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# Systematic Fuzzy Sliding Mode Approach Combined With Extended Kalman Filter for Permanent Magnet Synchronous Motor control

F. Benchabane, A.Titaouine

MSE Laboratory, University of Biskra, B.P.145, 07000, Biskra, Algeria.  
fateh\_benchabane@yahoo.fr.

O. Bennis

PRISME Institute, University of Orléans, France  
ouafae.bennis@univ-orleans.fr.

K. Yahia, D. Taibi

GEB Laboratory, University of Biskra, B.P.145, 07000, Biskra, Algeria.  
kd\_yahia@yahoo.fr.

**Abstract --** This paper proposes a new state observer based on an extended Kalman filter (EKF) for the sensorless systematic fuzzy sliding mode control of permanent magnet synchronous motors (PMSM). The observer can obtain a precise estimation of rotor speed and position. Load torque is also observed and used as the feed-forward compensation to improve the system performance during load torque changes. In this system, the initial rotor position is not necessary for the start up.

**Key words--** permanent magnet synchronous motor, extended Kalman filter, sensorless control, load torque observation, initial position, fuzzy, systematic sliding mode.

## I. INTRODUCTION

Permanent magnet synchronous motors (PMSM) are more and more used because of their good performances. High performance control of the PMSM requires identification of the rotor position which is usually measured by a mechanical rotor position sensor, which increases the cost and the instability of the system. As a result, sensorless control has recently become a research focus [1,2].

In this study, a fuzzy sliding-mode control system, which combines the merits of the sliding-mode control, the fuzzy inference mechanism and its algorithm, is proposed. In the sliding-mode controller a switching surface that includes an integral operation [3,4] is designed. When the sliding mode occurs, the system dynamic behaves as a robust state feedback control system. Furthermore, in the general sliding-mode control, the upper boundary of uncertainty, which include parameter variations and external load disturbance, must be available. However, the boundary of uncertainty is difficult to determine in advance for practical applications. A fuzzy sliding-mode controller is investigated to resolve this difficulty, in which a simple fuzzy inference mechanism is used to estimate the upper boundary of uncertainty. Furthermore, to reduce the control effort of the sliding-mode controller, the fuzzy inference mechanism is improved by adapting the centres of the membership functions to estimate the optimal boundary of uncertainty [5-7].

In this paper, a new state observer based on an extended Kalman filter is proposed. The precise rotor position and speed can be obtained by EKF. Then a sensorless speed control

system can be formed. The load torque is used as the feed-forward compensation of the reference torque at the output of the speed regulator. With the compensation, the speed fluctuations during load torque impacts can be greatly decreased.

## II. PMSM MODEL

With the simplifying assumptions relating to the PMSM, the model of the motor expressed in the Park reference frame is given in the suitable state form [8,9]:

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}) + \mathbf{G} \cdot \mathbf{U} \\ \mathbf{Y} = \mathbf{H}(\mathbf{X}) \end{cases} \quad (1)$$

where:

$$\mathbf{Y}(\mathbf{X}) = \begin{bmatrix} y_1(\mathbf{X}) \\ y_2(\mathbf{X}) \end{bmatrix} = \begin{bmatrix} h_1(\mathbf{X}) \\ h_2(\mathbf{X}) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} I_d \\ \Omega \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} I_d \\ I_q \\ \Omega \end{bmatrix}; \mathbf{U} = \begin{bmatrix} V_d \\ V_q \end{bmatrix}; \mathbf{G} = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{F}(\mathbf{X}) = \begin{bmatrix} f_1(\mathbf{X}) \\ f_2(\mathbf{X}) \\ f_3(\mathbf{X}) \end{bmatrix} = \begin{bmatrix} a_1 \cdot x_1 + a_2 \cdot x_2 \cdot x_3 \\ b_1 \cdot x_2 + b_2 \cdot x_1 \cdot x_3 + b_3 \cdot x_3 \\ c_1 \cdot x_3 + c_2 \cdot x_1 \cdot x_2 + c_3 \cdot x_2 - \frac{T_l}{J} \end{bmatrix}$$

and:

$$a_1 = -\frac{R_s}{L_d}; \quad a_2 = \frac{p \cdot L_q}{L_d}; \quad b_1 = -\frac{R_s}{L_q}; \quad b_2 = -\frac{p \cdot L_d}{L_q} \\ b_3 = -\frac{p \cdot \phi_f}{L_q}; \quad c_1 = -\frac{f}{J}; \quad c_2 = \frac{p \cdot (L_d - L_q)}{J}; \quad c_3 = \frac{p \cdot \phi_f}{J}$$

in  $f_1(\mathbf{X})$  the load torque  $T_l$  is removed from the state equations and will be considered as a perturbation.

### III. SPACE VECTOR MODULATION (SVM)

In space vector modulation techniques, the reference voltages are given by space voltage vector and the output voltages of the inverter are considered as space vectors. There are eight possible output voltage vectors, six active vectors  $u_1-u_6$ , and two zero vectors  $u_0, u_7$  (Figure. 1). The reference voltage vector is realized by the sequential switching of active and zero vectors. Figure 1 shows reference voltage vector  $u_{ref}$  and eight voltage vectors, which correspond to the possible states of inverter. The six active vectors divide a plane for the six sectors 1- 6. In each sector, the reference voltage vector  $u_{ref}$  is obtained by switching on, two adjacent vectors for an appropriate duration. As seen in figure 1 the reference vector  $u_{ref}$  can be implemented by the switching vectors of  $u_1, u_2$  and zero vectors  $u_0, u_7$  [10].

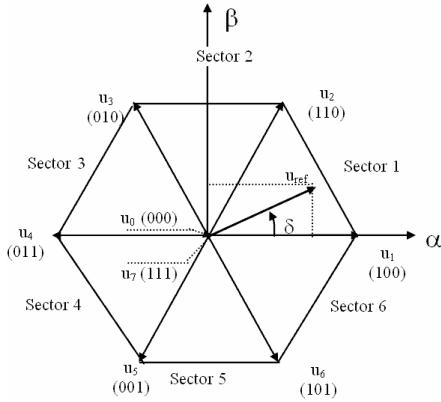


Figure. 1. Principle of space vector modulation.

The six active tension vectors can be formulated by:

$$\bar{u}_i = \sqrt{\frac{2}{3}} u_{dc} e^{j(i-1)\frac{\pi}{3}}, \quad i=1, 2, 3, 4, 5, 6. \quad (2)$$

Where:  $u_{dc}$  is a dc voltage.

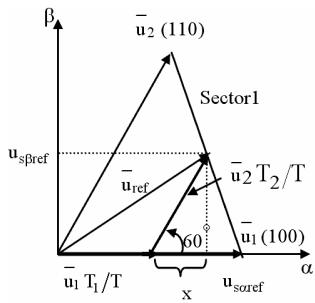


Figure. 2. Projection of the reference vector (sector 1)

Figure 2 shows the case when the reference vector is in sector 1. The times  $T_1$  and  $T_2$  are obtained by simple trigonometrical relationships and can be expressed in the following equations:

$$\begin{cases} T = T_1 + T_2 + T_0 \\ \bar{u}_{ref} = \frac{T_1}{T} \bar{u}_1 + \frac{T_2}{T} \bar{u}_2 \end{cases} \quad (3)$$

and:

$$\begin{cases} u_{s\beta ref} = \frac{T_2}{T} |u_2| \cos(30^\circ) \\ u_{s\alpha ref} = \frac{T_1}{T} |u_1| + x \\ x = \frac{u_{s\beta ref}}{\tan(60^\circ)} \end{cases} \quad (4)$$

Finally, according to (2), the application periods of every vector is given by:

$$\begin{cases} T_1 = \frac{T}{2u_{dc}} (\sqrt{6}u_{s\alpha ref} - \sqrt{2}u_{s\beta ref}) \\ T_2 = \sqrt{2} \frac{T}{u_{dc}} u_{s\beta ref} \end{cases} \quad (5)$$

### IV. SLIDING MODE CONTROLLER DESIGN

The advantages of the sliding mode controller can be summarized as follows [5,11]:

- 1) Fast response with no overshoot.
- 2) No steady state error.
- 3) Robustness, stability in a closed loop environment, insensitivity to parameter variations and load disturbances.

The switching patterns for a six-pulse inverter are generated directly by the sliding mode. The SMC is designed in  $d - q$  plane, and the voltages  $V_d^*, V_q^*$ , are taken as control inputs.

The following switching hypersurfaces are used:

$$S_1 = I_{dref} - I_d \quad (6)$$

$$S_2 = I_{qref} - I_q \quad (7)$$

with  $I_{dref} = 0$ .

These selections are based on the concept of vector control and instantaneous current control [5,11,12]. and are related to inner loop (current loop). In this case, the current components  $I_d$  and  $I_q$  are decoupled. The reference currents  $I_{dref}$  and  $I_{qref}$  are determined by the outer loop.

The control inputs are taken as follows:

$$V_d^* = K_d \text{sign}(S_1) \quad (8)$$

$$V_q^* = K_q \text{sign}(S_2) \quad (9)$$

the gains  $K_d, K_q$  are selected so as to satisfy the existence condition of the sliding mode:

$$\lim_{S_i \rightarrow 0} \dot{S}_i < 0, i=1,2 \quad (10)$$

Which is equivalent, using the Lyapunov stability, to

$$V = \sum_{i=1}^2 V_i \quad (11)$$

with

$$V_i = \frac{1}{2} S_i^2 \quad (12)$$

Then, for the control system to be stable, the time derivative of (11) must be negative, i.e.

$$V = \sum_{i=1}^2 S_i \dot{S}_i < 0 \quad (13)$$

$K_d$  selection: Hence, if we chose  $K_d$  as

$$K_d < -\max_{I_q, \omega} |L_q I_q \omega| \quad (14)$$

which yields the following inequality

$$\dot{S}_i S_i < -\frac{R_s}{L_d} S_i^2 < 0 \quad (15)$$

Only the information about the boundaries of  $I_q$  and  $\omega$  is needed before designing the SMC.

$K_q$  selection: Hence, we chose  $K_q$  as

$$S_i = I_{dref} - I_d \quad (16)$$

which yields the following inequality

$$S_i = I_{dref} - I_d \quad (17)$$

Only the information about the boundaries of  $I_{dref}$  and  $\omega$  is needed before designing the SMC.

## V. FUZZY LOGIC CONTROLLER

The general structure of a complete fuzzy control system is given in figure 3. The plant control  $u$  is inferred from the two state variables, error ( $e$ ) and change in error  $\Delta e$  [4].

The actual crisp inputs are approximates of the closer values of the respective universes of discourse. Hence, the fuzzyfied inputs are described by singleton fuzzy sets.

The design of this controller is based on the phase plan. The control rules are designed to assign a fuzzy set of the control input for each combination of fuzzy sets of  $e$  and  $\Delta e$  [5,13].

Table 1 shows one of possible control rule base. The rows represent the rate of the error change  $\Delta e$  and the columns represent the error ( $e$ ). Each pair  $(e, \Delta e)$  determines the output level NB to PB corresponding to  $u$ .

Here NB is negative big, NM is negative medium, ZR is zero, PM is positive medium and PB is positive big, are labels of fuzzy sets and their corresponding membership functions are depicted in figures 4, 5 and 6, respectively.

The continuity of input membership functions, reasoning method, and defuzzification method for the continuity of the

mapping  $u_{fuzzy}(e, \Delta e)$  is necessary. In this paper, the triangular membership function, the max-min reasoning method, and the center of gravity defuzzification method are used, as those methods are most frequently used frequently in the literature [4,13].

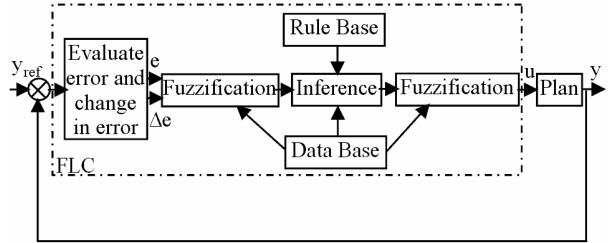


Figure. 3. The structure of a fuzzy logic controller.

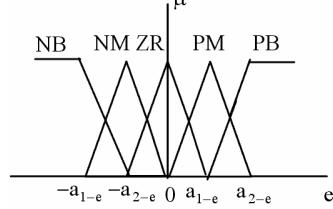


Figure. 4. Membership functions for input  $e$

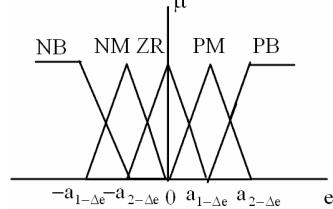


Figure. 5. Membership functions for input  $\Delta e$

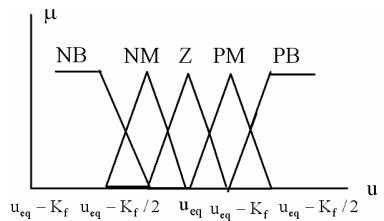


Figure. 6. Membership functions for output  $u$

|                |    | DE <sub>n</sub> |    |    |    |    |
|----------------|----|-----------------|----|----|----|----|
|                |    | NB              | NM | ZR | PM | PB |
| E <sub>n</sub> | NB | NB              | NB | NM | NM | ZR |
|                | NM | NB              | NM | NM | ZR | PM |
|                | ZR | NM              | NM | RZ | PM | PM |
|                | PM | NM              | ZR | PM | PM | GP |
|                | PB | ZR              | PM | PM | GP | GP |

Table. 1. Rules Base for speed control

## VI. DESIGN OF EKF OBSERVER

Accurate and robust estimation of motor variables which are not measured is crucial for high performance sensorless drives. A multitude of observers have been proposed, but only a few are able to sustain persistent and accurate wide speed range sensorless operation. At very low speed, theirs performances are poor. One of the reasons is the high sensitivity of the

observers to unmodeled nonlinearities, disturbance and model parameters detuning.

The Kalman filter provides a solution that directly cares for the effects of disturbance noises including system and measurement noises. The errors in parameters will also normally be handled as noise [14].

The dynamic state model for non non-linear stochastic machine is as follows where all symbols in the formulations denote matrices or vectors [12, 14]:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t) + w(t) \\ y(t) = h(x(t), t) + v(t) \end{cases} \quad (18)$$

$w(t)$ : System noise vector.

$v(t)$  : Measurement noise vector

$w, v$  : are unrelated and zero mean stochastic processes.

A recursive algorithm is presented for the discrete time case. For the given sampling time  $T_s$ , both the optimal estimate sequence  $x_{k/k}$  and its covariance matrix  $P_{k/k}$  generated by the filter go through a two step loop.

The first step (prediction) performs a prediction of both quantities based on the previous estimates  $x_{k-1/k-1}$  and the mean voltage vector actually applied to the system in the period from  $T_{k-1}$  to  $T_k$ .  $F$  is the system gradient matrix (Jacobian matrix).

$$F(\bar{x}(t), t) = \left. \frac{\partial f(x(t), u(t), t)}{\partial x^T(t)} \right|_{x(t)=\bar{x}(t)} \quad (19)$$

$$x_{k/k-1} = x_{k-1/k-1} + T_s \cdot F(x_{k-1/k-1}, u_{k-1}) \quad (20)$$

$$P_{k/k-1} = P_{k-1/k-1} + (F P_{k-1/k-1} + P_{k-1/k-1} F^T) \cdot T_s + Q \quad (21)$$

The second step (innovation) corrects the predicted state estimate and its covariance matrix trough a feedback correction scheme that makes use of the actual measured quantities; this is realized by the following recursive relations:

$$x_{k/k} = x_{k/k-1} + K_k (Y_k - H x_{k/k-1}) \quad (22)$$

$$P_{k/k} = P_{k/k-1} - K_k H P_{k/k-1} \quad (23)$$

Where the filter gain matrix is defined by:

$$K_k = P_{k/k-1} H^T (H P_{k/k-1} H^T + R)^{-1} \quad (24)$$

$H$  is transformation matrix.

$$H(\bar{x}(t), t) = \left. \frac{\partial h}{\partial x} \right|_{x(t)=\bar{x}(t)} \quad (25)$$

The proposed EKF observer is designed in rotor reference frame (d, q frame).

The state vector is chosen to be:

$$X = [I_d \ I_q \ \Omega \ \theta]^T$$

$$\text{Input: } U = [V_d \ V_q]^T;$$

$$\text{Output: } Y = [I_d \ I_q]$$

$I_d, I_q$  and  $V_d, V_q$  are motor stator currents, voltages in rotor reference frame.

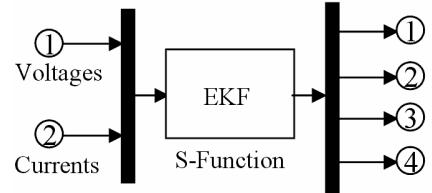


Figure 7. S-Function representation of the EKF.

The critical step in the EKF is the search for the best covariance matrices  $Q$  and  $R$  have to be set-up based on the stochastic properties of the corresponding noise. The noise covariance  $R$  accounts for the measurement noise introduced by the current sensors and quantization errors of the A/D converters [3,17]. Increasing  $R$  indicates stronger disturbance of the current. The noise is weighted less by the filter, causing also a slower transient performance of system.

The noise covariance  $Q$  reflects the system model inaccuracy, the errors of the parameters and the noise introduced by the voltage estimation [3,17].  $Q$  has to be increased at stronger noise driving the system, entailing a more heavily weighting of the measured current and a faster transient performance.

An initial matrix  $P_0$  represents the matrix of the covariance in knowledge of the initial condition. Varying  $P_0$  affects neither the transient performance nor the steady state condition of the system. In this study, the value of these elements is tuned "manually", by running several simulations. This is may be one of the major drawbacks of the Kalman filter.

The blocks CNL, SMC are regulators, the first is the nonlinear controller for speed, and the second is the sliding mode control regulator for rectifier.

Figure 5 shows the proposed Sensorless nonlinear control using EKF. In this study, the outputs of a PWM voltage source inverter are used as the control inputs for the EKF. These signals contain components at high frequencies, which are used as the required noise by the Kalman filter. Thus, no additional external signals are then needed.

$$f(x(k), u(k)) = \begin{bmatrix} I_d & I_q & \Omega & \theta \end{bmatrix}^T = \begin{bmatrix} (1 - T_s \frac{R_s}{L_d}) I_d + p \Omega T_s \frac{L_q}{L_d} I_q + T_s \frac{1}{L_d} V_d \\ (-p \Omega T_s \frac{L_d}{L_q}) I_d + (1 - T_s \frac{R_s}{L_q}) I_q - T_s \frac{\phi_{sf}}{L_q} p \Omega + T_s \frac{1}{L_q} V_q \\ p T_s \frac{L_d - L_q}{J} I_q I_d + p T_s \frac{\phi_{sf}}{J} I_q + (1 - T_s \frac{f}{J}) \Omega - T_s \frac{1}{J} T_l \\ \Omega \end{bmatrix} \quad (26)$$

where:

$$h = \begin{bmatrix} I_d & I_q \end{bmatrix}^T$$

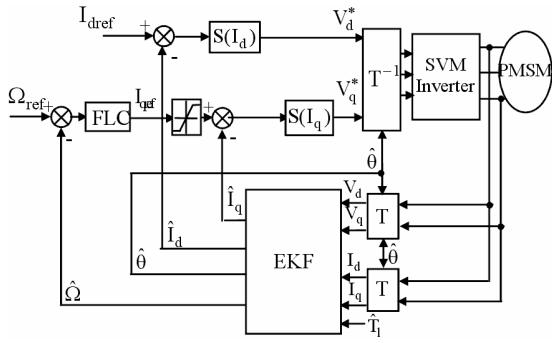


Figure 8. Speed control of a PMSM using the Fuzzy Sliding mode with an EKF for speed and position estimation.

## VII. LOAD TORQUE ESTIMATION

The load torque is not easily measurable so it has to be estimated. The method suggested by Le Pioufle allows estimating it in real-time [8]. Figure 9 illustrates the principle for estimation.

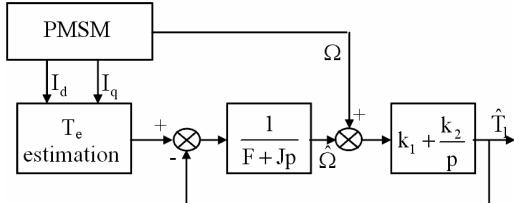


Figure 9. Estimation of load torque.

The error between measured speed and estimated speed is presented like the entry of the PI regulator:

$$\hat{T}_l = \frac{1 + \frac{k_1}{k_2} P}{1 + \frac{1 + k_1}{k_2} P + \frac{1}{k_2} P^2} T_l \quad (27)$$

$k_1$  et  $k_2$  are determined by poles imposition .

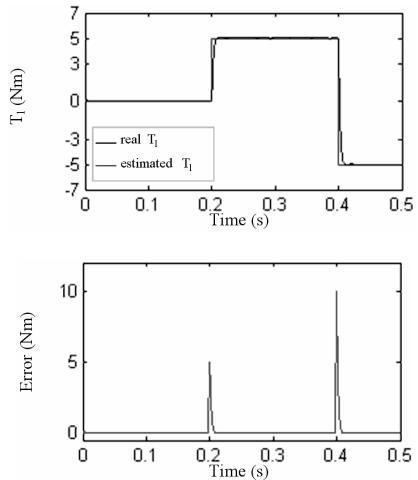


Figure 10. Estimation of load torque

## VIII. SIMULATION RESULTS

Extensive simulations have been performed using Matlab/Simulink Software to examine the control algorithm of the

sensorless fuzzy sliding mode control applied for PMSM presented in figure 8 and based on 3Kw motor parameters.

Figure 11 shows the responses of speed, position, currents  $I_d$  and  $I_q$ , load torque with errors between the actual and estimated states for step reference with 75% of rated load at  $t=0.1$  s. At  $t=0.2$  s the speed is reversed from  $+100\text{rad/s}$  to  $-100\text{rad/s}$  and at  $t=0.4$  s the reference speed becomes  $10\text{ rad/s}$ .

These responses illustrate high performances of the proposed EKF observer combined with Fuzzy Sliding mode during transient and steady states and the EKF works at very low speed, where many speed estimators or observers perform poorly.

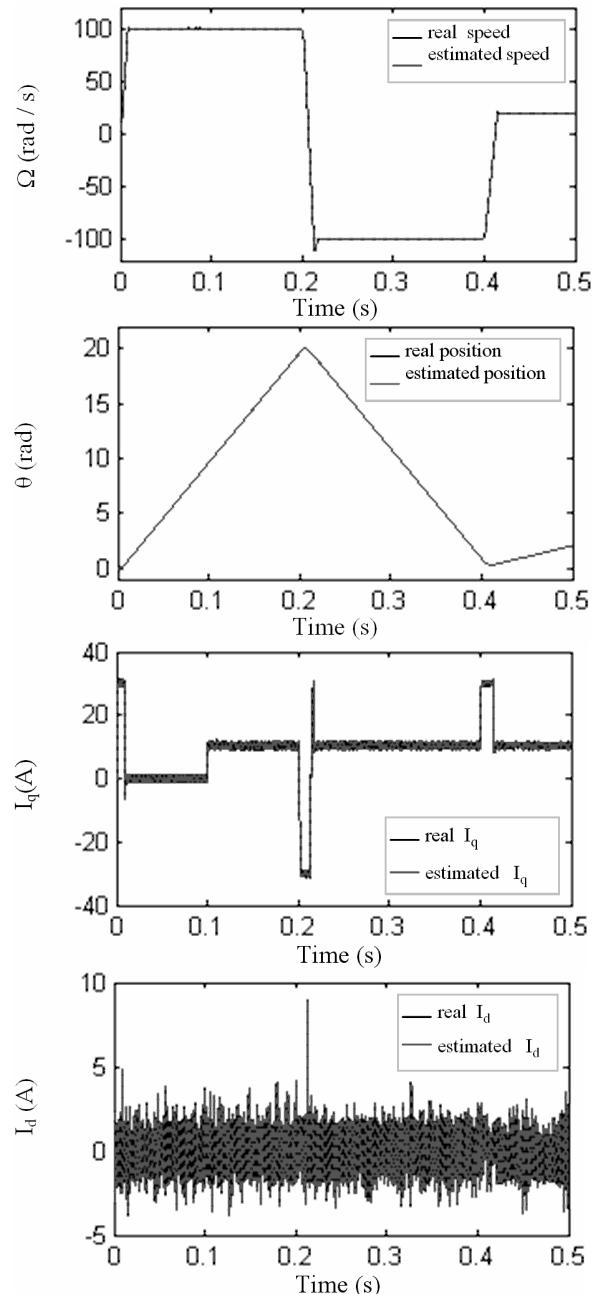


Figure 11 Drive response under reference speed inversion under load disturbances

## IX. CONCLUSION

In this paper, the authors determine fuzzy sliding mode control of a permanent magnet synchronous motor. The dynamic behavior and the control performances obtained are satisfactory. The perturbation is rejected.

The observer based on an extended Kalman filter can observe the exact rotor speed and position. The EKF observer is able to increase the performance of the fuzzy sliding mode control in terms of low-speed behavior, speed reversion and load rejection.

## X. APPENDIX

Parameters of the system used in simulation:

Parameters of the PMSM:

Rated power 1 kw

Rated speed 3000 rpm

Stator winding resistance 0.6  $\Omega$

Stator winding direct inductance 4 mH

Stator winding quadrature inductance 2.8 mH

Rotor flux 0.12 Wb

Viscous friction 1.4 e-3 Nm/rad/s

Inertia 1.1 e-3 kg m<sup>2</sup>

Pairs pole number 4

Nominal current line 20 A

Nominal voltage line 310 V

Machine type: Siemens 1FT6084-8SK71-1TGO

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