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Improvements of radiative transfer calculation for SF₆ thermal plasmas

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Abstract:

We present a comparison between an exact calculation of radiative transfer in SF₆ thermal plasma based on a fine description of the spectrum with 300 000 spectral points for each temperature value, for simplified conditions (1D and 2D geometries with imposed symmetrical temperature profiles and Local Thermodynamic Equilibrium) and two kinds of approximated calculations. The first is the classical Net Emission Coefficient largely used in arc modelling. The second one is based on a very simplified spectral description with only seven intervals assuming a grey body condition within each interval and using the Planck and the Rosseland averaging for deducing the mean absorption coefficient. At high temperature the use of the Rosseland averaging is not satisfactory. The other two approximated methods (Net emission and the Planck averaging) are acceptable but the radiative flux is in general not very accurate. The radiative transfer calculation can be improved following three ways: a better knowledge of the basic processes and in particular of the absorption coefficients for diatomic and polyatomic molecules; the use of different definitions of mean absorption coefficients (Planck averaging at high temperature and mean natural value at low temperature); a careful choice of the spectral intervals.

Keywords: thermal plasmas; radiation; radiative transfer; SF₆

1 – Introduction

Radiation plays an important role on the behaviour of electric arcs and of thermal plasmas and is often the main mechanism of energy transport within the plasma and between the plasma and the surrounding materials. In the case of SF₆ plasmas, radiative transfer has been calculated for more than 30 years because this gas is widely used in High Voltage Circuit Breaker (HVCB) where radiation has three important effects: the radiation losses tend to cool the plasma during the high current phase; the partial absorption of radiation by the cold surrounding SF₆ gas heats the gas and thus increases the pressure within the closed apparatus which has a big influence on the interruption capability; finally the radiative flux is highly responsible for the nozzle ablation which is used in modern HVCB to increase the efficiency of the interruption. The difficulty of radiative transfer calculation is related both to the geometry and to the spectral dependence of the radiation intensity. In practical conditions, exact radiative transfer calculation is impossible in a general modelling process with a lot of iterations. This fact obliges to make approximated calculations and the choice of these methods depends on the objectives.

The first objective is to compute the radiation escaping the hottest regions of the plasma which allows calculating with a good accuracy the temperature in these regions and then to have a realistic description of the plasma. The simplest and nevertheless quite accurate method is the previous calculation of the Net Emission Coefficient (NEC) followed by the direct use of this coefficient in thermal plasma modelling. We will present the bases of this method in part 3. This method is of interest because the complicated calculation of the net emission (net meaning emission minus absorption, which is mathematically represented by the divergence of the radiative flux) integrated over all the spectrum (VUV, UV, visible and IR parts) is previously calculated for isothermal conditions and then injected in the energy conservation equation by a simple source term. This NEC was initially proposed by Lowke [1] and first calculated for SF₆ by Liebermann and Lowke [2]. Since this pioneer work, other teams have been calculating the NEC for SF₆ [3-4] and several works demonstrated the interest in and accuracy of this coefficient for calculating the temperature field in arcs [5] introducing a multiplying factor [6-7].

However, by definition, the NEC cannot give an estimation of the radiation absorption in the plasma and cold gas surrounding the hot core of the arc. The first idea to take into account this absorption was to modify the NEC introducing an empirical absorption adjusted with

comparison with some experimental results [8]. A more sophisticated approach was proposed by Sevastyanenko introducing the Partial Characteristics Method (PCM) explained in [9]. This method is based on a previous and complicated calculation of source and sink terms (called Som and ΔSim) computed for segments of given lengths and functions of the two extreme points temperatures, assuming a linear variation of temperature along one segment. Then these data can be used for calculating the radiative transfer in a general modelling along various directions. For SF_6 the Som and ΔSim terms have been computed by Aubrecht and Lowke [10] and by Raynal et al [6; 11].

The NEC and PCM are based on a fine description of the spectrum needing a very high number of spectral points at each temperature value. On the contrary, there exist classical methods of radiative transfer calculation that are based on a simplified description of the spectrum (P1 model, discrete ordinates method, see for example the book of Modest [12]). The most popular simple spectral description for SF_6 plasma is to define a few spectral intervals (less than 10 in general) where the absorption coefficient is constant for a given temperature (grey body approximation by spectral intervals or Mean Absorption Coefficient (MAC)). It is well-known that this description may lead to very large errors because the line transfer is not well treated. Nevertheless, Gleizes et al proposed an original way to compute the MAC by calculating the Planck average coefficient and by using an escape factor for the lines [13]. For SF_6 the spectrum was divided in five intervals and the values of the MAC can be found in [14]; they have been used by Eby et al in a HVCB SF_6 arc modelling [15].

Having the possibility to use several methods, it is interesting to test them and to define the best compromise between the accuracy of the results and the difficulty (including CPU times) of the methods. In general, for SF_6 plasmas, the validation of the methods is realised either by comparing the modelling results (temperature field, variations of global pressure) obtained using various methods and/or by comparing these results with some experimental data. It is to be noted that a direct validation of the total flux or of the divergence of the flux is not possible in practical cases because a large part of the radiation occurs in the VUV part and is mainly caused by resonance lines. Only partial experimental validations on the limited part of the spectrum [16] and on the absorption by cold SF_6 [17] have been performed. Among the works including a comparison of simplified methods for radiative transfer calculation, we may refer to the papers of Verite et al [18] and Dixon et al [19], both including the NEC and PCM methods (MAC also in [19]). The indirect validation with temperature, voltage and pressure measurements, allows to define an uncertainty of the order of a factor of 2 for the divergence of the flux in the hottest regions (comparison with temperature in the hottest regions) and for

the absorbed radiation (comparison with pressure). The accuracy on the flux reaching the walls is difficult to evaluate.

Finally there are only few works on the direct validation of the calculation of radiative transfer obtained by simplified methods by comparison with an exact treatment of this calculation, for thermal plasmas. Indeed it is possible to perform an exact calculation taking into account a very fine description of the spectrum, and using simplified geometry conditions, i.e. imposing a temperature profile (1D calculation) or a temperature field (2D or 3D). For SF₆ Aubrecht and Protasevisch [20] have shown a comparison between the PCM and exact solution, whereas the recent work of Nordborg and Iordanidis [21] proposed an interesting analysis of various methods.

We have started a general project rather similar to the work of Nordborg and Iordanidis [21] i.e. a comparison between the results obtained by an exact treatment of the radiative transfer and those obtained by simplified methods, in SF₆ plasmas in Local Thermodynamic Equilibrium (LTE). Our main differences with [21] are the following:

- We calculate ourselves the fine spectral variations of the absorption coefficient and we plan to study the influence of various physical mechanisms such as the absorption due to the presence of diatomic and polyatomic molecules. We also can choose more refined description of the spectrum.
- We would like to optimise the number of spectral intervals for averaging the absorption coefficient. Nordborg and Iordanidis proposed a 28 bands model which is too important for plasma modelling.
- We also want to validate the radiative flux and not only the divergence of the flux, because it is very difficult to have simultaneous good values of the radiative flux and of its divergence, using simplified methods.

This paper presents a first part of this general project and is devoted to five points: a short presentation of the SF₆ plasma radiation as a function of frequency and temperature with a very fine spectral sampling, accounting for the mechanisms of radiation emission and absorption (part 2); the calculation of NEC showing the role of line overlapping that was not taken into account in our previous calculations (part 3); the exact calculation of radiative transfer in 1D and 2D, with imposed temperature profiles (part 4); the presentation of the simplified description of the spectrum using MAC (with various definitions of the mean absorption), and some improvements from our previous calculations (part 5); the comparison between the exact solution and the approximate results obtained using the NEC and the MAC methods for simple imposed temperature profiles (part 6).

2 – Basic data for the radiative spectrum of SF₆ plasmas

We consider that the plasma is in LTE and we use the equilibrium composition required for the radiation calculation. Furthermore, assuming that the plasma state is in LTE, the radiation can be described using either the emission coefficient ε_ν , or the absorption coefficient K_ν , because both are related by the Kirchoff's law (in fact we use K_ν taking into account induced emission):

$$\varepsilon_\nu = B_\nu K'_\nu \quad (1)$$

where B_ν is the blackbody intensity. Both line and continuum were considered in absorption coefficient calculation. The details of the studied continuum mechanisms (molecular dissociation and ionisation, Bremsstrahlung, electron-ion recombination, electron attachment) can be found in previous papers [3]. In SF₆ at atmospheric pressure the molecules are almost completely dissociated when the temperature is higher than 5000K. In these conditions the most important contribution to the continuum is due to the radiative recombination and we have calculated the Biberman-Schlüter coefficient using the work of Gongassian and Schlüter [22] based on semi-empirical solution of Schrödinger equation based on a scaled Thomas-Fermi potential. At low temperature (T<5000 K) the role of the molecules becomes important in the continuum part of the spectrum, because of photodissociation and photoionisation of these molecules. Nevertheless, in our calculation we do not take into account the radiation due to the rotational and vibrational bands emitted by the diatomic and polyatomic molecules, considering that in SF₆ arc plasmas the central temperature is much higher than 5000 K and that the radiation emitted in the external regions (where T < 5000 K) is negligible in front of the radiation emitted from the hottest regions. So the warm and cold gases play only a role of radiation absorption.

In thermal plasmas the atomic lines emitted from excited levels of neutral and ionised atoms represent in general the main contribution of the total radiation in a large range of temperature. In the case of SF₆ plasma, in the range of temperature between 300 and 30000K, we take into account the following atomic species S, S⁺, S⁺⁺, S⁺⁺⁺, F, F⁺, F⁺⁺ and F⁺⁺⁺. The atomic structures used are taken from NIST data tables [23]. We consider all the lines tabulated by the NIST [24] and by the calculation of Kurucs and Peytermann [25]; the oscillator strengths (or Einstein coefficients) of the lines are first chosen from NIST compilation [24] and then completed by the values of Kurucs and Peytermann. For SF₆ we consider 8000 lines. One difficult part of the study concerns the determination of line profile. In previous works we considered only the broadening mechanisms (line shifting was not

needed because each line was studied separately for a given temperature), but in this development, line shift must be taken into account. Basic data for line shift and broadening were taken from [26,27]. The total line profile is then given by:

$$P(\nu, T, p) = \frac{a}{\Delta\nu_D \pi \sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp\left[-\left(\frac{y}{\Delta\nu_D}\right)^2\right]}{a + (y - \Delta\nu)^2} dy \quad (2)$$

where:

$$a = \omega_L/2$$

$$\Delta\nu = \nu - \nu_0 - \delta$$

$$\Delta\nu_D = \frac{\omega_D}{2\sqrt{\ln 2}}$$

ω_D and ω_L represent the Doppler and Lorentz widths respectively, δ is the line shift. Expression (2) corresponds to a Voigt profile, convolution of Gaussian and Lorentzian profiles due to the broadening phenomena: Doppler, resonance, Van Der Waals and Stark effects. For an SF₆ plasma an example of spectral distribution of the absorption coefficient is given in figure 1 for two high values of temperature.

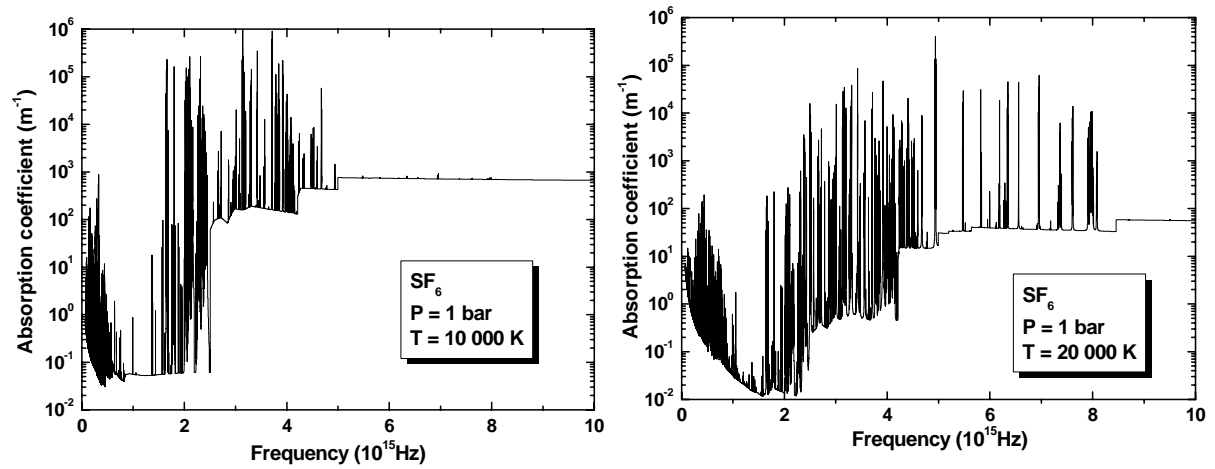


Figure 1: Absorption coefficient of SF₆ at atmospheric pressure for temperatures of 10 000 K and 20 000 K respectively.

Total absorption coefficient is clearly due to the continuum contribution (with several thresholds corresponding to the electronic structures of the components i.e. atoms and ions of fluorine and sulphur) and to the superposed lines. One of the numerical problems is the optimization of the spectral step to have a good description of the total spectrum. Decreasing this step tends to improve the accuracy but tends to exaggeratedly increase the number of spectral points.

The choice of the spectral step has been optimized following the following procedure:

- the spectral step $\Delta\nu$, is not constant over all the spectrum. We have first divided the total spectrum in five intervals considering the wavelengths of the resonance lines. Then we have calculated for a given $\Delta\nu$ for each interval (ν_1 - ν_2) the total emission corresponding to:

$$I = \int_{\nu_1}^{\nu_2} B_\nu K_\nu' d\nu \quad (3)$$

- this emission can also be calculated by integrating the continuum and summing the intensities of the lines located in this spectral interval.

$$I = \int_{\nu_1}^{\nu_2} B_\nu K_c' d\nu + \sum I_l \quad (4)$$

- the integration (3) is computed for various values of the integration step $\Delta\nu$ (with decreasing values of $\Delta\nu$) and compared to the value of expression (4) independent of the integration step. We consider that the integration step is correct when the relative difference between both values of the total emission is less than 1%. For all the temperature range between 5000 and 30000K at atmospheric pressure (at higher pressure the lines are more broadened than at atmospheric pressure). Tables 1 and 2 give the main results, i.e. the absolute values of the emission computed by (3) and compared to the values computed by (4) (table 1) and the relative difference (table 2), for various numbers of spectral integration points, and for 6 values of temperature. The final choice corresponds to 286802 points.

Nb of points	5000	10000	15000	20000	25000	30000
Relation (4)	5.7242D+05	1.1431D+09	5.3413D+09	2.1459D+10	9.2400D+10	2.4105D+11
160 802	5.6482D+05	1.1205D+09	5.2453D+09	2.1738D+10	-	-
286 802	5.7084D+05	1.1442D+09	5.3792D+09	2.1558D+10	9.2417D+10	2.4211D+11
538 801	5.7240D+05	1.1457D+09	5.3649D+09	2.1550D+10	9.2787D+10	2.4256D+11

Table 1 : Values of the intensities given by relations (4) (upper line) and (3). The first column indicates the number of spectral points used for computing relation (3)

Nb of points	5000	10000	15000	20000	25000	30000
160 802	-1.32D-02	-1.9D-02	1.79D-02	1.30D-02	-	-
286 802	-2.76D-03	9.62D-04	7.09D-03	4.61D-03	1.83D-04	4.39D-03
538 801	-3.49D-05	2.27D-03	4.41D-03	4.24D-03	4.18D-03	6.26D-03

Table 2 : Relative difference between the intensities computed by relations (3) and (4)

With this number of points the spectral resolution is fine enough to well describe the line profile and the possible line overlapping as illustrated in figure 2.

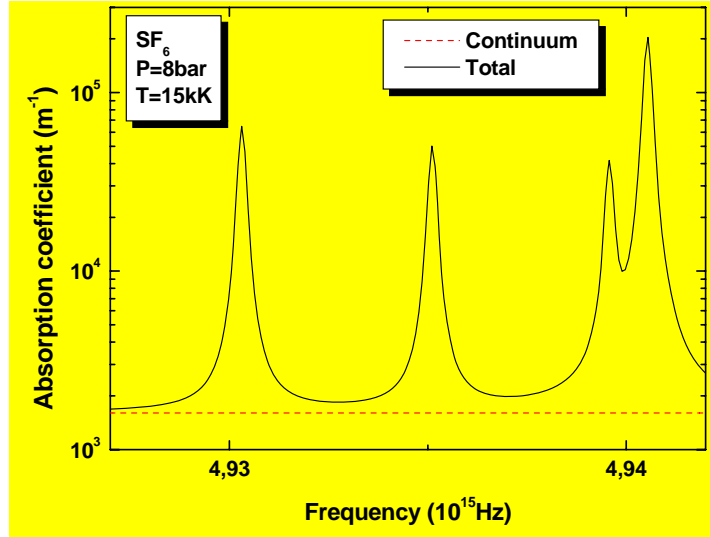


Figure 2: Example of line overlapping with the fine spectral description (dashed line corresponds to the continuum).

Finally we would like to focus on one possible weakness of the spectral description of SF_6 plasma. We will see that the absorption of radiation can be important on the edges of the plasma because of the presence of molecules that may absorb radiation by photodissociation and photoionisation.

We have plotted in figure 3 the photo-absorption cross sections of SF_6 , SF_4 and F_2 , given by literature and used in our calculation, versus the wavelength of the absorbed photon. For SF_6 we have made a compilation of 4 papers [28-31] for representing the total cross section including dissociation and ionisation. The first dissociation threshold of SF_6 ($\text{SF}_6 \rightarrow \text{SF}_5 + \text{F}$) corresponds to 3.995 eV (310.35 nm) but we can see from figure 3 that this dissociation phenomenon is not efficient for photoabsorption near the threshold. The same kind of remark can be noted for SF_4 (data from [31]) for which the first dissociation threshold ($\text{SF}_4 \rightarrow \text{SF}_3 + \text{F}$) occurs at 3.477 eV (356.59 nm) and the second threshold ($\text{SF}_4 \rightarrow \text{SF}_2 + \text{F}_2$) at 4.79 eV (258.79 nm) and for F_2 [32-33] for which the dissociation threshold occurs at 1.602 eV (773.41 nm).

So the literature data cover only a part of the spectrum and should be extrapolated. The extrapolation of SF_4 data to the lower wavelength (i.e. higher energy) is not problematic: a constant value around 10^{-17} cm^2 can be chosen. The extrapolation to the higher wavelengths is much more problematic. It is not correct to consider that the cross section becomes nil after the last point. Considering the dissociation energies of these 3 molecules we can perform an extrapolation but the shape of the curve is not known and the choice of this extrapolation may

have a rather important influence on the radiation absorption at high pressure ($p > 10$ bars), the condition which is usual in high voltage circuit-breaker. Apart from this aspect, we have no available data on photoabsorption cross section for other molecules as SF, S₂ and SF₂ and the presence of these molecules has probably an impact on the radiative transfer.

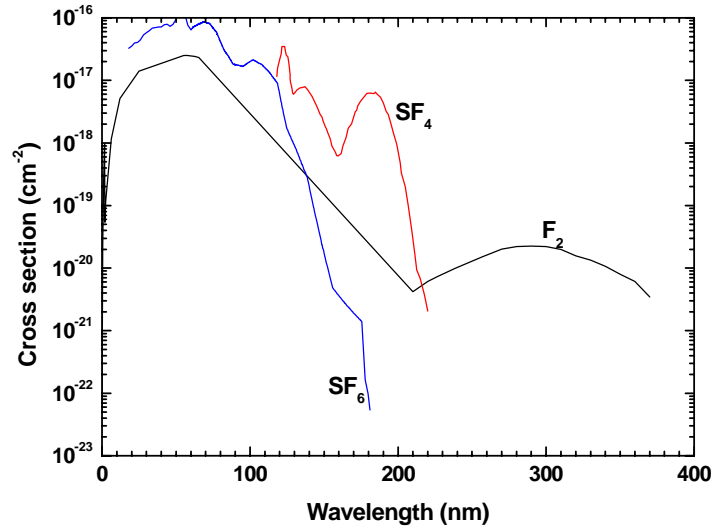


Figure 3: Photoabsorption cross section for 3 molecules

3 – Net Emission Coefficient: influence of line overlapping

As it has been written in the introduction, the NEC corresponds to an acceptable estimation of the radiation emitted from the hottest regions of an arc. This method, first proposed by Lowke [1] allows calculating the divergence of the flux at the centre of an isothermal spherical plasma:

$$\varepsilon_N = \frac{1}{4\pi} \vec{\nabla} q \quad (5)$$

such as:

$$\varepsilon_N = \int_0^\infty B_\nu K'_\nu \exp(-K'_\nu R_p) d\nu \quad (6)$$

where R_p is the radius of the sphere. NEC incorporates directly self-absorption. In previous calculations made by our research group, the lines were treated independently, i.e. neglecting line overlapping. For each line an escape factor A was introduced to represent the ratio between a real emitted power and the emitted power without absorption. This factor is equal to 1 when the line is not absorbed and tends to zero when self-absorption increases. This assumption of negligible line overlapping simplifies the calculation because for each line the

escape factor is previously computed as a function of the number density of the excited atomic level (deduced from the equilibrium composition) and of the broadening parameters, following the method of Drawin and Emard [34]. So the integration of the NEC along the frequency (Eq. 6) is reduced to the integration of the continuum absorption coefficient and the summation of the contributions of the lines. This assumption was made in order to reduce CPU time, knowing that the corresponding error on the NEC at atmospheric pressure was acceptable. For given values of pressure and temperature, it is important to note that the escape factor depends on the plasma size, and thus the NEC depends on R_p . This parameter that is the radius of the plasma sphere in the initial calculation, should be estimated in practical calculations (in general the plasma is not spherical) as the conduction radius of the plasma (in fact the distance from the axis where the temperature represents around 70-80% of the axis temperature).

Using the very fine description of the spectrum for each temperature, as illustrated in figure 1, with 300 000 frequency points, it is possible to calculate the NEC (Eq. 6) by direct integration. The results of the NEC function of temperature are plotted in figure 4 for two pressure values. In the fictitious case $R_p=0$ (in fact without any self-absorption) there is no difference between the values computed with the escape factors or without (in this latter case, that means integration over around 300 000 points in wavelength or frequency). This is normal and shows that the calculation with the total integration is correct. Then, figure 4 shows that about 90% of radiation is absorbed over the first millimetre of plasma, which is a well known result for the majority of thermal plasmas.

What is interesting in our results is the difference between the two kinds of results (with and without escape factors). We can see in figure 4 that the influence of line overlapping, weak at atmospheric pressure, is not totally negligible at higher pressure and tends to increase when temperature and pressure increase. This is due to the fact that several lines are very close (specifically triplets) and that the increase of temperature and pressure tends to broaden the lines (Stark and pressure effects) and then to favour the overlapping. The influence of overlapping may reach 40% of the NEC at a pressure of 8 bars.

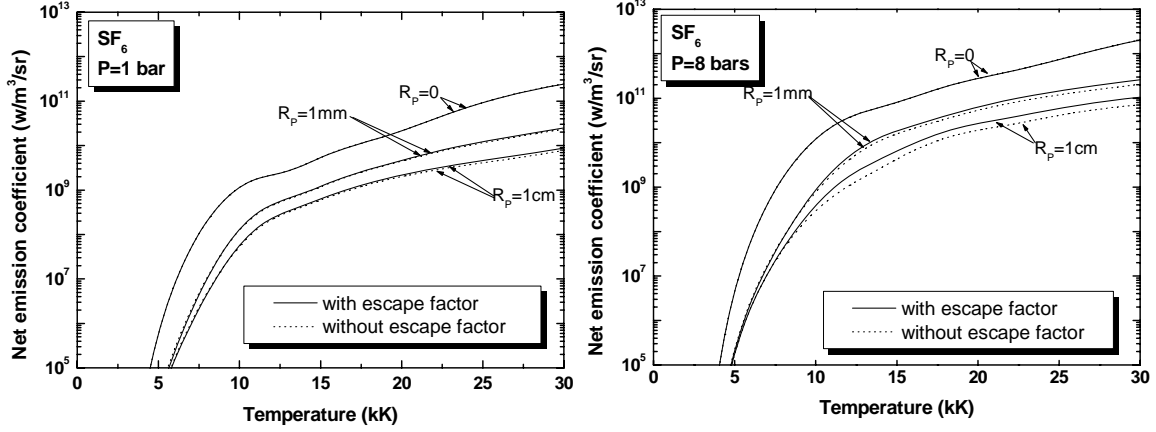


Figure 4: Influence of line overlapping on the net emission coefficient for SF₆ plasma, for 2 pressure values

4 – Exact calculation

Radiative transport equation is given by:

$$\hat{s} \cdot \nabla I_\nu(r, \hat{s}) = K'_\nu (B_\nu - I_\nu) \quad (7)$$

where I_ν is the spectral radiative intensity; r , \hat{s} the position and the direction, respectively.

Following one direction Ox , we obtain an ordinary differential equation:

$$\frac{dI_\nu}{dx} = K'_\nu (B_\nu - I_\nu) \quad (8)$$

with the solution :

$$I_\nu(x) = I_\nu(0) e^{-\int_0^x K'_\nu(l) dl} + \int_0^x K'_\nu(l) B_\nu(l) e^{-\int_l^x K'_\nu(\eta) d\eta} dl \quad (9)$$

In 1D configuration, knowing the temperature profile, the equation of radiative transfer can be numerically solved following Eq. (9). With a high number of mesh points, the solution can be considered as exact. In our calculation, we have imposed two kinds of temperature profiles, shown in figure 5, corresponding to the profiles along a radius. In 1D, we perform the calculation along a profile corresponding to a diameter (maximum temperature on the middle). In 2D, we calculate the radiative transfer in a plane with a circular geometry (section of cylinder plasmas, maximum on the centre) using discrete ordinates. Nordborg and Iordanidis [21] showed that this method gives very similar solutions as the direct integration given in their paper, provided the number of directions is high enough.

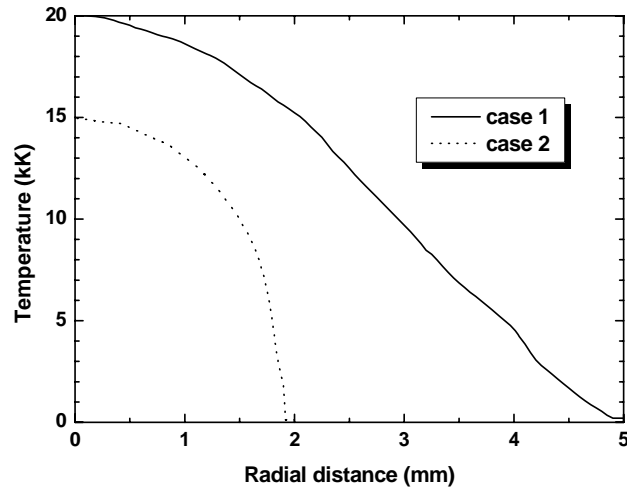


Figure 5: Imposed temperature profiles

In 1D, the accuracy depends on the number of spatial points N , and in 2D it depends also on the number of directions n . Figures 6 and 7 show the influence of these parameters on the divergence of the flux. For geometrical reason the divergence of the flux is symmetrical and we present it only on half a segment (1D) of along the radius (2D). Among the results we must notice that for the considered case the divergence becomes negative for a radial position higher than 2 mm which corresponds approximately to $T < 15\ 000\ \text{K}$ (see figure 5) and that a strong absorption occurs on the edge of the plasma when the temperature becomes lower than 3000K. The latter effect is mainly due to the absorption of radiation by the molecules which correspond to photodissociation and photoionisation of these molecules. From these preliminary tests, the general results presented in the last part were obtained using 200 spatial points in one direction, and using 36 directions for 2D computation.

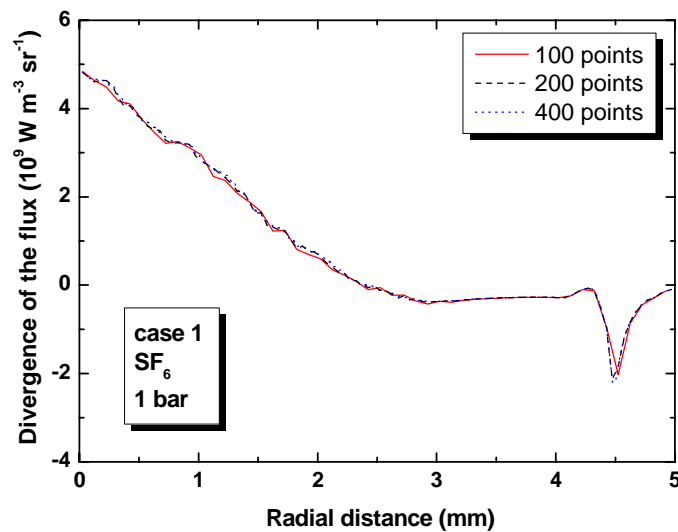


Figure 6: Calculation in 1D condition, of the divergence of the flux: influence of N , the number of spatial points

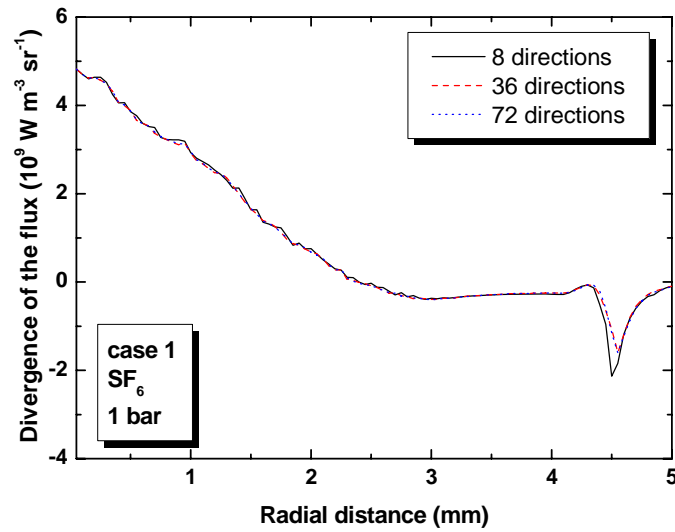


Figure 7: divergence of the flux as a function of the number of directions in 2D conditions

5 – Mean Absorption Coefficients

The use of the fine spectral description (100 000 points or more for each temperature) is not possible in thermal plasma modelling where the calculation of temperature field is performed iteratively for each spatial point. On the other hand, the NEC is unable to calculate the absorption in the cold regions or to estimate the radiative flux. In these conditions, a simplification of the spectral description is needed to perform the iterative calculation of the radiative and temperature fields. The simplest approximation is to consider that the total spectrum can be separated in a limited number of intervals and that the plasma can be considered as a grey body in each interval, i.e. for a given temperature the absorption coefficient is constant within each interval. Then these variations of the mean absorption coefficients versus temperature can be used in the general modelling by means of classical methods of radiative transport (P1 method for example). However it is to be noted that for each interval a partial derivative conservation equation can be solved and in order to limit the cpu time the number of intervals should be very limited (less than 10 for 3D models for example).

The procedure is the following:

- from the evolutions of the absorption coefficient K' versus frequency it is rather easy to select between 5 and 8 intervals where K' varies slowly within each interval for all the temperature range.
- For each interval we may calculate a MAC for the continuum. We may choose 3 kinds of definition for the MAC and we will discuss on them just below.

- Finally for each interval we must add the contribution of the lines.

We present in figure 8, the variations of the continuum absorption coefficient versus temperature at atmospheric pressure for various temperature values. As explained in previous papers (see [3] for example) the sudden variations of K' are due to the electronic structure of the atoms (neutral and ions for $T > 5000$ K) or of the molecules ($T < 3000$ K). We have also plotted in this figure the arbitrary limits of 7 spectral intervals (vertical black dotted lines), considering that the absorption coefficient is rather constant for a given temperature (grey body approximation). The total range of studied frequency is between 2.10^{14} Hz and 10^{16} Hz (wavelength range between 30 nm and 1.5 μ m).

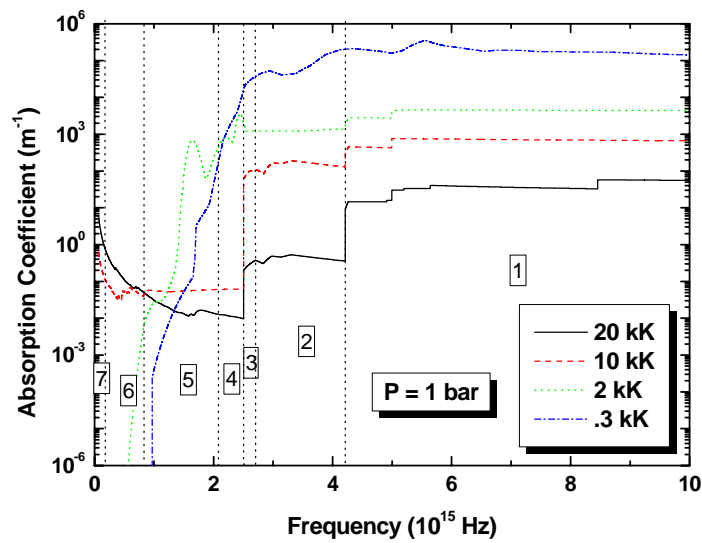


Figure 8: Evolutions with frequency of the continuum absorption coefficient for various temperature values. The limits of spectral intervals for the grey body approximation are plotted in vertical black dotted lines and the numbering of the intervals (called also bands) are indicated in the figure (Band 1 corresponds to higher frequency or photon energy)

In fact, for a frequency interval between ν_i and ν_{i+1} we may define three kinds of average. The Planck average is defined as:

$$\langle K_i' \rangle = \frac{1}{W_i} \int_{\nu_i}^{\nu_{i+1}} B_\nu K_\nu' d\nu \quad (10)$$

with $W_i = \int_{\nu_i}^{\nu_{i+1}} B_\nu d\nu \quad (11)$

The Rosseland average is defined as:

$$\frac{1}{\bar{K}'_i} = \frac{\int_{\nu_i}^{\nu_{i+1}} \frac{1}{K'_\nu} \frac{dB_\nu}{dT} d\nu}{\int_{\nu_i}^{\nu_{i+1}} \frac{dB_\nu}{dT} d\nu} \quad (12)$$

The natural average is:

$$\bar{K}'_i = \frac{\int_{\nu_i}^{\nu_{i+1}} K'_\nu d\nu}{\nu_{i+1} - \nu_i} \quad (13)$$

Of course when the absorption coefficient is really constant within a spectral range, all the definitions give the same mean value. The problem occurs when K'_ν is not constant and in particular when strong lines appear in the spectral range. It is well known that the Planck average is a good approximation when K'_ν is rather low, precisely when the optical depth is low so that the intensity as defined in relation (9) is almost reduced to the local emission $K'_\nu B_\nu$. The Rosseland averaging coefficient is in general more precise for high values of K'_ν , and presents the great advantage that the central part of the strong lines (that is totally reabsorbed in the medium) has no influence on the mean value because of the definition in $1/K$. Finally the natural average can be useful if we consider only the role of absorption (special conditions such as the local emission is much lower than the local absorption).

The previous calculation of radiation (for example through the NEC) have shown that the lines play in important role in thermal plasma radiation and it is well known that in these conditions the assumption of grey body intervals can lead to severe and unacceptable errors using the Planck averaging absorption coefficient [12, 21]. This is due to the fact that the most intense lines are strongly self-absorbed which is not taken into account if we sum directly the contribution of the lines in the interval $[\nu_i - \nu_{i+1}]$. In our previous works [13-14] we suggested to use the line escape factors, A_l , introduced in part 3 or the NEC. So in relation (10) the contribution to the integral, of a line centred at $\nu = \nu_0$ is written as:

$$B_{\nu_0} \Lambda_l \int K'_i(\nu) d\nu \quad (14)$$

The escape factor depends on the temperature and on the size of the plasma characterized by the radius R_p of the isothermal sphere used to compute the NEC. Thus our total MAC for a given interval depends on temperature and on R_p . The best interest in this approach is that for the hottest regions we obtain a good consistency between the NEC and the MAC:

$$\varepsilon_N = \sum_{i=1}^N W_i \langle K' \rangle_i \quad (15)$$

So in a general modelling we may first use the NEC (very fast) to have a good estimation of the temperature field and then engage a calculation of radiative transfer (P1 or discrete ordinates for example) using our values of MAC, without destabilizing the numerical code. A comparison of the values of the mean absorption coefficients for 7 bands in pure SF₆ at atmospheric pressure versus temperature, is given in figure 9 for R_p=2 mm. For bands 1 to 3 (high energy) the curves are superimposed at low temperature and the differences occur at high temperature (over around 15000K) due to the presence of ionic lines that have a very weak contribution in the Rosseland averaging. For band 4 the difference is very marked and is due to intense resonance lines emitted by excited neutral atoms (we have verified that the Planck averaging of the continuum is superimposed to the Rosseland averaging for the total radiation). Finally for bands 5-7 the differences between the two kinds of results (Planck and Rosseland) are mainly, but not totally, due to the lines.

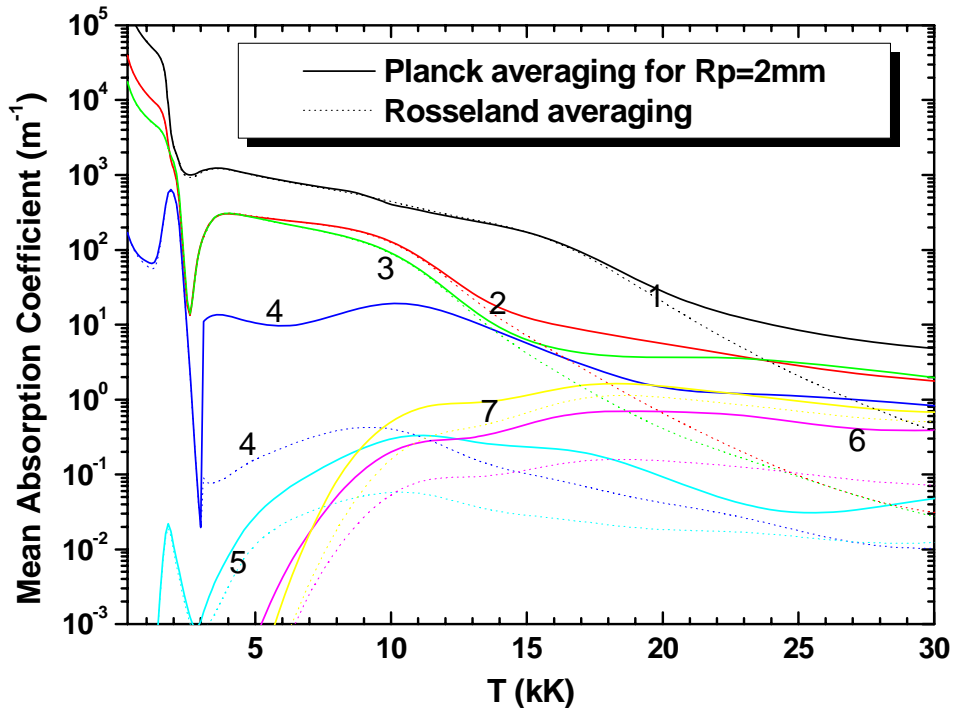


Figure 9: Evolutions of the mean absorption coefficient (the Planck and Rosseland averages) for the 7 bands, versus temperature and for R_p = 2 mm for the Planck averaging

6 – Comparison of methods and discussions

This general comparison will be made using the temperature profiles defined in figure 5. As the profile is considered symmetrical, the divergence of the flux is also symmetrical and will be presented between x= 0 and x=R; R is the profile radius equal to 5 mm in case 1 and to 2

mm in case 2. The calculation has been performed in 1D and 2D as explained previously. The flux is not symmetrical because it depends on the direction and will be given only in 1D geometry between $x=-R$ and $x=R$, corresponding to the diameter (the calculations were also performed for all the cords of the cylindrical section but will not be presented here because they gave similar results).

6.1 Net emission coefficient

Figure 10 presents the comparison of the divergence of the radiative flux obtained either by the net emission coefficient or by the exact method, for the case 1 (results very similar to case 2) and for a pressure of 1 bar. The NEC depends also on the parameter R_p , which is the radius of the isothermal sphere used for computing the NEC, and it corresponds in practical case to an equivalent plasma radius. In our case we show the results for two R_p values, 1 and 2 mm.

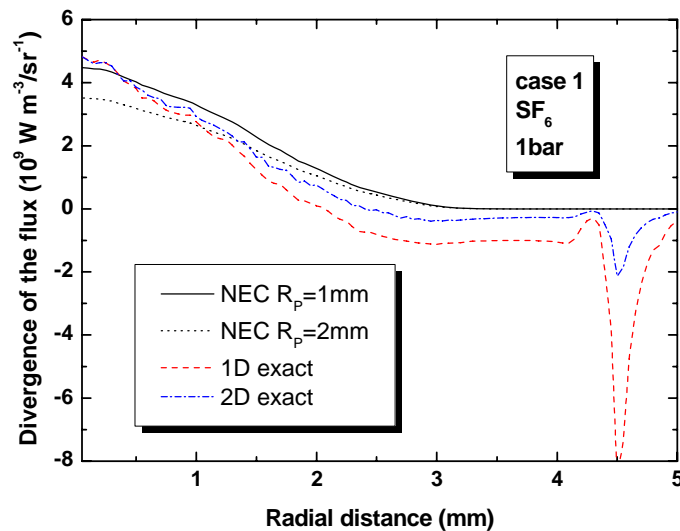


Figure 10: Comparison of the NEC with the exact solution for the radiative flux divergence

The results are classical. The NEC gives a rather satisfactory but not very good agreement in the hottest regions, whereas by definition the NEC is unable to compute a negative value of the divergence and thus is unable to treat the absorption of radiation in tedious and cold regions. It is also interesting to note another classical result showing the difference between 1D and 2D exact calculations: the 1D calculation performed following a diameter, always shows a more pronounced absorption in the outer regions than the 2D condition that averages this absorption along all the directions.

6.2 Mean Absorption coefficient: first results

Using the seven spectral intervals defined in figure 8 and the values of the Planck and Rosseland averaged mean absorption coefficients plotted in figure 9, we may calculate the radiative transfer for the given temperature profiles (defined in figure 5) and compare the results with the exact solutions in 1D and 2D conditions. This comparison on the divergence of the radiative flux (in 1D and 2D) is presented in figures 11a-b. From these figures it is clear that the use of the Rosseland averaging is not satisfactory because of a big and unacceptable underestimation of the divergence of the flux in the hottest regions. On the contrary we observe a qualitative agreement between the exact values (with almost 300 000 spectral points) and the approximated values using the Planck averaging (grey body with 7 spectral intervals): the differences are acceptable in the hottest regions whereas both methods show absorption of radiation in tedious and cold regions. Nevertheless we can see in figure 12 that in 1D condition the radiative flux near the wall (i.e. for the radial position equals to 5mm in case 1) depends rather strongly on the choice of R_p when using the Planck averaging method and can be different from the exact solution. Using the value of R_p giving the best agreement on the flux divergence near the centre ($R_p=1$ mm, see figure 12) the approximated value of the flux at the edge is twice the exact value. This result (overestimation of the flux) means also that the total absorption of radiation is underestimated by the approximated method based on Planck MAC which may have a strong influence on the pressure rise within a SF₆ HVCB. We also may note from figure 12 the errors induced by the use of the Rosseland MAC.

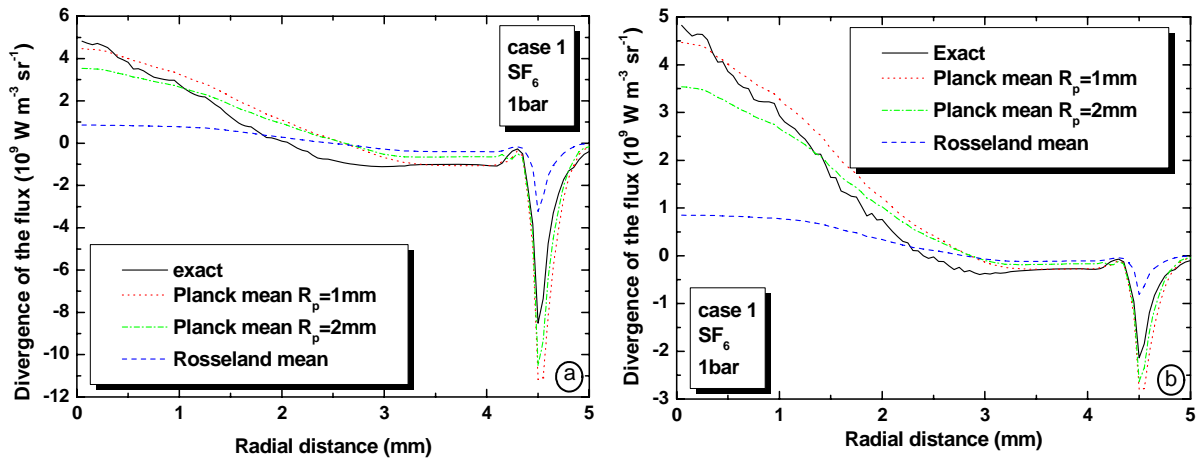


Figure 11: Comparison of the values of the divergence of the radiative flux between the exact solution and the approximated method based on 7 bands with the mean absorption coefficients. a) 1D calculation; b) 2D calculation

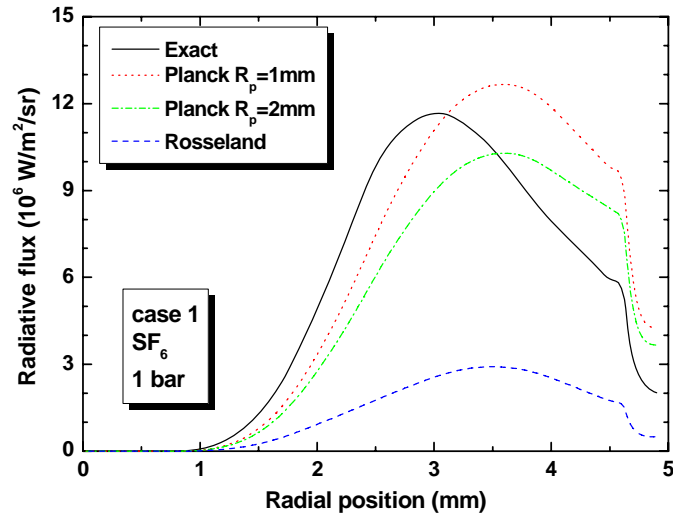


Figure 12: Evolution of the radiative flux in 1D condition; comparison between exact solution and approximated values obtained using 7 bands and the Planck mean absorption coefficients.

All the previous results were obtained using one temperature profile (case 1 in figure 5) corresponding to a maximum temperature of 20000K). Using the other temperature profile (case 2) the results are similar as it can be seen in figure 13, showing the divergence of the flux computed in 2D geometry..

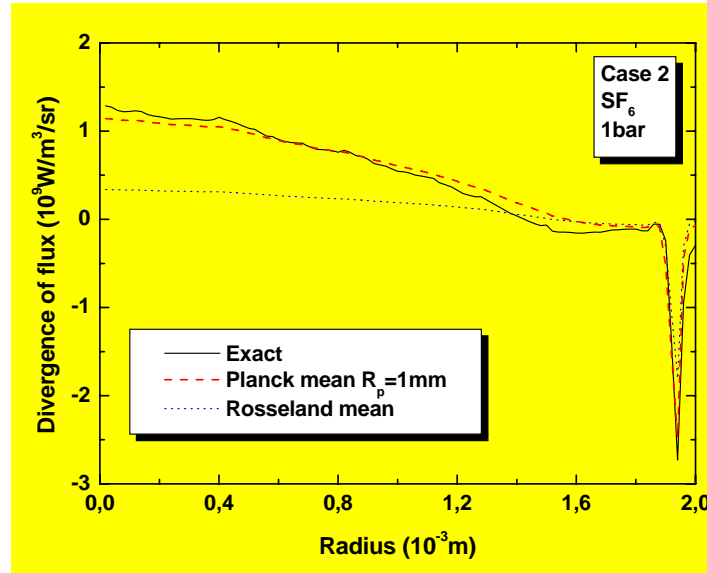


Figure 13: Divergence of the flux computed in 2D geometry for the temperature profile of case 2

6.3 Improvements on the description by spectral intervals

In order to evaluate the contribution of the various spectral intervals to the total radiative flux we have plotted in figures 14a-b the contributions of the seven bands in the Planck approximation (figure 14a) and the same corresponding contributions obtained by the exact solution (figure 14b). Intervals (or bands) 1-3 have a large contribution to the radiation, but the resulting flux for these high energy intervals at $r = 5\text{mm}$ is nil, because of the strong absorption in the edges. Band 6 contributes strongly to the final flux and is well treated with the approximated method. On the contrary the approximated method underestimates the absorption of radiation in bands 4 and 5. So the improvement of the calculation of the radiative flux on the plasma edges needs a better treatment for these two bands.

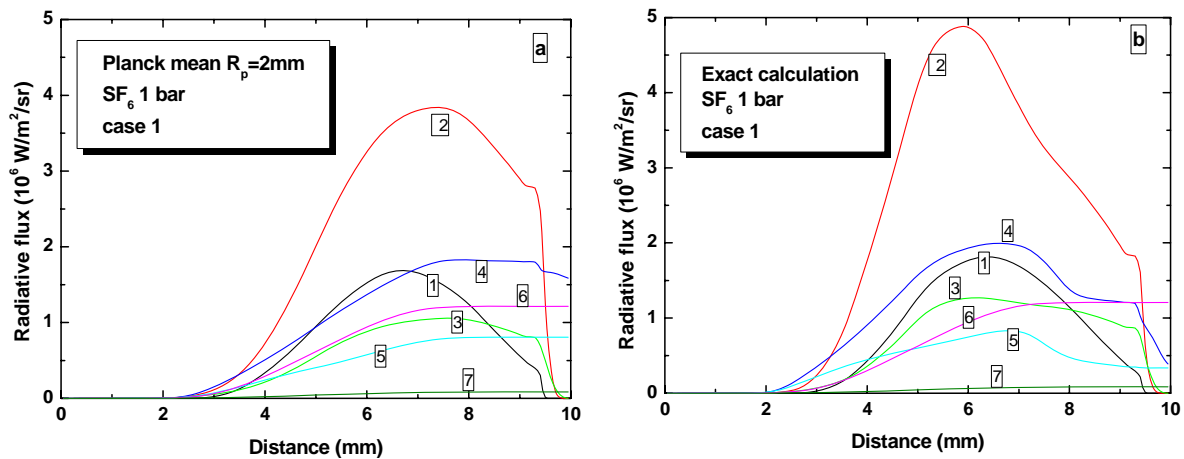


Figure 14: Contributions of the seven chosen intervals to the total flux in 1D condition. a) Approximated method using 7 intervals (marked 1 to 7 on the curves) with the Planck mean absorption coefficients. b) Exact contributions of these 7 intervals.

We have explained that the use of the Planck averaged absorption coefficient allows to have a rather good estimation of the net emission. But if we consider results presented in figures 11 we can see that the divergence of the flux is negative (i.e local absorption is greater than local emission) in regions where the temperature is lower than or of the order of 10000K, for case 1 of temperature profile. In any case when the central temperature is higher than 10000K, which is always the case in SF₆ arc, the regions where $T < 6000\text{K}$ are mainly absorbing regions. In this case is the Planck averaging useful? An answer to this question can be found analysing figure 15 representing the evolution of the absorption coefficient for Band 4 (zoom of figure 8 for this band or interval) at $P = 1\text{ bar}$ and $T = 300\text{ K}$, and indicating three values of mean absorption previously defined by relations (10, 12-13).

Obviously the Planck and Rosseland averages (almost superimposed) strongly underestimate the absorption coefficient. For the Rosseland approximation the low value comes from the definition of the averaging favouring the lowest values of the absorption coefficient. For the Planck approximation the underestimation is due to the fact that the Planck function at low temperature and high frequency ν varies very sharply with frequency and that the function $K_\nu B_\nu$ decreases strongly when ν increases even if the absorption coefficient increases by two orders of magnitude, as it is the case here. So at low temperature the Planck and Rosseland averages are not correct for representing the absorption.

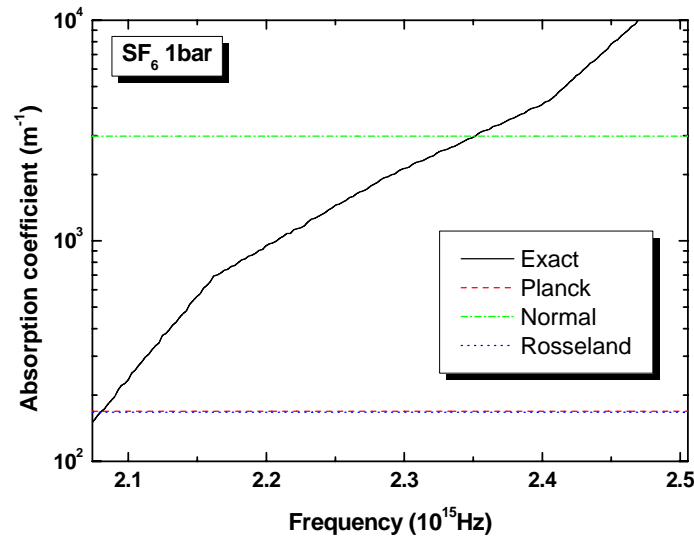


Figure 15: Evolutions of the absorption coefficient in Band 4 and comparison with the Planck, Rosseland and normal averaged values

Another improvement can be made if we observe on figure 8 the variations of the absorption coefficient in band 5 at 2000K. The partial maximum is due to the presence of SF_4 and was not taken into account in the previous choice of the intervals. Following the results we can group bands 2 and 3 and split into two new bands the old band 5. With this new choice of spectral intervals and replacing the Planck average by the normal average for $T < 6000$ K, we have calculated again the radiative transfer using the new 7 bands and one characteristic result is presented in figure 16 equivalent to figure 12. Obviously the final radiative flux at $d = 10$ mm obtained by the approximated method is now much better than the result obtained by the previous choice. The same kind of improvement has been noticed with the second temperature profile (case 2).

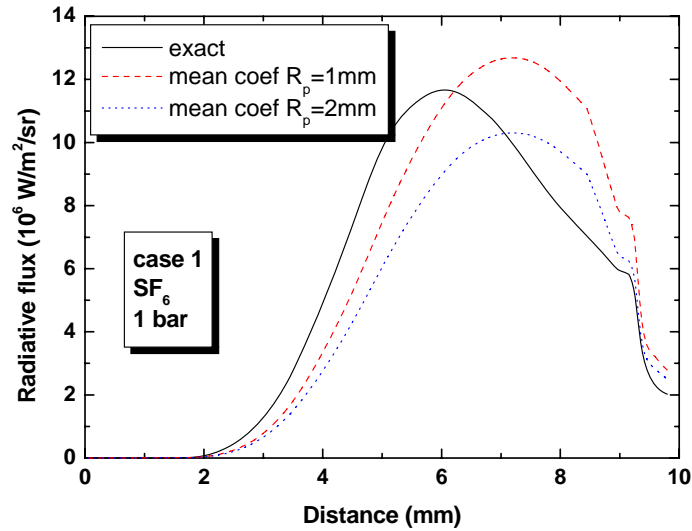


Figure 16: Evolution of the radiative flux in 1D condition; comparison between exact solution and approximated values obtained using 7 new bands with the Planck mean absorption coefficients for $T > 6000\text{K}$ and normal mean absorption coefficients for $T < 6000\text{K}$.

7 – Conclusion

The improvement of the calculation of radiative transfer in thermal plasmas established in molecular gases (here SF_6 but the method is general), can be made using two correlated ways. The first one that is physically complex consists in well treating the basic phenomena important in thermal plasma radiative transfer. The first kind of big difficulties, with rather high uncertainty, corresponds to the determination of atomic line profiles, in particular for resonance lines, responsible for a large part of the emission in spite of a strong self-absorption. The second kind of difficulties concerns the photodissociation and photoionisation of molecules, responsible for a large part of radiation absorption. In the case of SF_6 we have two main problems that are not yet solved and that should be studied in the future: the role of the presence of diatomic molecules S_2 and SF that is not taken into account; the knowledge of the absorption cross sections in all the UV part of the spectrum because data are limited to the VUV range in general. This unknown part may play a rather important role in the radiation absorption at high pressure (in HVCB pressure may reach several tens of bars).

The second way to improve the radiative transfer is to optimise the spectral simplified description. Nordborg and Iordanidis [21] presented a good approach and here we have proposed a method based on a lower number of spectral intervals that give satisfactory results using the Planck and the natural averaging for the mean absorption coefficients, whereas the use of the Rosseland averaging is not suitable. Other efforts should be developed to check the

method in more general conditions (3D and higher pressures) and using other usual methods (P1 and discrete ordinates) based on the simplified description of the radiation.

References

- [1] Lowke J J 1974 *J. Quant. Spectrosc. Radiat. Transfer* **14** 111
- [2] Liebermann R W, Lowke J J 1976 *J. Quant. Spectrosc. Radiat. Transfer* **16** 253
- [3] Gleizes A, Rahmani B, Gonzalez J-J, Liani B 1991 *J. Phys. D: Appl. Phys.* **24** 1300-09
- [4] Aubrecht V, Gross B 1994 *J. Phys.D: Appl. Phys.* **27** 95
- [5] Gleizes A, Gonzalez J-J, Razafinimanana M, Robert T 1992 *Plasma Sources Sci. Technol.* **1** 135
- [6] Raynal G, Gleizes A 1995 *Plasma Sources Sci. Technol.* **4** 152
- [7] Fang MTC, Zhuang Q. 1992 *J. Phys. D: Appl. Phys.* **25** 1192
- [8] Zhang J F, Fang M T C, Newland D B 1987 *J. Phys. D: Appl. Phys.* **20** 368
- [9] Soloukhin R.I. 1987 *Radiative Heat Transfer in High Temperature Gases* (London : Hemisphere)
- [10] Aubrecht V., Lowke J.J.1994 *J. Phys. D.: Appl. Phys.* **27** 2066
- [11] Raynal G., Vergne PJ, Gleizes A. 1995 *J. Phys. D.: Appl. Phys.* **28** 508
- [12] Modest M F 1993 *Radiative Heat Transfer* McGraw-Hill, Inc. ISBN 0-07-112742-9
- [13] Gleizes A, Erraki A, Naghizadeh-Kashani Y, Riad H 1997 *XXIII ICPIG Int. Conf. Phen. Ionized Gases* Toulouse (July 1997) II-106
- [14] Erraki A 1999 *Etude du transfert radiatif dans les plasmas thermiques : application au SF₆ et au mélange argon-fer* Thesis Doct. Univ. Paul Sabatier, Toulouse, N° 3447
- [15] Eby S D, Trepanier J Y, Zhang X D 1998 *J. Phys. D.: Appl. Phys.* **31** 1578
- [16] Yakomizu Y, Matsumura T 2008 *J. Phys. D.: Appl. Phys.* **41** 125203
- [17] Bouaziz M, Raynal G, Razafinimanana M, Gleizes A 1996 *J. Phys. D: Appl. Phys.* **29** 2885
- [18] Verite J C, Boucher T, Comte A, Delalondre C, Robin-Jouan P, Serres E, Texier V, Barrault M, Chevrier P, Fievet C 1995 *IEE Proc. Sc. Meas. Technol.* **142** 189
- [19] Dixon C M, Yan J D, Fang M T C 2004 *J. Phys. D: Appl. Phys.* **37** 3309
- [20] Aubrecht V, Protasevich E T 2000 *Radiative Transport of Energy in SF₆ Arc Plasmas* Review available at http://www.uvee.feec.vutbr.cz/~aubrecht/Main_book.pdf
- [21] Nordborg H, Iordanidis A A 2008 *J. Phys. D.: Appl. Phys.* **41** 135205
- [22] Gongassian M, Schlüter D 1986 *Z. Phys. D* **3** 7
- [23] NIST http://physics.nist.gov/PhysRefData/ASD/levels_form.html

- [24] NIST http://physics.nist.gov/PhysRefData/ASD/lines_form.html
- [25] Smith P L, Heise C, Esmond J R, Kurucz R L *Atomic spectral line database from CD-ROM 23 of R. L. Kurucz* <http://www.cfa.harvard.edu/amp/ampdata/kurucz23/sekur.html>
- [26] Traving G 1968 *Plasma diagnostics* Ed Lochte-Holtgreven North-Holland (Amsterdam)
- [27] Griem H R 1974 *Spectral line broadening by Plasmas*, Academic press NY, London
- [28] Hitchcock A P, Van der Wiel M J 1979 *J. Phys B: Atom. Mol. Phys.* **12** 2153-69
- [29] Lee L C, Phillips E, Judge D L 1977 *J. Chem. Phys.* **67** 1237-46
- [30] Bastien F, Chatterton P A, Marode E, Moruzzi J L 1985 *J. Phys. D: Appl. Phys.* **18** 1327
- [31] Pradayrol C, Casanovas A-M, Deharo I, Guelfucci J-P, Casanovas J J. *Phys. III* 1996 **6** 603-612
- [32] Bartfield W D, Koontz G D, Huebner W F 1973 *JQSRT* **12** 1409-33
- [33] Okabe H 1978 "The photochemistry of small molecules" Wiley - Interscience pub, New York
- [34] Drawin H W, Emard F. 1973 *Beitr. Plasmaphysik* **13** 143