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# Computing the Resilience of a Wastewater Treatment Bioreactor

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***Abstract*—Biological wastewater treatment reactor are designed to reduce the pollutant content of a wastewater to an acceptable level often fixed by wastewater discharge regulations. The reactor design is often based on average wastewater flow and composition patterns. However, industrial wastewater treatment reactors are often subject to unexpected perturbations (variations in wastewater flow, composition or shift in the microbial communities). Hence the capacity of the reactor to recover from these perturbations in reasonable times should also be considered as an important design criteria. In this work we illustrate how this property can be quantified by calculating reactor resilience using return time concept. We show that a reactor design that maximizes the productivity (in term of mass of pollutant removed per unit of time and unit of reactor volume) may not be optimal in term of resilience. Hence, a compromise between productivity and resilience should be targeted when designing biological wastewater treatment reactors.**

***Keywords*-return time; monod model; reactor design; reactor productivity**

## I. Introduction

The purpose of wastewater treatment is to remove pollutants that can harm the aquatic environment if they are discharged into it [1]. Among the techniques used for wastewater treatment, biological processes are the most widely used ones, especially for the treatment of large wastewater quantities produced by cities and industrial activities. In biological wastewater treatment processes, a community of microorganisms is cultivated in a mixed reactor fed with the wastewater. The microorganisms remove the pollutant compounds in the wastewater producing less harmful elements ($CO_2$, $N_2$, etc.), new cells (biomass) and if the process is anaerobic, some functional groups of microorganisms can transform a part of the pollution into methane, a gas that can be used in energy production.

Biological wastewater treatment reactors are often designed to reduce the pollutants content of a wastewater to an acceptable level that is usually fixed by wastewater discharge regulations. Based on averaged properties of the wastewater, engineers in charge of the design of wastewater treatment reactors often attempt to calculate the minimal reactor volume that allows the satisfaction of the discharge limit. A larger reactor would imply higher investment and operation costs. However, during the operation of industrial scale wastewater treatment systems, unexpected variation of the flow and composition of the wastewater or uncontrolled shifts in the microbial composition are common. Hence, the reactor capacity to respond and recover from these environmental and internal changes in reasonable times should also be considered during the design phase. In this paper, we propose to use the concept of engineering resilience [2] to quantify the speed at which the reactor recovers from a perturbation.

The paper is organized is five sections. Section 2 provides a short background on the biological treatment of wastewater and the concept of reactor resilience. In Section 3 we describe a simple dynamical model of a bioreactor and illustrate how this model can be used for the calculation of the reactor volume and resilience. In Section 4 we present simulation results that we discuss in Section 5.

## II. Background

### A. Biological reactor

A biological wastewater treatment reactor is basically a vessel filled with a wastewater (tank) and containing a microbial community, also called biomass. Figure 1 shows a simple example of a well-mixed and continuously-fed wastewater treatment reactor. The reactor operates at a constant volume $V$. However the flow rate and properties of the influent wastewater can varies and the system is not necessary at steady state. The biodegradation of pollution can schematically be represented using the following reaction

$$\begin{aligned} Pollutants + biomass \; &- > \; new\ biomass \\ &+ \; residual\ pollutant \end{aligned} \tag{1}$$

The microbes grow by consuming the pollutants content of the wastewater transforming a part of it to new cells. In the context of wastewater treatment a constraint is typically imposed on the residual pollutants concentration that should be lower than a limit value $S_{lim}$.

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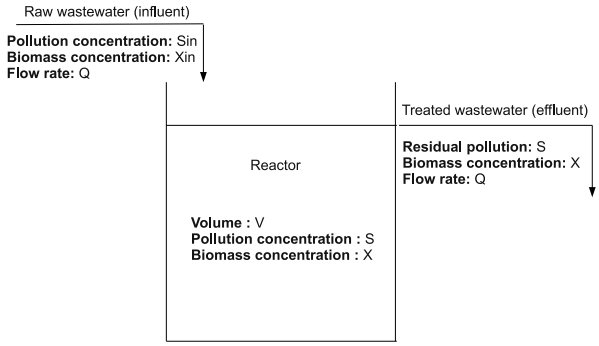


Figure 1: Simple example of a bioreactor. The reactor volume is often calculated such that the residual pollution concentration is lower than a limit value $S_{lim}$

### B. Reactor resilience

Reactor resilience can be defined as the speed with which the reactor recovers following a perturbation. This type of resilience is also called engineering resilience and assumes the existence of a global stability: only one equilibrium steady sate exists, or, if other operating states exist, they should be avoided by applying safeguards [2][3]. Perturbations like variation in the flow or composition of the influent wastewater can drive the system out of the equilibrium state inducing an increase of the residual pollution content beyond the limit value $S_{lim}$. The system is supposed to recover spontaneously in a more or less time. The faster the system recovers from this perturbation (the residual pollution concentration decreases below the limit value $S_{lim}$) the greater its resilience is said to be. Hence we use the return time, denoted $T_R$, to measure the reactor resilience. Shorter return times implies greater resilience.

## III. Reactor model

Let $S$ and $X$ denotes respectively the concentrations of pollutants and biomass in the reactor. A dynamical model of the reactor can be obtained by deriving the mass balances of each of these state variables following the generic equation:

$$\begin{aligned} Accumulation &= input - output \\ &+ production - consumption \end{aligned} \tag{2}$$

The mass balance of the biomass writes:

$$V\frac{dX}{dt} = QX_{in} - QX + V\mu(S,X) \tag{3}$$

where $V$ $\left([L^3]\right)$ is the reactor volume, $Q$ $\left([L^3T^{-1}]\right)$ is the influent wastewater flow rate, $X_{in}$ is the biomass concentration in the wastewater and $\mu(S,X)$ is the biomass growth function that depends on the state of the system. Analogously we can derive the mass balances of the pollutant:

$$V\frac{dS}{dt} = QS_{in} - QS - Y_{s/x}V\mu(S,X) \tag{4}$$

where $S_{in}$ is the pollutant concentration in the influent and $Y_{s/x}$ is the pollution consumption yield (mass of substrate consumed per unit of mass of produced biomass). Equations 3 and 4 can be rearranged by introducing the dilution rate $D = Q/V$:

$$\frac{dS}{dt} = DS_{in} - DS - Y_{s/x}\mu(S,X) \tag{5}$$

$$\frac{dX}{dt} = DX_{in} - DX + \mu(S,X) \tag{6}$$

The system can further be simplified by assuming that $X_{in} = 0$ (the concentration of microorganisms in the the inflow wastewater is neglected compared to the concentration of microorganisms in the reactor). A common form of the growth function $\mu(S,X)$ is the following Monod expression:

$$\mu(S,X) = \mu_{max}\frac{S}{S+k_s}X \tag{7}$$

where $\mu_{max}$ $([T^{-}1])$ is the maximum growth rate of the biomass and $k_s$ $\left([ML^{-3}]\right)$ is the half saturation constant. According to Equation 7 the growth rate of the biomass increases with the increase of the pollutant concentration and saturates at $\mu(S,X) = \mu_{max}$. The dynamical model defined by equations 5-7 (with $X_{in} = 0$) has two equilibrium states. A trivial unstable equilibrium given by $X^\star = 0$ and $S^\star = S_{in}$ and a stable equilibrium (for $\mu_{max} > D$) given by:

$$S^\star = \frac{k_s}{f-1} \tag{8}$$

$$X^\star = \frac{1}{Y_{x/s}}\left(S_{in} - \frac{k_s}{f-1}\right) \tag{9}$$

where $f = \mu_{max}/D$. The kinetic parameters $\mu_{max}$, $k_s$ and $Y_{s/x}$ depend on the microbial community, the chemical nature of the organic pollution and environmental conditions (temperature, pH, etc.). Assuming that these parameters are known, Equation 8 can be used to calculate the volume of the bioreactor for a target limit residual pollution value:

$$f = \frac{k_s}{S_{lim}} + 1 \quad (10)$$

$$\frac{V_{min}\mu_{max}}{Q} = \frac{k_s}{S_{lim}} + 1 \quad (11)$$

$$V_{min} = \frac{1}{Q\mu_{max}}\left(\frac{k_s}{S_{lim}} + 1\right) \quad (12)$$

where $V_{min}$ is the minimal reactor volume for which the residual pollution concentration is equal to the maximum allowed value $S_{lim}$. According to this simple model, reactor volumes higher than $V_{min}$ should satisfy the constraint $S < S_{lim}$. We propose to assess how the return time $T_R$ varies for different reactor volumes $V$ satisfying $V > V_{min}$.

## IV. Results

Using the parameters in Table I, the minimal reactor volume that satisfies the constraint on the residual pollution level can be calculated from Equation 12 for $S^\star = S_{lim} = 0.01kg/m^3$. This yields a reactor volume $V_{min} = 200m^3$. We explore how the return time varies when a perturbation pushes the system away from the stable equilibrium region. For this end we assume that a perturbation can potentially drives the system to any location in the system state space that we delimit to $S \in [S_{lim}, S_{in}]$, $X \in [0.005, 0.1]$. The distance between these points and the equilibrium state can be viewed as an indicator of the intensity of the perturbation. Implicitly a strong perturbation would drive the system far away from the equilibrium point while a small perturbation just slightly modifies the state of the system. To calculate the return time, we discretize state space of the system with a resolution $\Delta X = 0.005$, $\Delta S = 0.05$. For each point, we calculate the return time to the region $S < S_{lim}$ as illustrated in Figure 2. Each point in the state space is taken as a starting point to solve the reactor dynamical model. The return time corresponds than to the instant at which the system trajectory crosses the line $S = S_{lim}$.

Figures 3 shows the landscape of the return times in the case of a reactor with a minimal volume $V = 200m^3$. The return time are high ($T_R > 100hours$) for low $S$ and $X$ and low for high $S$ and $X$. This result is counterintuitive as the system seems to recover more rapidly from a strong perturbation that pushes it state faraway from the equilibrium than from small perturbations. Figure 4 shows two examples of trajectories starting from two points located at different distances from the equilibrium and confirm the previous result.

We explored how the return time evolves when the reactor volume is increased from $V = 200m^3$ to $V = 250m^3$. The comparison of Figure 3 and 5 clearly shows that the reactor resilience can be improved by increasing the reactor volume. The calculation of the return time can provide a mean to quantify what is gained in term of resilience when the volume of the reactor in increased.

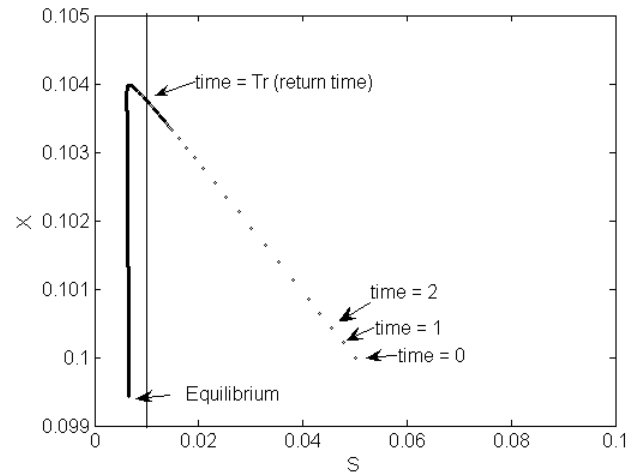


Figure 2: Example of trajectories starting from an initial condition $[S(t = 0) = 0.05, X(t = 0) = 0.1]$. The return time relative to this initial condition corresponds to the time at which the system trajectory in the phase plane crosses the constraint line $S_{lim} = 0.01$

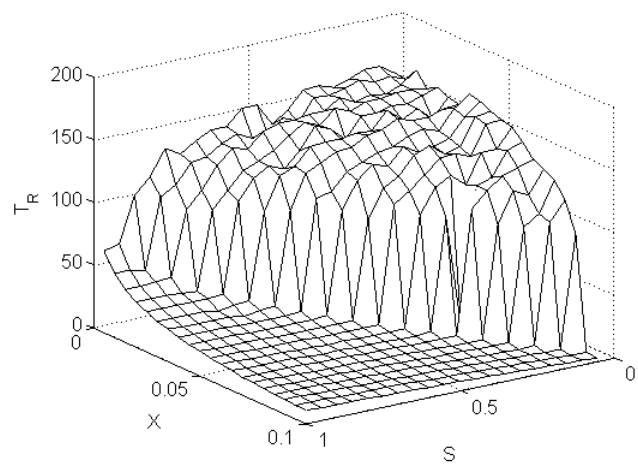


Figure 3: Map of the return time for $V = 200m^3$

## V. Discussion and conclusion

Wastewater treatment reactor design is usually based on maximizing reactor productivity by designing the smallest reactor that allows the removal of the wastewater pollutant to the required level. Our results illustrates that a reactor volume maximizing the productivity (expressed in mass of removed pollutant per unit of time and unit of reactor volume) is not necessarily optimal with regard to reactor resilience. In the context of biological wastewater treatment where significant unexpected fluctuations either in wastewater flow and quality are common we believe that finding a compromise between productivity and resilience during the

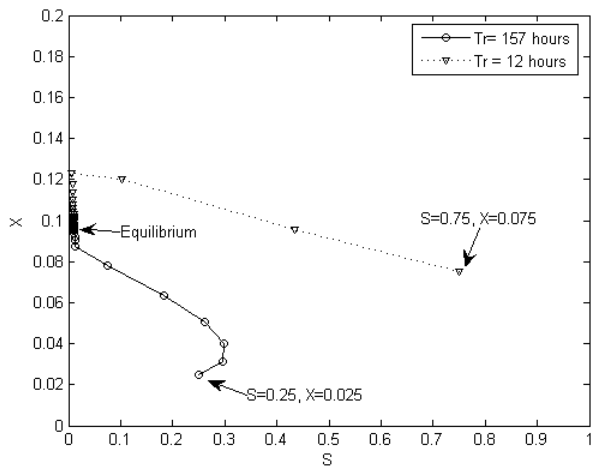


Figure 4: Example of trajectories starting from two points at different distances from the equilibrium state. The return time is not proportional to the distance to the equilibrium point. The state of the system is plotted for both simulations every 10 hours.

Table I: Model parameters values

| Parameter | Value | Unit |
|---|---|---|
| $\mu_{max}$ | 0.1 | 1/hour |
| $k_s$ | 0.01 | $kg\ pollution/m^3$ |
| $Y_{s/x}$ | 10 | $kg\ pollution/kg\ biomass$ |
| $S_{in}$ | 1.0 | $kg\ pollution/m^3$ |
| $X_{in}$ | 0.0 | $kg\ biomass/m^3$ |
| $Q$ | 10 | $m^3/hour$ |
| $S_{lim}$ | 0.01 | $kg\ pollution/m^3$ |

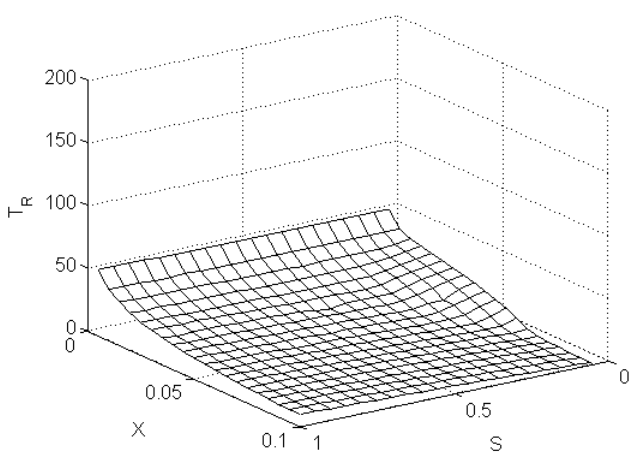


Figure 5: Map of the return time for $V = 250m^3$

reactor design phase is necessary.

The concept of resilience that we used in this paper is tightly linked to the properties of the dynamical system. The notion of return time implies that the system has a tendency to converge towards an equilibrium (attractor) that satisfies the imposed constraint. In our case the dynamical model has a unique stable equilibrium and the condition $V > V_{min} = 200$ is sufficient to ensure that the equilibrium satisfies the constraint $S < S_{lim}$. This concept of resilience can not be applied to a system with different attractors that may be reached by the system but do not necessary satisfy the constraint. In this case, the perturbation may drive the system dynamics to an unacceptable equilibrium, which implies an infinite return time. This problem can be solved by adding a mean to act on the system dynamics (a control). The problem of computing the reactor resilience is then better addressed using the framework of the viability theory [4][5]. In this case, the reactor is said to be resilient if there exit a control strategy that allows to drive the system back into the desired region. The resilience can then be quantified using a cost function that evaluate the performance of the control strategy.

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