



Ex nihilo ...

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Ex nihilo . . .

Alexander Afriat

July 26, 2011

Abstract

The Aharonov-Bohm effect has been called *topological*; it is also claimed to suggest a world made of loops. I will argue that if the effect is topological, so are electrostatics and Newtonian gravity (in fact just about anything involving radiation, propagation from a source); and also that if the effect suggests a world made of loops, then electrostatics and Newtonian gravity suggest a world made of membranes.

1 The Aharonov-Bohm effect

A wavefunction is split into two, which, having enclosed a (simply connected) region ω containing a solenoid, are made to interfere on a screen. The enclosing wavefunction is sensitive to any enclosed electromagnetism inasmuch as the electromagnetic four-potential A (a one-form) contributes a phase

$$\exp i \oint_{\partial\omega} A$$

to (the wavefunction along) the boundary $\partial\omega$ and hence to the interference pattern on the screen. The electromagnetism on ω is related to the circulation around the boundary by Stokes' theorem

$$\oint_{\partial\omega} A = \iint_{\omega} dA.$$

The electromagnetic field¹ $F = dA$ produced by the solenoid is circumscribed to a middle region $\lambda \subset \omega$ surrounded by an isolating region (say an annulus) $\lambda' = \omega - \lambda$ where F vanishes but not A . The full *Aharonov-Bohm effect* can be considered the 'differential' sensitivity of the interference pattern to variations in the current through the solenoid.

¹It is perhaps easiest to think of F as a magnetic field $\mathbf{B} = d\mathbf{A}$ produced by a three-current $\mathbf{J} = d*\mathbf{B}$ in the solenoid, where \mathbf{B} is a two-form, $*\mathbf{B}$ the one-form Hodge-dual to it (we're now in three dimensions), and the three-potential \mathbf{A} is three-quarters of the four-potential $A \leftrightarrow (\varphi, \mathbf{A})$.

2 The topological interpretation

The *topological interpretation*² of the effect can be formulated as follows: If A were closed throughout a simply connected region ω it would also be exact, and hence expressible as the gradient $A = d\mu$ of a zero-form μ (a real-valued function); the flux

$$\oint_{\partial\omega} d\mu = \iint_{\omega} d^2\mu$$

through the boundary would then vanish, since $d^2 = 0$. But here A is closed on λ' ; from $dA = 0|_{\lambda'}$ it does not follow that A is exact, nor that the flux through the enclosing loop vanishes: it may or may not. The existence of the source responsible for the effect is therefore ruled out by one topology (A closed throughout a simply connected region) but not another.

The same applies to a simply connected three-dimensional region Ω enclosed by a two-dimensional boundary $\partial\Omega$. If the two-form E were closed throughout Ω it would also be exact, and hence expressible as the curl $E = d\zeta$ of a one-form ζ ; the flux

$$\iint_{\partial\Omega} d\zeta = \iiint_{\Omega} d^2\zeta$$

through the boundary would then vanish. But if the region on which E is closed has a hole in it, the flux through the enclosing surface may or may not vanish.

This is precisely what we have in electrostatics, where the electric field $E = *d\varphi$ is (Hodge-dual to) the gradient $d\varphi$ of the scalar potential φ . The vanishing divergence dE expresses the conservation of electricity where none is created, away from the charges that produce E according to the Maxwell-Poisson equation $dE = d*d\varphi = \rho$ (the three-form ρ being the charge density). If the divergence dE vanished throughout the volume Ω , there would be no electricity produced and hence none radiated through the enclosing surface.³ But charges in Ω —say in a region $\Lambda \subset \Omega$ isolated by $\Lambda' = \Omega - \Lambda$ —prevent electricity from being conserved throughout Ω .

We have the same formalism in Newtonian gravity, where φ is the gravitational potential, $d\varphi$ and E both represent gravitational force, and ρ is the mass density. Gravity would therefore be another topological effect.

Again, the topology of the region where the ‘potential’ (A or E or whatever) is closed tells us relatively little: if the region were simply connected, conservation would be *general* within the enclosing surface since there could be no holes containing sources; and if nothing were created inside the enclosing surface, the total radiation through it would vanish. But if the topology does *not* allow the presence of holes to be ruled out, the presence of sources in them cannot either; and sources would produce a flux through the enclosing surface.

A non-trivial topology cannot, on its own at any rate, rule out the *absence* of sources either. Nor does it provide the *intensity* of the sources (which would, if known, tell us the intensity of the effect—the flux through the enclosing surface). So the full Aharonov-Bohm effect, which can be considered ‘differential,’ is hardly accounted for by topology.

²See Ryder (1996), Nash & Sen (2000), Nakahara (2003), Nounou (2003).

³Over and above any divergence-free electrical background that may or may not be present.

3 A loopy world made of membranes too

Since the electromagnetic field $F = dA = dA'$ is more directly measurable than A , it is customary to say that the freedom expressed by the substitution

$$(1) \quad A \mapsto A' = A + d\xi$$

(ξ being a zero-form) is unobservable—and that hence one doesn't know quite what to make of A . But even if the freedom (1) to bend and twist the level surfaces of A 's local potential⁴ γ can make one wonder about the physical meaning of A , all loops going around the solenoid once puncture the same number of level surfaces.

It is natural to be troubled by the ontological ambiguities of A and implications of (1). Encouraged by the more reassuring transformation properties of loops, Healey (2007) has proposed a world made of loops. I will neither claim that the world is not made of loops, nor that ontology and invariance are unrelated—only that in three dimensions we have the same thing: E is also subject to a freedom $E \mapsto E' = E + d\beta$ invisible to the divergence $dE = dE'$. But again, even if E 's local potential⁵ ζ can be freely transformed, any membrane containing the source will be punctured by the same number of field-lines of E , radiated by the source.

If the Aharonov-Bohm effect suggests a world of loops, a world of membranes is suggested by electrostatics and Newtonian gravity, in fact by just about any source radiating (heat or sound or electricity or gravity or whatever) in three dimensions.

References

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⁴For wherever A is closed it can be written locally as the gradient $A = d\gamma$ of a scalar potential γ .

⁵For wherever E is closed it can be written locally as the curl $E = d\zeta$ of a one-form ζ .