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A linear model for production management - Optimal solving policies

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A linear model for production management considering generic data on stocks, manufacturing processes, resource capacities and costs is presented. Any type of continuous or discrete production process also including assembly and disassembly tasks is taken into account. The model inversion issue is analysed and relevant joint production/inventory management policies are solved by optimizing linear and quadratic programs under resource and stock capacity constraints. **Beyond the single period inversion issue which is the focus of the paper, the model is tailored to support multi-period and multi-level planning approaches whose development is the subject of separate publications.**

Keywords: Manufacturing systems, production management, quadratic programming

1. Background

In manufacturing, the lean production and Just-In-Time concepts that appeared in the 70s are still in force to minimize as far as reasonable the stocks and to shorten the production cycles. **The last decade has seen the emergence of networked organizations so-called supply chains (Lee and Billington 1995), extended or virtual enterprises (Jagdev and Browne 1998) providing the enterprise network with the agility required by the market without taking the partners away from their own core business.** Combining the traditional need for production planning with cooperative environments leads to the **integration** of Production Management and Logistics, **and requires appropriate** advanced ERP and APS solutions. The role of the underlying information system is determining to offer the relevant contextual information to the decision makers and to synchronize partners activities despite the heterogeneity of the physical resources, processes and applications (Raghu-nathan 2003). Moreover, a focal enterprise commonly cumulates the activities resulting from various roles in the open market: those directly induced by customer orders, but also by external order makers in the surrounding supply chain(s). A consequence is that the networked production system is largely heterogeneous in terms of production management philosophy, as well as of IT solutions and the coherence of the global operating is most of the time far to be provable. The interoperability of the decision systems in networked manufacturing organizations is becoming a key topic in the research community (Chen *et al.* 2007). To **face** the growing mass of **surrounding** technical data, various multi-echelon (Mesarovic 1970), hierarchical or fractal (Warnecke 1993) decision making approaches **based** on aggregate views

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1 of the technical data were proposed (Tung *et al.* 1999, Axsäter 2001, Van der Vorst
2 *et al.* 2001, Fontan *et al.* 2005). Note that the aggregate planning approaches (Chen
3 and Liao 2003, Kogan and Portougal 2006) are mainly driven by local planning
4 solving issues and **are not designed to be applied to networked manufacturing or-**
5 **ganizations.** Production Management and Logistics are largely investigated as an
6 application field of Operations Research (Coughan and Coghlan 2002, Eom and
7 Kim 2006, Scholl and Becker 2006), more specifically addressing optimal inventory
8 management, Supply Chain management (Lee and Billington 1995), planning and
9 scheduling (Uzsoy 1999). Nevertheless most of the models supporting these opti-
10 mization issues are **case-specific** and the global modeling of a networked production
11 management is far to be achieved in the literature.

12
13 Among the generic issues addressed **in Production Management** are planning and
14 scheduling. Resource planning, as largely promoted by the MRP approach (Orlicky
15 1975), is the definition of the activity of a production unit seen as one aggregate re-
16 source (typically a workshop or a factory) in a sequence of future time periods, and
17 correlatively of a material procurement as well as product delivery **plan**. Planning
18 solutions appeal to optimization techniques **to implement** management policies un-
19 der capacity and cost constraints (Olhager and Selldin 2003). The scheduling issue
20 considers the internal resources (typically workstations) **of** a production system and
21 aims to allocate appropriate sub-activities to those resources, **according to** a com-
22 plex decision subject to logical (task precedence), technical (transformation and
23 transport resource capacities) and economical (work and stock costs) constraints.
24 **See e.g. (Tharumarajah 2001, Herroelen and Leus 2005) for a review of scheduling**
25 **techniques.** Actually, planning and scheduling are two **types of** decision to organize
26 the activity of networked production organizations **at** various levels of detail. The
27 need for a joint approach to planning and scheduling is emphasized in the literature
28 (Barták 1999, Smith *et al.* 2000).

29
30 The goal of our investigations was to develop a formal and generic model allowing
31 to support decision making in activity planning and scheduling within networked
32 organizations (Bourrières 1998, Lecompte *et al.* 2000). **As a first step towards this**
33 **objective,** this paper is focused on the **elaboration** of our generic production model
34 **and on its inversion over one single time-period under** combined production and
35 stock management strategies. The planning issue as such is not the subject of this
36 paper. Nonetheless the model presented here is tailored to support multi-period
37 planning and multi-level scheduling investigations that are the subject of separate
38 publications. **The last section indicates how to extend the results presented in this**
39 **paper to address the multi-period planning issue.**

40
41 The remainder of the paper is organized as follows: section 2 presents the generic
42 model or direct production model; section 3 focuses on model inversion and on op-
43 timal production solving policies; section 4 is dedicated to an illustrative example;
44 and section 5 is a conclusion.

45 46 47 48 49 **2. Direct production model**

50
51 In this section **are presented the parameters and variables to capture the technical**
52 **data,** then a production model is established that links the variations of the prod-
53 uct amounts to the works generating those variations. **We also define the resource**
54 **capacities and cost data.** All vectors in this paper are non-negative column-vectors.

55 56 *Product modeling*

57
58 **Parameters:** The parameters below define the nomenclature of all product items
59
60

traced in the manufacturing system: components, intermediate and finished products.

- O_u the set of **component** references
- O_x the set of **intermediate product** references
- O_y the set of **finished product** references
- O the global set of product references
- m_u the number of **component** references; $m_u = \|O_u\|$
- m_x the number of **intermediate product** references; $m_x = \|O_x\|$
- m_y the number of **finished product** references; $m_y = \|O_y\|$
- m the total number of product references ; $m = m_u + m_x + m_y$

Variables: The following vectors address quantities of products of any possible references:

- s_u the vector of **component** amounts with reference to set O_u
- s_x the vector of **intermediate product** amounts with reference to set O_x
- s_y the vector of **finished product** amounts with reference to set O_y
- s the global stock vector, i.e. the concatenation of vectors s_u, s_x, s_y

The vectors above depict the virtual stocks of products in the production system.

Work modeling

Parameters: We similarly introduce the parameters and variables referring to the transformation tasks:

- T the set of tasks
- n the length of task list of references; $n = \|T\|$

Variables: Vector w addresses quantities of transformation tasks and as such qualifies and quantifies the works carried out in the manufacturing system.

- w the vector of task amounts with reference to set T

Timing

In production management, time is divided into periods of constant duration and the concatenation of periods covers a finite time horizon. Below are introduced time-related notations.

- $s(k)$ the stock vector at the beginning of period k
- $s(k+1)$ the stock vector at the end of period k , i.e. beginning of period $k+1$
- $w(k)$ the vector of work performed during period k

In this paper, the production problem is considered over one period only, turning the stock vector from $s(k)$ to $s(k+1)$. In the conclusion we mention how to bridge the one-period optimization related in this paper with the multi-period planning issue.

Costing

Cost modelling for production management basically consists in the valuation of product transformation costs and of inventory costs. The data model is here

enhanced as follows:

$\beta(k)$ the cost vector of the work performed during period k
 $\gamma(k)$ the cost vector of stocks over period k

Over period k , the cost of work $w(k)$ is $\beta^T(k)w(k)$ and the cost of stock $s(k)$ is $\gamma^T(k)s(k)$.

Direct production model

Any product transformation requires specific input items and produces output items as well. Considering all item references and all product transformations feasible by the production system, the quantitative impact of a definite work $w(k)$ on stock vector $s(k)$ is defined by the linear model:

$$s(k+1) - s(k) = Cw(k) \quad w(k) \geq 0 \quad s(k) \geq 0 \quad s(k+1) \geq 0 \quad (1)$$

Notation $s(k)$ and $s(k+1)$ respectively refers to the state of the global stock before and after completion of work $w(k)$. The m -by- n matrix C defines the variation of product amounts generated by the achievement of work $w(k)$. The components of vectors $w(k)$ and $s(k)$ are non negative. They may be natural integer or real numbers, depending on the application case. Equation (1) links the working activities to the consumption and production of stocks, in other words identifies the direct production model, i.e. all feasible manufacturing processes. A graphical representation of the direct production model is obtained by considering equation (1) as the fundamental equation of a Petri net $\langle O, T, C \rangle$ where the set of places O is the concatenation of sets O_u , O_x and O_y of product references, the set of transitions T is the set of task references and C is the incidence matrix of the Petri net. The weighting of the arcs defines the number of product items consumed and produced by the occurrence of each task. Note that all types of product transformations (machining, assembly, cutting, etc) without any restriction can be modeled. Matrix C is composed of three blocks C_u , C_x and C_y defined with reference to sets O_u , O_x and O_y . The entries of C_u are non-positive since the input components can only be consumed by the production processes. The entries of C_y are non-negative as the finished products can only be generated by the production processes.

Figure 1 shows an example of production model with $O_u = \{o_1, o_2\}$ two references of components, $O_x = \{o_3, o_4, o_5\}$ three references of intermediate products, $O_y = \{o_6, o_7\}$ two references of finished products and $T = \{t_1, t_2, t_3, t_4\}$ four references of tasks.

Resource capacity model

The expression of resource capacities must be homogeneous with the expression of works, in order to be able to assess the load-capacity alignment. We define the capacity of a transformation resource for period k as the maximal work that the resource is able to perform during the period. The resource is potentially polyvalent and the way the resource is able to share its capacity upon the various types of work must also be modeled. One classically distinguishes disjunctive from cumulative resources (Mercier and Van Hentenryck 2007). A disjunctive resource is able to perform one task at once among a set of feasible

tasks, in other words is constrained by time-sharing. A cumulative resource is able to perform various tasks simultaneously and independently. In the latter case, the parallel tasks are commonly sharing a physical scalar (energy, space, weight, etc.). Nevertheless, if the resource is an aggregate and abstract resource, the various activities may be performed independently (e.g. a workshop can simultaneously and independently cumulate various activities allocated to distinct machines). To summarize, three basic categories of resource cover most of the situations: disjunctive resource, cumulative resource with dependent capacities, cumulative resource with independent capacities, but the two first categories can be merged as they both express the sharing of a scalar (time, energy, etc.). Hence, we finally distinguish two categories of transformation resources only: with dependent capacities or with independent capacities.

Below is defined the working capacity of a resource by a vector $\mathbf{a}(k)$, $\mathbf{a}(k) \geq 0$, the component $a_j(k)$, $j \in \{1, \dots, n\}$ of which is the inverse of the maximal number of j -type task occurrences that can be achieved during period k , assumed that the resource does not execute any other task. The work amount $\mathbf{w}(k)$ can be achieved by the resource during period k under the following conditions:

$$\mathbf{a}^T(k)\mathbf{w}(k) \leq 1 \quad \text{dependent capacities} \quad (2a)$$

$$\mathbf{A}(k)\mathbf{w}(k) \leq \mathbf{1} \quad \text{independent capacities} \quad (2b)$$

with $\mathbf{A}(k) = \text{diag}[a_1(k), \dots, a_n(k)]$.

The graphical representation of capacity models (2a) and (2b) for $n=2$ is shown on Figure 2.

The load rate of the resource can be characterized by a scalar $\rho(k) \in [0, 1]$ when the resource is with dependent capacities, and by a vector $\boldsymbol{\rho}(k) \in [0, 1]^n$ when the resource is with independent capacities, respectively defined by:

$$\rho(k) = \mathbf{a}^T(k)\mathbf{w}(k) \quad \rho(k) \in [0, 1] \quad \text{dependent capacities} \quad (3a)$$

$$\boldsymbol{\rho}(k) = \mathbf{A}(k)\mathbf{w}(k) \quad \boldsymbol{\rho}(k) \in [0, 1]^n \quad \text{independent capacities} \quad (3b)$$

A correlated issue is to identify the minimal duration $d[\mathbf{w}(k)]$ required to achieve a given load $\mathbf{w}(k)$. As the capacities are proportional to the duration of the period, an extrapolation (see Figure 3) can be made that yields:

$$d[\mathbf{w}(k)] = h\mathbf{a}^T(k)\mathbf{w}(k) \quad \text{dependent capacities} \quad (4a)$$

$$d[\mathbf{w}(k)] = h\text{Max}[a_j(k)w_j(k)]_{j=1,n} \quad \text{independent capacities} \quad (4b)$$

3. Inverse production model

3.1 Production capacity

A preliminary issue is to identify the maximal amount of products the system is able to achieve in the period, in other words to turn the resource capacities into product capacities : given the resource capacity vector $\mathbf{a}(k)$ and the current stock level vector $\mathbf{s}(k)$, find the maximal amount of j -type products that the system is able to manufacture in period k . The question may be relevant not only regarding the finished products (set O_y) but also any intermediate items (set O_x). Moreover, considering the particular case $\mathbf{s}_x(k) = 0$ leads to determine the number of j -type products the system is *per se* able to produce in the period, i.e. limited only by its capacity, assumed that the stocks of intermediate products are initially empty.

Let \mathbf{C}_i be the i -eth line of matrix \mathbf{C} . The work $\mathbf{w}(k)$ generates the quantity $\mathbf{C}_i\mathbf{w}(k)$ of items o_i . The maximal production of items o_i under the resource capacity constraint (2a) or (2b) is provided by solving the following linear program:

$$\text{Maximize } \mathbf{C}_i\mathbf{w}(k) \quad (5)$$

subject to :

$$\begin{array}{ll} \mathbf{w}(k) \geq 0 & \text{non negative solutions} \\ \mathbf{s}(k) + \mathbf{C}\mathbf{w}(k) \geq \mathbf{s}^*(k+1) & \text{stock floor constraint} \\ \mathbf{a}^T(k)\mathbf{w}(k) \leq 1 \quad \text{or} \quad \mathbf{A}(k)\mathbf{w}(k) \leq 1 & \text{resource capacity constraint} \\ \mathbf{w}(k) \in \mathbb{N}^n & \text{natural integer solutions (optional)} \end{array}$$

The last constraint is optional. The stock floor constraint $\mathbf{s}^*(k+1)$ is given non-negative. The initial stock $\mathbf{s}(k)$ is assumed to satisfy the floor and ceiling constraints. One may also add a stock ceiling constraint as shown in section 3.3.

The program above solves the maximal production problem. Nevertheless the solution $\mathbf{w}(k)$ may be not unique. It is necessary to search for the optimal achievement of the maximal production target stemming from program (5) by additionally considering one of the optimization programs presented in the next section.

3.2 Model inversion

The inversion of model (1) is interpreted as follows: given the current stock $\mathbf{s}(k)$, find any work vector $\mathbf{w}(k)$ producing the stock variation $\boldsymbol{\delta}(k) = \mathbf{s}(k+1) - \mathbf{s}(k)$ expected. Practically, the production target $\boldsymbol{\delta}(k)$ is only partially specified, addressing at least the items of O_y (finished products) but also potentially some items of O_x (intermediate products) when a stock regulation policy is applied. The unspecified part of $\boldsymbol{\delta}(k)$, i.e. the consumption of components and the unspecified stock variations of intermediate products will result from the solution $\mathbf{w}(k)$ chosen. We consequently share set O in two subsets: \bar{O} the list of item references for which a production target is given, and \hat{O} the list of item references for which no production target is specified. Accordingly, the stock cost vector $\boldsymbol{\gamma}$ is split into vectors $\bar{\boldsymbol{\gamma}}$ and $\hat{\boldsymbol{\gamma}}$, the variation vector $\boldsymbol{\delta}(k)$ into $\bar{\boldsymbol{\delta}}(k)$ and $\hat{\boldsymbol{\delta}}(k)$, and matrix \mathbf{C} into blocks $\bar{\mathbf{C}}$ and $\hat{\mathbf{C}}$ whose dimensions are \bar{m} -by- n and \hat{m} -by- n respectively, with \bar{m} the cardinal of \bar{O} and $\hat{m} = m - \bar{m}$ the cardinal of \hat{O} . Consequently, the production model (1) is refined as follows:

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$$\bar{\delta}(k) = \bar{C}w(k) \tag{6}$$

$$\hat{\delta}(k) = \hat{C}w(k) \tag{7}$$

Then, to invert the linear form (6) i.e. to identify the work $w(k)$ required by the achievement of the specified stock variation $\bar{\delta}(k)$. Each solution $w(k)$ will thanks to relation (7), allow to determine the free stock variation $\hat{\delta}(k)$, in particular the component requirement.

Matrix \bar{C} being non square, **the inversion of the linear model (6) falls under classical discussion**. The inversion problem has solutions only if all of the $\bar{m} - rank\bar{C}$ characteristic determinants of equation (6) all equate to zero. Under these conditions, the dimension of the solution space, i.e. the order of multiplicity of the solution, is $n - rank\bar{C}$. We here assume that \bar{C} is regular, i.e. $rank\bar{C} = Min(\bar{m}, n)$.

If $\bar{m} > n$, i.e. $rank\bar{C} = n$, the system (6) is overdetermined. The system is invertible when all of the $\bar{m} - n$ characteristic determinants are equal to zero. Under this condition, there is a unique solution $w(k) = \bar{C}^+\bar{\delta}(k)$ where $\bar{C}^+ = \bar{C}^T(\bar{C}\bar{C}^T)^{-1}$ is the Moore-Penrose inverse (Penrose 1955), also called pseudo-inverse, of \bar{C} . The corresponding free stock variation is $\hat{\delta}(k) = \hat{C}\bar{C}^+\bar{\delta}(k)$. Note that as the target $\bar{\delta}(k)$ is implicated in the characteristic determinants, the existence of solutions results from the compatibility of the production target with the logical structure of the manufacturing process expressed by matrix \bar{C} . Should the system be non invertible, in other words when the production target cannot be exactly achieved, then $w(k) = \bar{C}^+\bar{\delta}(k)$ is the 'best fit' (least square) solution. This solution generates a stock variation that is as close as possible to the target $\bar{\delta}(k)$ expected.

If $\bar{m} = n$, i.e. $rank\bar{C} = \bar{m}$, equation (6) is a Cramer system whose unique solution is $w(k) = \bar{C}^{-1}\bar{\delta}(k)$, therefore $\hat{\delta}(k) = \hat{C}\bar{C}^{-1}\bar{\delta}(k)$.

If $\bar{m} < n$, i.e. $rank\bar{C} = \bar{m}$, the solution is not unique and an optimization criterion is required to identify good solutions within the $n - \bar{m}$ dimensional solution space. In production management, the objective is obviously to minimize both stock and working costs. The stock management policy is here expressed *i)* by the specification of 'hard' targets $\bar{s}(k + 1)$ for the items whose stock level at the end of the period is strictly defined and *ii)* by the definition, in the optimization function, of 'soft' attracting targets $\hat{s}^*(k + 1)$ for all other items. Note that the number \bar{m} of controlled stocks must not overpass the number n of tasks, otherwise the system (6) is over-determined and conditionally invertible only.

Below is discussed how to solve the optimization problem, using first a linear programming approach then quadratic programming.

3.3 Optimal inverse model

An economical way to reach the target $\bar{\delta}(k)$ is defined by the following linear program:

$$\text{Minimize } J_1 = \beta^T w(k) + \hat{\gamma}^T [\hat{s}(k+1) - \hat{s}^*(k+1)] \quad (8)$$

subject to :

$w(k) \geq 0$	non negative solutions
$\bar{C}w(k) = \bar{\delta}(k)$	achieve specified targets
$\hat{s}(k+1) = \hat{s}(k) + \hat{C}w(k)$	production model
$\hat{s}(k+1) \geq \hat{s}^*(k+1)$	stock floor constraint
$a^T(k)w(k) \leq 1$ or $A(k)w(k) \leq 1$	respect resource capacity
$\hat{s}(k+1) \leq \hat{s}_{Max}$	stock ceiling constraint (optional)
$w(k) \in \mathbb{N}^n$	natural integer solutions (optional)

The two last constraints are optional. The stock floor $\hat{s}^*(k+1)$ and ceiling \hat{s}_{Max} are given non-negative. Should some stock levels remain free of control, the corresponding weights in vector $\hat{\gamma}$ will be set to weak values.

The linear program formulated above is easily solved using a Simplex algorithm. The solving is made in real or natural integer numbers, depending on the semantics of the production model. Given the initial stock $s(k)$ and the target $\bar{\delta}(k)$, the optimization process finally provides the optimal work load $w^\circ(k)$, the corresponding load rate according to (3a) or (3b), the minimal cost J_1° and the stock variation $\hat{\delta}(k) = \hat{C}w^\circ(k)$ among which is in particular the component requirement $\hat{\delta}_u(k) = C_u w^\circ(k)$.

Instead of solving numerically the optimal production problem, a parametric approach may be also expected to provide a deeper understanding of the problem solving. We will focus on the following quadratic optimization problem:

$$\text{Minimize } J = w^T \Psi w \quad \Psi \text{ symmetric positive definite} \quad (9)$$

subject to:

$\bar{C}w(k) = \bar{\delta}(k)$	\bar{C} regular	\bar{m} -by- n	$\bar{m} < n$
$w(k) \geq 0$			
$a^T(k)w(k) \leq 1$	dependent capacities		
or			
$A(k)w(k) \leq 1$	independent capacities		

Matrix Ψ will allow us to express definite production strategies.

Minimizing a quadratic criterion subject to underdetermined linear equality constraints is a classical issue, see e.g. (Ben-Israel 2002):

$$J = x^T \Psi x \quad \text{s.t.} \quad Mx = N \text{ is minimal for } x = \Psi^{-1}M^T[M\Psi^{-1}M^T]^{-1}N \quad (10)$$

assumed that Ψ is a symmetric positive definite matrix. Adding the inequality constraints (the optimal solution $w^\circ(k)$ must be non-negative and satisfy the capacity constraints) the program has a unique solution when the Karush-Kuhn-Tucker (KKT) conditions are satisfied, see e.g. (Jensen and Bard 2002, Shinnars 1998). Logarithmic barriers may also be introduced in the objective function to consider

positive solutions (Frisch 1955). Nevertheless, solving the KKT conditions leads to set up a new linear program itself requiring Simplex-like numerical algorithms. The solving process is thus not fully parametric and there is no more argument to prefer the quadratic optimization approach to the linear program (8). Here we bypass the difficulty of quadratic programming under inequality constraints by previously identifying the ‘feasible’ production targets for which we are sure that the optimal solution is non-negative and capacity complying. We hereafter make a two-fold study consisting in *i*) identifying the optimal solution $\mathbf{w}^\circ(k)$ for some relevant quadratic objective functions subject to equality constraints only and *ii*) identifying the conditions of target feasibility which ensure that the inequality constraints are satisfied.

Let first equation (6) be considered. An analytical description of the solution space is provided by the generalised inverse formalism:

$$\mathbf{w}(k, \mathbf{z}) = \bar{\mathbf{C}}^+ \bar{\boldsymbol{\delta}}(k) + (\mathbf{I} - \bar{\mathbf{C}}^+ \bar{\mathbf{C}}) \mathbf{z} \quad (11)$$

where \mathbf{z} is an arbitrary n -dimensional vector, \mathbf{I} the identity matrix and $\bar{\mathbf{C}}^+$ the pseudo-inverse of $\bar{\mathbf{C}}$ (Mayer 1973, Varga 2001, Ben-Israel 2002). According to (7), the corresponding free stock variation is:

$$\hat{\boldsymbol{\delta}}(k, \mathbf{z}) = \hat{\mathbf{C}} \bar{\mathbf{C}}^+ \bar{\boldsymbol{\delta}}(k) + \hat{\mathbf{C}} (\mathbf{I} - \bar{\mathbf{C}}^+ \bar{\mathbf{C}}) \mathbf{z} \quad (12)$$

Within the solution domain (11) we identify the solutions that optimize some specific quadratic criteria expressing relevant production policies, as shown in Table 1. Hereafter is discussed each of the optimization policies.

(J_2) *Minimize work amount.* Minimizing criterion $J_2 = \|\mathbf{w}(k)\|^2$ simply ensures that the target is achieved with the minimal quadratic work amount. The optimal solution is $\mathbf{w}^\circ(k) = \bar{\mathbf{C}}^+ \bar{\boldsymbol{\delta}}(k)$, provided by (10) with $\boldsymbol{\Psi} = \mathbf{I}$, $\mathbf{M} = \bar{\mathbf{C}}$ and $\mathbf{N} = \bar{\boldsymbol{\delta}}(k)$.

(J_3) *Minimize working cost.* We have to minimize criterion $J_3 = \|\mathbf{E}\mathbf{w}(k)\|^2$ with $\mathbf{E} = \text{diag}(\beta_1, \dots, \beta_n)$ where β_j is the cost of task j . Let \mathbf{v} be a vector such that $\mathbf{w}(k) = \mathbf{E}^{-1} \mathbf{v}$. According to (10), minimizing $J_3 = \|\mathbf{v}\|^2$ s.t. $\bar{\mathbf{C}} \mathbf{E}^{-1} \mathbf{v} = \bar{\boldsymbol{\delta}}(k)$ leads to solution $\mathbf{v} = (\bar{\mathbf{C}} \mathbf{E}^{-1})^+ \bar{\boldsymbol{\delta}}(k)$. Therefore J_3 is minimal for $\mathbf{w}^\circ(k) = \mathbf{E}^{-1} (\bar{\mathbf{C}} \mathbf{E}^{-1})^+ \bar{\boldsymbol{\delta}}(k)$. If more particularly the task costs are all identical, then $\mathbf{E} = \mathbf{I}$ and $\mathbf{w}^\circ(k) = \bar{\mathbf{C}}^+ \bar{\boldsymbol{\delta}}(k)$.

(J_{4a}) *Load rate control* (dependent capacities). Assumed that the resource has dependent capacities defined by (2a), the objective function J_{4a} aims to engage the resource capacities at a specified level $\rho^\circ(k)$ while achieving the production target $\bar{\boldsymbol{\delta}}(k)$. Here we have to minimize

$$\|\rho(k) - \rho^\circ(k)\|^2 = [\mathbf{w}(k) - \mathbf{w}^\circ(k)]^T \mathbf{a}(k) \mathbf{a}(k)^T [\mathbf{w}(k) - \mathbf{w}^\circ(k)]$$

where $\mathbf{w}^\circ(k)$ is so that $\mathbf{a}(k)^T \mathbf{w}^\circ(k) = \rho^\circ(k)$.

Unfortunately, matrix $\mathbf{a}(k) \mathbf{a}(k)^T$ is singular thus solution (10) is not applicable. To minimize J_{4a} we follow (Fletcher 1985) by identifying a \bar{m} -by- n matrix \mathbf{S} and a \bar{m} -by- $(n-\bar{m})$ matrix \mathbf{Z} complying with $\bar{\mathbf{C}} \mathbf{S} = \mathbf{I}$ and $\bar{\mathbf{C}} \mathbf{Z} = \mathbf{0}$ and $[\mathbf{S} | \mathbf{Z}]$ invertible. Note that $\mathbf{Z}^T \mathbf{a}(k) \mathbf{a}(k)^T \mathbf{Z}$ is positive definite on the solution space defined by the equality

constraint. With simplified notations, we minimize the following Lagrangian :

$$L(\mathbf{w}, \boldsymbol{\lambda}) = (\mathbf{w} - \mathbf{w}^\circ)^T \mathbf{a} \mathbf{a}^T (\mathbf{w} - \mathbf{w}^\circ) + \boldsymbol{\lambda}^T (\bar{\mathbf{C}} \mathbf{w} - \bar{\boldsymbol{\delta}})$$

which requires to solve :

$$\nabla_{\mathbf{w}} = 2 \mathbf{a} \mathbf{a}^T (\mathbf{w} - \mathbf{w}^\circ) - \bar{\mathbf{C}}^T \boldsymbol{\lambda} = 0$$

$$\nabla_{\boldsymbol{\lambda}} = \bar{\mathbf{C}} \mathbf{w} - \bar{\boldsymbol{\delta}} = 0$$

By introducing a vector \mathbf{v} such as $\mathbf{w} = \mathbf{S} \bar{\boldsymbol{\delta}} + \mathbf{Z} \mathbf{v}$ we still have $\bar{\mathbf{C}} \mathbf{w} - \bar{\boldsymbol{\delta}} = 0$ since $\bar{\mathbf{C}} \mathbf{S} = \mathbf{I}$ and $\bar{\mathbf{C}} \mathbf{Z} = \mathbf{0}$. By left multiplying $\nabla_{\mathbf{w}}$ by \mathbf{Z}^T we eliminate $\boldsymbol{\lambda}$:

$$\mathbf{Z}^T \mathbf{a} \mathbf{a}^T (\mathbf{S} \bar{\boldsymbol{\delta}} + \mathbf{Z} \mathbf{v} - \mathbf{w}^\circ) = 0$$

As $\mathbf{Z}^T \mathbf{a} \mathbf{a}^T \mathbf{Z}$ is regular, the unique solution is :

$$\mathbf{v} = (\mathbf{Z}^T \mathbf{a} \mathbf{a}^T \mathbf{Z})^{-1} \mathbf{Z}^T \mathbf{a} \mathbf{a}^T (\mathbf{w}^\circ - \mathbf{S} \bar{\boldsymbol{\delta}})$$

Finally, $\mathbf{w} = \mathbf{S} \bar{\boldsymbol{\delta}} + \mathbf{Z} \mathbf{v}$ provides the optimal solution:

$$\tilde{\mathbf{w}} = \mathbf{Z} (\mathbf{a}^T \mathbf{Z})^{-1} \rho^\circ + [\mathbf{I} - \mathbf{Z} (\mathbf{a}^T \mathbf{Z})^{-1} \mathbf{a}^T] \mathbf{S} \bar{\boldsymbol{\delta}}$$

Note that solution $\tilde{\mathbf{w}}$ leads $\|\rho(k) - \rho^\circ(k)\|^2$ to be exactly equal to zero hence $\tilde{\mathbf{w}} = \mathbf{w}^\circ$. The reason is that the quadratic program is semi-definite. This can be verified by left multiplying $\tilde{\mathbf{w}}$ by \mathbf{a}^T , which leads to $\mathbf{a}^T \tilde{\mathbf{w}} = 1 \rho^\circ + \mathbf{0} \bar{\boldsymbol{\delta}} = \rho^\circ$.

(J_{4b}) *Load rate control* (independent capacities). If the resource has independent capacities, see (2b), one may wish to spare the resource capacities by minimizing $J_{4b} = \|\boldsymbol{\rho}(k)\|^2 = \|\mathbf{A} \mathbf{w}(k)\|^2$ with $\mathbf{A}(k) = \text{diag}[a_1(k), \dots, a_n(k)]$. The optimal solution is here $\mathbf{w}^\circ(k) = \mathbf{A}^{-1} (\bar{\mathbf{C}} \mathbf{A}^{-1})^+ \bar{\boldsymbol{\delta}}(k)$. As matrix \mathbf{A} is regular, the quadratic programme is definite and does not bring J_{4b} to zero but to its minimal positive value.

(J_5) *Stock and/or work control*. The component $\|\mathbf{E} \mathbf{w}(k)\|^2$ of criterion J_5 aims to minimize work cost whereas $\|\hat{\mathbf{F}}[\hat{\boldsymbol{\delta}}(k) - \hat{\boldsymbol{\delta}}^*(k)]\|^2$ is to attract the parasitic production $\hat{\boldsymbol{\delta}}(k)$ to a predefined value $\hat{\boldsymbol{\delta}}^*(k)$ expressing a ‘soft’ stock target $\hat{\mathbf{s}}^*(k+1) = \hat{\mathbf{s}}(k) + \hat{\boldsymbol{\delta}}^*(k)$.

Noting that $\hat{\boldsymbol{\delta}}(k) - \hat{\boldsymbol{\delta}}^*(k) = \hat{\mathbf{s}}(k+1) - \hat{\mathbf{s}}^*(k+1)$, the expression $\|\hat{\mathbf{F}}[\hat{\boldsymbol{\delta}}(k) - \hat{\boldsymbol{\delta}}^*(k)]\|^2$ is the weighted quadratic error on the final stock and the entries of the diagonal matrix $\hat{\mathbf{F}}$ are the stock costs. The minimization of J_5 with simplified notations is as follows:

$$J_5 = \|\mathbf{E} \mathbf{w}\|^2 + \|\hat{\mathbf{F}}(\hat{\boldsymbol{\delta}} - \hat{\boldsymbol{\delta}}^*)\|^2 = \mathbf{w}^T \boldsymbol{\Phi} \mathbf{w} - 2 \mathbf{w}^{*T} \hat{\mathbf{C}}^T \hat{\mathbf{F}} \hat{\mathbf{C}} \mathbf{w} + \mathbf{w}^{*T} \hat{\mathbf{C}}^T \hat{\mathbf{F}} \hat{\mathbf{C}} \mathbf{w}^*$$

where $\boldsymbol{\Phi} = \mathbf{E}^2 + \hat{\mathbf{C}}^T \hat{\mathbf{F}} \hat{\mathbf{C}}$.

Here \mathbf{w}^* is the work generated by the variation $\hat{\delta}^*$. Using the Lagrange multiplier technic, we have $\mathbf{L}(\mathbf{w}, \boldsymbol{\lambda}) = J_5 + \boldsymbol{\lambda}^T(\bar{\mathbf{C}}\mathbf{w} - \bar{\boldsymbol{\delta}})$ and consequently:

$$\nabla_{\mathbf{w}} = 2\Phi\mathbf{w} - 2\hat{\mathbf{C}}^T\hat{\mathbf{F}}^2\hat{\mathbf{C}}\mathbf{w}^* - \hat{\mathbf{C}}^T\boldsymbol{\lambda} = 0$$

$$\nabla_{\boldsymbol{\lambda}} = \bar{\mathbf{C}}\mathbf{w} - \bar{\boldsymbol{\delta}} = 0$$

whose explicit solution after some calculation is :

$$\mathbf{w}^\circ = \Phi^{-1}\hat{\mathbf{C}}^T(\bar{\mathbf{C}}\Phi^{-1}\hat{\mathbf{C}}^T)^{-1}\bar{\boldsymbol{\delta}} + [\mathbf{I} - \Phi^{-1}\hat{\mathbf{C}}^T(\bar{\mathbf{C}}\Phi^{-1}\hat{\mathbf{C}}^T)^{-1}\bar{\mathbf{C}}]\Phi^{-1}\hat{\mathbf{C}}^T\hat{\mathbf{F}}^2\hat{\delta}^*$$

Hence the error on the final stock is $\hat{\mathbf{s}}^*(k+1) - \hat{\mathbf{C}}\mathbf{w}^\circ(k)$. The particular case $\mathbf{E} = 0$ minimizes stock costs only, whereas program J_5 tends to program J_3 (work minimization) when $\hat{\mathbf{F}}$ tends to $\mathbf{0}$.

Taking into account inequality constraints : target feasibility domain

To ensure that the optimal solutions given by Table 1 are non-negative (real or natural integer) vectors satisfying the capacity constraints, we here identify the restrictions on the production targets.

Looking for real solutions, the optimal solutions from Table 1 must additionally satisfy the following constraints depending on the capacity model (2a) or (2b):

$$\mathbf{w}^\circ(k) \geq 0 \quad \mathbf{a}^T(k)\mathbf{w}^\circ(k) \in [0, 1] \quad (\text{dependent capacities}) \quad (13a)$$

$$\mathbf{w}^\circ(k) = \mathbf{A}^{-1}\boldsymbol{\rho}^\circ(k) \quad \boldsymbol{\rho}^\circ(k) \in [0, 1]^n \quad (\text{independent capacities}) \quad (13b)$$

The condition (13a) or (13b) defines the target feasibility domain making $\bar{\boldsymbol{\delta}}(k)$, $\hat{\delta}^*(k)$ and $\boldsymbol{\rho}^\circ(k)$ linearly dependent. Note that the target feasibility domain is a linear inequality defining a convex set and may be turned into a standard LMI form to benefit from LMI-based optimization techniques (Boyd *et al.* 1994). For example looking for the real solutions of quadratic program J_{4a} (load rate control), the conditions (13a) applied to J_{4a} define the target feasibility domain as follows:

$$\mathbf{Z}(\mathbf{a}^T\mathbf{Z})^{-1}\boldsymbol{\rho}^\circ(k) + [\mathbf{I} - \mathbf{Z}(\mathbf{a}^T\mathbf{Z})^{-1}\mathbf{a}^T]\mathbf{S}\bar{\boldsymbol{\delta}}(k) \geq 0 \quad \boldsymbol{\rho}^\circ(k) \in [0, 1] \quad (14a)$$

whereas (13b) combined with (6) defines the following feasibility domain:

$$\bar{\boldsymbol{\delta}}(k) = \bar{\mathbf{C}}\mathbf{A}^{-1}\boldsymbol{\rho}^\circ(k) \quad \boldsymbol{\rho}^\circ(k) \in [0, 1]^n \quad (14b)$$

Looking for natural integer solutions, the optimal solutions from Table 1 must satisfy the following constraints depending on the capacity model (2a) or (2b):

$$\mathbf{w}^\circ(k) \in \mathbb{N}^n \quad \mathbf{a}^T(k)\mathbf{w}^\circ(k) \in [0, 1] \quad (\text{dependent capacities}) \quad (15a)$$

$$\mathbf{w}^\circ(k) = \mathbf{A}^{-1}\boldsymbol{\rho}^\circ(k) \quad [\boldsymbol{\rho}^\circ(k)]_j = \eta_j a_j \quad (\text{independent capacities}) \quad (15b)$$

with $\eta_j = 0, 1, \dots, a_j^{-1}$

1 Finally, an appropriate approach to solve the quadratic programs J_2 to J_5
 2 submitted to both equality and inequality constraints is the following:
 3

4 1) prior define the feasible targets under inequality constraints using (13a),
 5 (13b), (15a) or (15b), then
 6

7
 8 2) apply the optimal solution provided by Table 1. Programs J_2, J_3, J_5 are usable
 9 with both capacity models (2a) and (2b). Program J_{4a} suits for the dependent
 10 capacity model (2a) whereas J_{4b} suits for the independent capacity model (2b).
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 13 **4. Application**
 14

15 This section illustrates the use of our linear model in the determination of single-
 16 period production scenarios according to the control policies presented in Table
 17 1. We consider the production model shown on Figure 1. We associate the ‘hard’
 18 production target with items o_4, o_6 and o_7 whereas the ‘soft’ target is on items o_3
 19 and o_5 . We consequently note:
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$$\bar{\delta}(k) = (\delta_4 \delta_6 \delta_7)^T \quad \hat{\delta}(k) = (\delta_1 \delta_2 \delta_3 \delta_5)^T$$

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24 Let moreover the following data be considered:
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 27 $s(k) = (300 \ 300 \ 50 \ 50 \ 50 \ 100 \ 200)$ initial stock level
 28 $\alpha(k) = 10^{-3}(10 \ 2 \ 10 \ 20)$ resource capacity
 29 $\mathbf{E} = \text{diag}(30 \ 25 \ 10 \ 20)$ working cost
 30 $\hat{\mathbf{F}} = \text{diag}(20 \ 10 \ 10 \ 20)$ stock cost
 31

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 33 We have :
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$$\bar{O} = \{o_4, o_6, o_7\} \quad \bar{\mathbf{C}} = \begin{pmatrix} 1 & 1 & -2 & -1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix} \quad \hat{O} = \{o_1, o_2, o_3, o_5\} \quad \hat{\mathbf{C}} = \begin{pmatrix} -2 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 0 & 1 & 0 & -3 \end{pmatrix}$$

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$$\bar{\mathbf{C}}^+ = \frac{1}{6} \begin{pmatrix} 3 & 1.5 & 1.5 \\ 3 & 1.5 & 1.5 \\ 0 & 2 & 0 \\ 0 & -1 & 3 \end{pmatrix} \quad \mathbf{I} - \bar{\mathbf{C}}^+ \bar{\mathbf{C}} = \frac{1}{2} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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50 Here $\bar{m} = 3, n = 4, \text{rank} \bar{\mathbf{C}} = 3$ and the dimension of the solution space is
 51 $n - \text{rank} \bar{\mathbf{C}} = 1$. To solve the semi-definite program J_{4a} the following matrices may
 52 be used:
 53

54
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$$\mathbf{S} = \frac{1}{6} \begin{pmatrix} 6 & 0 & 3 \\ 0 & 3 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 3 \end{pmatrix} \quad \mathbf{Z} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

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which verify $\bar{C}S = I$ and $\bar{C}Z = 0$.

Maximal production The linear program (5) applied to items o_6 and o_7 provides the production capacities shown in Table 2. Here the constraint $w(k) \in \mathbb{N}^n$ is additionally considered. The production capacities are both computed for the initial stock $s(k)$ previously given and for $s_x(k) = 0$ (the intermediate stocks are empty and the resource has consequently to achieve the whole manufacturing processes). The results are obtained for dependent and for independent resource capacities.

Feasible targets Assumed first that the resource is with dependent capacities, we solve the load rate control problem by previously identifying the feasible targets according to (13a):

$$\begin{pmatrix} 125 \\ -125 \\ 0 \\ 0 \end{pmatrix} \rho^\circ(k) + \frac{1}{24} \begin{pmatrix} -6 & -3 & -33 \\ 30 & 15 & 45 \\ 0 & 8 & 0 \\ 0 & -4 & 12 \end{pmatrix} \bar{\delta}(k) \geq 0 \quad \rho^\circ(k) \in [0, 1]$$

The LMI above defines the domain of feasibility of the production target $\bar{\delta}(k)$ and ensures that the corresponding work vector $w(k)$ is non negative and satisfies the capacity constraint. The domain is ρ° -homothetic. It is shown on Figure 4a. Setting the load rate to 1, the production of achieved items is maximal at point P_1 ($\bar{\delta}_6=300$; $\bar{\delta}_7=100$), assumed that the stock level of o_4 is sufficient ($\bar{\delta}_4=-200$). If the production of o_7 only is expected, one can obtain up to $\bar{\delta}_7=100$ with $\bar{\delta}_4=-50$, see point P_2 . One may also expect to concentrate the activity on the production item o_4 up to $\bar{\delta}_4=500$, see P_3 .

If the resource is with independent capacities, we consider (13b):

$$\bar{\delta}(k) = \begin{pmatrix} 100 & 500 & -200 & -50 \\ 0 & 0 & 300 & 0 \\ 0 & 0 & 100 & 100 \end{pmatrix} \rho^\circ(k) \quad \rho^\circ(k) \in [0, 1]^4$$

The target feasibility domain is shown on Figure 4b.

Optimal production policies Let e.g. the ‘hard’ target $\bar{\delta}(k)=(0 \ 70 \ 40)^T$ be specified i.e. to produce 70 o_6 and 40 o_7 while strictly maintaining the stock o_4 unchanged. To be feasible, the target must additionally comply with (13a) or (13b), which yields:

$$0.51 \leq \rho^\circ(k) \leq 0.95 \quad (\text{dependent capacity})$$

$$20\rho_1^\circ + 100\rho_2^\circ = 11 \quad \rho_3^\circ = 0.23 \quad \rho_4^\circ = 0.17 \quad (\text{independent capacity})$$

The optimized solutions to reach the target $\bar{\delta}(k) = (0 \ 70 \ 40)^T$ with reference to criteria J_1 to J_5 are presented in Table 3. A Simplex solver was used to minimize J_1 . The other criteria were optimized analytically, applying the results from Table

2. For criterion J_{4a} we chose $\rho^\circ = 0.51$, the minimal possible value of the load rate identified above. The 'soft' target $\hat{\delta}^*(k) = (-100 \ -100 \ 0 \ 0)^T$ was used for J_5 . The optimization of J_5 was made for the matrix \mathbf{E} previously given (work and stock cost minimization) and for $\mathbf{E}=0$ (stock cost minimization only) as well. For each optimization problem, we have computed the optimal solution $\mathbf{w}^\circ(k)$ and the corresponding production $\delta(k)$. The 'hard' production target $\hat{\delta}(k)$ is strictly reached, whereas the 'free' production $\hat{\delta}(k)$ -of which the component requirements stem from the particular economical and/or stock level control policy expressed by the objective function. The optimization of J_2 is very easily computerized and minimizes the quantity of operations to be performed. Nevertheless the resource capacity is also partly engaged to generate a parasitic production. Minimizing J_3 provides the most economical solution with regard to the working cost. But the parasitic production of items o_3 is very high and over consumes the resource capacity ($\rho^\circ=0.86$). The programs J_{4a} and J_{4b} allow to spare the resource capacity by engaging the minimal load rate. The counterpart is the consumption of the intermediate stock o_3 . The program J_1 and J_5 give the priority to working and stocking costs minimization rather than sparing the load rate.

5. Conclusion

The linear model presented in this paper is applicable to a large range of discrete or continuous production systems performing various transformation tasks, including assembly/disassembly. The boundaries of the article are the consideration of single-period production and stock management policies formalized as optimization problems subject to linear equalities (the production model) and inequalities (resource capacity constraints). Appealing to linear criteria leads to easy problem solving approaches using popular Simplex algorithms. Being numerical, those algorithms nevertheless do not provide any deep understanding of the solutions obtained. For this reason, we have performed the analytical solving of some quadratic programs expressing the most relevant policies. The inequality constraints were integrated through prior analysis of the target feasibility domain.

On the basis of the model presented in this paper, our investigations are in two directions : *i*) multi-period resource planning and *ii*) multi-level scheduling.

The multi-period planning model is derived from the one-period production problem by extending vector $\mathbf{w}(k)$ to a vector $\tilde{\mathbf{w}}(k)$ such that:

$$\tilde{\mathbf{w}}^T(k) = [\mathbf{w}^T(k) \ \boldsymbol{\pi}^T(k) \ \boldsymbol{\xi}^T(k)]$$

with $\boldsymbol{\pi}^T(k)$ the procurement vector in period k and $\boldsymbol{\xi}^T(k)$ the expedition vector in period k . We consequently state an under-determined linear model over a planning horizon model $k = 1, H$ with $(\mathbf{w}^T(k), k = 1, H)$ as decision variables whose inversion under capacity and cost constraints leads to formulate linear or quadratic programs similar to those studied in this paper.

The formulation of the multi-level scheduling problem addresses simultaneously two levels of data : *i*) aggregate data related to the capacity of a mother resource as well as to the corresponding product/task/processes and *ii*) detailed data related to the network of daughter-resources encapsulated in the mother resource. The interrelations between the aggregate and detailed views can be formulated in a linear

1 framework and lead to identify aggregation/disaggregation consistency rules. Based
2 on the linear model presented in this paper, our investigations aim to promote uni-
3 fied planning and scheduling generic primitives in a multi-level plan breaking-up
4 approach to iteratively turn a product order book into detailed activities.
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Table 1. Optimal solutions s.t. equality constraint $\bar{\mathbf{C}}\mathbf{w}(k) = \bar{\boldsymbol{\delta}}(k)$

Optimization criterion	Optimal solution
$J_2 = \ \mathbf{w}(k)\ ^2$	$\mathbf{w}^\circ(k) = \bar{\mathbf{C}}^+ \bar{\boldsymbol{\delta}}(k)$
$J_3 = \ \mathbf{E}\mathbf{w}(k)\ ^2$	$\mathbf{w}^\circ(k) = \mathbf{E}^{-1}(\bar{\mathbf{C}}\mathbf{E}^{-1})^+ \bar{\boldsymbol{\delta}}(k)$
$J_{4a} = \ \rho(k) - \rho^\circ(k)\ ^2$	$\mathbf{w}^\circ(k) = \mathbf{Z}(\mathbf{a}^T\mathbf{Z})^{-1}\rho^\circ(k) + [\mathbf{I} - \mathbf{Z}(\mathbf{a}^T\mathbf{Z})^{-1}\mathbf{a}^T]\mathbf{S}\bar{\boldsymbol{\delta}}(k)$
$J_{4b} = \ \boldsymbol{\rho}(k)\ ^2$	$\mathbf{w}^\circ(k) = \mathbf{A}^{-1}(\bar{\mathbf{C}}\mathbf{A}^{-1})^+ \bar{\boldsymbol{\delta}}(k)$
$J_5 = \ \mathbf{E}\mathbf{w}(k)\ ^2 + \ \hat{\mathbf{F}}(\hat{\boldsymbol{\delta}}(k) - \hat{\boldsymbol{\delta}}^*(k))\ ^2$	$\mathbf{w}^\circ(k) = \Phi^{-1}\hat{\mathbf{C}}^T(\bar{\mathbf{C}}\Phi^{-1}\hat{\mathbf{C}}^T)^{-1}\bar{\boldsymbol{\delta}}(k) + [\mathbf{I} - \Phi^{-1}\hat{\mathbf{C}}^T(\bar{\mathbf{C}}\Phi^{-1}\hat{\mathbf{C}}^T)^{-1}\bar{\mathbf{C}}]\Phi^{-1}\hat{\mathbf{C}}^T\hat{\mathbf{F}}^2\hat{\boldsymbol{\delta}}^*(k)$

Notes: $\mathbf{E} = \text{diag}(\beta_1, \dots, \beta_n)$ $\mathbf{A} = \text{diag}(a_1, \dots, a_n)$ $\hat{\mathbf{F}}$ diagonal $\Phi = \mathbf{E}^2 + \hat{\mathbf{C}}^T\hat{\mathbf{F}}^2\hat{\mathbf{C}}$

J_{4a} and J_{4b} for respectively dependent and independent resource capacities

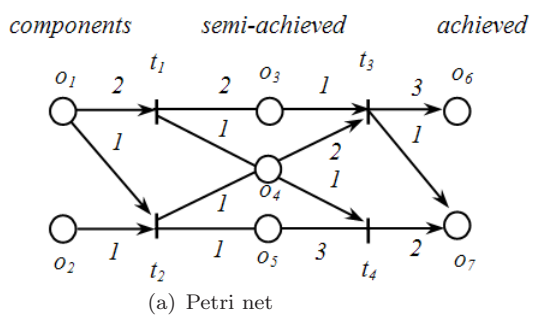
Table 2. Production capacity

Resource capacity	dependent	dependent	independent	independent
Initial stock	$\mathbf{s}(k)$	$\mathbf{s}_x(k)=0$	$\mathbf{s}(k)$	$\mathbf{s}_x(k)=0$
Max δ_6	216	165	300	300
Max δ_7	90	76	200	200

Table 3. Optimal production scenarios for target $\bar{\delta}(k) = (0 \ 70 \ 40)^T$

Policy	Optimal work				Load rate			Stock variation					
	$w_1(k)$	$w_2(k)$	$w_3(k)$	$w_4(k)$	$\rho(k)$ $\rho^T(k)$	$\hat{\delta}_1(k)$	$\hat{\delta}_2(k)$	$\hat{\delta}_3(k)$	$\bar{\delta}_4(k)$	$\hat{\delta}_5(k)$	$\bar{\delta}_6(k)$	$\bar{\delta}_7(k)$	
resource with dependent capacities													
J_1	30	25	23.33	8.33	0.75	-85	-25	36.66	0	0	70	40	
J_{4a}	0	55	23.33	8.33	0.51	-55	-55	-23.33	0	30	70	40	
resource with independent capacities													
J_1	55	0	23.33	8.33	[0.55 0.00 0.23 0.17]	-85	-25	36.66	0	0	70	40	
J_{4b}	2.12	52.88	23.33	8.33	[0.02 0.11 0.23 0.17]	-57.12	-52.88	-19.10	0	27.88	70	40	
resource with dependent or independent capacities													
J_2	27.5	27.5	23.33	8.33	0.73 [0.28 0.06 0.23 0.17]	-82.5	-27.5	31.66	0	2.5	70	40	
J_3	44	11	23.33	8.33	0.86 [0.44 0.02 0.23 0.17]	-99	-11	-64.66	0	-14	70	40	
J_5	35.12	19.88	23.33	8.33	0.79 [0.35 0.04 0.23 0.17]	-90.12	-19.88	46.92	0	5.12	70	40	
$J_5 (\mathbf{E} = 0)$	6.31	48.69	23.33	8.33	0.56 [0.06 0.10 0.23 0.17]	-61.31	-48.69	-10.71	0	23.69	70	40	

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$$C = \begin{pmatrix} C_u \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} -2 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ \hline 2 & 0 & -1 & 0 \\ 1 & 1 & -2 & -1 \\ \hline 0 & 1 & 0 & -3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

(b) Incidence matrix

Figure 1. Production model

For Peer Review Only

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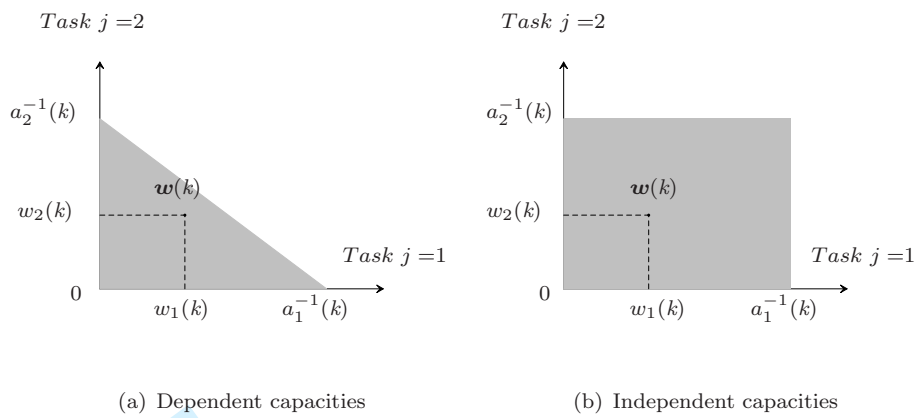


Figure 2. Resource capacity model

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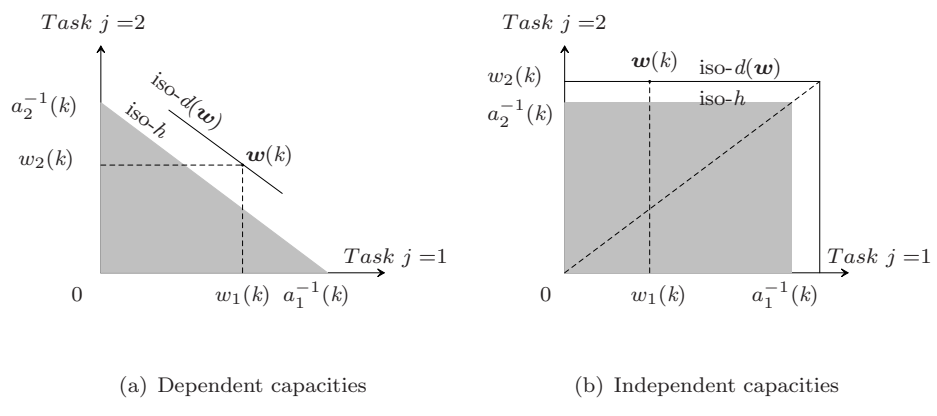


Figure 3. Delay to achieve work $w(k)$

For Peer Review Only

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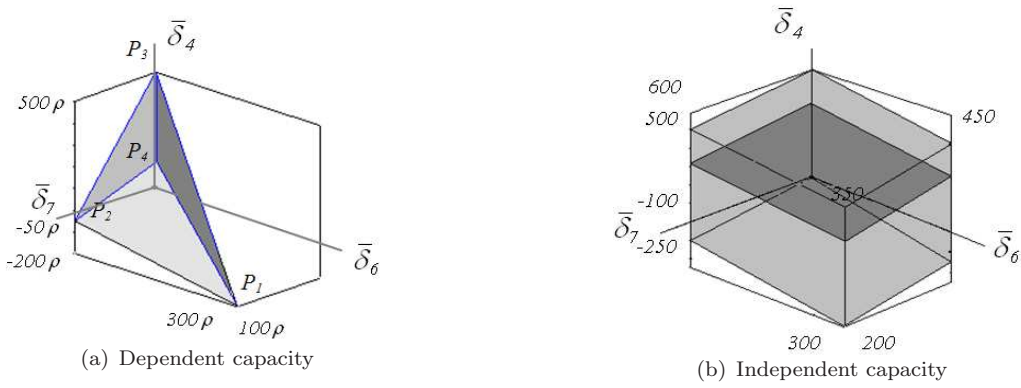


Figure 4. Target feasibility domain

For Peer Review Only

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