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# Ex nihilo . . .

Alexander Afriat

July 26, 2011

## Abstract

The Aharonov-Bohm effect has been called *topological*; it is also claimed to suggest a world made of loops. I will argue that if the effect is topological, so are electrostatics and Newtonian gravity (in fact just about anything involving radiation, propagation from a source); and also that if the effect suggests a world made of loops, then electrostatics and Newtonian gravity suggest a world made of membranes.

## 1 The Aharonov-Bohm effect

A wavefunction is split into two, which, having enclosed a (simply connected) region  $\omega$  containing a solenoid, are made to interfere on a screen. The enclosing wavefunction is sensitive to any enclosed electromagnetism inasmuch as the electromagnetic four-potential  $A$  (a one-form) contributes a phase

$$\exp i \oint_{\partial\omega} A$$

(the wavefunction along) the boundary  $\partial\omega$  and hence to the interference pattern on the screen. The electromagnetism on  $\omega$  is related to the circulation around the boundary by Stokes' theorem

$$\oint_{\partial\omega} A = \iint_{\omega} dA.$$

The electromagnetic field<sup>1</sup>  $F = dA$  produced by the solenoid is circumscribed to a middle region  $\lambda \subset \omega$  surrounded by an isolating region (say an annulus)  $\lambda' = \omega - \lambda$  where  $F$  vanishes but not  $A$ . The full *Aharonov-Bohm effect* can be considered the 'differential' sensitivity of the interference pattern to variations in the current through the solenoid.

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<sup>1</sup>It is perhaps easiest to think of  $F$  as a magnetic field  $\mathbf{B} = d\mathbf{A}$  produced by a three-current  $\mathbf{J} = d*\mathbf{B}$  in the solenoid, where  $\mathbf{B}$  is a two-form,  $*\mathbf{B}$  the one-form Hodge-dual to it (we're now in three dimensions), and the three-potential  $\mathbf{A}$  is three-quarters of the four-potential  $A \leftrightarrow (\varphi, \mathbf{A})$ .

## 2 The topological interpretation

The *topological interpretation*<sup>2</sup> of the effect can be formulated as follows: If  $A$  were closed throughout a simply connected region  $\omega$  it would also be exact, and hence expressible as the gradient  $A = d\mu$  of a zero-form  $\mu$  (a real-valued function); the flux

$$\oint_{\partial\omega} d\mu = \iint_{\omega} d^2\mu$$

through the boundary would then vanish, since  $d^2 = 0$ . But here  $A$  is closed on  $\lambda'$ ; from  $dA = 0|_{\lambda'}$  it does not follow that  $A$  is exact, nor that the flux through the enclosing loop vanishes: it may or may not. The existence of the source responsible for the effect is therefore ruled out by one topology ( $A$  closed throughout a simply connected region) but not another.

The same applies to a simply connected three-dimensional region  $\Omega$  enclosed by a two-dimensional boundary  $\partial\Omega$ . If the two-form  $E$  were closed throughout  $\Omega$  it would also be exact, and hence expressible as the curl  $E = d\zeta$  of a one-form  $\zeta$ ; the flux

$$\iint_{\partial\Omega} d\zeta = \iiint_{\Omega} d^2\zeta$$

through the boundary would then vanish. But if the region on which  $E$  is closed has a hole in it, the flux through the enclosing surface may or may not vanish.

This is precisely what we have in electrostatics, where the electric field  $E = *d\varphi$  is (Hodge-dual to) the gradient  $d\varphi$  of the scalar potential  $\varphi$ . The vanishing divergence  $dE$  expresses the conservation of electricity where none is created, away from the charges that produce  $E$  according to the Maxwell-Poisson equation  $dE = d*d\varphi = \rho$  (the three-form  $\rho$  being the charge density). If the divergence  $dE$  vanished throughout the volume  $\Omega$ , there would be no electricity produced and hence none radiated through the enclosing surface.<sup>3</sup> But charges in  $\Omega$ —say in a region  $\Lambda \subset \Omega$  isolated by  $\Lambda' = \Omega - \Lambda$ —prevent electricity from being conserved throughout  $\Omega$ .

We have the same formalism in Newtonian gravity, where  $\varphi$  is the gravitational potential,  $d\varphi$  and  $E$  both represent gravitational force, and  $\rho$  is the mass density. Gravity would therefore be another topological effect.

Again, the topology of the region where the ‘potential’ ( $A$  or  $E$  or whatever) is closed tells us relatively little: if the region were simply connected, conservation would be *general* within the enclosing surface since there could be no holes containing sources; and if nothing were created inside the enclosing surface, the total radiation through it would vanish. But if the topology does *not* allow the presence of holes to be ruled out, the presence of sources in them cannot either; and sources would produce a flux through the enclosing surface.

A non-trivial topology cannot, on its own at any rate, rule out the *absence* of sources either. Nor does it provide the *intensity* of the sources (which would, if known, tell us the intensity of the effect—the flux through the enclosing surface). So the full Aharonov-Bohm effect, which can be considered ‘differential,’ is hardly accounted for by topology.

<sup>2</sup>See Ryder (1996), Nash & Sen (2000), Nakahara (2003), Nounou (2003).

<sup>3</sup>Over and above any divergence-free electrical background that may or may not be present.

### 3 A loopy world made of membranes too

Since the electromagnetic field  $F = dA = dA'$  is more directly measurable than  $A$ , it is customary to say that the freedom expressed by the substitution

$$(1) \quad A \mapsto A' = A + d\xi$$

( $\xi$  being a zero-form) is unobservable—and that hence one doesn't know quite what to make of  $A$ . But even if the freedom (1) to bend and twist the level surfaces of  $A$ 's local potential<sup>4</sup>  $\gamma$  can make one wonder about the physical meaning of  $A$ , all loops going around the solenoid once puncture the same number of level surfaces.

It is natural to be troubled by the ontological ambiguities of  $A$  and implications of (1). Encouraged by the more reassuring transformation properties of loops, Healey (2007) has proposed a world made of loops. I will neither claim that the world is not made of loops, nor that ontology and invariance are unrelated—only that in three dimensions we have the same thing:  $E$  is also subject to a freedom  $E \mapsto E' = E + d\beta$  invisible to the divergence  $dE = dE'$ . But again, even if  $E$ 's local potential<sup>5</sup>  $\zeta$  can be freely transformed, any membrane containing the source will be punctured by the same number of field-lines of  $E$ , radiated by the source.

If the Aharonov-Bohm effect suggests a world of loops, a world of membranes is suggested by electrostatics and Newtonian gravity, in fact by just about any source radiating (heat or sound or electricity or gravity or whatever) in three dimensions.

### References

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<sup>4</sup>For wherever  $A$  is closed it can be written locally as the gradient  $A = d\gamma$  of a scalar potential  $\gamma$ .

<sup>5</sup>For wherever  $E$  is closed it can be written locally as the curl  $E = d\zeta$  of a one-form  $\zeta$ .