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# Multi-objective optimization by genetic algorithms in $H_\infty/LPV$ control of semi-active suspension

A. L. Do, O. Sename, L. Dugard and B. Soualmi

*GIPSA-lab, Control Systems Dept, CNRS-Grenoble INP, ENSE3, BP 46, F-38402 St Martin d'Hères cedex, France (e-mail: {anh-lam.do, olivier.sename, luc.dugard}@gipsa-lab.grenoble-inp.fr).*

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**Abstract:** In semi-active suspension control, comfort and road holding are two essential but conflicting performance objectives. In a previous work, the authors proposed an  $LPV$  formulation for semi-active suspension control of a realistic nonlinear suspension model where the nonlinearities (i.e the bi-viscous and the hysteresis) were taken into account; an  $H_\infty/LPV$  controller to handle the comfort and road holding was also designed. The present paper aims at improving the method of Do et al. (2010) by using Genetic Algorithms (GAs) to select the optimal weighting functions for the  $H_\infty/LPV$  synthesis. First, a general procedure for the optimization of the weighting functions for the  $H_\infty/LPV$  synthesis is proposed and then applied to the semi-active suspension control. Thanks to GAs, the comfort and road holding are handled using a single high level parameter and illustrated via the Pareto optimality. The simulation results performed on a nonlinear vehicle model emphasize the efficiency of the method.

*Keywords:* Genetic algorithms,  $H_\infty$  control,  $LPV$  systems, semi-active suspensions

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## 1. INTRODUCTION

The  $H_\infty$  control design approach is an efficient tool to improve the performances of a closed-loop system in pre-defined frequency ranges. The key step of the  $H_\infty$  control design relies on the selection of the weighting functions which depends on the engineer skill and experience. In many real applications, the difficulty in choosing the weighting functions increases because the performance specification is not accurately defined, i.e., it is simply to achieve the best possible performances (optimal design) or to achieve an optimally joint improvement of more than one objective (multi-objective design). So it is still interesting to optimise the selection of the weighting functions to get the desired closed-loop performances, as studied in Beaven et al. (1996), Hu et al. (2000) ... They propose to consider each system, no matter how complex it is, as a combination of sub-systems of the first and second order, for which it is easy to find the good weighting functions to be used in the  $H_\infty$  control methodology. However, there is no explicit method to find these functions in the general case. The usual way is to proceed by trial-and-error. Recently, the idea to use an optimization tool was proposed in Alfaro-Cid et al. (2008). The choice of GAs seems natural because their formulation is well suited for this type of problematic.

The semi-active control problem has been studied during the recent decades (see Savaresi et al. (2010) and reference therein) and  $H_\infty$  control is one of the most investigated approaches. This paper aims at using GAs to obtain optimal weighting functions for the  $H_\infty/LPV$  control of semi-

active suspension systems. First, from the frequency based industrial performance criteria of suspension systems as defined in Sammier et al. (2003), an appropriate cost function is formulated. This cost function is in fact a convex combination of two specifications: passenger comfort and road holding, allowing to tune the suspension system using a single high level parameter. Second, the weighting functions are then tuned by GAs in such a way that the cost function is minimized. Finally, the best weighting functions are obtained are used for the synthesis of  $H_\infty/LPV$  controller. For illustration, the performances of the closed-loop semi-active suspension system associated with the controller obtained with GAs will be analyzed to emphasize the efficiency of the approach. It is important to note that this methodology provides a new efficient and generic tool for suspension control design, where the industrial performance objectives are included in the design procedure (and not evaluated a posteriori).

The outline is as follows. In Section 2, a brief introduction about multi-objective optimization using GAs is presented. The procedure for the optimization of the weighting functions for  $H_\infty/LPV$  problem using GAs is then introduced in Section 3. In Section 4, the  $H_\infty/LPV$  control synthesis for semi-active suspensions, presented in Do et al. (2010), is recalled. In Section 5, the application of weighting function selection using GAs on the  $H_\infty/LPV$  control problem formulated in Section 4 is presented. The results obtained in simulations with a nonlinear quarter car model are discussed in Section 6. Finally, some conclusions and perspectives are given in Section 7.

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## 2. GENETIC ALGORITHMS AND MULTI-OBJECTIVE OPTIMIZATION

### 2.1 Genetic algorithms

Genetic algorithms have now grown strongly from the first study of Holland (1975), a popular theory-oriented book of Goldberg (1989) and an application-oriented book of Davis (1991). The algorithms are based on the mechanism of the natural selection and have been proven to be very effective in optimization with many real applications such as in finance and investment strategies, robotics, engineering design, telecommunications... They are likely global optimization techniques (despite the high computational expense) (see Marler and Arora (2004)) using probabilistic, multi-points search, random combination (crossover, mutation) and information of previous iterations to evaluate and improve the population. A great advantage of GAs compared to other searching methods (for example gradient methods) is that they search regardless of the nature of the objective functions and constraints.

GAs initialize with a random population. Through the genetic operation: *selection*, *crossover* and *mutation*, a new population is obtained. By using the selection process, the fittest individuals based on their *fitness* values will be chosen; crossover and mutation will be then applied to create the new population. The genetic operation on individuals of population continues until the optimization criterion is satisfied or a certain number of generations is reached.

### 2.2 Multi-objective optimization

One well-known application of the GAs is to find the optimal solution for the multi-objective optimization problem involving multiple and conflicting objectives. This is a very popular problem in practice and can be described as follows

$$\min_{x \in C} F(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_{n_{obj}}(x) \end{bmatrix}, \quad n_{obj} \geq 2, \quad (1)$$

where  $x$  is called the decision vector,  $C$  the set of possible decision vectors (or the searching space), and  $F(x)$  the objective vector. The existence of an ideal solution  $x^*$  that can minimize simultaneously all objective functions  $f_1, f_2 \dots f_{n_{obj}}$  is in fact rarely feasible. Hence, in this case, the concept of *Pareto-optimal* or *Pareto-set* is usually used to describe the solution of multi-objective optimization problem (see Marler and Arora (2004)).

There are many formulations to solve the problem (1) like weighted min-max method, weighted global criterion method, goal programming methods... see Marler and Arora (2004) and references therein. One of the most popular and simple approaches is the weighted sum method which converts the multi-objective problem into a single objective problem. In this paper, a particular case of the weighted sum method is used, where the multi-objective function  $F$  is replaced by the convex combination of objectives

$$\min J = \sum_{i=1}^{n_{obj}} \alpha_i f_i(x), \quad s.t \quad x \in C \quad (2)$$

where  $\sum_{i=1}^{n_{obj}} \alpha_i = 1$ . The vector  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_{n_{obj}})$  represents the gradient of function  $J$ . By using various sets of  $\alpha$ , one can generate several points in the Pareto set.

## 3. OPTIMIZATION OF WEIGHTING FUNCTIONS FOR LPV/ $H_\infty$ SYNTHESIS USING GENETIC ALGORITHM

### 3.1 General $H_\infty$ /LPV problem

Consider a general LPV system

$$\begin{aligned} \dot{x} &= A(\rho(t))x + B_1(\rho(t))w + B_2(\rho(t))u \\ z &= C_1(\rho(t))x + D_{11}(\rho(t))w + D_{12}(\rho(t))u \\ y &= C_2(\rho(t))x + D_{21}(\rho(t))w + D_{22}(\rho(t))u \end{aligned} \quad (3)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^m$ ,  $w \in \mathbb{R}^q$ ,  $z \in \mathbb{R}^r$  and  $y \in \mathbb{R}^p$  are the state, the input, the disturbance vectors, the control output and the measured output, respectively.  $\rho(t) \in \mathbb{R}^k$  is a vector of scheduling parameters and assumed to be known (measured or estimated). From now on,  $\rho(t)$  is simply denoted as  $\rho$ .

It is well-known that, to satisfy some required closed-loop performance, one must select suitable weighting functions. While the weights on disturbance inputs are always defined thanks to the knowledge about their bandwidth and magnitude, the selection of weighting functions on control outputs is more difficult and play a key role in the synthesis procedure. Without loss of generality, it is assumed that there is no weight on disturbance inputs. The control outputs are weighted as  $z_w(s) = W(s)z(s)$  where the weighting function matrix  $W(s)$  has the following state-space representation

$$\begin{aligned} \dot{x}_w &= A_w(\nu)x_w + B_w(\nu)z \\ z_w &= C_w(\nu)x_w + D_w(\nu)z \end{aligned} \quad (4)$$

where  $\nu$  is the vector of the weighting functions parameters which need to be found through optimization using GAs.

From (3) and (4), the augmented system used for the controller synthesis is written as

$$\begin{aligned} \dot{\xi} &= \mathcal{A}(\rho, \nu)\xi + \mathcal{B}_1(\rho, \nu)w + \mathcal{B}_2(\nu)u \\ z_w &= \mathcal{C}_1(\rho, \nu)\xi + \mathcal{D}_{11}(\rho, \nu)w + \mathcal{D}_{12}(\nu)u \\ y &= \mathcal{C}_2\xi + \mathcal{D}_{21}w \end{aligned} \quad (5)$$

where  $\xi = (x^T \ x_w^T)^T$

$$\mathcal{A}(\rho, \nu) = \begin{bmatrix} A(\rho) & 0 \\ B_w(\nu)C_1(\rho) & A_w(\nu) \end{bmatrix}, \quad \mathcal{B}_1(\rho, \nu) = \begin{bmatrix} B_1(\rho) \\ B_w(\nu)D_{11}(\rho) \end{bmatrix},$$

$$\mathcal{B}_2(\nu) = \begin{bmatrix} B_2 \\ B_w(\nu)D_{12} \end{bmatrix}, \quad \mathcal{C}_1(\rho, \nu) = [D_w(\nu)C_1(\rho) \ C_w(\nu)], \quad \mathcal{C}_2 = C_2$$

$$D_{11}(\rho, \nu) = D_w(\nu)D_{11}(\rho), \quad D_{12}(\nu) = D_w(\nu)D_{12}, \quad D_{21} = D_{21}.$$

The LPV controller is defined as follows

$$K(\rho) : \begin{pmatrix} \dot{x}_c \\ u \end{pmatrix} = \begin{pmatrix} A_c(\rho) & B_c(\rho) \\ C_c(\rho) & D_c(\rho) \end{pmatrix} \begin{pmatrix} x_c \\ y \end{pmatrix} \quad (6)$$

where  $x_c$ ,  $y$  and  $u$  are respectively the state, the input and output of the controller, respectively, of the controller associated with the system (5).  $A_c \in \mathbb{R}^{n \times n}$ ,  $B_c \in \mathbb{R}^{n \times n_y}$ ,

$C_c \in \mathbb{R}^{n_u \times n}$  and  $D_c \in \mathbb{R}^{n_u \times n_y}$ .

*H<sub>∞</sub>/LPV problem* - The objective of the synthesis is to find an LPV controller  $K(\rho)$  of the form (6) such that the closed-loop system is quadratically stable and that, for a given positive real  $\gamma$ , the  $L_2$ -induced norm of the operator mapping  $w$  into  $z_w$  is bounded by  $\gamma$  i.e

$$\frac{\|z_w\|_2}{\|w\|_2} \leq \gamma \quad (7)$$

for all possible trajectories of  $\rho$ .

In this paper, the polytopic approach with quadratic Lyapunov function is employed for the synthesis of a  $H_\infty$ /LPV controller. The following is assumed:  $\rho \in \text{conv}\{\rho_1, \dots, \rho_k\}$ , the matrices  $B_2$ ,  $D_{12}$ ,  $C_2$ ,  $D_{21}$  are parameter independent and  $D_{22} = 0$ . For known weighting functions and a suitable pre-defined real positive scalar  $\gamma$ , the sufficient condition that solves the  $H_\infty$ /LPV problem is given by Eq. (8)-(9) (see the detail of the solution in Scherer et al. (1997)). It is worth noting that, unlike the original solution, the weighting function parameters are present in the LMIs problem and will be optimized by genetic algorithm.

$$\begin{bmatrix} M_{11}(\rho_i, \nu) & * & * & * \\ M_{21}(\rho_i, \nu) & M_{22}(\rho_i, \nu) & * & * \\ M_{31}(\rho_i, \nu) & M_{32}(\rho_i, \nu) & -\gamma I_m & * \\ M_{41}(\rho_i, \nu) & M_{42}(\rho_i, \nu) & M_{43}(\rho_i, \nu) & -\gamma I_p \end{bmatrix} \prec 0 \quad (8)$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \succ 0 \quad (9)$$

for  $i = 1 : k$

where

$$M_{11}(\rho_i, \nu) = \mathcal{A}(\rho_i, \nu)X + X\mathcal{A}(\rho_i, \nu)^T + \mathcal{B}_2\hat{C}(\rho_i) + \hat{C}(\rho_i)^T\mathcal{B}_2^T$$

$$M_{21}(\rho_i, \nu) = \hat{A}(\rho_i) + \mathcal{A}(\rho_i, \nu)^T + \mathcal{C}_2^T\hat{D}(\rho_i)^T\mathcal{B}_2^T$$

$$M_{22}(\rho_i, \nu) = Y\mathcal{A}(\rho_i, \nu) + \mathcal{A}(\rho_i, \nu)^TY + \hat{B}(\rho_i)\mathcal{C}_2 + \mathcal{C}_2^T\hat{B}(\rho_i)^T$$

$$M_{31}(\rho_i, \nu) = \mathcal{B}_1(\rho_i, \nu)^T + \mathcal{D}_{21}^T\hat{D}(\rho_i)^T\mathcal{B}_2^T$$

$$M_{32}(\rho_i, \nu) = \mathcal{B}_1(\rho_i, \nu)^TY + \mathcal{D}_{21}^T\hat{B}(\rho_i)^T$$

$$M_{41}(\rho_i, \nu) = \mathcal{C}_1(\rho_i, \nu)X + \mathcal{D}_{12}\hat{C}(\rho_i)$$

$$M_{42}(\rho_i, \nu) = \mathcal{C}_1(\rho_i, \nu) + \mathcal{D}_{12}\hat{D}(\rho_i)\mathcal{C}_2$$

$$M_{43}(\rho_i, \nu) = \mathcal{D}_{11}(\rho_i, \nu) + \mathcal{D}_{12}\hat{D}(\rho_i)\mathcal{D}_{21}$$

The controller  $K_{c_i}$  at vertex  $i$  is then reconstructed as

$$D_c(\rho_i) = \hat{D}(\rho_i)$$

$$C_c(\rho_i) = (\hat{C}(\rho_i) - D_c(\rho_i)\mathcal{C}_2X)M^{-T}$$

$$B_c(\rho_i) = N^{-1}(\hat{B}(\rho_i) - Y\mathcal{B}_2D_c(\rho_i)) \quad (10)$$

$$A_c(\rho_i) = N^{-1}(\hat{A}(\rho_i) - Y\mathcal{A}(\rho_i)X - Y\mathcal{B}_2D_c(\rho_i)\mathcal{C}_2X)M^{-T} - B_c(\rho_i)\mathcal{C}_2XM^{-T} - N^{-1}Y\mathcal{B}_2C_c(\rho_i)$$

where  $M$ ,  $N$  are defined such that  $MN^T = I_n - XY$  which can be solved through a singular value decomposition and a Cholesky factorization. The global  $H_\infty$ /LPV controller is the convex combination of these local controllers.

### 3.2 Optimizing the weighting functions

*Decision vector* - Certainly  $\nu$  is one element of the decision vector. Besides, the authors observed that the real positive scalar  $\gamma$  must be taken into account in the decision vector as well. Indeed one aims at optimizing a predefined multi-objective function  $J$  (see below) and not  $\gamma$ .

To avoid the infeasibility of the LMIs (8)-(9) resulting from the bad (i.e too small) value of  $\gamma$  generated by GAs,  $\gamma$  will be split into two positive real elements  $\gamma_{min}$  and  $\gamma_{ga}$

where  $\gamma_{min}$  is the minimal  $\gamma$  satisfying the LMIs (8)-(9), and  $\gamma_{ga}$  is tuned by GAs. Due to the convexity of the LMIs problem, the existence of  $\gamma_{min}$  will ensure the feasibility of LMIs (8)-(9) with  $\gamma = \gamma_{min} + \gamma_{ga}$  for any positive real  $\gamma_{ga}$ . The minimal value  $\gamma_{min}$  can be found by using LMIs toolbox like Yalmip & Sedumi.

*Objective functions* - In this paper, the frequency-based objective functions are considered. With the remarks above, the performance of the closed-loop system depends on the choice of weighting functions and the real positive scalar  $\gamma$ . Hence, to refer to these implicit dependencies, the objective function can be written as  $J = J(\nu, \gamma)$ .

*Proposed weighting function optimization procedure for H<sub>∞</sub>/LPV synthesis*

- Step 1: Initialize with random positive weighting functions  $\nu = \nu^0$  and random positive real  $\gamma_{ga} = \gamma_{ga}^0$ .
- Step 2: Solve the minimization problem of  $\gamma$  subject to the LMIs (8)-(9) to compute the minimal real scalar  $\gamma_{min}$ . Solve again the the LMIs (8)-(9) with the couple  $(\nu, \gamma)$  where  $\gamma = \gamma_{min} + \gamma_{ga}$ . At the end of this step, compute the objective function  $J(\nu, \gamma)$ .
- Step 3: Select the individuals.
- Step 4: Apply crossover and mutation to generate a new generation:  $\nu = \nu^{new}$  and  $\gamma_{ga} = \gamma_{ga}^{new}$ .
- Step 5: Evaluate the new generation: If the criteria of interest (for example, reaching the limit number of generation) are not satisfied, go to Step 2 with  $\nu = \nu^{new}$  and  $\gamma_{ga} = \gamma_{ga}^{new}$ ; else, stop and save the best individual  $\nu^{opt} = \nu^{new}$  and  $\gamma^{opt} = \gamma_{ga}^{new}$ .

## 4. SEMI-ACTIVE SUSPENSION CONTROL

### 4.1 System description

Consider a simple quarter vehicle model (see Fig. 1) made up of a sprung mass ( $m_s$ ) and an unsprung mass ( $m_{us}$ ) connected by a spring with the stiffness coefficient  $k_s$  and a semi-active damper. The wheel tire is modeled by a spring with the stiffness coefficient  $k_t$ . In this model,  $z_s$  (respectively  $z_{us}$ ) is the vertical position of  $m_s$  (respectively  $m_{us}$ ) and  $z_r$  is the road profile. It is assumed that the wheel-road contact is ensured. The dynamical equations of a quarter

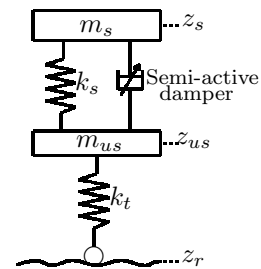


Fig. 1. Model of quarter vehicle with a semi-active damper.

vehicle are governed by:

$$\begin{cases} m_s \ddot{z}_s = -F_{spring} - F_{mr} \\ m_{us} \ddot{z}_{us} = F_{spring} + F_{mr} - k_t(z_{us} - z_r) \end{cases} \quad (11)$$

where  $F_{spring} = k_s z_{def}$  is the spring force,  $z_{def} = z_s - z_{us}$  the damper deflection (assumed to be measured or estimated), and  $\dot{z}_{def} = \dot{z}_s - \dot{z}_{us}$  the deflection velocity (can be directly computed from  $z_{def}$ ).

In this paper, the behavior of the semi-active suspension is modeled by the following nonlinear equation, as in Do et al. (2010):

$$F_{mr} = a_2(\dot{z}_{def} + \frac{v_0}{x_0} z_{def}) + a_1 \tanh(a_3(\dot{z}_{def} + \frac{v_0}{x_0} z_{def})) \quad (12)$$

where  $a_2$ ,  $a_3$ ,  $v_0$  and  $x_0$  are constant parameters and  $a_1$  is varying according to the electrical current in coil ( $0 < a_{1min} \leq a_1 \leq a_{1max}$ ). This model allows fulfilling the dissipativity constraint of the semi-active damper. The quarter vehicle used in this paper is the "Renault Mégane Coupé" model (see Zin et al. (2004)) whose specific parameters are:  $m_s = 315$  kg,  $m_{us} = 37.5$  kg,  $k_s = 29500$  N/m,  $k_t = 210000$  N/m. The damper model parameters have been chosen according to the MR damper in Nino-Juarez et al. (2008):  $a_2 = 1500$  Ns/m,  $a_3 = 129$  s/m,  $v_0 = 0.788 \times 10^{-3}$  m/s,  $x_0 = 1.195 \times 10^{-3}$  m,  $a_{1min} = 0$  N,  $a_{1max} = 400$  N,  $F_1 = (a_{1max} - a_{1min})/2 = 200$ .

After some mathematical transformations as in Do et al. (2010), a control oriented LPV model with input saturation is obtained with two scheduling parameters  $\rho_1^*$  and  $\rho_2^*$

$$\begin{cases} \dot{x} = A(\rho_1^*, \rho_2^*)x + Bu_c + B_1w \\ y = Cx \end{cases} \quad (13)$$

where

$$x = \begin{pmatrix} x_s \\ x_f \end{pmatrix}^T, A(\rho_1, \rho_2) = \begin{pmatrix} A_s + \rho_2 B_{s2} C_{s2} & \rho_1 B_s C_f \\ 0 & A_f \end{pmatrix},$$

$$B = \begin{pmatrix} 0 \\ B_f \end{pmatrix}, B_1 = \begin{pmatrix} B_{s1} \\ 0 \end{pmatrix}, C = \begin{pmatrix} C_s \\ 0 \end{pmatrix}^T$$

$$\rho_1^* = \tanh(C_{s2} x_s) \tanh\left(\frac{C_f x_f}{F_1}\right) \frac{F_1}{C_f x_f}, \rho_2^* = \frac{\tanh(C_{s2} x_s)}{C_{s2} x_s}.$$

$x_s$ ,  $A_s$ ,  $B_s$ ,  $B_{s1}$ ,  $C_s$  are state and matrices of a state-space representation of the plant (11);  $x_f$ ,  $A_f$ ,  $B_f$ ,  $C_f$  are state and matrices of a representation of the following low-pass filter  $W_{filter} = \frac{\Omega_f}{s + \Omega_f}$ . This filter is added to the plant to make the control input matrices parameter independent. Notice also that  $\rho_1^*$  and  $\rho_2^*$  are not independent and the relation between them is presented in Fig. 2. For the polytopic approach, the polytope  $P_1 P_2 P_3 P_4$  that contains the set of  $(\rho_1^*, \rho_2^*)$  will be considered. While  $P_3$  and  $P_4$  are related to the real set of  $(\rho_1^*, \rho_2^*)$ ,  $P_1$  and  $P_2$  in fact do not belong to the set.

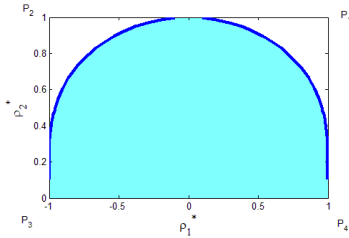


Fig. 2. Set of  $(\rho_1^*, \rho_2^*)$  (shaded area + curved line) and set of  $(\rho_1, \rho_2)$  (curved line).

#### 4.2 Suspension Performance Objectives

The performance objectives for comfort and road holding are obtained following ideas from Sammier et al. (2003).

- Comfort in high frequencies ( $[3 - 10]$  Hz):  $\min \int_3^{10} \ddot{z}_s / z_r(f) df$ .
- Comfort in low frequencies ( $[0 - 4]$  Hz):  $\min \bar{\sigma}(z_s / z_r)(f)$ .
- Road holding ( $[0 - 20]$  Hz):  $\min \bar{\sigma}(z_{us} / z_r(f))$  (Rebound of wheel) and  $\int_0^{20} (z_{us} - z_r) / z_r(f) df$  (Contact force).

#### 5. SEMI-ACTIVE SUSPENSION CONTROL USING GENETIC ALGORITHMS

The procedure for optimizing the weighting functions in Sec. 3.2 is now applied to the LPV semi-active suspension system (13) with the following specifications.

##### 5.1 Parameterizing weighting functions

Along with the bounded real positive scalar  $\gamma$ , the following weighting functions are used for the  $H_\infty/LPV$  synthesis

$$W_{filter} = \frac{\Omega_f}{s + \Omega_f} \quad (14)$$

$$W_{\ddot{z}_s} = k_{\ddot{z}_s} \frac{s^2 + 2\xi_{11}\Omega_{11}s + \Omega_{11}^2}{s^2 + 2\xi_{12}\Omega_{12}s + \Omega_{12}^2} \quad (15)$$

$$W_{z_{us}} = k_{z_{us}} \frac{s^2 + 2\xi_{21}\Omega_{21}s + \Omega_{21}^2}{s^2 + 2\xi_{22}\Omega_{22}s + \Omega_{22}^2} \quad (16)$$

The decision vector is as follows

$$\nu = [\gamma \ \Omega_f \ \Omega_{11,12} \ \xi_{11,12} \ k_{\ddot{z}_s} \ \Omega_{21,22} \ \xi_{21,22} \ k_{z_{us}}]^T \quad (17)$$

It is worth noting that the structures of these weighting functions are the same as in Do et al. (2010).

The parameter (i.e. each element in the decision vector) encoding is defined as follows

$$\text{Parameter Value} = (a + 0.1b + 0.01c + 0.001d) \times 10^{e-2} \quad (18)$$

where  $a, b, c, d, e$  are natural number and  $(a, b, c, d, e) \in [0; 9]^5$ . This encoding covers the range  $(0; 100)$ .

##### 5.2 Objective and fitness function

*Objective function* - The objective function is as follows

$$J = \alpha J_{\text{Comfort}} + (1 - \alpha) J_{\text{Road Holding}} \quad (19)$$

where  $J_{\text{Comfort}}$  and  $J_{\text{Road Holding}}$  are performance indices corresponding to vertex  $P_3$  (Note that solutions at  $P_3$  and  $P_4$  are identical because of the symmetric property of the polytope  $(P_1, P_2, P_3, P_4)$  and defined as follows (according to Sec. 4.2)

$$J_{\text{Comfort}} = \frac{\int_1^{10} \ddot{z}_s / z_r(f) df}{\int_1^{10} (\ddot{z}_s / z_r)_{\text{Passive}}(f) df} \quad (20)$$

$$J_{\text{Road Holding}} = \frac{\bar{\sigma}_{f \in [10-20]}(z_{us} / z_r(f))}{\bar{\sigma}_{f \in [10-20]}(z_{us} / z_r(f))_{\text{Passive}}} \quad (21)$$

Note that the criteria with "passive" are computed for the open-loop system (13) with  $u_c = 0$ .

*Remark 1:* The feasibility of the new generated elements is not guaranteed because of the possible infeasibility of the LMIs (8)-(9). The problem can be overcome by repeating the crossover or mutation until the feasible solution is obtained. However, a simpler way is to assign a large objective value (for instant  $J = \infty$ ) to these infeasible solutions and as a consequence, they will be eliminated by the selection procedure after some generations.

*Remark 2:* In many cases, to preserve the performance of the closed-loop system with input saturation, a stable stabilizing controller is required. A stable *LPV* controller can be obtained by eliminating the “unstable solutions” corresponding to at least one unstable local controller during the synthesis.

To sum up, the objective function is chosen as follows.

**Algorithm 1: Objective value assignment**

```

if (8)-(9) is feasible then
  if all local controllers  $K_{c_i}$  are stable then
    Calculate  $J$  using Eq. 19, 20 and 21
  else
     $J = \infty$ 
  end
else
   $J = \infty$ 
end.

```

*Fitness function* - Because the working principle of GAs is the maximization of fitness, the following fitness function is used

$$F_{\text{fitness}} = 10 - 10 \frac{1 - 5e^{-12(J-0.6)}}{1 + 5e^{-12(J-0.6)}} \quad (22)$$

### 5.3 Genetic operation

The selection method used in this paper is the *proportionate selection* developed by Holland (1975). This method assigns to each individual a probability of selection proportional to its relative fitness. Proportionate selection can be illustrated by a roulette wheel. The *crossover* happens with a probability of 0.9 and the *mutation* happens with a very small probability 0.095.

### 5.4 Design results

The result of the optimization procedure using GAs is given in Fig. 3. Some points in the Pareto-Set are depicted. In the next section, we validate this linear design result on a nonlinear quarter car model.

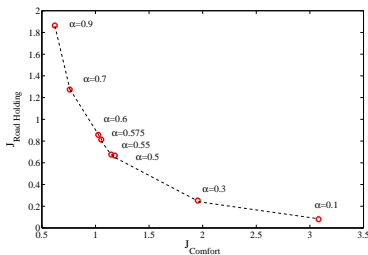


Fig. 3. Performance indices of the closed-loop linear system corresponding to vertex  $P_3$  (with different values of  $\alpha$ ).

## 6. SIMULATION RESULTS

The nonlinear frequency responses using the “variance gain” algorithm by Savaresi et al. (2005) are presented in Fig. 5 - 7.

In Fig. 4, the performance indices  $J_{\text{Comfort}}$  and  $J_{\text{Road Holding}}$  are computed for a nonlinear quarter car model with the  $H_\infty/LPV$  controllers corresponding to each value of  $\alpha$  with different values of  $\alpha$  in the range of  $[0.1; 0.99]$ . As expected, with  $\alpha = 0.99$ , the smallest  $J_{\text{Comfort}}$  is obtained; with  $\alpha = 0.1$ , the smallest  $J_{\text{Road Holding}}$  is obtained.

Three stable  $H_\infty/LPV$  controllers  $C_1, C_2, C_3$  corresponding to  $\alpha = 0.99, \alpha = 0.55, \alpha = 0.1$  (respectively) synthesized with the optimized weighting functions (whose parameters are presented in Tab. 1) are chosen for the analysis. Their performances are compared with those of the  $H_\infty/LPV$  controller obtained in Do et al. (2010), of the passive soft MR Damper ( $a_1 = a_{1min} = 0 N$ ) and of the passive hard MR Damper ( $a_1 = a_{1max} = 400 N$ ).

- The comfort oriented controller  $C_1$  is the best one for comfort but not good for road holding.
- The road holding (rebound of wheel) oriented controller  $C_3$  is the best for preventing the rebound of wheel in high frequency. It can be seen that this controller has the same behavior as that of the hard MR damper. However, a good comfort in high frequency.
- The intermediate controller  $C_2$  turns out to be the best one. It guarantees a road holding, a good comfort improvement in high frequency and a medium comfort improvement in low frequency.

To summarize, the overall evaluation of performance based on the criteria given in Sec. 4.2 is depicted in Fig. 8.

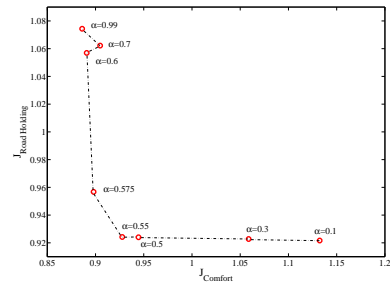


Fig. 4. Performance indices of the closed-loop nonlinear system (with different values of  $\alpha$ ).

## 7. CONCLUSION

In this paper, a weighting function optimization using GAs for  $H_\infty/LPV$  semi-active control optimization problem has been proposed. The input used for this optimization procedure is a frequency-based objective function, instead of the disturbance attenuation level  $\gamma$  as in conventional  $H_\infty/LPV$  design. The simulation results have shown the efficiency of GAs in finding a suitable controller to satisfy some performance objectives. For future work, powerful GAs in multi-objective optimization like SPEA2 or NSGA-II will be considered and compared with the simple weighted sum of objectives method used in this paper.

Controllers	bounded real scalar $\gamma_{min} - \gamma_{ga}$	Filter $\Omega_f$	$W_{\ddot{z}_s}$						$W_{z_{us}}$				
			$\Omega_{11}$	$\Omega_{12}$	$\xi_{11}$	$\xi_{12}$	$k_{\ddot{z}_s}$	$\Omega_{21}$	$\Omega_{22}$	$\xi_{21}$	$\xi_{22}$	$k_{z_{us}}$	
<b>ACC 2010</b>	363-0	50	125.7	125.7	20	1	1	69.1	69.1	20	1	1	
<b>C<sub>1</sub></b> ( $\alpha = 0.99$ )	456-463	51.1	33.29	28.54	52.32	5.26	1.95	1.29	0.072	4.09	0.136	0.152	
<b>C<sub>2</sub></b> ( $\alpha = 0.55$ )	320-163	8.3	0.48	4.66	37.95	0.76	3.77	0.87	3.3	0.19	1.8	8.1	
<b>C<sub>3</sub></b> ( $\alpha = 0.1$ )	93.9-0.11	1.4	2.373	0.66	0.222	33.87	0.22	24.81	0.49	7.14	5.75	0.343	

Table 1. Parameters for  $H_\infty/LPV$  semi-active suspension design.

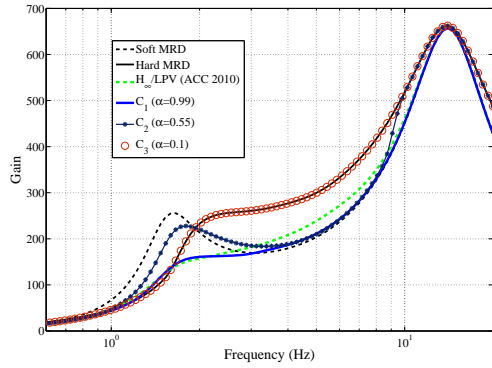


Fig. 5. Comfort in high frequencies  $\ddot{z}_s/z_r$ .

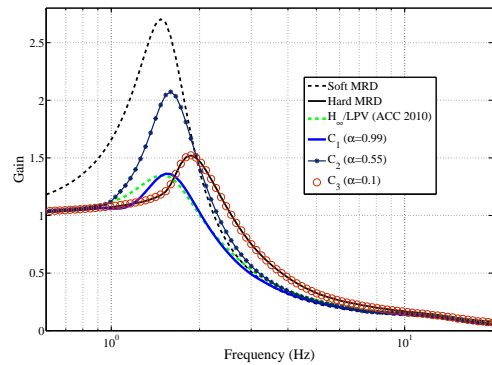


Fig. 6. Comfort in low frequencies  $z_s/z_r$ .

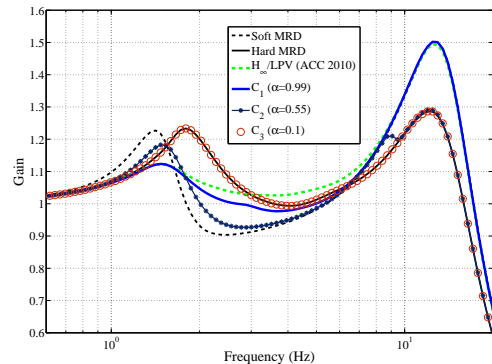


Fig. 7. Road holding  $z_{us}/z_r$ .

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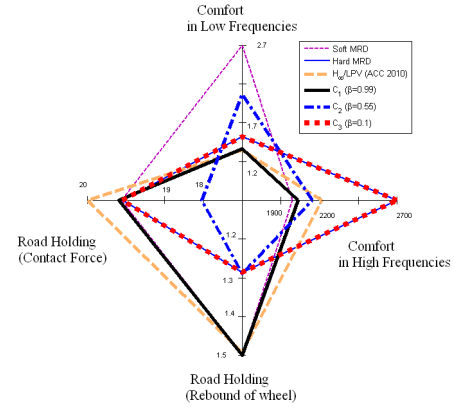


Fig. 8. Overall performance comparison.

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