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Horst Tempelmeier and Sascha Herpers

Department of
Supply Chain Management and Production
University of Cologne, Germany

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Abstract

This paper deals with the dynamic multi-item capacitated lot-sizing problem (CLSP) with random demands over a finite discrete time horizon. Unfilled demands are backordered. It is assumed that a fillrate constraint is in effect. We propose a heuristic solution procedure called ABC_β that extends the A/B/C heuristic introduced by Maes and Van Wassenhove (1986) for the deterministic CLSP to the case of random demands.

Keywords: lot-sizing, capacity, dynamic random demand, fillrate

1 Introduction

We consider the stochastic version of the dynamic multi-item capacitated lot sizing problem (CLSP). The problem is to determine production quantities to satisfy demands for multiple products over a finite discrete time horizon such that the sum of setup and holding costs is minimized, whereby a capacity constraint of a resource must be taken into consideration. Setup times are assumed to be zero. In contrast to the deterministic CLSP, it is assumed that for every period t and product k the demand D_{kt} ($k = 1, 2, \dots, K; t = 1, 2, \dots, T$) is specified by its mean and its coefficient of variation. The period demand may be non-stationary (to permit dynamic effects such as seasonal variations, promotions, or general mixtures of known customer orders with random portions of period demands). Demand that cannot be filled immediately from stock on hand is backordered. As the precise quantification of shortage penalty costs which involve intangible factors such as loss of customer goodwill is very difficult, if not infeasible, we assume that management has set a target service level. In particular, we assume that the fillrate criterion (β service level) is in effect, as this criterion is very popular in industrial practice [see Tempelmeier (2006)].

In industry, mainly two different planning approaches to dynamic production planning under uncertainty are used [Boulaksil et al. (2009)]. The first approach is to use results from stochastic inventory theory. In this case, a production order is triggered through the occurrence of a random demand that reduces the inventory position below a given reorder point which is computed based on assumptions concerning the demand during the replenishment lead time.

The second approach, which is implemented in many industrial material requirements planning (MRP) systems, uses a forecasting procedure that provides a deterministic time series of future demands. Uncertainty is taken into consideration by reserving a fixed amount of inventory as safety stock. The amount of this reserve stock is usually computed with simple rules of thumb, e.g. the standard deviation of the demand during the risk period is multiplied with a quantile of the standard normal distribution, or even a multiple of the average demand. In the MRP calculations the safety stock is added to the net demand which is then used in the deterministic lot size calculations performed with a dynamic uncapacitated single-item lot sizing procedure [see Wijngaard and Wortmann (1985)].

Whether the safety stock is set once for the complete planning horizon or is adjusted from time to time, in both approaches the actual timing and size of the replenishments are the outcome of the observed demand process, which is random. This directly translates into random resource requirements which from a practical point of view is undesirable.

2 Literature

The deterministic multi-item capacitated lot sizing problem has been studied for a long time. For recent overviews see Karimi et al. (2003) and Buschkühl et al. (2009). However, only a relative limited number of researchers have considered the capacitated lot sizing problem under random demand. A literature overview is presented in Sox et al. (1999). Depending on the modeling of the time axis, two groups of stochastic lot sizing models can be differentiated. Continuous time models are the stochastic counterpart of the Economic Lot Scheduling Problem (ELSP) which assumes stationary demands for all products. Discrete time models are extensions of the Capacitated Lot Sizing Problem (CLSP). These models assume time-varying dynamic demands. While some research has been done on continuous time models, only very few papers have been presented that treat capacitated lot sizing under uncertainty in discrete time. This is surprising, as many industrial planning environments such as Material Requirements Planning (MRP) systems or Advanced Planning Systems (APS) are based on a discrete time structure and randomness is a major issue in most practical planning situations. Sox and Muckstadt (1997) solve a variant of the stochastic dynamic CLSP, where item- and period-specific backorder costs as well as extendible production capacities are considered. The authors propose a Lagrangean heuristic to solve the resulting non-linear integer programming problem that is repeatedly applied in a dynamic planning environment. Martel et al. (1995) proposed a branch-and-bound procedure for the solution of a similar problem formulation. Brandimarte (2006) considers the stochastic CLSP where the uncertainty of the demand is represented through a scenario tree and unfilled demand is lost. As to our knowledge, no results are available for the dynamic capacitated lot sizing problem under random demand, when the performance is measured in terms of a fillrate. Considering the popularity of the fillrate in industrial practice, this is an open problem that deserves being studied.

The majority of papers that consider dynamic lot sizing problems under random demand

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3
4 refer to single-item problems with unlimited capacity. For our work the most relevant is
5 the paper of Bookbinder and Tan (1988). These authors studied several so-called stra-
6 tegies which define different modes of planning the timing and size of the production
7 quantities. According to the „static-dynamic uncertainty strategy”, at the beginning of
8 the planning horizon all production periods are fixed in advance. The actual production
9 quantities, however, are determined with the help of order-up-to levels that are computed
10 based on a target service level. As the actual production quantities are the result of the
11 observed demands, the „static-dynamic uncertainty strategy” leads to random resource
12 requirements which impedes the precise consideration of capacity constraints.

13
14 A second strategy discussed by Bookbinder and Tan (1988) is the so-called „static uncer-
15 tainty strategy”. Here, the timing and the size of all production quantities are fixed in
16 advance for the complete planning horizon. This strategy, although being inferior as far as
17 the costs are concerned, has the significant advantage that the production quantities are
18 deterministic decision variables and it is therefore possible to respect limited capacities in
19 the presence of dynamic random demands.

20
21 Compared to the rather limited amount of literature on dynamic stochastic capacitated
22 lot sizing, there exists a large number of publications on the stochastic economic lot
23 scheduling problem with stationary demands. These are reviewed in Sox et al. (1999) and
24 Winands et al. (2005).

25
26 In this paper we present a formulation of the stochastic dynamic capacitated lot sizing
27 problem (SCLSP) that implements the „static uncertainty” strategy of Bookbinder and
28 Tan (1988) and takes into account random demand and a fillrate constraint. We propose
29 a heuristic solution procedure to this problem. The result is a production plan for the
30 entire planning horizon that respects the available production capacity in every period,
31 despite the randomness of demands.

32
33 In Section 3 the mathematical model is formulated. In Section 4 we present a solution
34 approach that is based on the A/B/C-Heuristic originally introduced by Maes and Van
35 Wassenhove (1986) for the solution of the deterministic CLSP. In Section 5 the results of
36 a numerical experiment are given. The last section contains some conclusions.

3 Problem formulation

We consider K products that are produced to stock on a single resource. The planning situation is completely identical with the classical dynamic capacitated lot sizing problem without setup times (CLSP) with one exception: For each product k , the period demands D_{kt} are random variables with given expected values $E\{D_{kt}\}$ and variances $V\{D_{kt}\}$. These moments, which may vary over time, are the outcome of a forecasting procedure. Unfilled demands are backordered and the amount of backorders is controlled by imposing a fillrate (β service level) constraint. We define the fillrate as the ratio of the expected demand observed during the coverage time of a production order that is routinely filled from stock on hand and the actual lot size. More precisely, let τ be a production period of product k , let t be the period immediately before the next production of product k and let $q_{k\tau t}$ be the lot size produced in τ covering the demand up to t . Finally, let $F_{kt}(q_{k\tau t})$ be the product k backorders that occur in period t . Then for a target service level β_k^* it is required, that

$$1 - \frac{E\left\{\sum_{i=\tau}^t F_{ki}(q_{k\tau t})\right\}}{E\left\{\sum_{i=\tau}^t D_{ki}\right\}} \geq \beta_k^* \quad k = 1, 2, \dots, K \quad (1)$$

This constraint is equivalent to the fillrate definition under stationary conditions which relates the average backorders per cycle to the average replenishment quantity. At the beginning of the planning horizon there is a known initial inventory I_{k0} ($k = 1, 2, \dots, K$) which may be zero.

The mathematical formulation of the considered dynamic multi-item Stochastic Capacitated Lot-Sizing Problem with fixed production quantities and fillrate constraint is as follows:

Model SCLSP $_{\beta}^q$

$$\text{Minimiere } Z = \sum_{k=1}^K \sum_{t=1}^T (s_k \cdot \gamma_{kt} + h_k \cdot E\{[I_{kt}]^+\}) \quad (2)$$

s. t.

$$I_{k,t-1} + q_{kt} - D_{kt} = I_{kt} \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \quad (3)$$

$$q_{kt} - M \cdot \gamma_{kt} \leq 0 \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \quad (4)$$

$$\sum_{k=1}^K tb_k \cdot q_{kt} \leq b_t \quad t = 1, 2, \dots, T \quad (5)$$

$$I_{kt}^{f,prod} = - [I_{k,t-1} + q_{kt}]^- \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \quad (6)$$

$$I_{kt}^{f,end} = - [I_{kt}]^- \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \quad (7)$$

$$F_{kt} = I_{kt}^{f,end} - I_{kt}^{f,prod} \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \quad (8)$$

$$l_{kt} = (l_{k,t-1} + 1) \cdot (1 - \gamma_{kt}) \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \quad (9)$$

$$l_{k,0} = -1 \quad k = 1, 2, \dots, K \quad (10)$$

$$\omega_{kt} = \gamma_{k,t+1} \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T - 1 \quad (11)$$

$$\omega_{kT} = 1 \quad k = 1, 2, \dots, K \quad (12)$$

$$1 - \frac{E \left\{ \sum_{j=t-l_{kt}}^t F_{kj} \right\}}{E \left\{ \sum_{j=t-l_{kt}}^t D_{kj} \right\}} \geq \beta_k^* \quad k = 1, 2, \dots, K; t \in \{t \mid \omega_{kt} = 1\} \quad (13)$$

$$q_{kt} \geq 0 \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \quad (14)$$

$$\gamma_{kt} \in \{0, 1\} \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \quad (15)$$

Legend:

b_t	capacity in period t (time units)
β_k^*	target fillrate for product k
D_{kt}	demand for product k in period t
F_{kt}	backorders of product k in period t
γ_{kt}	binary setup variable for product k in period t
h_k	inventory holding cost per time period per unit of product k
I_{kt}	net inventory for product k at the end of period t
$I_{kt}^{f,\text{end}}$	backlog for product k at the end of period t (random variable)
$I_{kt}^{f,\text{prod}}$	backlog for product k after production in period t , but before demand satisfaction
l_{kt}	number of periods since the last setup for product k in period t
K	number of products
M	sufficiently large number
ω_{kt}	indicator variable: $\omega_{kt} = 1$, if production of product k takes place in period $t+1$; $\omega_{kt} = 0$, otherwise
q_{kt}	lot size for product k in period t
s_k	setup costs for product k
tb_k	capacity usage to produce one unit of product k
T	length of planning horizon

The objective function (2) minimizes the total setup costs and expected inventory holding costs, where $[I_{kt}]^+$ is the inventory on hand at the end of period t for product k with $[x]^+ = \max\{0, x\}$. Equations (3) are the standard inventory balance equations. Constraint

(4) forces the setup indicator γ_{kt} to 1, whenever there is a positive production quantity q_{kt} , according to the assumptions of a big-bucket lot sizing model. Constraint (5) requires that the available capacity b_t per period must not be exceeded. Equation (6) defines the backlog in period t immediately after a production has taken place and all outstanding backorders, if any, have have been filled as much as possible before the new demand of period t is filled. Equation (6) describes the backlog at the end of period t . Equation (8) defines the backorders that newly occurred in period t .

The remaining equations are use for book-keeping. In order to calculate the average fillrate during an order cycle, we count the number of periods covered by a production quantity with the help of variable l_{kt} . Equations (11) and (12) set the indicator variable ω_{kt} to 1, if either period $t + 1$ is a setup period or the planning horizon ends in period t . In addition, the length of an order cycle (the number of periods between two consecutive setups) must be known. This is computed with the help of equations (9) and (10). l_{kt} is reset to zero whenever $\gamma_{kt} = 1$, i. e., when t is a setup period for product k . Otherwise, l_{kt} is incremented by one to $(l_{k,t-1} + 1)$. Equation (13) defines the expected fillrate within the actual production cycle since the last production of product k .

The expected inventory of product k at the end of period t is calculated as follows. Let $Q_k^{(t)}$ be the sum of the initial inventory I_{k0} and the production quantities produced up to period t and let $Y_k^{(t)}$ denote the cumulated demands from period 1 to t . Then the expected inventory at the end of period t is equal to $E\{[Q_k^{(t)} - Y_k^{(t)}]^+\}$. This can be written as

$$\begin{aligned} E\{I_{kt}^p\} &= \int_0^{Q_k^{(t)}} (Q_k^{(t)} - y) \cdot f_{Y_k^{(t)}}(y) \cdot dy \\ &= Q_k^{(t)} - E\{Y_k^{(t)}\} + G_{Y_k^{(t)}}^1(Q_k^{(t)}) \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \end{aligned} \quad (16)$$

where $G_Y^1(Q)$ is the first-order-loss function with respect to the random variable Y and the quantity Q .

Note that with model $SCLSP_{\beta_c}^q$ the timing and the size of replenishments are determined in advance. The model and the underlying „static uncertainty” planning strategy (see Bookbinder and Tan (1988)) are easily applicable within the context of a forecast-driven planning system such as an MRP system. It is noteworthy that there is no need to differentiate between cycle stock and safety stock, as with the replenishment quantity there is only one decision variable that ensures the fulfillment of the expected period demands as

well as their random deviations.

4 Solution Approach

Model $SCLSP_{\beta}^q$ describes a stochastic dynamic optimization problem. Up to now, there is no exact solution procedure available for this model. In the sequel we propose a heuristic solution approach which is based on the A/B/C heuristic presented by Maes and Van Wassenhove (1986) for the solution of the deterministic CLSP. The A/B/C heuristic is a period-by-period procedure which transforms a matrix of demands d_{kt} into a matrix of production quantities q_{kt} ($k = 1, 2, \dots, K; t = 1, 2, \dots, T$). It consists of three parts, namely the lot sizing step, a feasibility routine and an improvement step.

In the lot sizing step the demands are accumulated to lots. Thereby three parameters are used to govern the sequence in which future demands are considered for inclusion into a production lot:

- A With parameter A the selection sequence of the products is influenced. Here, criteria like the average time-between-orders (TBO), the ratio of setup and holding costs, and four other criteria are used.
- B Once a product has been selected, parameter B defines the criterion for deciding whether the extension of the current production lot is economically favorable. Here, the criteria known from the dynamic single-item lot sizing heuristics such as the Silver-Meal criterion, the least unit cost criterion, the least total cost criterion or the absolute cost criterion are used.
- C Finally, parameter C defines how the search through the demand matrix is performed, for example, product by product (east), period by period (south) or a combination thereof.

Table 1 provides an overview over the abbreviations used in the sequel.

Parameter A (Sorting)	
TBO	average time between orders: $\sqrt{\frac{2 \cdot s_k}{h_k \cdot \bar{d}_k}}$, $\bar{d}_k =$ average demand
SH	setup costs over holding costs: $\frac{s_k}{h_k}$
SHC	setup costs over holding costs and average capacity requirement: $\frac{s_k}{h_k \cdot tb_k \cdot \bar{d}_k}$
EC	expected average costs per period: $\frac{s_k}{TBO_k} + \frac{h_k \cdot \bar{d}_k \cdot TBO_k}{2}$
ES	expected savings when combining demand over TBO periods: $(s_k - h_k \cdot \bar{d}_k) + (s_k - 2 \cdot h_k \cdot \bar{d}_k) + \dots + (s_k - (TBO_k - 1) \cdot h_k \cdot \bar{d}_k)$
ESC	ES divided by average capacity requirement $tb_k \cdot \bar{d}_k$
Parameter B (Cost criterion)	
SM	Silver-Meal criterion $\frac{s_k + h_k \cdot \sum_{t=\tau}^j (t-\tau) d_{kt}}{j-\tau+1}$
LUC	Least unit cost criterion: $\frac{s_k + h_k \cdot \sum_{t=\tau}^j (t-\tau) \cdot d_{kt}}{\sum_{t=1}^j d_{kt}}$
LTC	Least total cost criterion: $\left s_k - h_k \cdot \sum_{t=\tau}^j (t-\tau) \cdot d_{kt} \right $
AC	Absolute cost criterion: $h_k \cdot \sum_{t=\tau+1}^j (t-\tau) \cdot d_{kt} \leq \frac{s_k}{h_k}$
Parameter C (Direction)	
E	East
S	South
SE	South-east

Table 1: Abbreviations

Whenever a production lot is about to be fixed and before the actual planning period is increased, a look-ahead routine is used to ensure feasibility. In particular, the amount of capacity caused by future demands (from periods $t+1, t+2, \dots, T$) still to be met in the actual planning period t is computed as

$$CF_t = \max \left\{ 0, \sum_{k=1}^K tb_k \cdot d_{k,t+1} - b_{t+1} + CF_{t+1} \right\} \quad t = T-1, T-2, \dots, 1 \quad (17)$$

with $CF_T = 0$.

A specific heuristic is defined through the combination of the parameters A, B and C. As computation times are very small, Maes and Van Wassenhove (1986) propose to select the best production plan found after applying all 72 possible combinations of the parameters. The reported computational results show that for the deterministic CLSP the A/B/C heuristic can be expected to find high-quality solutions with low computational effort (Maes and Van Wassenhove (1986); Maes and Van Wassenhove (1988)). Moreover, due

to its structural flexibility, the A/B/C heuristic can be used as the basis for solving the stochastic version of the CLSP, as stated by model SCLSP $^q_\beta$.

In the following we propose a heuristic called ABC $_\beta$ that includes the adjustments required to use the principle structure of the A/B/C heuristic for the solution of model SCLSP $^q_\beta$.

These adjustments refer to

- the calculation of the lot sizes,
- the cost criteria used, and
- the feasibility routine.

Calculation of the lot sizes. While in the deterministic case the lot sizes result from the cumulation of consecutive period demands, with random demand the target fillrates β_k^* must be taken into consideration. If the production lot for product k in period τ is extended to cover the demand of an additional period t , then the lot size is recalculated according to the fillrate constraint as follows:

$$q_{k\tau t}^* = \min \left\{ q_{k\tau t} \mid 1 - \frac{E \left\{ \sum_{i=\tau}^t F_{ki}(q_{k\tau t}) \right\}}{E \left\{ \sum_{i=\tau}^t D_{ki} \right\}} \geq \beta_k^* \right\} \quad t = \tau, \tau + 1, \tau + 2 \dots \quad (18)$$

The optimum lot size $q_{k\tau t}^*$ can be found with a standard search procedure.

Cost criteria. With deterministic demands, a typical dynamic lot sizing rule increases the production quantity in period τ until $C_{\tau, \tau+1} > C_{\tau t}$, where $C_{\tau t}$ denotes the 'costs' that result if the quantity produced in period τ covers the demands from period τ up to period t . As with random demand the development of the inventory over time is random, too, the cost criteria known from the single-item dynamic lot sizing heuristics are adjusted as follows [see Tempelmeier and Herpers (2009)]. For simplicity, in the following the product index k is omitted.

If the *Silver-Meal criterion* is used as the basis, we obtain

$$E\{C_{\tau t}\} = \frac{s + h \cdot \sum_{\ell=\tau}^t E \left\{ \left[I_{\tau-1}(P_{\tau-1}) + q_{\tau t}^* - \sum_{i=\tau}^{\ell} D_i \right]^+ \right\}}{t - \tau + 1} \quad \tau = 1, 2, \dots; t = \tau + 1, \tau + 2, \dots \quad (19)$$

where $P_{\tau-1}$ is the actual production plan (sequence of lots) from period 1 up to period

($\tau - 1$) and τ denotes the current production period for which the lot sizes are determined. For the adjusted *Least unit cost criterion* the criterion is

$$E\{C_{\tau t}\} = E \left\{ \frac{s + h \cdot \sum_{\ell=\tau}^t \left[I_{\tau-1}(P_{\tau-1}) + q_{\tau t}^* - \sum_{i=\tau}^{\ell} D_i \right]^+}{\sum_{i=\tau}^t D_i} \right\} \quad \tau = 1, 2, \dots; t = \tau + 1, \tau + 2, \dots \quad (20)$$

For the adjusted *Least total cost criterion* we get

$$E\{C_{\tau t}\} = E \left\{ s + h \cdot \sum_{\ell=\tau}^t \left[I_{\tau-1}(P_{\tau-1}) + q_{\tau t}^* - \sum_{i=\tau}^{\ell} D_i \right]^+ \right\} \quad \tau = 1, 2, \dots; t = \tau + 1, \tau + 2, \dots \quad (21)$$

For the adjusted *Absolute cost criterion* we obtain

$$\max \left\{ t \left| h \cdot \sum_{\ell=\tau+1}^t E \left\{ \left[I_{\tau-1}(P_{\tau-1}) + q_{\tau t}^* - \sum_{i=\tau}^{\ell} D_i \right]^+ \right\} \leq s \right. \right\} \quad \tau = 1, 2, \dots; t = \tau, \tau + 1, \dots \quad (22)$$

Feasibility routine. Under deterministic demand equation (17) is sufficient to ensure feasibility when proceeding from period τ to $\tau + 1$, as the total capacity requirements in the future are constant. In the case of random demand, however, the total production quantities depend on the lot sizes which are influenced by the target fillrates. Therefore, the total capacity requirements in periods $\tau + 1, \tau + 2, \dots, T$ and consequently the feasibility depend on the lot sizes which are determined only in a later planning step. To circumvent this problem, we take a conservative view and compute the capacity requirements based on a lot-for-lot policy for the periods $(t + 1, t + 2, \dots, T)$, where each production lot ($q_{k,t+1,t+1}^*$)

in the future is determined with equation (18). Thus, equation (17) is modified as follows:

$$CF_t = \max \left\{ 0, \sum_{k=1}^K tb_k \cdot q_{k,t+1,t+1}^* - b_{t+1} + CF_{t+1} \right\} \quad (23)$$

In any planning period τ the feasibility check may require that future demand must be produced in period τ in order to avoid a capacity overload in future periods $\tau < t$. If this infeasibility is avoided through a new setup for a product that is not produced in period τ , then the length of the last order cycle before period τ of this product decreases. This in turn requires the adjustment of the corresponding lot size which now covers a smaller number of period demands. In our implementation, this adjustment is made in a post-processing step at the end of the heuristic.

Finally, in the computation of the TBO criterion (parameter A) used for product selection, the average period demand is replaced by the average lot size that would result for a lot-for-lot policy.

After the lot sizing step is completed, in the same way as for the deterministic CLSP an improvement step tries to eliminate lots by combining them with earlier production quantities (Maes and Van Wassenhove (1986); Dixon and Silver (1981)).

5 Numerical Results

In order to test the quality of the proposed ABC_β heuristic we conducted a numerical experiment including a large number of invented problem instances. Unfortunately, at present for the considered stochastic lot sizing problem there are no benchmark solutions available. Therefore, the objective of the study is to find out which parameter combination is likely to perform best and which type of parameter (A – sorting, B – cost criterion, C – search direction) has the dominating influence on the solution quality.

We constructed a set of test instances based on deterministic problems from the literature. The first set of 360 problem instances including 12 products and 12 periods was created with an implementation of the problem generator described by Maes and Van Wassenhove (1986). The problem instances were generated through the variation of the five control factors studied in the experimental design used by these authors, namely variability of average demands, capacity absorption, average time between orders, tightness of capacity

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4 and demand lumpiness, whereby the experimental values given in Maes and Van Was-
5 senhove (1986) were used. The second set of problem instances included 540 instances
6 with 20 periods and 4, 6 and 8 products taken from Sürie (2002), which summed up to a
7 total of 900 deterministic instances. For both problem sets, we added the control factors
8 'standard deviation of period demand' and 'fillrate'. We considered four different standard
9 deviations of the period demands ($\sigma \in \{0.1, 0.2, 0.3, 0.4\}$) and four different target fillra-
10 tes ($\beta^* \in \{0.875, 0.925, 0.95, 0.98\}$) resulting in 14400 problem instances. For the problem
11 instances with $\sigma < 0.3$ we assumed normal demands while for larger values of σ gamma-
12 distributed demands were used. Each instance was solved using all 72 combinations of the
13 heuristic's parameters. 1093 problem instances were skipped as no feasible solution was
14 found. The remaining 13307 problem instances are the basis for the following analysis.

15
16 As noted above, no benchmark solutions are available for the problem at hand. Therefore,
17 we cannot make precise statements concerning the solution quality of the proposed heuri-
18 stic. Instead, we study the relative performance of the different parameter combinations.
19 In order to accomplish this, for each problem instance we proceed as follows. Let w_i be the
20 highest (worst) solution value found with any of the 72 parameter combinations. Let x_{ij}
21 be the solution value of parameter combination j ($j = 1, 2, \dots, 72$) for problem instance i .
22 Then the average relative deviation of parameter combination j over all 13307 problem
23 instances is $\Delta_j = \frac{1}{13307} \cdot \sum_{i=1}^{13307} \frac{w_i - x_{ij}}{w_i}$. Δ_j can be interpreted as a relative performance
24 measure for the solution quality of parameter combination j .

25
26 Figure 1 shows the Δ_j -values for all parameter combinations. In each triple group of
27 vertical bars, the left bar denotes the east direction (parameter C), while the other bars
28 stand for the south and the south-east direction, respectively.
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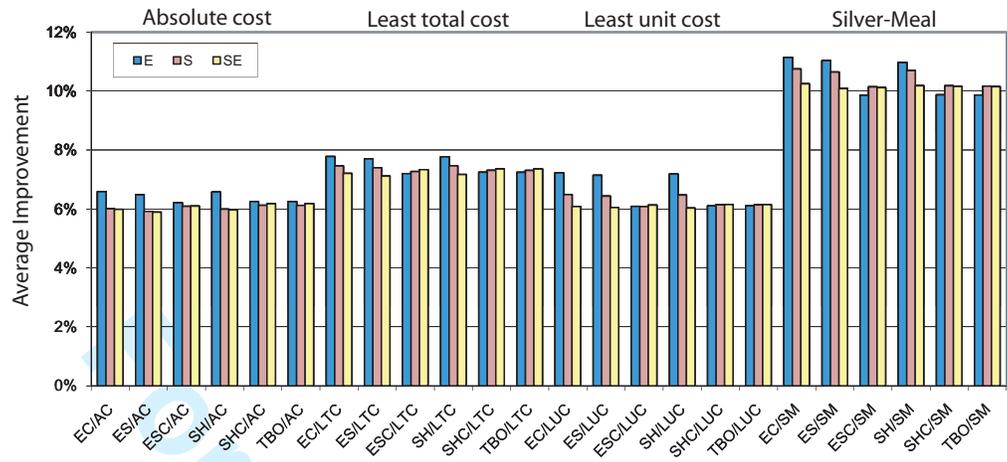


Figure 1: Average relative improvements for all parameter combinations

It appears that on the average the parameter combinations using the Silver-Meal criterion (parameter B) perform best, while the specification of the other parameters (A and C) plays only a minor role. Nevertheless, the 'east' search direction outperforms 'south' and 'south-east'.

The most effective parameter combinations are EC/SM/E, ES/SM/E and SH/SM/E, whereas many combinations based on the AC criterion and the LUC criterion show inferior results.

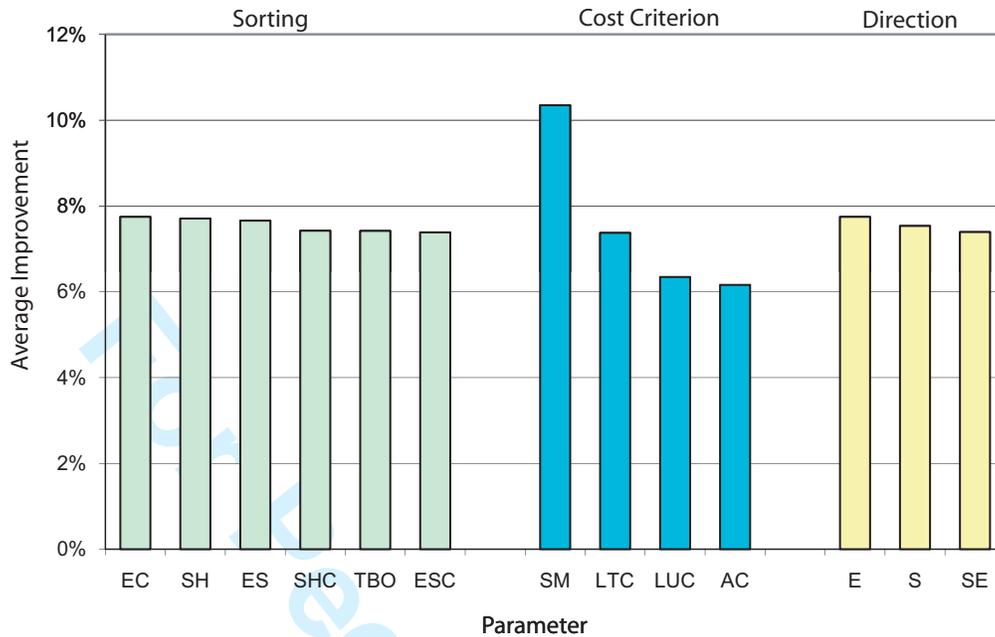


Figure 2: Average relative improvement per parameter type

Figure 2 shows the relative improvement for each parameter value averaged over all problem instances. The isolated view on the three parameter groups reveals that the sorting (parameter A) has a minor impact on the relative solution quality. This is also true for parameter C (direction). The strongest influence on the solution quality is observed for the cost criterion (parameter B), whereby, as noted above, the Silver-Meal criterion dominates the other criteria.

The computation times of the proposed heuristic are quite moderate. For the problem group with 20 periods it took 2.1, 4.1 and 5.6 seconds on the average on a standard PC (Windows XP, 2.5 Ghz, 2 GB RAM) to perform one variant of the heuristic. It appears that the number of periods has a stronger influence on the computation times than the number of products. In addition, the computation time increases with the target fillrate, as finding the required production lot sizes for low target fillrates is easier than for high target fillrates. Nevertheless, in a practical application, the run time should not be a problem, as the stochastic capacitated lot sizing problem is a big bucket problem applied in operative production planning. According to our practical experience for this type of

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4 planning problem, the typical length of the planning horizon will rarely exceed three
5 months (12 weeks).
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8 **6 Conclusions and directions for future research**

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11 In this paper we introduced a model for the single level capacitated lot sizing problem
12 with stochastic demand and a fillrate constraint. The underlying strategy that fixes the
13 timing and the sizes of all production quantities in advance has the positive characteristic,
14 that limited capacities can be respected with certainty. Alternative strategies that wait
15 until demand is observed and react on the demand do not have this advantage.
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18 We proposed a heuristic procedure that is based on the A/B/C heuristic published by Maes
19 and Van Wassenhove (1986). Our heuristic is as flexible as the deterministic variant. While
20 our numerical study shows that the Silver-Meal criterion dominates other cost criteria in
21 the same way as under deterministic demand conditions, the least unit cost criterion does
22 not.
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25 Further research will include other service levels (e. g. **the probability distribution of the**
26 **customer** waiting time) and the consideration of setup times. In addition, different solution
27 techniques for the current problem formulation $SCLSP_{\beta}^q$ are required so that the absolute
28 performance of the heuristic could be judged. This is the subject of our future research
29 agenda. **Further, another direction of research could be the introduction of the „static**
30 **uncertainty strategy” into other dynamic lotsizing models.**
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36 **References**

- 37
38
39 Bookbinder, J. and J.-Y. Tan (1988). Strategies for the probabilistic lot-sizing problem
40 with service-level constraints. *Management Science* 34, 1096–1108.
41
42 Boulaksil, Y., J. Fransoo, and E. van Halm (2009). Setting safety stocks in multi-stage
43 inventory systems under rolling horizon mathematical programming models. *OR*
44 *Spectrum* 31, 121–140.
45
46 Brandimarte, P. (2006). Multi-item capacitated lot-sizing with demand uncertainty. *In-*
47 *ternational Journal of Production Research* 44(15), 2997–3022.
48
49
50
51
52
53
54
55
56
57
58
59
60

- 1
2
3
4 Buschkühl, L., F. Sahling, S. Helber, and H. Tempelmeier (2009). Dynamic capacitated
5 lot-sizing problems: a classification and review of solution approaches. *OR Spectrum*.
6
7 Dixon, P. and E. Silver (1981). A heuristic solution procedure for the multi-item single-
8 level limited capacity lot-sizing problem. *Journal of Operations Management* 2(1),
9 23–39.
10
11 Karimi, B., S. M. T. Fatemi Ghomi, and J. M. Wilson (2003). The capacitated lot sizing
12 problem: A review of models and algorithms. *Omega* 31, 365–378.
13
14 Maes, J. and L. Van Wassenhove (1986). A simple heuristic for the multi item single level
15 capacitated lotsizing problem. *OR Letters* 4, 265–273.
16
17 Maes, J. and L. Van Wassenhove (1988). Multi-item single-level capacitated dynamic lot-
18 sizing heuristics: A general review. *Journal of the Operational Research Society* 39,
19 991–1004.
20
21
22 Martel, A., M. Diaby, and F. Boctor (1995). Multiple items procurement under stochastic
23 nonstationary demands. *European Journal of Operational Research* 87(1), 74 – 92.
24
25 Sox, C., P. Jackson, A. Bowman, and J. Muckstadt (1999). A review of the stochastic lot
26 scheduling problem. *International Journal of Production Economics* 62, 181–200.
27
28 Sox, C. and J. Muckstadt (1997). Optimization-based planning for the stochastic lot-
29 scheduling problem. *IIE Transactions* 29, 349–357.
30
31
32 Sürie, C. (2002). Description of clspl test instances. [http://www.bwl.tu-darmstadt.](http://www.bwl.tu-darmstadt.de/bwl1/forschung/ti_clspl/TI_CLSPL_Description.pdf)
33 [de/bwl1/forschung/ti_clspl/TI_CLSPL_Description.pdf](http://www.bwl.tu-darmstadt.de/bwl1/forschung/ti_clspl/TI_CLSPL_Description.pdf) (visited: April, 17th
34 2007).
35
36 Tempelmeier, H. (2006). *Inventory management in supply networks — problems, models*
37 *solutions*. Norderstedt: Books on Demand GmbH.
38
39 Tempelmeier, H. and S. Herpers (2009). Dynamic uncapacitated lot sizing with random
40 demand under a fillrate constraint. Technical report, Universität zu Köln, Seminar
41 für Supply Chain Management und Produktion.
42
43
44 Wijngaard, J. and J. Wortmann (1985). Mrp and inventories. *European Journal of*
45 *Operational Research* 20, 218–293.
46
47 Winands, E., I. Adan, and G. v. Houtum (2005). The stochastic economic lot scheduling
48 problem: A survey. Technical report, Technische Universiteit Eindhoven.
49
50
51
52
53
54
55
56
57
58
59
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