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Fault diagnosis for a class of Descriptor Linear Parameter Varying systems

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SUMMARY

In this paper, a model-based fault estimation method for a particular class of discrete-time Descriptor Linear Parameter Varying (D-LPV) systems is developed. The main contribution of this work consists in the design of an observer that performs simultaneously both, the states estimation and the fault magnitude vectors, considered as unknown inputs. The conditions for the existence of such observer are given. Such conditions guarantee the observer stability and they are proved through a Lyapunov analysis combined with a Linear Matrix Inequalities (LMI) formulation. The fault estimation scheme is evaluated through numerical simulations. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: fault estimation; observers; LPV systems; descriptor systems

1. INTRODUCTION

Descriptor systems (also known as singular systems) are a special class of systems that include ordinary algebraic equations, (differential) state-space equations, combinations of algebraic and differential equations and noncausal systems [1]. From the point of view of process modeling, descriptor systems can be considered as a generalization of state-space linear systems where algebraic relations resulting from the interaction of the process dynamics exist. Several works dealing with linear and nonlinear descriptor systems exist in literature (see for instance [2] for linear systems and [4] for the nonlinear case). Some recent works dealing with descriptor systems are [5, 6, 7] where different observer approaches are used for this kind of systems. Different fault diagnosis techniques for descriptor systems are recently synthesized in [8, 9].

On the other hand, Linear Parameter Varying (LPV) systems (which can be considered as a particular case of Linear Time Varying systems LTV) can be used to approximate nonlinear systems, hence systematic and generic available theoretical results for LPV systems

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can be applied to derive nonlinear control laws for nonlinear systems. For instance, in [10] the authors use the LPV approach to model and control diesel engines. In [11] a method for designing structured fault diagnosis filters (with guaranteed H_∞ nuisance attenuation and fault transmission H_- gain) for polytopic LPV systems is developed and simulated for a secondary circuit of a nuclear power plant. An application of fault detection and isolation to a Boeing 747-100/200 aircraft is presented in [12].

In recent years, fault detection and isolation for LPV systems has been investigated for several authors. The case of robust fault detection and isolation of multiple simultaneous faults for affine LPV systems affected by disturbances is presented in [13], where the authors design a residual generator filter that maximizes the DC-transmission gain from a specific fault to the residual while attenuating the effect caused by the nuisances. The authors in [14] present an H_∞ approach for fault detection and isolation (and estimation), where a linear fractional transformation (LFT) representation of the LPV system is adopted. In [15, 16], failure detection for LPV systems is investigated by using a geometric approach, where a procedure was derived to obtain the detection filter gain via the construction of a suitable family of invariant subspaces. Parameter-dependent observers with pole assignment are used to synthesize the residual generators for LPV systems in [17, 18, 19]. A robust state-based approach to estimate faults in uncertain LPV systems can be found in [20].

Although the idea of merging descriptor and LPV systems is not new (see for instance [21]), there are very few works dealing with observer and controller synthesis, stabilization, fault diagnosis and other topics on automatic control for this kind of systems. Some recent papers on this topic dealing with the problem of control and stabilization are [22, 23], whereas the authors in [24] addresses the problem of observer-based controllers.

The aim of this work is to develop a fault estimation method for discrete-time descriptor linear parameter varying systems (in the sequel referred as D-LPV systems), described by a polytopic system representation. In order to achieve this objective, an observer for D-LPV systems is synthesized. The observer is based on the observer proposed in [25] where a method to design full-order observers for LTI descriptor systems is considered. The existence conditions of the observer are given. Such conditions guarantee the observer convergence proved through a combined method based on the original approach proposed by [25] and a Lyapunov analysis.

By taking into account some simple matrix manipulations, this observer can estimate both, the states and the unknown inputs, simultaneously. In order to achieve a robust fault detection and identification, the unknown input estimation feature is used to build a fault diagnosis scheme based on a bank of observers. The effectiveness and performance of the proposed scheme are illustrated through a numerical example.

2. PROBLEM FORMULATION

Consider the following discrete-time polytopic D-LPV system where the terms k and $k+1$ are the simplified form to write kT_s and $kT_s + T_s$ respectively, and T_s is the sampling time:

$$\begin{aligned} \bar{E}\tilde{x}(k+1) &= \sum_{i=1}^{M} \varepsilon_i(\rho(k)) \left(\bar{A}_i\tilde{x}(k) + \bar{B}_i u(k)\right) + \bar{F}v(k) \\ y(k) &= \bar{C}\tilde{x}(k) \end{aligned} \tag{1}$$

where $\tilde{x}(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^q$, $y(k) \in \mathbb{R}^p$, $v(k) \in \mathbb{R}^d$ are the state, the measured input, the measured output and the unmeasured input vectors which can be classically considered as fault vectors, respectively, $\bar{E} \in \mathbb{R}^{m\times n}$, $\bar{A}_i \in \mathbb{R}^{m\times n}$, $\bar{B}_i \in \mathbb{R}^{n\times q}$, $\bar{C} \in \mathbb{R}^{p\times n}$, $\bar{F} \in \mathbb{R}^{n\times d}$, are constant matrices, $\rho(k)$ is a time-varying bounded parameter vector assumed to be measured on-line and M is the total number of weighting functions $\varepsilon_i(\rho(k))$ defined as

$$\sum_{i=1}^{M} \varepsilon_i(\rho(k)) = 1, \;\; \varepsilon_i(\rho(k)) \geq 0 \tag{2}$$

Classicaly, the term polytopic comes from the fact that the vector $\varepsilon(\rho(k))$ evolves over the convex set:

$$\Omega = \left\{ \begin{array}{c} \mathsf{col} \\ i \end{array} : \sum_{i=1}^{M} \varepsilon_i(\rho(k)) = 1, \; \varepsilon_i(\rho(k)) \geq 0 \right\}$$

The following assumptions are considered ([25]):

(**A1**) $\mathsf{rank}\ \bar{E} = r < n$.

(**A2**) $\mathsf{rank} \begin{pmatrix} \bar{E} \\ \bar{C} \end{pmatrix} = n$.

Assumption **A1** simply states that $\bar{E}$ is a singular matrix. Assumption **A2**, guarantees the existence of a nonsingular matrix $\bar{\Gamma}$:

$$\bar{\Gamma} = \begin{pmatrix} \bar{\alpha} & \bar{\beta} \\ \bar{\gamma} & \bar{\xi} \end{pmatrix}$$

such that

$$\bar{\alpha}\bar{E} + \bar{\beta}\bar{C} = I_n \tag{3}$$

$$\bar{\gamma}\bar{E} + \bar{\xi}\bar{C} = 0 \tag{4}$$

where $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ and $\bar{\xi}$ are constant matrices of appropriate dimensions which can be found by the singular value decomposition of $\begin{pmatrix} \bar{E} \\ \bar{C} \end{pmatrix}$.

The problem is to synthesize an observer for system (1) such that the estimation error $\tilde{e}(k) = \tilde{x}(k) - \hat{\tilde{x}}(k)$ converges to zero as $k \to \infty$.

In this paper bias are considered constant faults, called bias, are considered, i.e.

(**A3**) $v(k) = v(k-1)$.

This assumption may be considered as restrictive, however in many practical situations, once a fault appears, it remains constant during a significant time period that is long enough to estimate the fault. In this way, the system (1) can be rewritten in the following form:

$$\begin{array}{rcl} Ex(k+1) & = & \displaystyle\sum_{i=1}^{M} \varepsilon_i(\rho(k)) \left(A_i x(k) + B_i u(k)\right) \\ y(k) & = & Cx(k) \end{array} \tag{5}$$

where

$$E = \begin{pmatrix} \bar{E} & -\bar{F} \\ 0_{d\times n} & I_d \end{pmatrix} \tag{6}$$

$$A_i = \begin{pmatrix} \bar{A}_i & 0_{m\times d} \\ 0_{d\times n} & I_d \end{pmatrix} \tag{7}$$

$$B_i = \begin{pmatrix} \bar{B}_i \\ 0_{d\times q} \end{pmatrix} \tag{8}$$

$$C = \begin{pmatrix} \bar{C} & 0_{p\times d} \end{pmatrix} \tag{9}$$

Following the idea presented by Darouach [7] for state and unknown input estimation, the expanded state vector is formed by the usual state variables and the faults at the precedent time instant

$$x(k) = \begin{pmatrix} \tilde{x}(k) \\ v(k-1) \end{pmatrix} \in \mathbb{R}^{(m+d)\times(n+d)} \tag{10}$$

In this way, the faults can be estimated simultaneously with the state variables. The problem becomes to design an adequate observer for the D-LPV system (5).

Assumption **A2** becomes

(**A4**) $\mathsf{rank}\begin{pmatrix} E \\ C \end{pmatrix} = n + d.$

Considering Assumption **A4**, it exists a nonsingular matrix Γ:

$$\Gamma = \begin{pmatrix} \alpha & \beta \\ \gamma & \xi \end{pmatrix}$$

such that

$$\alpha E + \beta C = I_{n+d} \tag{11}$$

$$\gamma E + \xi C = 0 \tag{12}$$

where α, β, γ and ξ are constant matrices of appropriate dimensions which can be found by the singular value decomposition of $\begin{pmatrix} E \\ C \end{pmatrix}$.

The following additional assumption is considered:

(**A5**) The pair $\left(\alpha A_i, \begin{pmatrix} \gamma A_i \\ C \end{pmatrix}\right)$ is detectable.

As proposed in [25], if Assumptions A4 and A5 holds, an observer for the system (5) should have the following form:

$$\begin{aligned} z(k+1) &= \sum_{i=1}^{M} \varepsilon_i(\rho(k)) \left[N_i z(k) + L_{1i} y(k) + G_i u(k) + L_{2i} y(k)\right] \\ \hat{x}(k) &= z(k) + \beta y(k) + K\xi y(k) \end{aligned} \tag{13}$$

where $z(k) \in \mathbb{R}^{n+d}$ is the observer state vector; N_i, L_{1i}, L_{2i}, G_i and K, $i = 1, \ldots, M$ are constant matrices of appropriate dimensions.

In the next section, sufficient conditions are given, such that system (13) is an observer for system (5).

3. THE OBSERVER SYNTHESIS

For the sake of simplicity, the following notation is used:

$$\Omega(\rho_k) = \sum_{i=1}^{M} \varepsilon_i(\rho(k))\Omega_i \quad i = 1, \ldots, M \tag{14}$$

Thus, system (13) can be rewritten in the simplified form:

$$\begin{aligned} z(k+1) &= N(\rho_k)z(k) + L_1(\rho_k)y(k) + G(\rho_k)u(k) + L_2(\rho_k)y(k) \\ \hat{x}(k) &= z(k) + \beta y(k) + K\xi y(k) \end{aligned} \tag{15}$$

Let the observer error $e(k) \in \mathbb{R}^{n+d}$ be defined as:

$$e(k) = x(k) - \hat{x}(k) \tag{16}$$

The difference equation of the error is

$$\begin{aligned} e(k+1) &= \left[(\alpha + K\gamma)\, A(\rho_k) - N(\rho_k)\,(\alpha + K\gamma)\, E - L_1(\rho_k)C - L_2(\rho_k)C\right] x(k) \\ &+ \left[(\alpha + K\gamma)\, B(\rho_k) - G(\rho_k)\right] u(k) + N(\rho_k) \underbrace{\left[(\alpha + K\gamma)\, Ex(k) - z(k)\right]}_{e(k)} \end{aligned} \tag{17}$$

If the following conditions are fulfilled

$$G(\rho_k) = (\alpha + K\gamma)\, B(\rho_k) \tag{18}$$

$$L_1(\rho_k) = N(\rho_k)\,(\beta + K\xi) \tag{19}$$

and

$$N(\rho_k) = K\gamma A(\rho_k) + \alpha A(\rho_k) - L_2(\rho_k)C \tag{20}$$

then (17) reduces to

$$e(k+1) = N(\rho_k)e(k) \tag{21}$$

Equation (21) can be written as:

$$e(k+1) = \sum_{i=1}^{M} \varepsilon_i(\rho(k))(\tilde{A}_i + \tilde{K}_i\tilde{C}_i)e(k) \tag{22}$$

where $\tilde{A}_i = \alpha A_i$, $\tilde{K}_i = (K \quad -L_{2i})$ and $\tilde{C}_i = \begin{pmatrix} \gamma A_i \\ C \end{pmatrix}$.

Equation (22) confirms the necessity of Assumption A5. The following subsection is devoted to demonstrated the stability and convergence of the error dynamics given in Equation (22).

3.1. Observer stability

Theorem 3.1. *Considering Assumptions A4-A5, the system* (15) *is a stable observer for the system* (5) *if there exist symmetric positive definite matrices* H_i*, nonsingular matrices* J_i*, and matrices* L_{2i}*, and* K *such that*

$$\begin{pmatrix} J_i^T + J_i & N_i^T & J_i^T \\ N_i & H_j & 0 \\ J_i & 0 & H_i^{-1} \end{pmatrix} > 0 \tag{23}$$

$\forall\ i = 1, ..., M,\ j = 1, ..., M$ *, where* $N_i = K\gamma A_i G_i + \alpha A_i G_i - L_{2i} C G_i$.

Proof:

Consider the following Parameter Dependent Lyapunov Function (PDLF)

$$V(e(k), \xi(\rho(k))) = e^T(k) P(\rho_k) e(k) \tag{24}$$

where according to notation (14):

$$P(\rho_k) = \sum_{i=1}^{M} \varepsilon_i(\rho(k)) P_i \tag{25}$$

P_i are symmetric positive definite constant matrices. According to the Lyapunov stability theorem, this PDLF must satisfy:

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= e^T(k)(N^T(\rho_k) P(\rho_{k+1}) N(\rho_k) - P(\rho_k)) e(k) \leq 0 \end{aligned} \tag{26}$$

It can be seen that the function $\Delta V(k)$ is negative definite if

$$N^T(\rho_k) P(\rho_{k+1}) N(\rho_k) - P(\rho_k) < 0 \tag{27}$$

The following notation is used for $P(\rho_{k+1})$:

$$P(\rho_{k+1}) = \sum_{i=1}^{M} \varepsilon_i(\rho(k+1)) P_i = \sum_{j=1}^{M} \varepsilon_j(\rho(k)) P_j \tag{28}$$

By substituting (25) and (28) in (27), it can be deduced that the negative definiteness of (27) is guaranteed if [26]:

$$N_i^T P_j N_i - P_i < 0,\ \forall\ i = 1, \ldots, M,\ j = 1, \ldots, M. \tag{29}$$

The authors in [26] have demonstrated that poly-quadratically stability of the system $e(k+1) = N(\rho_k) e(k)$ (which ensures the stability of the observer) is guaranteed if and only if

there exist symmetric positive definite matrices H_i, and matrices (of appropriate dimensions) Ψ_i, $i = 1, \ldots, M$, such that

$$\begin{pmatrix} \Psi_i + \Psi_i^T - H_i & \Psi_i^T N_i^T \\ N_i \Psi_i & H_j \end{pmatrix} > 0 \tag{30}$$

$\forall\, i = 1, \ldots, M,\ j = 1, \ldots, M$, with

$$P(\rho_k) = \sum_{i=1}^{M} \varepsilon_i(\rho(k)) H_i^{-1} \tag{31}$$

It is worth to note from equations (25), (28) and (31) that $H_i = P_i^{-1}$ and $H_j = P_j^{-1}$. In this way, the feasibility problem of inequalities (29) can be replaced for the feasibility problem of inequality (30). If so, then (26) is a negative definite and decreasing sequence.

Replacing N_i from (20) into (30):

$$\begin{pmatrix} \Psi_i + \Psi_i^T - H_i & (\star)^T \\ K\gamma A_i \Psi_i + \alpha A_i \Psi_i - L_{2i} C \Psi_i & H_j \end{pmatrix} > 0 \tag{32}$$

$\forall\, i = 1, \ldots, M,\ j = 1, \ldots, M$. It can be seen that the main difficulty to solve the inequality (32) is the bilinear term $K\gamma A_i \Psi_i$, where K and Ψ_i are unknown matrices.

Assuming that Ψ_i is nonsingular, the positive definiteness of (30) is equivalent to the positive definiteness of the following congruent matrix

$$\begin{pmatrix} \left(\Psi_i^{-1}\right)^T & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \Psi_i + \Psi_i^T - H_i & \Psi_i^T N_i^T \\ N_i \Psi_i & H_j \end{pmatrix} \begin{pmatrix} \Psi_i^{-1} & 0 \\ 0 & I \end{pmatrix} > 0 \tag{33}$$

or

$$\begin{pmatrix} \left(\Psi_i^{-1}\right)^T + \Psi_i^{-1} - \left(\Psi_i^{-1}\right)^T H_i \Psi_i^{-1} & N_i^T \\ N_i & H_j \end{pmatrix} > 0 \tag{34}$$

Equation (34) is the Schur complement of the following positive definite matrix

$$\begin{pmatrix} J_i^T + J_i & N_i^T & J_i^T \\ N_i & H_j & 0 \\ J_i & 0 & H_i^{-1} \end{pmatrix} > 0 \tag{35}$$

with $J_i = \Psi_i^{-1}$. This completes the proof. □

The inequality (23), implies that $H_j H_i^{-1} = I$ when $i = j$, or simply

$$H_i P_i = I \tag{36}$$

(where $P_i = H_i^{-1}$). Thus, inequality (23) can be written as:

$$\begin{pmatrix} J_i^T + J_i & N_i^T & J_i^T \\ N_i & H_j & 0 \\ J_i & 0 & P_i \end{pmatrix} > 0,\ H_i P_i = I. \tag{37}$$

Among various methods to solve LMI problem under a LME constraint, the cone complementary technique has been considered. The following facts are used to give a solution procedure to solve the inequality (37):

Fact 3.1. *The feasibility of the LMI*

$$\begin{pmatrix} H_i & I \\ I & P_i \end{pmatrix} \geq 0,\ H_i > 0;\ P_i > 0\ i = 1, \ldots, M$$

implies that $\mathsf{Tr}(H_i P_i) \geq n$*, where the equality* $\mathsf{Tr}(H_i P_i) = n$ *holds if and only if* $H_i P_i = I$ *(see [27]).*

The following matrices are introduced

$$\mathbb{H} = \mathsf{diag}\,(H_1, \ldots, H_M) \tag{38}$$

$$\mathbb{P} = \mathsf{diag}\,(P_1, \ldots, P_M) \tag{39}$$

By taking into account Fact 3.1, it is easy to deduce the following:

Fact 3.2. *The feasibility of the LMI*

$$\begin{pmatrix} \mathbb{H} & I \\ I & \mathbb{P} \end{pmatrix} \geq 0,\ \mathbb{H} > 0;\ \mathbb{P} > 0$$

implies that $\mathsf{Tr}(\mathbb{HP}) \geq nM$*, where the equality* $\mathsf{Tr}(\mathbb{HP}) = nM$ *holds if and only if* $\mathbb{HP} = I$

In this way, the problem of finding a solution of (37) is equivalent to solve the following minimization problem:

$$\mathsf{min}\ \mathsf{Tr}(\mathbb{HP})\ \textit{subject to} \begin{cases} \begin{pmatrix} J_i^T + J_i & N_i^T & J_i^T \\ N_i & H_j & 0 \\ J_i & 0 & P_i \end{pmatrix} > 0 \\ \begin{pmatrix} \mathbb{H} & I \\ I & \mathbb{P} \end{pmatrix} > 0 \\ \mathbb{H} > 0 \\ \mathbb{P} > 0 \end{cases} \tag{40}$$

$\forall\ i = 1 \ldots, M$ and $\forall\ j = 1 \ldots, M$. The bilinear objective problem: $\mathsf{min}\ \mathsf{Tr}(\mathbb{HP})$, in (40) is solved by using the cone complementary algorithm proposed in [28].

The following notation is introduced: $H^{[\mathsf{k}]}$ is used to denote the k-th element of the matrix sequence $H^{[0]},\ H^{[1]}, \ldots, H^{[\mathsf{k}_{opt}]}$.

The cone complementary algorithm propose to transform the bilinear objective problem given in (40) by a linear objective problem of the form

$$\min \mathsf{Tr}(\mathbb{H}^{[\mathsf{k}-1]}\mathbb{P}^{[\mathsf{k}]} + \mathbb{P}^{[\mathsf{k}-1]}\mathbb{H}^{[\mathsf{k}]}) \; subject\ to \begin{cases} \begin{pmatrix} \left(J_i^{[\mathsf{k}]}\right)^T + J_i^{[\mathsf{k}]} & \left(N_i^{[\mathsf{k}]}\right)^T & \left(J_i^{[\mathsf{k}]}\right)^T \\ N_i^{[\mathsf{k}]} & H_j^{[\mathsf{k}]} & 0 \\ J_i^{[\mathsf{k}]} & 0 & P_i^{[\mathsf{k}]} \end{pmatrix} > 0 \\ \begin{pmatrix} \mathbb{H}^{[\mathsf{k}]} & I \\ I & \mathbb{P}^{[\mathsf{k}]} \end{pmatrix} > 0 \\ \mathbb{H}^{[\mathsf{k}]} > 0 \\ \mathbb{P}^{[\mathsf{k}]} > 0 \end{cases} \tag{41}$$

for $\mathsf{k} = 1, \ldots, \mathsf{k}_{opt}$, and to find recursively the values of $\mathbb{H} = \mathbb{H}^{[\mathsf{k}_{opt}]}$, $\mathbb{P} = \mathbb{P}^{[\mathsf{k}_{opt}]}$, $J_i = J_i^{[\mathsf{k}_{opt}]}$, $N_i = N_i^{[\mathsf{k}_{opt}]}$, where k_{opt} represents the iteration number where the sequence converges to $\min \mathsf{Tr}(\mathbb{H}^{[\mathsf{k}-1]}\mathbb{P}^{[\mathsf{k}]} + \mathbb{P}^{[\mathsf{k}-1]}\mathbb{H}^{[\mathsf{k}]}) = 2nM$ or equivalently $\min \mathsf{Tr}(\mathbb{H}\mathbb{P}) = nM$, in other words $\min \mathsf{Tr}(H_i^{[\mathsf{k}-1]}P_i^{[\mathsf{k}]} + P_i^{[\mathsf{k}-1]}H_i^{[\mathsf{k}]}) = 2n$ or equivalently $\min \mathsf{Tr}(H_iP_i) = n$.

A small enough value of a constant stopping criterion $\varepsilon > 0$ should be considered with the purpose of reduce the convergence time of the algorithm, i.e., once the value of $\mathsf{Tr}(\mathbb{H}^{[\mathsf{k}]}\mathbb{P}^{[\mathsf{k}]}) < nM + \varepsilon$, the algorithm stops. As indicated [28], each step of the algorithm has a simple LMI problem. The recursive sequence is bounded and decreases if and only if $\mathbb{H}^{[\mathsf{k}]}\ \mathbb{P}^{[\mathsf{k}]}$ is at the optimum. For more details concerning the used of such method, the authors recommend to read carefully [28] and associated papers.

The initial matrices $\mathbb{H}^{[0]}$ and $\mathbb{P}^{[0]}$, are obtained by computing an initial feasible point of the simple LMIs

$$\begin{pmatrix} \left(J_i^{[0]}\right)^T + J_i^{[0]} & \left(N_i^{[0]}\right)^T & \left(J_i^{[0]}\right)^T \\ N_i^{[0]} & H_j^{[0]} & 0 \\ J_i^{[0]} & 0 & P_i^{[0]} \end{pmatrix} > 0$$
$$\begin{pmatrix} \mathbb{H}^{[0]} & I \\ I & \mathbb{P}^{[0]} \end{pmatrix} > 0 \tag{42}$$
$$\mathbb{H}^{[0]} > 0$$
$$\mathbb{P}^{[0]} > 0$$

$\forall\, i = 1 \ldots, M$ and $\forall\, j = 1 \ldots, M$.

3.2. Observer gain design based on a pole placement

The observer should generate an accurate state and unknown inputs estimations based on an appropriate gain design. Among various kind of LMI regions, such as disk, cone, sub-plane, a pole placement through a disk as proposed by [29] has been considered. In fact, any region included inside the disk $D(0, 1)$ can be chosen. However, if pole placement design is not considered, then the algorithm used to solve (41) could compute any H_i, J_i, L_{2i} and K satisfying the LMI (23), such that the matrices N_i, $i = 1, ..., M$ have eigenvalues anywhere

inside the unit circle, disregarding if they are near from the origin or near from the border of the unit circle. Then, in order to ensure an adequate pole placement of matrices N_i, $i = 1, \ldots, n$, the following inequalities can be taken into account instead of the inequalities shown in (29):

$$(N_i - \lambda_i I)^T P_j (N_i - \lambda_i I) - \delta_i^2 P_i < 0, \ \forall\, i = 1, \ldots, M, \ j = 1, \ldots, M. \tag{43}$$

By doing so, the poles of matrices N_i are assigned in a disk $D(\lambda_i, \delta_i)$ defined by the radius $\delta_i > 0$ and the real number $\lambda_i \in \mathbb{R}$ such that $\delta_i \leq 1 - |\lambda_i|$, $\forall\, i = 1, \ldots, M$. The parameters associated to a disk region are chosen from the nominal dynamic behavior of the system in order to stabilize and to guarantee an efficient state estimation.

The inequality shown in (43) can be written in the simplified form:

$$\bar{N}_i^T P_j \bar{N}_i - \bar{P}_i < 0, \ \forall\, i = 1, \ldots, M, \ j = 1, \ldots, M. \tag{44}$$

which have the same form of inequality (29) with $\bar{N}_i = (N_i - \lambda_i I)$ and $\bar{P}_i = \delta_i^2 P_i$. According to Theorem 3.1, the feasibility of inequality (44) is equivalent to the feasibility of finding symmetric positive definite matrices $\bar{H}_i$, and matrices (of appropriate dimensions) Ψ_i, $i = 1, \ldots, M$, such that

$$\begin{pmatrix} \Psi_i + \Psi_i^T - \bar{H}_i & \Psi_i^T \bar{N}_i^T \\ \bar{N}_i \Psi_i & H_j \end{pmatrix} > 0 \tag{45}$$

$\forall\, i = 1, \ldots, M, \ j = 1, \ldots, M$, with

$$\bar{P}(\rho_k) = \sum_{i=1}^{M} \varepsilon_i(\rho(k)) \bar{H}_i^{-1} \tag{46}$$

where $\bar{H}_i = \bar{P}_i^{-1} = \delta_i^{-2} P_i^{-1} = \delta_i^{-2} H_i$

Replacing $\bar{H}_i$ and $\bar{N}_i$ in (45), and by following an analogous procedure to that described in the proof of Theorem 3.1, the following corollary can be easily derived

Corollary 3.1. *Considering Assumptions A4-A5, the system* (15) *is a stable observer for the system* (5) *if there exist symmetric positive definite matrices H_i and nonsingular matrices J_i such that*

$$\begin{pmatrix} J_i^T + J_i & (N_i - \lambda_i I)^T & J_i^T \\ (N_i - \lambda_i I) & H_j & 0 \\ J_i & 0 & \delta_i^2 H_i^{-1} \end{pmatrix} > 0 \tag{47}$$

$\forall\, i = 1, \ldots, M, \ j = 1, \ldots, M$, where $N_i = K\gamma A_i + \alpha A_i - L_{2i} C$ and L_{2i}. The matrices $G(\rho_k)$, $L_1(\rho_k)$ and $N(\rho_k)$ are deduced from (18)-(20). *The poles of N_i are located in a disk $D(\lambda_i, \delta_i)$ defined by the radius $\delta_i > 0$ and the real number $\lambda_i \in \mathbb{R}$ such that $\delta_i \leq 1 - |\lambda_i|$.*

Finally, the cone complementary algorithm transforms the problem of feasibility of the inequality shown in (47) in the following linear objective minimization problem:

$$\min \mathsf{Tr}(\mathbb{H}^{[\mathsf{k}-1]}\mathbb{P}^{[\mathsf{k}]}+\mathbb{P}^{[\mathsf{k}-1]}\mathbb{H}^{[\mathsf{k}]}) \; subject\ to \begin{cases} \begin{pmatrix} \left(J_i^{[\mathsf{k}]}\right)^T + J_i^{[\mathsf{k}]} & (N_i^{[\mathsf{k}]} - \lambda_i I)^T & \left(J_i^{[\mathsf{k}]}\right)^T \\ (N_i^{[\mathsf{k}]} - \lambda_i I) & H_j^{[\mathsf{k}]} & 0 \\ J_i^{[\mathsf{k}]} & 0 & \delta_i^2 P_i^{[\mathsf{k}]} \end{pmatrix} > 0 \\ \begin{pmatrix} \mathbb{H}^{[\mathsf{k}]} & I \\ I & \mathbb{P}^{[\mathsf{k}]} \end{pmatrix} > 0 \\ \mathbb{H}^{[\mathsf{k}]} > 0 \\ \mathbb{P}^{[\mathsf{k}]} > 0 \end{cases} \tag{48}$$

which is solved as described above. If (48) is feasible, then the poles of N_i are located in a disk $D(\lambda_i, \delta_i)$ defined by the radius $\delta_i > 0$ and the real number $\lambda_i \in \mathbb{R}$ such that $\delta_i \leq 1 - |\lambda_i|$.

3.3. Extended case

In this subsection, the design of a full-order observer for descriptor systems with measurement noises and perturbations is presented.

Let us consider the signal $w(t)$ characterized as the output of the following linear discrete-time system:

$$\begin{aligned} x_w(k+1) &= A_w x_w(k) + B_w u_w(k) \\ w(k) &= C_w x_w(k) \end{aligned} \tag{49}$$

where $x_w(k) \in \mathbb{R}^w$, $u_w(k) \in \mathbb{R}$, $w(k) \in \mathbb{R}$ are the state, the input and the output of the perturbation model. A_w, B_w and C_w are constant matrices of appropriate dimensions.
The signal $w(k)$ can represent either a perturbation signal, or a noise signal. The output $y_m(k)$ is given by

$$y_m(k) = y(k) + w(k) = [C \;\; C_w] \begin{bmatrix} x(k) \\ x_w(k) \end{bmatrix}$$

The state-space models of systems (5) and (49) can be rewritten in the compact form:

$$\begin{aligned} \tilde{E}\dot{\bar{x}}(k+1) &= \tilde{A}(\rho_k)\bar{x}(k) + \tilde{B}(\rho_k)\bar{u}(k) \\ y_m(k) &= \tilde{C}\bar{x}(k) \end{aligned} \tag{50}$$

where

$$\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & I_w \end{bmatrix} \; \tilde{A}(\rho_k) = \begin{bmatrix} A(\rho_k) & 0 \\ 0 & A_w \end{bmatrix} \; \tilde{B}(\rho_k) = \begin{bmatrix} B(\rho_k) & 0 \\ 0 & B_w \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} C & C_w \end{bmatrix} \; \bar{x}(k) = \begin{bmatrix} x(k) \\ x_w(k) \end{bmatrix} \; \bar{u}(k) = \begin{bmatrix} u(k) \\ u_w(k) \end{bmatrix}$$

The procedure to design an observer for system (50) is equivalent to design an observer for system (5) extended to a classical robust residual generators method [3] .

4. ILLUSTRATIVE EXAMPLE

Consider a descriptor LPV system of the form (1) in the presence of two structured faults with unknown fault magnitudes given by:

$$\bar{A}_1 = \begin{pmatrix} 2.018 & 1.326 & -0.08691 & -0.1644 \\ 2.277 & 2.018 & -0.1359 & -0.2908 \\ 0.4259 & 0.1666 & 0.5714 & 0.1732 \\ 0.6659 & 0.4259 & -0.4822 & -0.152 \end{pmatrix} \bar{A}_2 = \begin{pmatrix} 1 & 1.4 & -0.07 & -0.2 \\ 1.5 & 2 & -0.15 & -0.3 \\ 0.4 & 0.1 & 0.5 & 0.2 \\ 0 & 0 & -0.5 & -0.2 \end{pmatrix} \quad (51)$$

$$\bar{B}_1 = \begin{pmatrix} 0.6047 \\ 1.644 \\ -1.709 \\ -1.732 \end{pmatrix} \bar{B}_2 = \begin{pmatrix} 0.5 \\ 1 \\ -1.5 \\ -1 \end{pmatrix} \bar{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad (52)$$

$$\bar{E} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} F = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (53)$$

where matrices A_1, B_1, E and C were taken from [22] with structured fault matrix F.

The system (1) can be rewritten in the form of system (5), where the augmented matrices E and C are:

$$E = \begin{pmatrix} \bar{E} & -F \\ 0_{2\times 4} & I_2 \end{pmatrix} C = \begin{pmatrix} \bar{C} & 0_{2\times 2} \end{pmatrix} \quad (54)$$

It can be verified that

$$\mathsf{rank} \begin{pmatrix} E \\ C \end{pmatrix} = n + d = 6 \quad (55)$$

The matrices α, β, γ and ξ, satisfying (11) and (12) are computed by using the Singular Value Decomposition property of $(E \;\; C)^T$:

$$\alpha = \begin{pmatrix} 0.333 & 0 & 0 & 0 & 0.333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -0.333 & 0 & 0 & 0 & 0.667 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \beta = \begin{pmatrix} 0.667 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0.333 & 0 \\ 0 & 0 \end{pmatrix} \quad (56)$$

$$\gamma = \begin{pmatrix} -0.471 & 0.577 & 0 & 0 & -0.471 & 0 \\ 0.333 & 0.817 & 0 & 0 & 0.333 & 0 \end{pmatrix} \xi = \begin{pmatrix} 0.471 & 0 \\ -0.333 & 0 \end{pmatrix} \quad (57)$$

At this point, it is worth to note that the pair $\left(\alpha A_i, \begin{pmatrix} \gamma A_i \\ C \end{pmatrix}\right)$ is detectable $\forall\, i = 1, \ldots, 2$. An observer of the form (13) can be designed. The disks to place the poles of the matrices N_i are $D(0.2, 0.8)$ for $i = 1, 2$.

Following the procedure described in Section 3, the following matrices are obtained:

$$P_1 = \begin{pmatrix} 1.0895 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0895 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6681 & 0.3240 & -0.0135 & -1.0236 \\ 0 & 0 & 0.3240 & 6.6398 & 0.0825 & 1.1574 \\ 0 & 0 & -0.0135 & 0.0825 & 1.0909 & 0.0296 \\ 0 & 0 & -1.0236 & 1.1574 & 0.0296 & 2.2746 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 1.0895 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0895 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6943 & 0.5733 & 0.0165 & -0.9729 \\ 0 & 0 & 0.5733 & 5.9887 & 0.0929 & 0.4898 \\ 0 & 0 & 0.0165 & 0.0929 & 1.0882 & 0.0002 \\ 0 & 0 & -0.9729 & 0.4898 & 0.0002 & 2.0350 \end{pmatrix}$$

$$J_1 = \begin{pmatrix} 0.6129 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6129 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3758 & 0.1823 & -0.0076 & -0.5757 \\ 0 & 0 & 0.1823 & 3.7349 & 0.0464 & 0.6510 \\ 0 & 0 & -0.0076 & 0.0464 & 0.6136 & 0.0167 \\ 0 & 0 & -0.5757 & 0.6510 & 0.0167 & 1.2795 \end{pmatrix}$$

$$J_2 = \begin{pmatrix} 0.6129 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6129 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3905 & 0.3225 & 0.0093 & -0.5473 \\ 0 & 0 & 0.3225 & 3.3686 & 0.0523 & 0.2755 \\ 0 & 0 & 0.0093 & 0.0523 & 0.6121 & 0.0001 \\ 0 & 0 & -0.5473 & 0.2755 & 0.0001 & 1.1447 \end{pmatrix}$$

$$K = \begin{pmatrix} 0.4714 & -0.3333 \\ 00 & \\ 5.4512 & 7.7220 \\ -2.5181 & -3.5357 \\ 1.1644 & -0.0398 \\ 2.7433 & 3.8936 \end{pmatrix} \quad L_{21} = \begin{pmatrix} -0.1000 & 0 \\ -0 & -0.1000 \\ 21.9574 & 19.2470 \\ -9.2008 & -8.3224 \\ -0.3503 & 0.1037 \\ 10.8547 & 9.6179 \end{pmatrix} \quad L_{22} = \begin{pmatrix} -0.1000 & 0 \\ -0 & -0.1000 \\ 14.5827 & 19.0106 \\ -6.5026 & -8.6696 \\ 0.0642 & 0.0259 \\ 7.1491 & 9.5325 \end{pmatrix}$$

By using these gain matrices the following eigenvalues of N_i are obtained:

$$\mathsf{eig}(N_1) = \{0.8499,\ 0.3747,\ 0.1677,\ 0.1076,\ 0.1,\ 0.1\}$$

and

$$\mathsf{eig}(N_2) = \{0.6422 \pm 0.0401i,\ -0.0983,\ 0.1008,\ 0.1,\ 0.1\}$$

Finally, matrices $G(\rho)$, $L_1(\rho)$ and $N(\rho)$ are computed by means of Eqs. (18)-(20).

The effectiveness of the proposed observer scheme is illustrated by the system studied in open-loop. In this simulation, the time-varying parameter $\rho(k)$ is the measured input, and it is shown in Fig. 1. The weighting functions $\varepsilon_i(\rho(k))$ $i = 1, 2$ of the LPV system (1) are shown

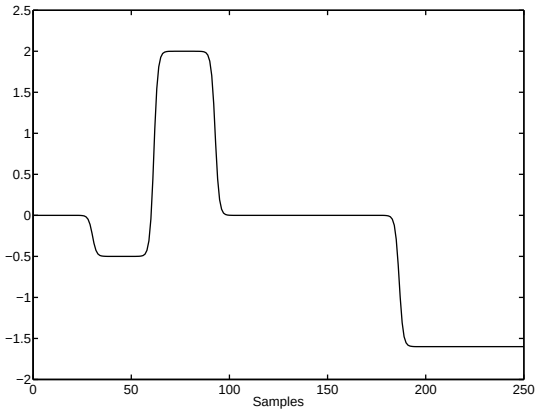

Figure 1. Dynamic behaviour of the input

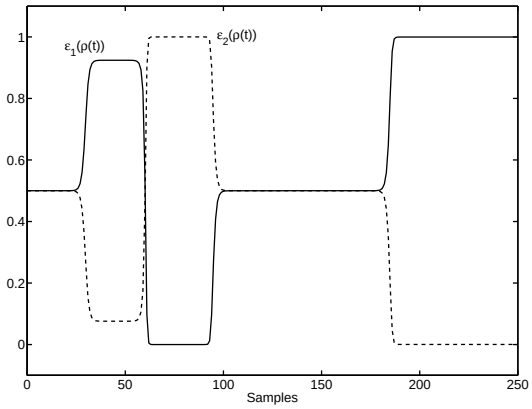

Figure 2. Dynamic behaviour of the weighting functions ε_1 (solid line) and ε_2 (dashed line)

in Fig. 2. The time evolution of these functions are in accordance with (2). It can be seen that the system is evaluated over the entire operating envelope. Fault-free dynamic behaviour of the outputs is shown in Fig. 3. It can be noted that a gaussian noise is added to the measured outputs.

Two severe faults were simulated. The first fault occurs at the sampled time $t_k = 50$ and the second fault occurs at $t_k = 130$. The faults acts as two unknown inputs to the system, affecting the dynamics of the outputs and the states (see Figs. 4-5). The state observer tracks well both, the system states, despite the variations on the input (and consequently the changing operating points), the noisy measurements (see Fig. 4), and the faults. In Fig. 6, it can be appreciated

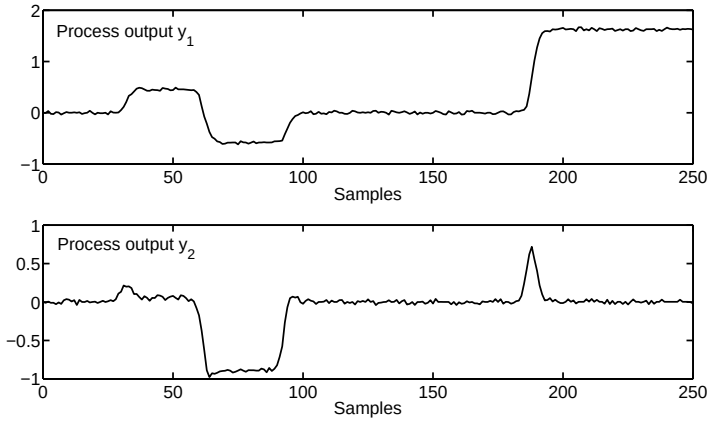

Figure 3. Fault-free dynamic behaviour of the outputs y_1 and y_2

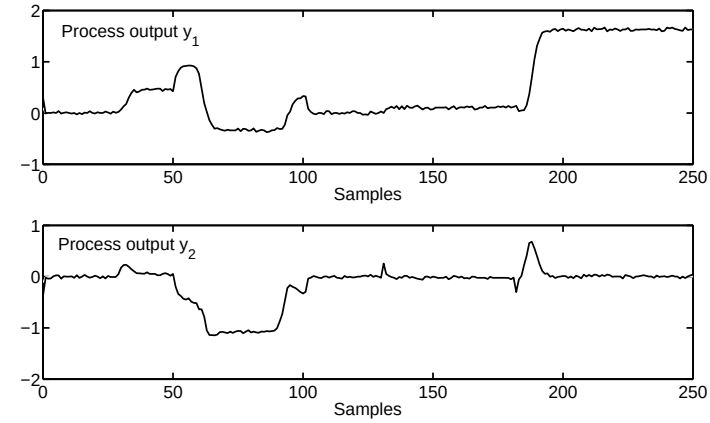

Figure 4. Faulty dynamic behaviour of the outputs y_1 and y_2

that the observer detects these faults.

As expected, the observer is noise sensitive. However, these simulation results show that the observer is very effective in detecting and isolating the fault over the entire operating envelope. The estimation of the faults should be used to generate the residuals which can be evaluated through classical statistical methods with the purpose of generate alarms. The fault diagnosis scheme constructed in this way is able to detect, to isolate and to estimate the fault and it represents an efficient tool in the operator's decision.

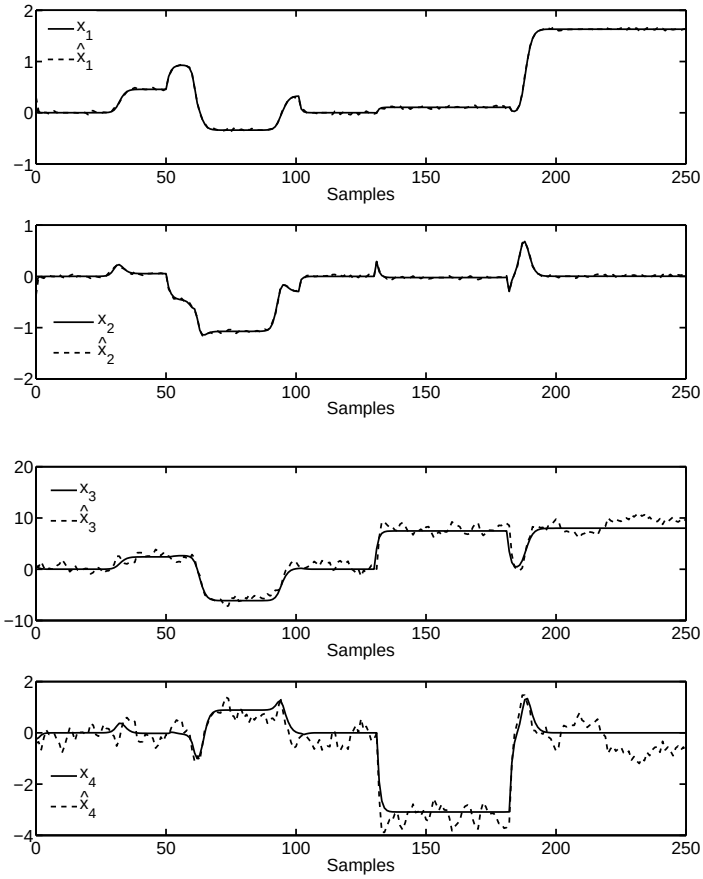

Figure 5. Dynamic behaviour of the states x_1, x_2, x_3 and x_4 and their estimated values $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$ and $\hat{x}_4$

5. CONCLUSION

In this paper, a model-based fault diagnosis method for discrete-time D-LPV systems is introduced. For that, an observer that performs fault detection and fault magnitude estimation over the entire operating envelope of the system. The observer synthesis is an extension of the work presented in [25] where an observer for LTI descriptor systems is reported. By taking into account some simple matrix manipulations, this observer can estimate simultaneously both, the states and the fault magnitude vectors, considered as unknown inputs. Sufficient conditions are stated to ensure the existence and the stability of the proposed observer by using a combined Lyapunov analysis based on LMI formulation. An observer gain synthesis has been also proposed to guarantee the efficiency of the model-based fault diagnosis technique. The proposed method is evaluated successfully via numerical simulations.

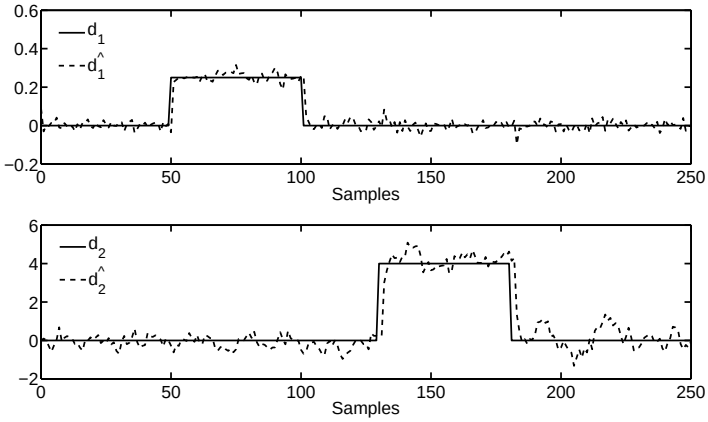

Figure 6. Dynamic behaviour of the faults d_1 and d_2 and their estimated values $\hat{d}_1$ and $\hat{d}_2$

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