathbf{G}	Trace
4	Rassool <i>et al.</i>	$\begin{bmatrix} 0 & 2 & 1 & 2 \\ 1 & 2 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 2 & 1 & 2 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 3 \end{bmatrix}$	32
	New 4	$\begin{bmatrix} 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 0 \\ 2 & 1 & 3 & 2 \\ 2 & 1 & 3 & 2 \\ 2 & 3 & 2 & 0 \\ 2 & 3 & 1 & 2 \end{bmatrix}$	32
16	Rassool <i>et al.</i>	$\begin{bmatrix} 1 & 2 & 2 & 0 & 3 & 2 \\ 3 & 2 & 3 & 2 & 1 & 2 \\ 2 & 0 & 1 & 2 & 2 & 0 \\ 1 & 2 & 2 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 & 2 & 3 \\ 2 & 3 & 0 & 2 & 2 & 1 \end{bmatrix}$	48
	New 5	$\begin{bmatrix} 0 & 2 & 2 & 1 & 0 & 2 \\ 0 & 2 & 2 & 1 & 2 & 3 \\ 2 & 1 & 0 & 2 & 0 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 \\ 2 & 3 & 0 & 2 & 2 & 1 \\ 2 & 3 & 2 & 1 & 2 & 1 \end{bmatrix}$	48
32	Rassool <i>et al.</i>	$\begin{bmatrix} 1 & 2 & 3 & 1 & 2 & 0 & 0 & 3 \\ 2 & 2 & 3 & 2 & 2 & 3 & 0 & 0 \\ 0 & 2 & 2 & 1 & 1 & 2 & 0 & 2 \\ 2 & 0 & 3 & 2 & 2 & 1 & 0 & 0 \\ 2 & 3 & 2 & 1 & 0 & 1 & 0 & 3 \\ 2 & 1 & 2 & 0 & 3 & 0 & 0 & 2 \end{bmatrix}$	52
	New 6	$\begin{bmatrix} 0 & 2 & 2 & 1 & 0 & 2 & 0 & 2 \\ 0 & 2 & 2 & 1 & 2 & 3 & 0 & 1 \\ 2 & 1 & 2 & 3 & 0 & 0 & 0 & 2 \\ 2 & 1 & 0 & 2 & 2 & 3 & 0 & 1 \\ 2 & 3 & 2 & 1 & 2 & 1 & 0 & 3 \\ 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 \end{bmatrix}$	52

Table 5:

n_T	States	Code	\mathbf{G}	Trace
7	16	New 7	$\begin{bmatrix} 0 & 2 & 2 & 1 & 2 & 1 \\ 0 & 2 & 0 & 2 & 2 & 1 \\ 2 & 3 & 0 & 2 & 2 & 3 \\ 2 & 1 & 2 & 3 & 2 & 3 \\ 2 & 1 & 2 & 3 & 2 & 1 \\ 2 & 3 & 0 & 2 & 0 & 2 \\ 2 & 3 & 2 & 1 & 0 & 2 \end{bmatrix}$	56
8	16	New 8	$\begin{bmatrix} 0 & 2 & 2 & 3 & 2 & 3 \\ 0 & 2 & 2 & 1 & 2 & 3 \\ 0 & 2 & 0 & 2 & 2 & 3 \\ 2 & 1 & 2 & 1 & 2 & 3 \\ 2 & 1 & 2 & 3 & 2 & 1 \\ 2 & 1 & 0 & 2 & 0 & 2 \\ 2 & 1 & 2 & 1 & 0 & 2 \\ 2 & 1 & 2 & 3 & 0 & 2 \end{bmatrix}$	64

Table 6:

n_T	States	Code	\mathbf{G}	Trace
5	8	New 9	$\begin{bmatrix} 0 & 0 & 4 & 4 & 0 & 0 \\ 0 & 4 & 2 & 5 & 4 & 6 \\ 4 & 2 & 1 & 6 & 0 & 4 \\ 4 & 2 & 1 & 1 & 4 & 6 \\ 4 & 2 & 5 & 1 & 4 & 6 \end{bmatrix}$	20.58
6	8	New 10	$\begin{bmatrix} 4 & 6 & 5 & 0 & 0 & 4 \\ 4 & 6 & 5 & 4 & 0 & 6 \\ 4 & 6 & 1 & 4 & 0 & 2 \\ 0 & 4 & 6 & 6 & 4 & 1 \\ 0 & 4 & 2 & 2 & 4 & 3 \\ 0 & 0 & 4 & 2 & 4 & 7 \end{bmatrix}$	25.17

Figure 1:

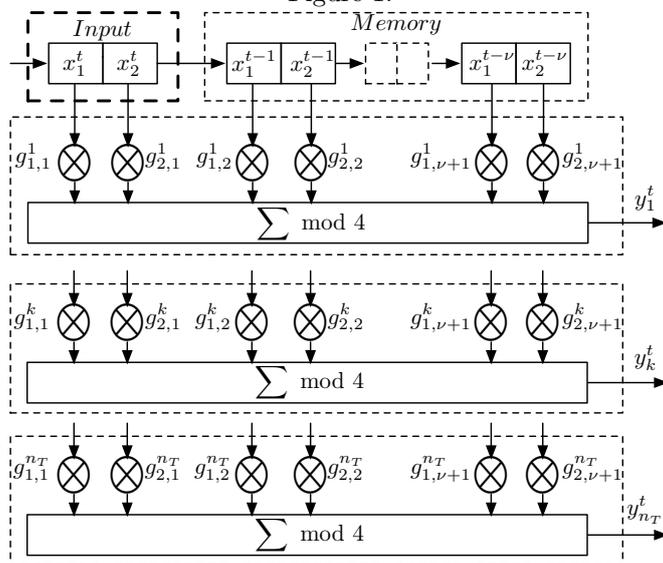


Figure 2:

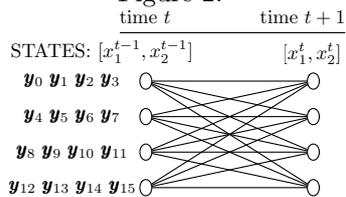


Figure 3:

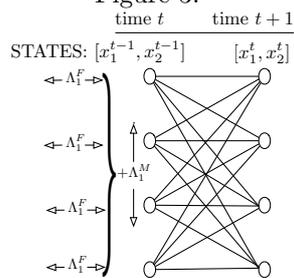


Figure 4:

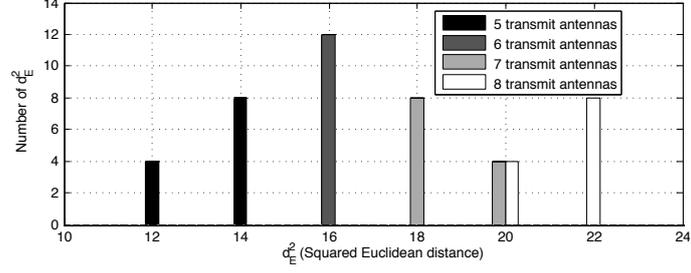


Figure 5:

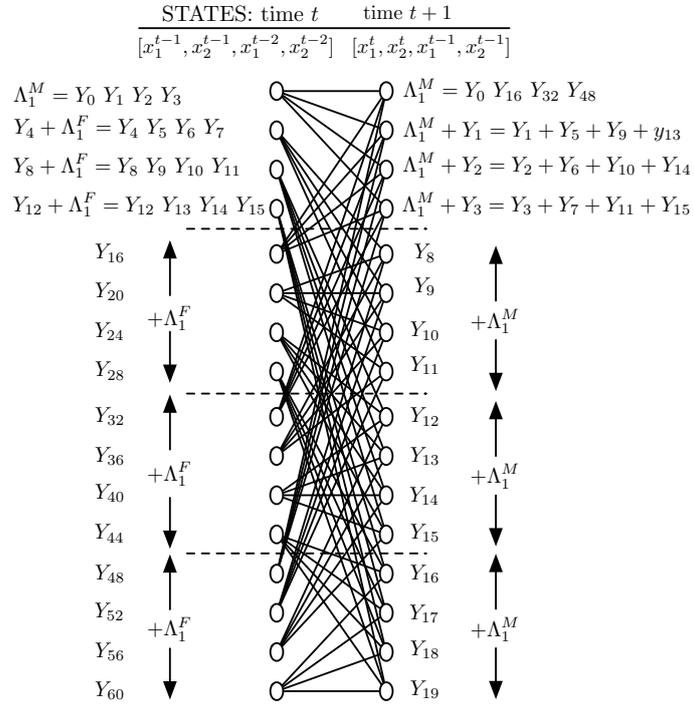


Figure 6:

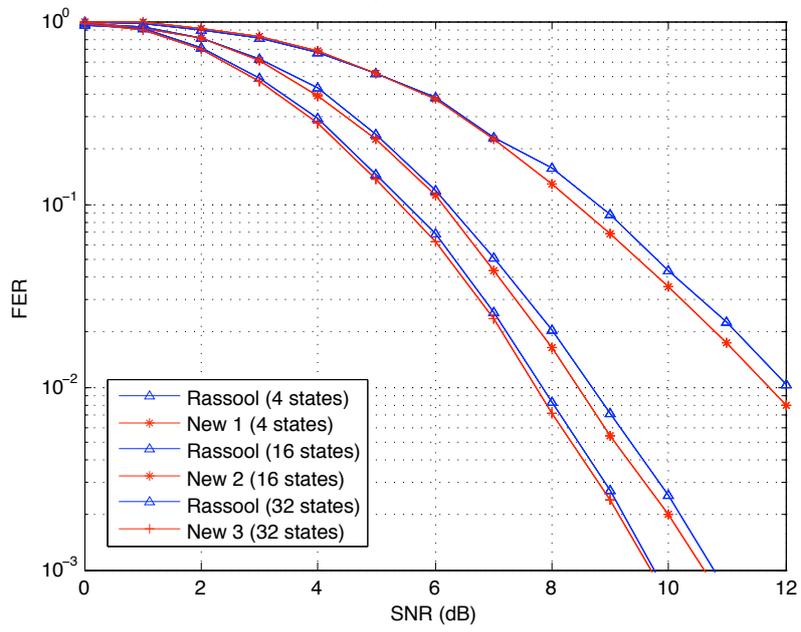


Figure 7:

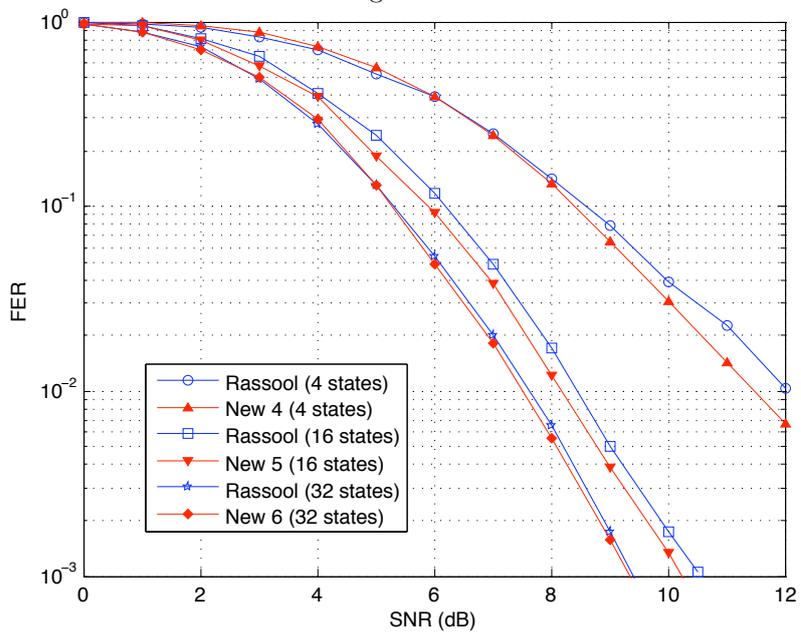


Figure 8:

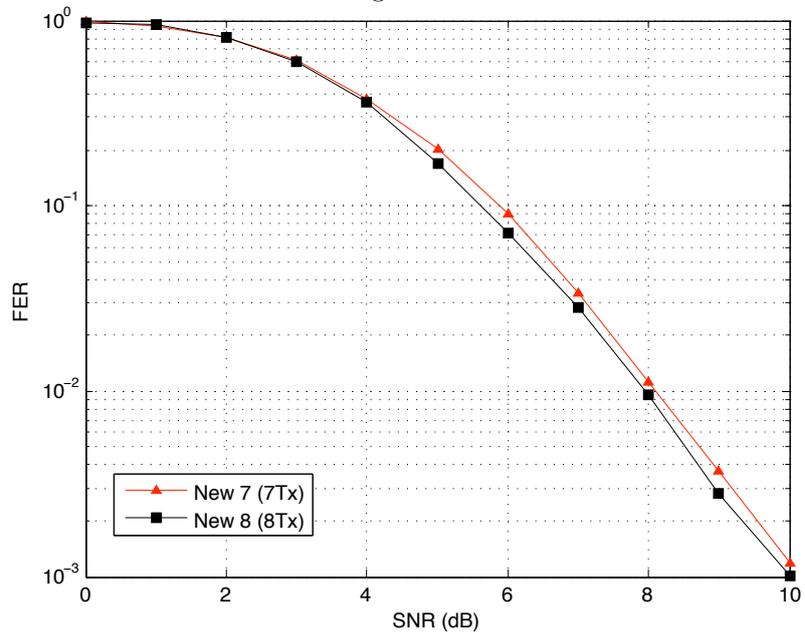


Figure 9:

