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# Enhancement of the $E(J,B)$ power law characterization for superconducting wires from electrical measurements on a coil

J. Leclerc, K. Berger, B. Douine, and J. Lévêque

**Index Terms**—Superconductors, characterization, tape, wire, coil, magnet.

**Abstract**—We propose an original method for the characterization of superconducting tapes and wires from electrical measurements on a test coil. The principle is to measure the voltage-current characteristic of the solenoid for different exterior applied flux density. A computer program then allows finding parameters of a model of the  $E(J,B)$  law that best fit the experimental curves. This method has many advantages compared to the conventional measurements on short samples: voltages are higher therefore easier to measure, the self flux density of the coil is taken into account and, tests are performed under conditions of the subsequent use of the conductor, e.g., wire is wound. This method has been employed experimentally to characterize a BiSCCO tape, manufactured by Trithor GmbH. Permanent magnets have been used to impose the external flux density to the coil. Parameters of three different models have been obtained and compared. Moreover, this process can be used to quantify the degradation of the wire's characteristics, according to the winding process, e.g., the bending radius and the tensile strength.

## I. INTRODUCTION

THE design of superconducting applications requires the characteristics knowledge of the wire used, in particular the variation law of the electric field  $E$  as a function of the current density  $J$  and of the flux density  $B$ , i.e., the  $E(J,B)$  law. It is usually measured on short sample by the transport method [1]: the voltage  $V_{tr}$  is recorded for different currents  $I_{tr}$  by a 4 wires measurement and for several external flux density applied  $B_{ext}$ . The  $E(J,B_{ext})$  law is then deduced using the simple relationship  $J = I_{tr}/S$  and  $E = V_{tr}/L$ , where  $S$  and  $L$  are respectively the section of the conductor and the length between the voltage leads. However, this method has three major drawbacks. First, the measured voltages are only of a few microvolts, therefore very difficult to obtain. Secondly, tests are not performed under conditions of the subsequent use of the conductor, e.g., wire is not wound. Thirdly, the self magnetic field of the sample is usually neglected [2]. The characteristic thus obtained is the  $E(J,B_{ext})$  law and not the  $E(J,B)$  law, which depends on the true flux density. To overcome these problems, we propose an original characterization method.

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The principle is to make measurements of the current-voltage characteristic  $V_{exp}(I_{exp})$  of a superconducting coil, made from a few meters of the wire to characterize. These measures are performed for different values of external flux density  $B_{ext}$  applied to the solenoid. A model should be chosen to represent the  $E(J,B)$  law of the superconductor. A computer program then determines the parameters of the model that best fit the experimental curves, taking into account the self flux density of the coil  $B_{self}$ . Therefore, the wire is characterized in its terms of use, taking into account the effect of  $B_{self}$ . In addition, the measured voltages on the coil are higher than for a short sample, thus easier to obtain.

An estimation of the error made on the parameters determination was performed. We have shown that it is better to fit at the same time several curves, measured for different values of  $B_{ext}$ , to reduce the error produced by measurement noise.

This method was used to characterize a BiSCCO tape. The flux density was applied to the solenoid by two NdFeB magnets. They avoid the use of a field coil which is more expensive and requires an extra power supply.

## II. METHOD DESCRIPTION

In this section the method is described in the general case, for a coil made with a superconducting wire.

### A. Numerical Calculation

The operation of the method relies on numerical computation. We want to find parameters of a model that best fit measures. For this, a program was developed in Matlab. The  $E(J,B)$  characteristic of the superconducting wire has been modeled by a power law (1). Different models can be used to represent the flux density dependence of  $J_c$  and  $n$  such as the Model 1 presented in Table I, based on the Kim model [3].

$$E(J,B) = E_c \left( \frac{J}{J_c(B)} \right)^{n(B)} \quad (1)$$

The theoretical current-voltage curve of the coil, noted  $V_{th}(I_{th})$ , can be calculated, from (2), for a given set of model parameters, and for a certain applied field. Correspondence of abbreviations used in (2) are given in the Table II, column 1. The total flux density  $B$  must be calculated at any point on the coil. It is the sum of the self and superimposed flux density (respectively  $B_{self}$  and  $B_{ext}$ ). The self field is calculated using the method described in [4]. This turns out to be more efficient

TABLE I  
DIFFERENT MODELS USED TO REPRESENT THE  $J_c(B)$  AND  $n(B)$  LAWS OF SUPERCONDUCTING WIRES (MODEL 1) AND TAPES (MODELS 2, 3 AND 4)

Model name	$J_c(B)$	$n(B)$
Model 1	$\frac{J_{c1}}{1 +  B /B_{J1}}$	$\frac{n_1}{1 +  B /B_{n1}}$
Model 2	$\frac{J_{c2}}{1 +  B_{\perp} /B_{J2}}$	$\frac{n_2}{1 +  B_{\perp} /B_{n2}}$
Model 3	$\frac{J_{c3}}{(1 +  B_{\perp} /B_{J3})^{\beta_3}}$	$\frac{n_3}{1 +  B_{\perp} /B_{n3}}$
Model 4	$\frac{J_{c4}}{(1 + \sqrt{k B_{//}^2 + B_{\perp}^2}/B_{J4})^{\beta_4}}$	$\frac{n_4}{1 +  B_{\perp} /B_{n4}}$

than direct use of the of Biot-Savart law, due to a shorter computing time.

$$U_{th}(I_{th}, B_{ext}) = \frac{4\pi N}{L(Re - Ri)} \times \int_0^{\frac{E_p}{2}} \int_{Ri}^{Re} E \left( \frac{I_{th}}{S}, B_{ext}(r, z) + B_{self}(r, z) \right) r dr dz \quad (2)$$

The double integral that appears in (2) is calculated using the *dblquad* function available in Matlab. Assuming that the current is evenly distributed across the section of the superconductor, i.e.,  $I_{th} = J/S$ , it is possible to obtain the voltage across the coil for any value of current. The  $V_{th}(I_{th})$  curve can be reconstituted, and this for different values of  $B_{ext}$ .

The minimization function *lsqnonlin* of Matlab is then used to find model parameters that best fit the experimental curves, at the least squares sense. As it will be shown in Subsection II-B, it is better to have several  $V_{exp}(I_{exp})$  curves, each measured with a different superimposed flux density and to perform the minimization on all measured points. By performing the minimization of (3),  $n_p$  being the number of points per curve,  $n_c$  the number of curves and  $U_c$  the critical voltage of the coil with the  $1 \mu\text{V}/\text{cm}$  criterion, the computation time is very high. The minimization of (4) gives a shorter calculation time. To show that, a curve representing measurements at zero superimposed flux density was generated using Matlab, for a coil whose characteristics are presented in Table II. The model used to produce these curves is the one that is presented in Table I - Model 1. Then, 500 parameters searches were carried out for each of the two functions that can be minimized, setting, such as stopping criterion, a maximum number of iteration. The starting point of *lsqnonlin* was randomly generated at each iteration. The obtained solutions for each parameter are therefore an array (in a statistical sense). The probability density function was then calculated for each parameter using the *ksdensity* Matlab

TABLE II  
CHARACTERISTICS OF THE COIL USED IN COMPUTER SIMULATIONS

Inner radius ( $Ri$ )	30 mm	$J_{c1}$	60 A/mm <sup>2</sup>
Outer radius ( $Re$ )	50 mm	$B_{J1}$	0.108 T
Length ( $L$ )	100 mm	$n_1$	16.52
Number of turns ( $N$ )	1000	$B_{n1}$	0.205 T
Wire section ( $S$ )	1 mm <sup>2</sup>		

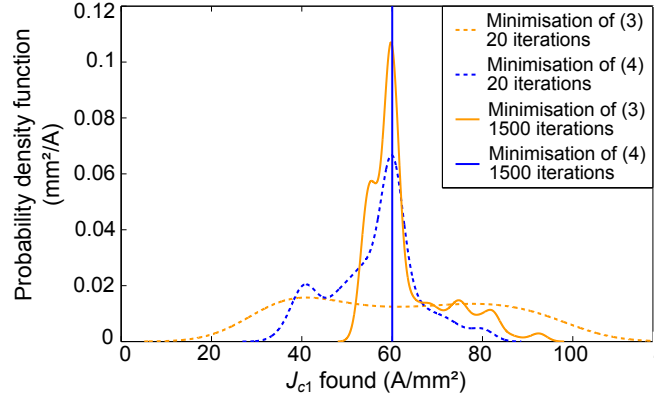


Fig. 1. Probability density function of the solutions found for  $J_{c1}$ . The starting point was randomly generated for each of the 500 parameters searches. When (3) is used for the minimization, we can see that, for a given number of iteration, the probability density functions (orange curves) are more scattered around the right solution (60 A/mm<sup>2</sup>) than when (4) is used (blue curves). This shows that (4) allows converging more rapidly to the right solution.

function.

$$err_1 = \sum_{k=1}^{n_c} \sum_{i=1}^{n_p} \left( \frac{U_{th(i,k)}}{U_c} - \frac{U_{exp(i,k)}}{U_c} \right)^2 \quad (3)$$

$$err_2 = \sum_{k=1}^{n_c} \sum_{i=1}^{n_p} \left( \ln \left( \frac{U_{th(i,k)}}{U_c} \right) - \ln \left( \frac{U_{exp(i,k)}}{U_c} \right) \right)^2 \quad (4)$$

The probability density function of the solutions found for  $J_{c1}$  is provided in Fig. 1. It show that, for a given number of iterations, the solutions found when minimizing (3), are more scattered around the correct solution (60 A/mm<sup>2</sup> for  $J_{c1}$ ) as when (4) is used. Using (4) thus allows the algorithm to converge faster. Moreover, the probability density function when minimizing (4) with 1500 iterations is a pulse that goes up to  $8.10^6 \text{ mm}^2/\text{A}$ , placed at 60 A/mm<sup>2</sup>, the correct solution. This means that, in this case, the found solution is the right and is unique. Similar results were obtained for the other parameters.

### B. Error Produced by the Measurement Noise

We studied the influence of noise and number of curves used, on the error made on determined model parameters. For this, curves representing measurements were generated using Matlab, as in the previous sub-section, with the same coil characteristics and with the same model but for five different superimposed flux densities. At each point ( $V_{th(i,k)}, I_{th(i,k)}$ ) is added a voltage representing the noise. Its value is randomly generated, between  $-ns_{max}$  and  $+ns_{max}$ , according to a uniform law. To make comparisons, research of parameters

should be performed on noised curves, for several numbers of curves and for different noise levels. The model use for the parameters search must be the same as the one that was used to construct curves in order to know the real parameters of the wire, and to be able to calculate the deviation with those who were actually obtained. The *lsqnonlin* algorithm stops when the final change in the sum of squares relative to its initial value is less than the specified value of TolX which is in our case  $10^{-6}$ .

The deviation obtained on a parameter  $X$  for a number of curve  $n_c$  and a maximum noise  $ns_{max}$ , where  $X$  may be in our case  $J_{c1}$ ,  $B_{j1}$ ,  $n_1$  or  $B_{n1}$ , is denoted as  $DEV_X(n_c, ns_{max})$ . However, it depends on the added noise. But, this last is produced randomly. So,  $DEV_X(n_c, ns_{max})$  is an aleatory variable. It is therefore necessary to insert a loop in the program and to perform, for each pair  $(n_c, ns_{max})$ , a large number of parameters search, generating the noise at each iteration. The set of calculation points for a pair  $(n_c, ns_{max})$ , constitutes a population. We compared the distribution of the deviation within each population. 100 parameters searches per pair  $(n_c, ns_{max})$  were performed. Data for  $X=J_{c1}$  are plotted as box plots for  $n_c = 1$  to 5 and for  $ns_{max} = 0.5$  and  $2.5 \mu V$  (Fig. 2). They show that, of course, the higher the noise is, the more the deviation with the desired parameters is high too. However, we also observed that, for a given noise level, if we increase  $n_c$ , then the expected value, the quartiles and the maximum of all the data (furthest solution found) decreases. This means that the more the number of curves is high, the more the parameter determination is robust against noise. Moreover, the method cannot be used with a single curve because a slight noise produces a too large error. The same behavior was observed for the 3 other parameters.

### III. EXPERIMENTAL

#### A. Applying the Method to a Superconducting Tape

This method has been experimentally tested to characterize a BiSCCO tape, manufactured by Trithor GmbH. The characteristics of the coil used are presented in Table III. We chose to use permanent magnets to impose the flux density to the coil. This avoids the use of a field coil, device that would be more expensive and more restrictive. Flux density produced by magnets was numerically calculated using the Ampère's model [5] to obtain the value of  $B_{ext}$  at each points of the coil.

The model that was used at first for the parameters search is the one described in Table I - Model 2. This is the model used in Section II, slightly modified to represent more accurately the tape behavior. Thus, only the flux density perpendicular to the tape  $B_{\perp}$  is taken into account. Indeed, it degrades much more the tape characteristics than the parallel flux density  $B_{//}$  [6]. The effect of this latter is consequently neglected. Therefore, the flux density applied to the coil will have a strong perpendicular component on the tape so that its influence on measures could be seen.

#### B. Experimental Apparatus

The experimental apparatus used for measurements is shown in Fig. 3. Two NdFeB magnets, with two identical poles facing

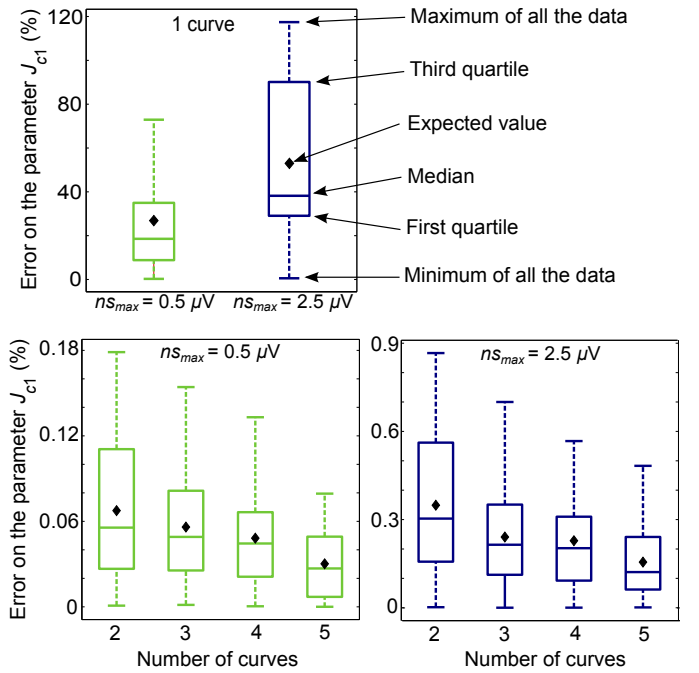


Fig. 2. Box plot of the obtained solutions for  $J_{c1}$  by performing 100 parameters searches. Noise was generated randomly between  $-ns_{max}$  and  $+ns_{max}$  using a uniform law and then added to the theoretical curves. Researches of parameters were performed by minimizing (4) for a number of curve ranging from 1 to 5. We can see that, the more  $n_c$  is high, the more the error on the determined parameter is low.

TABLE III  
CHARACTERISTICS OF THE COIL USED FOR EXPERIMENTAL MEASUREMENTS

Winding type	Double pancake
Inner radius ( $R_i$ )	35 mm
Outer radius ( $R_e$ )	56.5 mm
Length ( $L$ )	8.2 mm
Number of turns ( $N$ )	206
Tape section ( $S$ )	0.814 mm <sup>2</sup>
Superconductor material	BiSCCO

each other, are placed on either side of the superconducting coil. They impose a flux density which is mainly radial to the coil, i.e., perpendicular to the tape. All is held by aluminum flanges which, screwed on a brass threaded rod, also allow adjusting the distance  $d$  between the two magnets. It's by varying this distance that different values of flux density can be applied to the coil. The assembly was cooled into liquid nitrogen, at 77 K. Electrical measurements were performed by the 4 wires method. A power supply Xantrex XFR 7.5-300 regulated in current was used to power the coil. The voltage was measured differentially using a Nanovoltmeter Keithley 2182 connected to two voltage leads.

#### C. Experimental Results

Measurements were performed for 5 different magnets positions, i.e., for 5 different applied fields. The parameters search was carried out by minimizing (4). The obtained parameters, using the Model 2, are shown in Table IV. The experimental

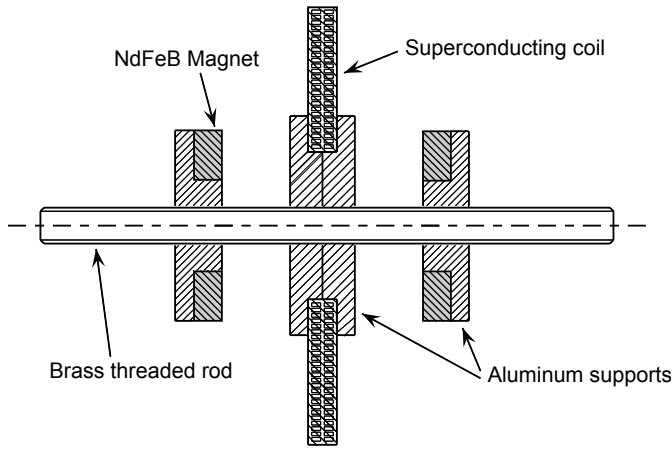


Fig. 3. Schematic of the support used experimentally. It allows maintaining and positioning the coil and the two magnets.

TABLE IV  
OPTIMAL PARAMETERS FOUND FOR EACH OF THE THREE  
MODELISATION OF THE SUPERCONDUCTING TAPE

	Model 2	Model 3	Model 4
$J_{c2}$	57.2 A/mm <sup>2</sup>	$J_{c3}$ 36.0 A/mm <sup>2</sup>	$J_{c4}$ 37.30 A/mm <sup>2</sup>
$B_{J2}$	0.0350 T	$B_{J3}$ 0.379 T	$B_{J4}$ 0.307 T
$n_2$	9.17	$n_3$ 12.0	$n_4$ 11.7
$B_{n2}$	0.275 T	$B_{n3}$ 0.124 T	$B_{n4}$ 0.129 T
		$\beta_3$ 3.66	$\beta_4$ 3.15
			$k$ 0.0951

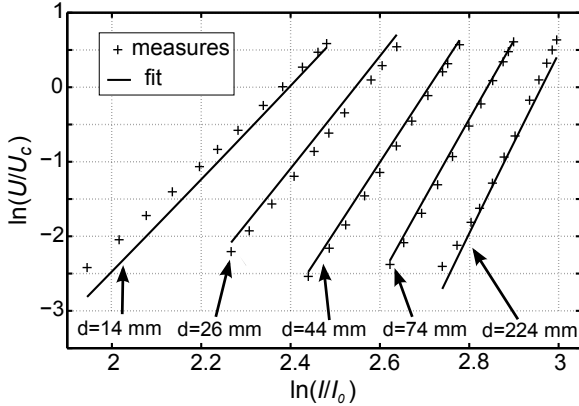


Fig. 4. Comparison between experimental measurements and their fit using the Model 2. The normalization values of voltage and current are respectively  $U_c = 5.6$  mV (critical voltage of the coil) and  $I_0 = 1$  A. We note that there remains a slight gap between the measures and their fit.

curves and their fit using this model are presented in Fig. 4. We can notice that there remains a slight gap between the model curves and measurements. The residual obtained from this minimization (final value reached by *err2*) is 1.30.

We therefore tested other models and compared the residuals to determine which best represent the behavior of the superconductor. Changes to the programs have been minimal and have taken only a few minutes. Two models were selected: models 3 and 4, shown in Table I. The optimal parameters are presented in Table IV. The residual reached during the minimization of *err2*, for models 3 and 4, are respectively

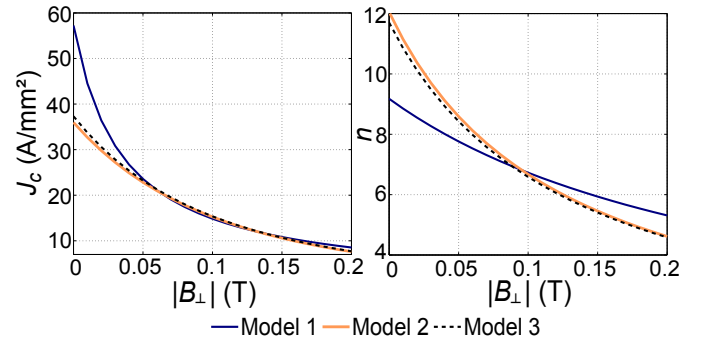


Fig. 5. Plot of the critical current density  $J_c$  and of the exponent  $n$  as function of  $|B_{\perp}|$  for each of the three models obtained for the superconducting tape. Models 3 and 4 are very close, while Model 2 is quite far from the other.

0.183 and 0.180. We can therefore deduce that these models represent more accurately the behavior of the superconductor than the previous one. However, they have more parameters and can therefore be, in some cases more complicated to implement. We also note that the Model 4 takes into account the effect of the parallel flux density  $B_{\parallel}$ . However, the low value obtained for the parameter  $k$  (Table IV) shows clearly that it can be neglected.

The  $J_c(B_{\perp})$  and  $n(B_{\perp})$  curves (Fig. 5) of the three models show that the Model 2 is quite distant from the other two. By cons, the latter are similar. This reinforces the idea that the real  $E(J, B)$  law of the superconducting tape must be very close to the models 3 and 4.

#### IV. CONCLUSION

The method for the determination of the  $E(J, B)$  law of superconducting wires or tape presented in this paper offers several advantages. In addition to facilitated measures compared to the method on short samples, the conductor is characterized in its future conditions of use. Moreover, the self flux density of the coil is taken into account. This method is based on a function minimization carried out through a computation. A theoretical study has highlighted that it is preferable to perform the minimization of (4) rather than (3), due to a shorter computing time. In addition, the more the number of curves used, measured with different superimposed flux density, is high, the more the parameter determination is robust against noise.

This method has been successfully tested to characterize a BiSCCO tape. Permanent magnets have been used to apply the flux density to the coil. They can impose up to 200 mT to the tape. This avoids the use of a field coil, coupled with its power supply, device which would be more restrictive and more expensive. Three models were tested to represent the superconducting tape behavior. When Model 2 is used, the experimental curves are not perfectly fitted. However, with regard to models 3 and 4, the results are very good. They represent more accurately the superconductor behavior. This method is applicable to any type of superconducting wire or tape. We can imagine use it to obtain the characteristics of these conductors as function of the bending radius and the tensile strength.

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