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**Stochastic approach to estimate the arterial pressure**

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**Abstract**

The aim of this paper is to illustrate the application of a stochastic approach to estimate the human common carotid arterial pressure. The analysis took into account the possible random uncertainties of the problem inputs such as geometric information and mechanical model parameters so that it is called a probabilistic parametric approach. Based on the only available information reported in literature, entropy maximum principle was used to develop probabilistic density functions for every random variable. In addition, in vivo human experimental data were considered for the determination of the so-called mean or deterministic model. Furthermore, numerical simulations of Monte Carlo were carried out involving the dispersion of all the uncertain parameters. Results showed that uncertainty of 5% led to error up to 20% in the arterial pressure estimation. Convergence was proved and a region with a confidence probability of 95% was constructed to allow the prediction of the random response of the arterial pressure. Eventually, we managed numerous calculations to analyze the influence of each random variable of the problem inputs over the arterial pressure evolution.

**Keywords:** Stochastic, Arterial pressure, Hyperelastic material, Maximum entropy principle

## 1 Introduction

It is well known that arteries are subjected to many diseases such as atherosclerosis, aneurysm or hypertension (Humphrey, 2002). The main purpose of current researches is to establish constitutive laws which describe the hyperelastic behavior of the arterial tissue. In this context, authors propose several mechanical models including a variable number of geometric and material parameters (Humphrey, 2002; Baek et al., 2007) which are often determined by performing *in vitro* tests on the arterial specimens (Humphrey, 2002).

Accordingly, it is important that the obtained parameters (problem data) are well calibrated to ensure the accuracy of the problem solution in order to investigate the mechanical behavior of arteries. Note that a similar problem for the arterial geometric parameters in the reference configuration is reached (Alastrué et al., 2007).

Several identification methods have been recently developed but few studies have reported parameters for human arteries, particularly in the *in vivo* case. (Stalhand et al., 2004) showed that *in vivo* measurable data can be used directly for parameter identification for a circular, hyperelastic, anisotropic and incompressible model that accounts for the residual stresses in the unloaded arterial wall. The model parameters are determined by stating a minimization problem for the model pressure. (Olsson et al., 2006) proposed a general method to compute the parameters of a residual stress field in the mechanical modelling of soft tissues. The authors also suggested a method to obtain this parameterization from *in vivo* measurements by a nonlinear parameter identification problem. More recently, (Masson et al., 2008) demonstrated the feasibility of material identification and wall stress computation for human common carotid arteries based on non-invasive *in vivo* clinical data. The reader also could consult (Humphrey, 2002) where the problem of the identification of the vascular tissue parameters is described.

Hence, it would be desirable to develop methods that allow parameter estimation in mechanical arterial models. However, estimating parameters is not straightforward in the *in vivo* cases and their values can be assessed within a certain range.

In this paper, we demonstrate the importance to include the stochastic uncertainties in the modeling of the arterial mechanical behavior through a parametric approach. For that, we used *in vivo* human data obtained from non-invasive measurements. The influence of the uncertain problem data on the arterial pressure response was investigated by considering a finite elastodynamical model. Thus, we focused on the probabilistic feature of the theoretical problem data to quantify the induced final error in the arterial pressure calculation.

## 2 Methods

### 2.1 Arterial measurements

In vivo experimental measurements have been performed to study the mechanical behavior of the human common carotid arteries (CCA). During 4 to 6 cardiac cycles, the CCA medial diameter (MD), considered as the outer diameter, and the intima-media thickness (IMT) were recorded using a high spatial resolution ultrasound echotracking system (Wall Track System, Pie Medical) (Hoeks et al., 1997; Boutouyrie et al., 1999) over a line perpendicular to the longitudinal and transversal axes of the artery, selected on the 2D B-mode image. In addition, the CCA intraluminal pressure was also measured non-invasively with a pencil-type probe that incorporated a high-fidelity Millar strain-gauge transducer coupled with tonometry by applanation (Boutouyrie et al., 1999). The intraluminal blood pressure increases up to the systolic peak which indicates the maximum arterial pressure, then the CCA pressure curve decreases during the diastolic phase of the cardiac cycle. The bend in the descending part corresponds to the dicrotic wave which matches with the closure of the aortic valves. Data used in this study are from a 39-year old healthy male subject.

### 2.2 Theoretical model

Consider a sector of a circular and thick cylinder representing the CCA defined by the angle  $\Theta_0$  (Figure 1). The dynamic deformation of the artery over the cardiac cycle may be described by two successive deformations. Starting from the stress free state ( $\Omega_0$ ), the artery is closed inducing residual stresses. The obtained configuration is the unloaded state ( $\Omega_1$ ). Then the CCA is subjected to dynamic radial expansion and axial stretch ( $\Omega_2$ ). In the cylindrical coordinate system relative to the basis  $(\bar{\mathbf{e}}_r, \bar{\mathbf{e}}_\theta, \bar{\mathbf{e}}_z)$ , the mapping is described by

$$r = r(R,t), \quad \theta = \left( \frac{\pi}{\Theta_0} \right) \Theta, \quad z = \lambda \Lambda Z, \quad (1)$$

where  $(R, \Theta, Z)$  and  $(r, \theta, z)$  are respectively the reference and the deformed coordinates of a material particle,  $\Theta_0$  is the opening angle,  $\Lambda$  and  $\lambda$  are axial stretches ratios respectively from  $(\Omega_0)$  to  $(\Omega_1)$  and from  $(\Omega_1)$  to  $(\Omega_2)$ . Let  $R_i$  and  $r_i(t)$  denote respectively the inner radii of the artery in the reference state and in the deformed state ( $R_e$  and  $r_e(t)$  are the outer radii). The inner radius  $r_i(t)$  is derived from the experimental measurements by  $r_i(t) = MD(t)/2 - IMT(t)$ .

Using (1), it follows that the physical components of the deformation gradient tensor  $\mathbf{F}$  are

$$\mathbf{F} = \text{diag} \left[ \frac{\partial r}{\partial R}, \frac{\pi r}{\Theta_0 R}, \lambda \Lambda \right] = \text{diag} [\lambda_r, \lambda_\theta, \lambda_z], \quad (2)$$

where  $\lambda_r, \lambda_\theta, \lambda_z$  are respectively the radial, circumferential and axial stretch ratios. Consequently, the left and the right Cauchy-Green strain tensors, denoted  $\mathbf{B}$  and  $\mathbf{C}$ , have the same matrix representation

$$\mathbf{B} = \mathbf{C} = \text{diag} \left[ \left( \frac{\partial r}{\partial R} \right)^2, \left( \frac{\pi r}{\Theta_0 R} \right)^2, (\lambda \Lambda)^2 \right] = \text{diag} [\lambda_r^2, \lambda_\theta^2, \lambda_z^2]. \quad (3)$$

Furthermore, the arterial material is considered as incompressible. The local volume ratio is given by  $J = \det \mathbf{F} = \lambda_r \lambda_\theta \lambda_z = 1$  and leads to

$$r(\mathbf{R}, t) = [r_i^2(t) + \frac{\Theta_0}{\pi \lambda \Lambda} (\mathbf{R}^2 - \mathbf{R}_i^2)]^2. \quad (4)$$

Let us assume that the mechanical behavior of the artery is described by the following stored strain-energy density function (Tang et al., 2004)

$$W = a(I_1 - 3) + b(I_2 - 3) + \frac{c}{d} [\exp(\frac{d}{2}(I_1 - 3)) - 1], \quad (5)$$

where  $I_1 = \text{Tr}(\mathbf{B})$  and  $I_2 = \text{Tr}[\det(\mathbf{B})\mathbf{B}^{-1}]$  are strain invariants and  $(a, b, c, d)$  the material parameters.

The corresponding response equation for the Cauchy stress tensor  $\boldsymbol{\sigma}$  is

$$\boldsymbol{\sigma} = -p\mathbf{1} + 2(W_1\mathbf{B} + W_2\mathbf{B}^{-1}), \quad (6)$$

where  $\mathbf{1}$  denotes the identity tensor and  $W_k = \frac{\partial W}{\partial I_k}$  ( $k = 1, 2$ ).

From (6), the equations of motion in the absence of body forces are reduced to

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \ddot{r}, \quad (7)$$

the dot denotes the time differentiation and  $\rho$  denotes the current mass density of the arterial material.

From (4), it easily follows that the second time differentiation of  $r(\mathbf{R}, t)$  is

$$\ddot{r}(\mathbf{R}, t) = \frac{\ddot{h}(t)}{2r(\mathbf{R}, t)} - \frac{\dot{h}^2(t)}{4r^3(\mathbf{R}, t)}, \quad (8)$$

where  $h(t) = r_i^2(t)$ .

Consequently, integrating (7) between the inner radius  $r_i(t)$  and the outer radius  $r_c(t)$ , and using (8) leads to the following relation (Humphrey et al., 2002)

$$P_i(t) = P_c(t) + \int_{r_i(t)}^{r_c(t)} \frac{\sigma_{\theta\theta}(r, t) - \sigma_{rr}(r, t)}{r} dr + \frac{\rho}{2} \left[ \ddot{h}(t) \ln \left( \frac{r_c(t)}{r_i(t)} \right) + \frac{\dot{h}^2(t)}{4} \left( \frac{1}{r_c^2(t)} - \frac{1}{r_i^2(t)} \right) \right], \quad (9)$$

where  $P_i(t)$  and  $P_e(t)$  are respectively the inner and the outer pressure applied to the CCA.

Note that the outer pressure can be expressed as an exponential form [Masson et al., 2008; Humphrey et al., 2002)

$$P_e(t) = e \cdot \exp\left(f \frac{r_e(t)}{r_e(t_0)}\right), \quad (10)$$

where  $t_0$  is the diastolic time over the cardiac cycle and  $e$  and  $f$  are material parameters.

### 3 Probabilistic approach

In this section, we present the followed methodology to take into account the random uncertainties of geometric and mechanical parameters for the estimation of the arterial pressure  $P_i(t)$ . The random variables considered here are the opening angle  $\Theta_0$ , the axial stretches  $\lambda$  and  $\Lambda$ , the inner radius of the artery  $R_i$ , the density  $\rho$ , the six parameters (a,b,c,d,e,f) governing the strain-energy density function and the outer pressure  $P_e(t)$ . As a result, we have 11 random variables to examine.

The construction of the probabilistic approach requires the description of random variables by probabilistic density functions (pdf for short). The pdf allows to carry out numerous and independent realizations of each random variable and consequently of the arterial pressure. The entropy maximum principle is used here to construct the pdf (Soize, 2001; Jaynes, 1957). Let  $X$  be a real random variable which represent the previous uncertain parameters, we assume that  $X$  is a stochastic modeling of the variable  $x$ . The uncertainty (or dispersion) measurement of  $X$  is given by the entropy of the probabilistic density function  $p_X$

$$S(X) = - \int_{-\infty}^{+\infty} p_X(x) \log(p_X(x)) dx \quad (11)$$

The entropy maximum principle allows to determine the pdf by searching the maximum of  $S(p_X)$  among the potential density functions under the only available (or used) information. This latter can be, for instance, the support of random variable, the mean, the standard deviation, higher moments, etc. Each type of information defines a constraint for the optimization problem given by



$$\dots \quad (12)$$

The first equation of the system (12) is the normalization condition which must be satisfied on the random variable support, whereas the second equation describes the available information. For example, if the given information is the mean of  $X$ ,  $g_i(x) = x$ .

To solve the optimization problem, we introduce  $(1+n)$  Lagrange multipliers  $\lambda_i$  associated to each constraint. It was shown in (Kapur et al., 1992) that  $\lambda_i$  can be obtained by minimizing a strictly convex function  $H$  given by

$$H(\lambda_0, \lambda_1, \dots, \lambda_n) = \lambda_0 + \sum_{i=1}^n f_i + \int_{\mathbf{R}} \Pi_{[a', b']} (x) \exp(-\lambda_0 - \sum_{i=1}^n \lambda_i g_i(x)) dx, \quad (13)$$

where  $[a', b']$  is the support of the pdf of the random variable  $X$  and  $\Pi_{[a', b']} (x)$  is the indicator function which is equal to 1 if  $x$  belongs to  $[a', b']$  and 0 if not.

Eventually, maximizing the entropy  $S(p_X)$  leads to minimizing the function  $H(\lambda_i)$ . The pdf is completely known once  $\lambda_i$  is known and it is written as

$$p_X(x) = \Pi_{[a', b']} (x) \exp(-\lambda_0 - \sum_{i=1}^n \lambda_i g_i(x)) \quad (14)$$

The probabilistic approach leads to the construction of a confidence region whose bounds are defined by

$$p^- = \zeta(1 - p_c) \quad ; \quad p^+ = \zeta p_c \quad (15)$$

where  $p^-$  and  $p^+$  are respectively the lower and upper limits and  $\zeta$  is the fractile of order  $p_c$  defined for a random variable  $X$  as

$$\zeta(p) = \{x; p(X < x) = p_c\} \quad (16)$$

The reader could consult (Kree et al., 1983) where the probabilistic approach is more detailed.

#### 4 Results and discussion

To construct the probabilistic density functions of random variables, let us consider the 5 random variables  $(\Theta_0, \lambda, \Lambda, \rho, R_i)$ . As no statistic data is available to our knowledge, we have been based on the literature to construct the information that we have to use to determine the pdf. The different values found throughout different researches have permitted to define an

average and a support for each random variable. For instance, we have considered that the mean of the opening angle  $\Theta_0$  is equal to  $130^\circ$  (Delfino et al., 1997) and that  $\Theta_0$  belongs to  $[60^\circ, 180^\circ]$ . Therefore, the pdf associated to  $\Theta_0$  is given by

$$p_X(x) = \Pi_{[a',b']}(x) \exp(-\lambda_0(a', b', m_X) - x\lambda_1(a', b', m_X)), \quad (17)$$

where  $m_X$  is the mean of the random variable  $X$ .

Concerning the six remaining random variables (a,b,c,d,e,f), we have only defined a support for each parameter, since there was not enough information about these values to also define an average for each one of them regarding the common carotid artery application. In this case, the obtained pdf is uniform on the considered support and given as follows for a random variable  $X$

$$p_X(x) = \Pi_{[a',b']}(x) \frac{1}{b' - a'}. \quad (18)$$

In Table (1), we summarize the available information of the eleven random variables used in the modeling.

Considering the random variable of the opening angle and setting  $\alpha = 180/\Theta_0$ , the minimization of  $H(\lambda_{\alpha 0}, \lambda_{\alpha 1})$  leads to:  $\lambda_{\alpha 0} = -3.4564$  and  $\lambda_{\alpha 1} = 2.5216$  where  $\lambda_{\alpha i}$  ( $i = 0,1$ ) are the Lagrange multipliers associated to  $\alpha$ .

Similarly, the minimization of  $H(\lambda_{\lambda 0}, \lambda_{\lambda 1})$  leads to:  $\lambda_{\lambda 0} = -5.0982$  and  $\lambda_{\lambda 1} = 4.4664$ . Figure 2 presents the pdf of each random variable of the series ( $\Theta_0, \lambda, \rho, R_i$ ) as well as their realizations according to the pdf. We can notice that all the obtained pdf have a similar form except the one associated with the random variable  $\rho$ . This difference can be explained by the support of this variable which is smaller than the four other supports. The given support influences the minimization result. For this case, we obtain  $\lambda_{\rho 0} = -3.9121$  and  $\lambda_{\rho 1} = 0.0001$ . Consequently, the pdf given by (17) is constant.

The eleven random variables are considered to manage numerous arterial pressure simulations, in total 2000 simulations were carried out. The first results given by the probabilistic approach showed a great dispersion in the estimation of the internal pressure which is essentially due to the large supports given by the available information. In fact, the interval bounds were chosen from different values reported in literature in order to construct a potential support for each random variable. Since the arterial pressure is sensitive to several random variables, the definition of the interval bounds should be more accurate to avoid great results deviation. Thus, we have made use of the first results in order to initialize the available information of random variables so that final results are more realistic and convincing.

The starting up of the stochastic approach is based on random variable values obtained from the best realization which here corresponds to the selection number 1881 and presents the minimum error with the experimental data. Figure 3 depicts the evolution of the arterial pressure obtained with this realization. Both stochastic and experimental curves are close except around  $t = 0.35$  s where the error is nearly 7%. The global error, noted between the stochastic model and the data, is equal to 2.28%. The results are recapitulated in the table 2.

These values are next considered as the averages of random variables whereas the support bounds are obtained by adding  $\pm 5\%$  to the average in order to take into account the uncertainty of parameters. The associated stochastic arterial pressure is the mean (or deterministic) model since it uses the averages of all the random variables as input data. Thus, if  $L^-$  and  $L^+$  are respectively the lower and upper bounds of the support of the random variable  $X$ , then  $L^- = 0.95m_X$  and  $L^+ = 1.05m_X$ . The new averages and supports represent the updated available information to construct the pdf according (17) and (18).

Next, we present the results of Monte Carlo simulations (Kalos et al., 1992) of the arterial pressure taking into consideration the updating available information of the random variables. Let recall that the mean model is known since experimental data are available and it was constructed in order to match them. Suppose that we have no data to suggest the good averages of the random variables, one may consider a percentage uncertainty for the description of the problem entries. In our case, we have considered a small uncertainty of 5% for each of the eleven random variables.

1500 simulations of Monte Carlo were carried out by generating random values of the uncertain variables according to the pdf that is constructed after the starting up step. Convergence of the Monte Carlo simulations can be observed by the calculation of the mean and the standard deviation (SD) of the mean arterial pressure random response. The mean arterial pressure can be defined as  $MAP = (SAP + 2 DAP)/3$  where SAP and DAP are respectively the systolic and diastolic arterial pressure.

The convergence of the stochastic method is reached in the 500th realization (Figure 4). The mean of MAP appears to be equal to 0.0102 MPa (76.5 mmHg) whereas the SD is equal to 0.0024 MPa. On the other hand, the MAP corresponding to the experimental data is 0.0093 MPa (69.8 mmHg). Note that, according to the strong law of large numbers theory (Andrews, 1988) the obtained arterial pressure at convergence is the same as that found by the mean model, the obtained set of parameters is therefore equal to that previously given by the deterministic case.

The different independent trials enable us to define a predicted confidence region in which the probability to have a realization is equal to  $p_c = 95\%$ . This can be done by ordering the MAP from the lower value  $p^-$  to the upper value  $p^+$ . For example, in our case  $p^- = 0.0062$  MPa and  $p^+ = 0.0176$  MPa. It should be emphasized that a physiological constraint was taken into account in our modeling to ensure the limitation of the systolic and diastolic pressures to physiological values (McDonald, 1958). This non violation condition allows the final solution to be realistic and concluding.

In Figure 5, we present the evolution of the maximal and minimal arterial pressures in the confidence region for a confidence probability equal to 95%. The line and dashed curves are respectively associated with the data and the mean model whereas the dotted curves represent the boundaries of the confidence region.

Eventually, we can notice that an uncertainty of 5% in the problem input description can lead to an error estimated to 21.42%. This error is obviously higher when the uncertainty percentage is 10% or 15% for example. This remark justifies the use of stochastic approach for the modeling of arterial mechanical behavior.

The different carried out realizations can be summarized graphically by a histogram to describe the MAP distribution. Here, we split the range of the pressure into 40 classes in abscissa, and in vertical axis, we note the frequency to have a value of the MAP. When the number of classes increases, the bars become less large and therefore the curve fitting of the pdf is more continuous. This enables us to have information about the pdf corresponding to the global random response. In Figure 6, we present the real pdf of the MAP in comparison with the Gaussian pdf constructed by using the mean and standard deviation obtained from Monte Carlo convergence. We can notice that the real pdf describing the MAP evolution is similar to Gaussian pdf.

On the other hand, we present some results of the stochastic approach by considering the only uncertainty of each random variable in order to analyze the influence of every parameter independently of the other uncertain parameters. During simulations, the considered random variable is described by its associated pdf constructed as aforementioned, whereas the 10 others parameters are maintained constant to their values obtained by the deterministic model. In Figure 7, we show the distribution of the arterial pressure for each random case. The bold curve represents the mean model whereas the other curves describe the stochastic simulations. We can notice that the arterial pressure random response is more influenced by some data than others. The vertical arrow in the first figure indicates the range width of the pressure variation. We can see that the arterial pressure is much more sensitive to the opening angle

and the inner radius variation than the axial stretches ( $\lambda$ ,  $\Lambda$ ). Concerning the mechanical parameters, the random variable  $f$  has more influence on the solution than the others. Nevertheless, the effect of the parameters  $a$  and  $b$  is negligible in the considered support. This analysis illustrates the necessity to take into consideration the uncertainties of the parameters to have a good prediction of the region confidence of the random final result.

## 5 Conclusion

Through a simple case study, we have demonstrated that a parametric analysis should be considered to take into account the random uncertainties of the problem inputs. The probabilistic approach allows the evaluation of the arterial pressure dispersion and provides a 95% confidence interval which covers the potential random responses. This approach is worth to be adapted for more accurate biomechanical constitutive relations such as anisotropic models by considering the contribution part of the collagen fibers in the artery to improve the description of the arterial mechanical behavior. To our knowledge, this is the first time that material and geometric parameters of artery have been estimated using in vivo human data based on stochastic model.

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Figure 1

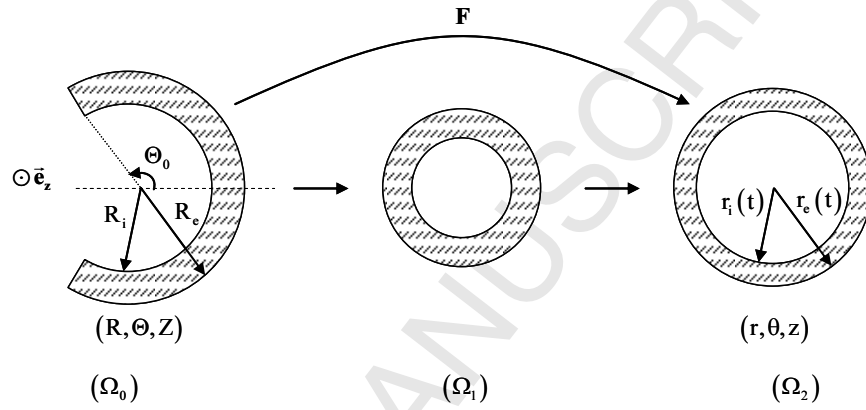




Figure 2

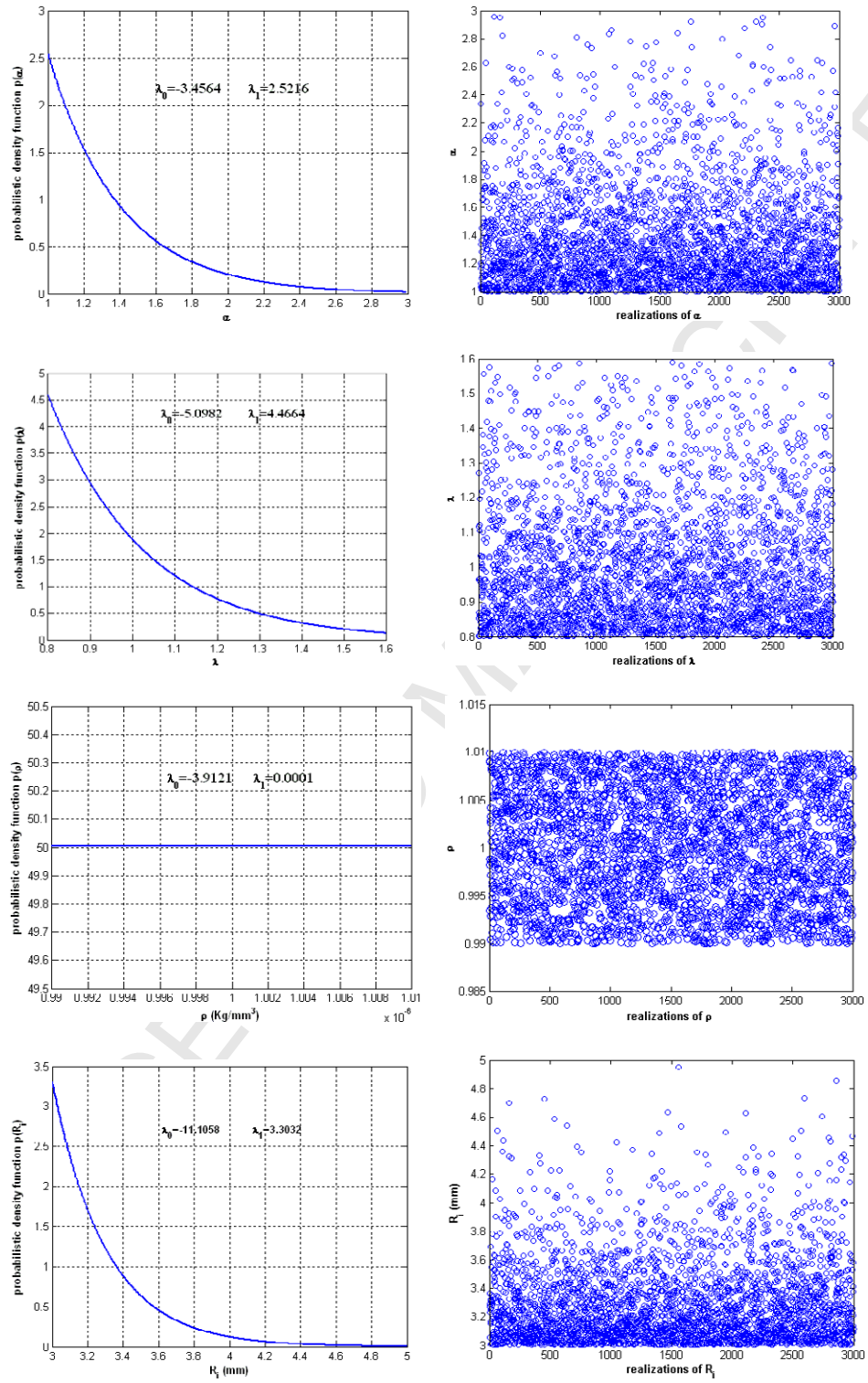


Figure 3

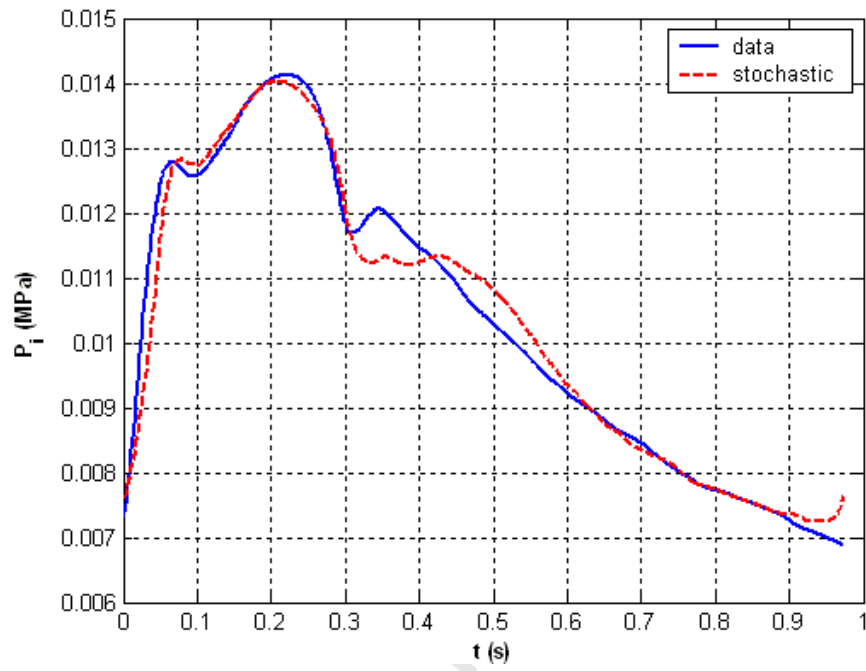


Figure 4

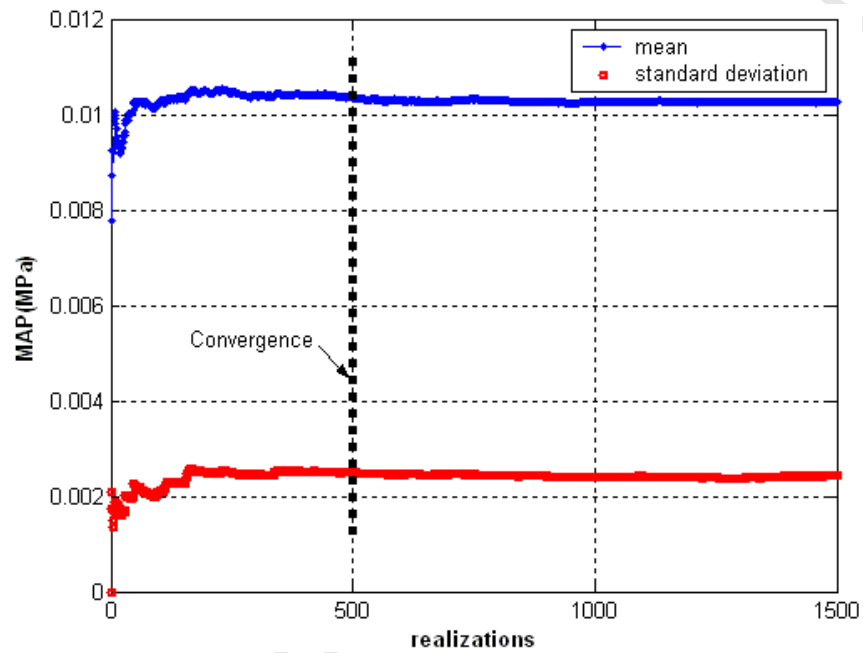


Figure 5

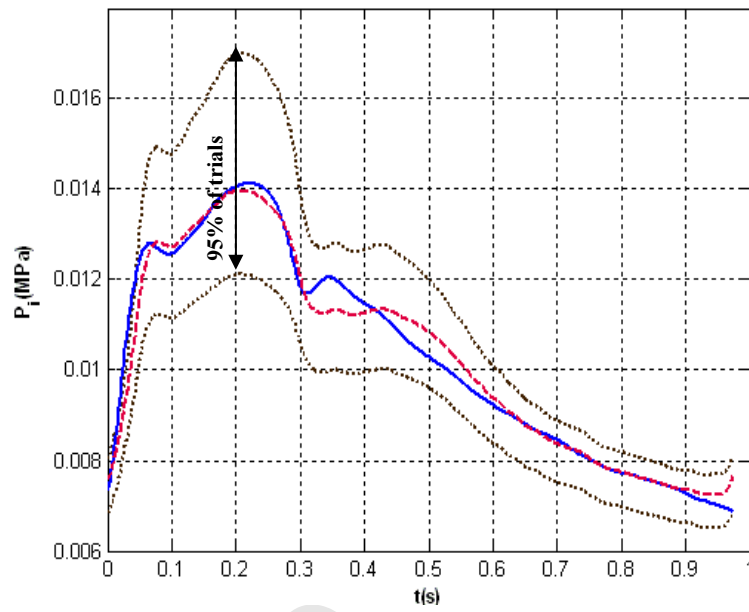


Figure 6

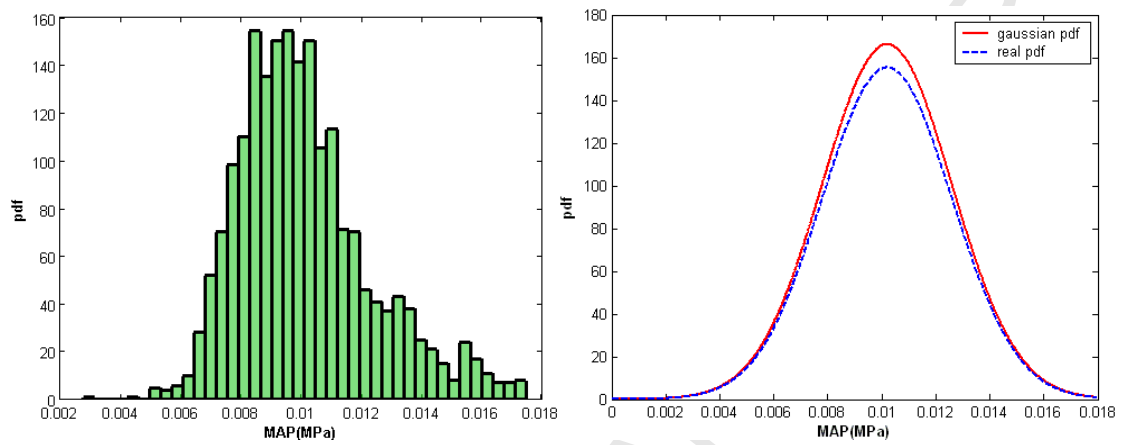
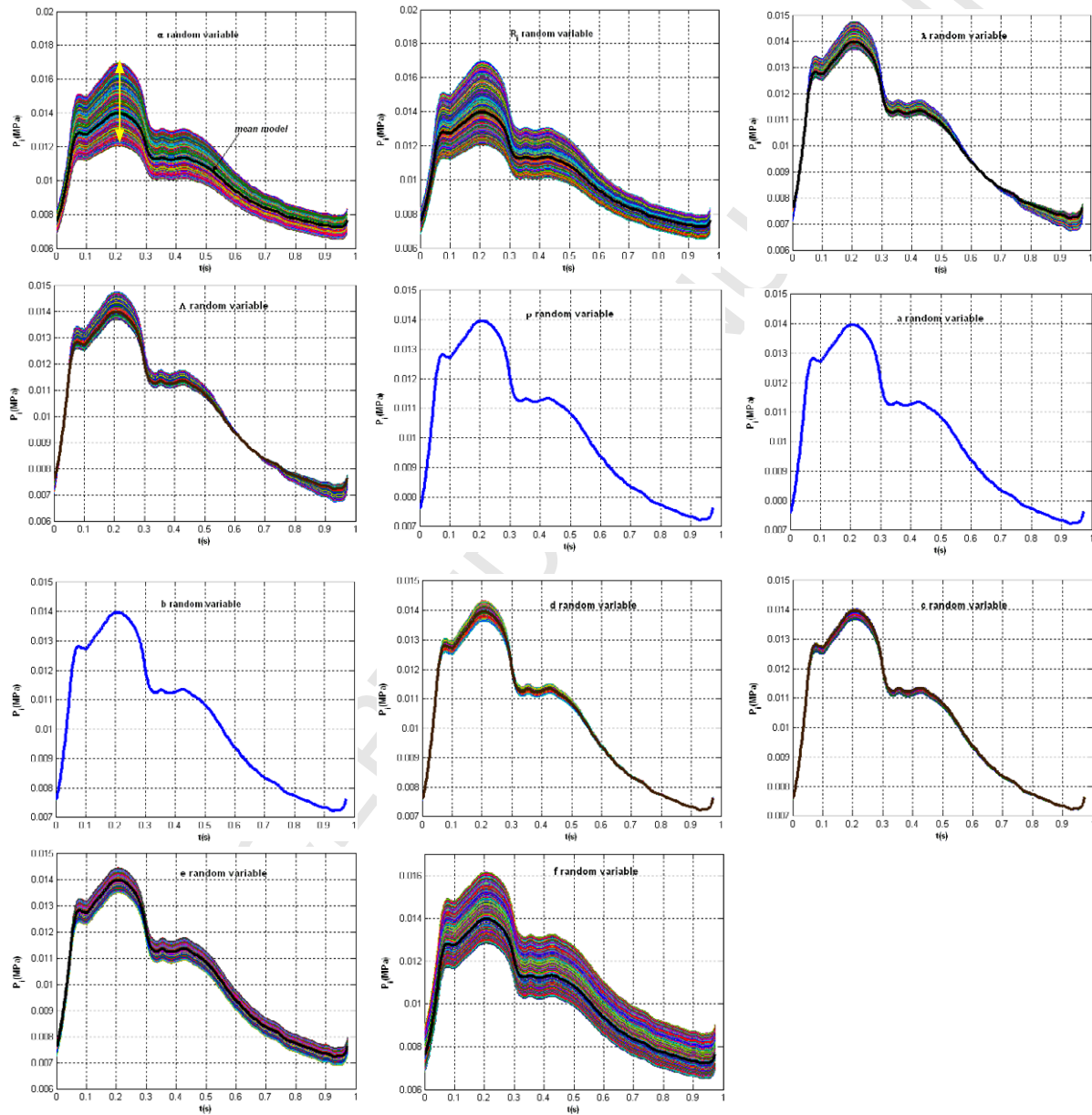


Figure 7



**List of figures**

**Figure 1.** Kinematics of the arterial wall relative to a nearly stress-free reference state ( $\Omega_0$ ), the unloaded state ( $\Omega_1$ ) and the in vivo loaded state ( $\Omega_2$ ).

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**Table 1.** Available information for the random variables.

Random Variable	Average (mean)	Support
$\Theta_0(^{\circ})$	130	[60 , 180]
$\lambda$	1	[0.8, 1.6]
$\Lambda$	1	[0.8, 1.6]
$\rho(\text{Kg}/\text{mm}^3)$	$1e^{-6}$	$[0.99e^{-6}, 1.01e^{-6}]$
$R_x(\text{mm})$	3.3	[3, 5]
a (MPa)	#	$[9.2e^{-6}, 1.8e^{-2}]$
b (MPa)	#	$[1e^{-12}, 1e^{-10}]$
c (MPa)	#	$[3.6e^{-6}, 1.4e^{-2}]$
d	#	[1, 17]
e (MPa)	#	$[8e^{-5}, 1.8e^{-4}]$
f	#	[3, 4]



**Table 2.** Mean model results.

$\Theta_0$ (degree)	$\lambda$	$\Lambda$	$\rho$ [Kg.mm <sup>-3</sup> ]	$R_i$ [mm]	$a$ [MPa]	$b$ [MPa]	$c$ [MPa]
138.846°	0.985	0.826	1.002e <sup>-6</sup>	3.609	0.0014	1.103e <sup>-12</sup>	0.0114
$d$	$e$ [MPa]	$f$					
16.438	1.45e <sup>-4</sup>	3.856					