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A Stochastic Model of Trip End Disaggregation in Traffic Assignment to a Transportation Network

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Abstract

So far, traffic assignment to a network has been modelled by aggregating trip ends into zone centroids. The paper introduces a disaggregate model of trip ends, by associating to each zone a set of ‘anchor nodes’ for network access and a random vector of ‘terminal costs’ (or time) between trip ends and zonal anchors. These can be specified by OD pair. Assuming that terminal costs and network costs are independent, the system assignment can be performed in two stages, respectively network and terminal. A probit model based on Clark’s formulas is recommended. A computation scheme is provided that loads OD flows by anchor pair onto the network in an efficient way. A binary case is addressed as illustration. Also reported is an application to the roadway network of the Paris area.

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Keywords: Traffic assignment; trip disaggregation; discrete choice model

1. Introduction

Traffic assignment models, since their basic foundation by Wardrop (1952) and Beckmann (1956), have been developed in diverse ways and applied in many problems of transportation network planning. Among the developments achieved so far, let us quote: multiclass models, stochastic assignment, transit assignment, multimodal models, integrated models that deal with route choice together with mode choice or destination choice or even location choice, dynamic models of congestion phenomena and trip timing... However little attention has been paid to the assumption of trip end zones: i.e. that within a given zone especially designed for traffic assignment (hence a TAZ) all the trip endpoints are assimilated to the zone ‘centroid’ (focal point), which is connected to the network by ad hoc links called ‘connectors’. Thus the zoning system influences the route choice by origin-destination pair (OD pair), with little if any behavioral basis and under little control by the study analyst. The bad effects of the centroid approximation in traffic assignment have been known for a long time. Ortúzar and Willumsen (2004) recommend considering zones small enough to satisfy approximately the following assumption: that all of the activity within a zone takes place at its centroid. Solutions mainly consist in detailed zoning and centroid positioning. Crevo (1981) and Baass (1991) investigated the impact of zone design on transit assignment; the latter used an algorithm that starts from small spatial units and merges those with the minimum difference in certain socioeconomic variables: the main effect is to decrease the number of intrazonal trips. Chang et al. (2002) notice that selecting centroids on the basis of weighted averages of population and density yields better assignment results. More recently, Constantin and Florian (2010) suggest splitting any trip flow passing by a given network node between several likely next links rather than one link only, with probabilities that stem from a logit model: this can be applied to the choice of the connector.

Our purpose here is to develop a traffic assignment model that deals with individual trip ends in a disaggregate manner. This model enables the analyst to get the zoning system and the associated connecting system under

control. A mathematical formulation is provided, including a decomposition principle in two stages respectively network and terminal. Also provided is a related computational scheme.

The approach is both spatial and probabilistic. First, the detailed locations within a given TAZ are modeled in a probabilistic way. Then, “anchor nodes” are identified on the transportation network, together with zonal sets of anchor nodes: such a set contains the most likely passage points to access to and from that zone. Next, the terminal time between the detailed locations within the zone and a given anchor node is modeled as a random variable. The vector of these RVs (possibly dependent) describes the terminal times between the trip ends within the zone and the network. Lastly, route choice is modeled at the level of the individual trip as the selection of a path of minimum cost including network time plus the terminal times at both origin and destination, in a probabilistic way by OD pair.

All in all, this model belongs to the family of stochastic assignment models. It is innovative in that its stochastic features pertain to the terminal times, whereas only the network time has been subjected to stochasticity in previous assignment models. We avail ourselves of the well-known properties of stochastic assignment (eg. Abraham, 1961, Burrell, 1968, Dial, 1971, Beilner and Jacobs, 1971, Daganzo and Sheffi, 1977, Sheffi and Powell, 1982) to develop first a general formulation, second a logit formulation and third a probit formulation. All of them require a separate treatment of network and terminal times, successively. The logit model is fairly simple but its outreach is limited to independent, homoskedastic, Gumbel distributed terminal times to anchor nodes. The probit model is much more powerful and flexible as it deals with heteroskedastic, dependent terminal times to anchors, which can even be correlated between the origin and destination parts of the trip; the Clark formula (1961) is useful to evaluate the path choice probability by OD pair and the distribution of the minimum OD travel time, as suggested by Maher and Hughes (1998) but in a specific way to address terminal times.

The body of the paper is organized into five sections. Section 2 sets out the basic model formulation, for a general zoning system and a private transportation network under given travel conditions. Section 3 addresses a classroom case with one OD pair, one or two anchor nodes by zone and one link by anchor pair: this amounts to a binary discrete choice model; closed-form formulae are provided for a logit and a probit model. Section 4 states the assignment algorithm, including a shortest path search by pair of anchor points and an elaborate algorithm to load OD trip flows onto the network in an efficient way. Section 5 is devoted to numerical illustration and experiment, based on the Paris roadway network which includes 15,000 nodes, 40,000 arcs and 1,300 TAZs. A fixed-time assignment and a time-dependent assignment are carried out for the novel assignment model as well as the classic, aggregate model. Finally, Section 6 concludes about potential developments and applications.

2. Model setting

Let us focus on a set of trips, Ω_{od} , on a given origin-destination (OD) pair (o, d) , at a given time period, with homogeneous trip purpose and travel behavior.

2.1. Disaggregate locations

Let us consider that the trip ends vary with the individual trip-maker, denoted as $\omega \in \Omega_{od}$. Within the origin zone o , the disaggregate trip ends are located at points $M(\omega)$. Within the destination zone d , the disaggregate trip ends are located at points $M'(\omega)$. Thus the disaggregate OD pair of trip ends is the couple $(M(\omega), M'(\omega))$.

As most trip ends are associated with buildings for residence or another activity, it may be thought that the distribution of $M(\omega)$ (resp. $M'(\omega)$) would depend only on the zone of origin (resp. destination). However this could be misleading in some cases, notably so when the OD trips are mostly carried by a main network route, in which case the proximity of each endpoint to that route is likely to exert an important influence.

2.2. Anchor nodes and terminal times

In a traditional, zone aggregated traffic assignment model, it is assumed first that all trips take their endpoint at the zone centroid – a kind of centre of mass – and second that the centroid is connected to the transportation network by connectors, i.e. fictive links that represent average travel conditions for terminal access to and from the network.

To extend that representation to disaggregate trip ends, we shall focus primarily on the network nodes which make the connector endpoint other than the centroid: define an ‘anchor node’ as a network node that trips are likely

to go through in the vicinity of their endpoint. Let the ‘anchoring set’ of a zone be the group of its anchor nodes for a given OD pair. Denote by A_{od} the anchoring set of origin zone o with respect to destination zone d , and B_{od} that of destination zone d with respect to origin node o .

The *terminal cost* (or time) from endpoint $M(\omega) \in Z_o(\Omega_{od})$ to anchor node $\alpha \in A_{od}$ is a random variable on Ω_{od} denoted by $\theta_\alpha^{od}(\omega)$, or $\theta_\alpha(\omega)$ for simplicity. Similarly, from anchor node $\beta \in B_{od}$ to endpoint $M'(\omega) \in Z_d(\Omega_{od})$ let $\theta_\beta'^{od}(\omega)$ or $\theta_\beta'(\omega)$ denote the destination terminal cost, a random variable on Ω_{od} . Any pair $(\theta_\alpha, \theta_\beta')$ may exhibit stochastic dependence, including correlation. Furthermore, stochastic dependence is expected to exist between the terminal costs of the anchor nodes that pertain to a given zone, since the disaggregate endpoint has particular access conditions to each of the anchor nodes. Let then $\Theta_{A(od)} = [\theta_\alpha : \alpha \in A_{od}]$ denote the random vector of terminal travel conditions in origin zone o with respect to destination zone d , and symmetrically let $\Theta_{B(od)}' = [\theta_\beta' : \beta \in B_{od}]$ stand for the random vector of terminal travel conditions in destination zone d with respect to origin zone o . Stochastic dependency is expected to hold within and between the two random vectors.

2.3. On trip and network paths and costs

On a disaggregate basis, the detailed paths from origin endpoint to destination endpoint are likely to include not only some main network links but also some minor links, particularly so near of the trip endpoints (Bovy and Stern, 1990). Here it is assumed that the main links are explicitly modeled in the transportation network, i.e. the pair (N, L) of node set N and link set L of directed links between two nodes in N ; and that the remaining, very minor links make up a set of ‘subnetwork features’ which are sufficiently accounted for in the stochastic description of the terminal conditions.

Thus a disaggregate path r_ω is expressed as a threefold sequence of first a terminal subnetwork path, r_ω^+ from origin endpoint to an anchor node $\alpha = n_{r(\omega)}^+$, second a network path $r_{\alpha\beta}$ from anchor α to anchor node $\beta = n_{r(\omega)}^-$ of the destination zone, third a terminal subnetwork path r_ω^- from β to destination endpoint:

$$r_\omega = (r_\omega^+, r_{\alpha\beta}(\omega), r_\omega^-). \quad (1)$$

Letting $t_{r(\alpha\beta)}$ denote the travel cost (or time) along route $r_{\alpha\beta}$, the disaggregate trip cost is modelled as the addition of the partial costs along the route:

$$T_{r(\omega)} = \theta_\alpha(\omega) + t_{r(\alpha\beta)} + \theta_\beta'(\omega). \quad (2)$$

2.4. Disaggregate route choice

Every trip-maker is assumed to be a rational, cost-minimizing decision-maker in his choice of route from origin to destination. As an individual user he behaves in a self-optimizing way; thus he selects a route $r^*(\omega)$ that achieves the minimum cost $T_{r(\omega)}$ among the set R_{od} of available routes.

At the aggregate level, the interest lies in the use of the anchor nodes and of the network elements and paths:

$$p_\alpha^{od} = \Pr\{\omega \in \Omega_{od} : \alpha = n_\omega^+(r^*(\omega))\} \text{ for } \alpha \in A_{od}.$$

$$p_\beta'^{od} = \Pr\{\omega \in \Omega_{od} : \beta = n_\omega^-(r^*(\omega))\} \text{ for } \beta \in B_{od}.$$

$$p_r^{od} = \Pr\{\omega \in \Omega_{od} : r = r^*(\omega)\} \text{ for any network path } r.$$

$$p_{\alpha\beta}^{od} = \Pr\{\omega \in \Omega_{od} : \alpha = n_\omega^+(r^*(\omega)) \text{ and } \beta = n_\omega^-(r^*(\omega))\} \text{ for } \alpha \in A_{od}, \beta \in B_{od}.$$

There are some obvious properties linking these choice probabilities, for instance:

$$p_{\alpha}^{od} = \sum_{r \in R_{od}, n_r^+ = \alpha} p_r^{od},$$

$$p_{\beta}^{\prime od} = \sum_{r \in R_{od}, n_r^- = \beta} p_r^{od}.$$

However, in most cases $p_{\alpha\beta}^{od}$ cannot be expected to be equal to the product of p_{α}^{od} and $p_{\beta}^{\prime od}$.

Denoting by q_{od} the trip volume on the OD pair, the product of it by a given probability yields the partial trip volume that meets the associated condition: for instance

$$x_{\alpha}^{od} = q_{od} \cdot p_{\alpha}^{od}.$$

2.5. Posterior cost variables

Also of interest are the cost variables that stem from the assignment of the OD pairs to the transportation system. At the zone level, let θ_{α}^* (resp. θ_{β}^*) denote the terminal cost optimized at the disaggregate trip level, conditional on α (resp. β) being selected as the anchor node in the optimal path: this is a random variable defined on a sub-population of trips. It differs from the original, unconditional variable θ_{α}^{od} (resp. $\theta_{\beta}^{\prime od}$). Remark that only the conditional terminal costs may be observed by trip inspection in a field survey.

At the network level, if there is no dependency between the terminal and the network costs, then $r_{\alpha\beta}^*(\omega)$ must be a shortest path from α to β along the network, since if there were a shorter network path it would have been selected instead of $r_{\alpha\beta}^*(\omega)$. Then:

Proposition 1. *Along the network between an OD pair of anchor nodes, all of the chosen network paths have the same and minimum cost.*

Thus the user-optimized disaggregate assignment is also user-optimized at the level of the OD pair, though aggregated by anchor node only.

2.6. Two-stage assignment model

Proposition 1 enables us to perform traffic assignment with disaggregate trip ends in two stages: first, a shortest path search on the transportation network between any pair of origin and destination anchor nodes; second, by OD pair at the zone level, a discrete choice model (DCM) in which the choice option is a pair of anchor nodes $(\alpha, \beta) \in A_{od} \times B_{od}$ with random disutility function as follows:

$$T_{\alpha\beta}^{od}(\omega) = \theta_{\alpha}^{od}(\omega) + t_{\alpha\beta}^* + \theta_{\beta}^{\prime od}(\omega), \quad (3)$$

wherein $t_{\alpha\beta}^*$ is the shortest path cost from α to β along the network.

Proposition 2. (i) *The problem of disaggregate assignment and the DCM by OD pair of traffic zones have identical solution sets.* (ii) *Then the option probabilities $[\tilde{p}_{\alpha\beta}^{od}]_{\alpha\beta}$ and the choice probabilities $[p_{\alpha\beta}^{od}]_{\alpha\beta}$ are equivalent.*

Proof. (i) Let us demonstrate that for every pair (α, β) the respective solution sets are mutually inclusive:

$$\Omega_{\alpha\beta}^* = \{\omega \in \Omega_{od} : \exists r = (r_{\alpha}^+, r_{\alpha\beta}, r_{\beta}^-) \in R_{od}, T_r(\omega) \leq T_{r'}(\omega) \forall r' \in R_{od}\},$$

$$\tilde{\Omega}_{\alpha\beta} = \{\omega \in \Omega_{od} : T_{\alpha\beta}(\omega) \leq T_{\alpha'\beta'}(\omega) \forall (\alpha', \beta') \in A_{od} \times B_{od}\}.$$

On one hand, if $\omega \in \Omega_{\alpha\beta}^*$ then $T_{\alpha\beta}(\omega) = T_r(\omega) \leq T_{r'}(\omega) \forall r' \in R_{od}$. But $\forall (\alpha', \beta') \in A_{od} \times B_{od}$, $T_{\alpha'\beta'}(\omega) = T_{r'}(\omega)$ for some route $r' = (r_{\alpha'}^+, r_{\alpha'\beta'}, r_{\beta'}^-) \in R_{od}$ hence $T_{\alpha\beta}(\omega) \leq T_{\alpha'\beta'}(\omega)$, yielding that $\omega \in \tilde{\Omega}_{\alpha\beta}$ hence $\Omega_{\alpha\beta}^* \subset \tilde{\Omega}_{\alpha\beta}$.

On the other hand, if $\omega \in \tilde{\Omega}_{\alpha\beta}$ then there exists $r = (r_{\alpha}^+, r_{\alpha\beta}, r_{\beta}^-) \in R_{od}$ that supports $T_r(\omega) = T_{\alpha\beta}(\omega)$ so that $T_r(\omega) \leq T_{\alpha'\beta'}(\omega) \forall (\alpha', \beta') \in A_{od} \times B_{od}$. But $\forall r' \in R_{od}$, $r' = (r_{\alpha'}^+, r_{\alpha'\beta'}, r_{\beta'}^-)$ hence $T_{r'}(\omega) \geq T_{\alpha'\beta'}(\omega)$, implying that $T_{r'}(\omega) \geq T_r(\omega)$ hence $\omega \in \Omega_{\alpha\beta}^*$ and $\tilde{\Omega}_{\alpha\beta} \subset \Omega_{\alpha\beta}^*$.

(ii) The two vectors of choice probabilities measure an identical system of sets. When the sets are disjoint the probability vectors must be equal. If there is some partial intersection then the vectors belong to admissible sets which are identical for both problems.

3. Binary model

Let us illustrate the trip end disaggregated assignment model in a simple case with two travel options. This occasion will give us the opportunity to introduce the logit and probit models of discrete choice.

3.1. Case setting

Consider two zones linked by two network routes only, say $r \in \{1,2\}$ from origin anchor nodes α_1 and α_2 to destination anchor nodes β_1 and β_2 , respectively. The disaggregation of trip ends on the origin side yields random terminal cost θ_r to α_r . That on the destination side yields random terminal costs θ'_r from β_r . Thus each route has disutility function as follows, letting t_r denote the network cost from α_r to β_r :

$$T_r(\omega) = \theta_r(\omega) + t_r + \theta'_r(\omega). \quad (4)$$

The main result in the binary model is the choice probability of the first route:

$$p_1 = \Pr\{\omega \in \Omega : T_1(\omega) \leq T_2(\omega)\} = \Pr\{T_1 - T_2 \leq 0\},$$

which involves the cumulative distribution function of $T_1 - T_2$.

Square zones with uniform distribution of trip ends may be taken as reference, on assuming independence between origin and destination. Assuming further that a subnetwork for terminal access is a dense Manhattan grid, then each θ_r (resp. θ'_r) is the sum of two independent real random variables distributed uniformly over a real interval along an axis of coordinates, X or Y. Thus θ_r (resp. θ'_r) has a triangular distribution, of which the mean μ_r (resp. μ'_r) depends mostly on the location of α_r (resp. α'_r) in the subnetwork grid and the variance σ_r^2 (resp. $\sigma_r'^2$) depends both on the anchor location w.r.t. the zone and the zone size. There is covariance v_{12} (resp. v'_{12}) between the terminal access costs of the two routes, coming from the trip ends:

$$\chi_{12} = v_{12} + v'_{12}. \quad (5)$$

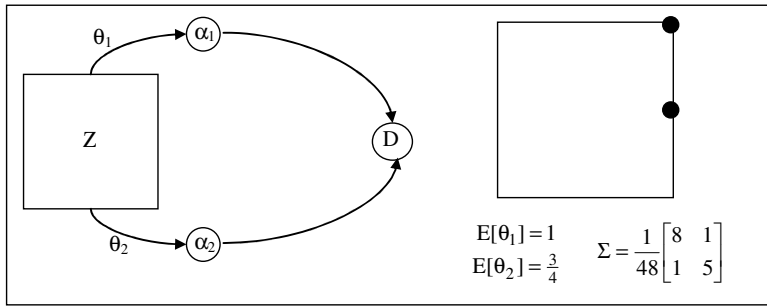


Fig. 1. Instance setting (a) Network, (b) Zone, anchors and distribution of terminal times.

3.2. Multinomial logit model

The multinomial logit model is by far the most widely used DCM, owing to its analytical simplicity (Ortuzar and Willumsen, 2004). Let us recall its main assumptions:

- by option, the utility function has a Gumbel distribution with cumulated function $F(x) = \exp(-\exp(\psi(m-x)))$, mean $m - \gamma$ (γ being Euler's constant) and standard deviation $\sigma = \pi/(\psi\sqrt{3})$.
- between distinct options, the variances σ^2 must be identical (i.e. homoskedasticity) and the utility functions must be independently distributed.

So its application to our setting requires to specify some average variance for route cost, and to neglect the covariance: $\hat{\sigma}^2 = (\sigma_1^2 + \sigma_2^2)/2$ and $\hat{v}_{12} = 0$. Thus we can avail ourselves of the well-known formula for the choice probability of a given option r , say $r = 1$: letting $\psi \equiv \pi/\hat{\sigma}\sqrt{3}$,

$$\hat{p}_r = \exp(-\psi\bar{T}_r) / \sum_{r'} \exp(-\psi\bar{T}_{r'}), \text{ or in the binary case } \hat{p}_1 = 1/[1 + \exp(-\psi(\bar{T}_2 - \bar{T}_1))]. \quad (5)$$

3.3. Probit model

In the probit model of discrete choice, it is only assumed that the vector of options (dis)utilities is a multivariate Gaussian vector, of which the mean vector and the variance-covariance matrix are unconstrained. Thus each option utility function is a random Gaussian variable with unconstrained mean and variance. These assumptions are less requiring than the logit ones to apply our assignment model, although the Gaussian shape does not fit to perfection a given distribution of access cost and nor does the Gaussian-type dependence fit perfectly a given joint distribution of access costs.

In the binary case the probit model is endowed with a closed form formula for option probability, namely

$$\tilde{p}_1 = \Phi\left(\frac{1}{\tilde{\sigma}}(\bar{T}_2 - \bar{T}_1)\right), \tag{6}$$

wherein Φ is the cumulative distribution function of a reduced Gaussian variable (i.e. with null mean and unit variance) and $\tilde{\sigma}^2 \equiv \text{var}(T_2 - T_1) = \sigma_1^2 + \sigma_2^2 - 2\nu_{12}$.

3.4. Model application and assessment

Let us apply to our instance system the following assignment models: (a) door to door, (b) aggregate, (c) logit, (d) probit without covariance, (e) probit with covariance. On varying the difference in network travel time, $\Delta t \equiv \bar{T}_2 - \bar{T}_1$, it turns out that the probit model with covariance matches the disaggregate assignment quite well, whereas the logit and probit without covariance are as far from these ‘best models’ as the aggregate model (Fig. 2). This emphasizes the assignment sensitivity to the location of anchor nodes and the disaggregate conditions of terminal access.

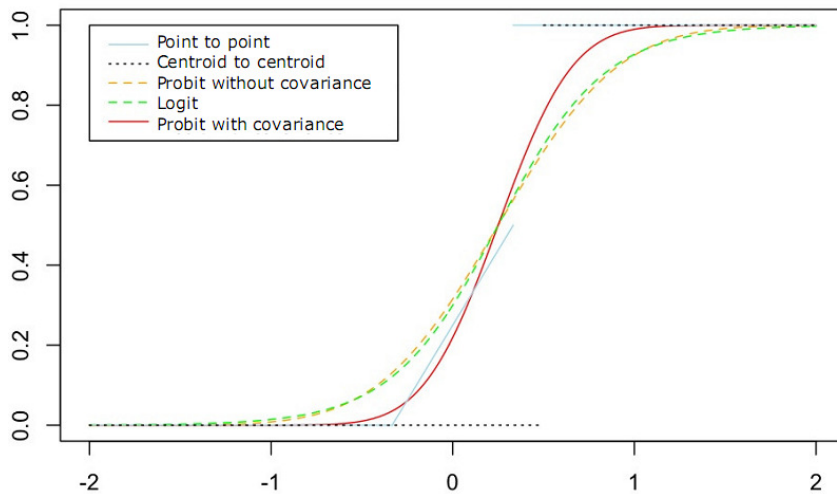


Fig. 2. Share of Route 1 with respect to Δt

4. Computational scheme

Let us now sketch out a method to compute the disaggregate trip end assignment in any application.

4.1. Scheme overview

Our computation scheme is based on the two-stage decomposition and a probit model for trip end disaggregation. It involves the following steps:

1. The characterization of terminal times as random vectors, at the zone level.
2. By OD pair of zones, the search of shortest paths $r_{\alpha\beta}$ on the main network from any anchor $\alpha \in A_{od}$ to any anchor $\beta \in B_{od}$, yielding optimum network costs $t_{\alpha\beta}^*$.
3. By OD pair of zones, the evaluation of choice probabilities $p_{\alpha\beta}^{od}$ by anchor pair (α, β) .
4. Network local loads at the level of nodes and links (and even turns) are determined by local superposition of route flows $q_{od} p_{\alpha\beta}^{od}$ along the network elements that make up $r_{\alpha\beta}$.

To expedite the loading of route flows onto the network elements, it is recommended to use a cascade algorithm by destination anchor β from all anchors α of all origin zones.

4.2. The characterization of terminal times

This step depends on which disaggregate information is available to the study analyst. Today, detailed data disaggregated up to the individual building have become available as information about trip ends, at least in the main cities of the developed world. Similarly, exhaustive databases have become available for transport networks, which are impractical for assignment but amenable to a given, one-shot analysis by using a GIS. The task then involves:

1. to select a zoning system (as usual).
2. to identify the anchor nodes relevant for each zone.
3. then, within each zone, to characterize the terminal vector by disaggregate trip end by computing its terminal cost to each anchor. The resulting vector $(t_{\omega\alpha})_{\alpha \in o}$ with appropriate statistical weight w_{ω} must be incorporated into provisional accounts of $\sum_{\omega} w_{\omega} t_{\omega\alpha}$ and $\sum_{\omega} w_{\omega} t_{\omega\alpha} t_{\omega\alpha'}$ by pair (α, α') of origin anchors. After dealing so with all trip ends, then the mean vector and the matrix of variance-covariance for the distribution of terminal times may be recovered straightforwardly.

4.3. DCM by OD pair

Given an OD pair (o, d) and associated anchoring sets A_{od} and B_{od} , the treatment of trip ends provides a vector $\mu_o = [\mu_{\alpha}^o : \alpha \in A_{od}]$ and a variance-covariance matrix $\Sigma^o = [v_{\alpha\alpha'}^o]$ of the origin terminal costs, as well as their counterparts μ_d' and Σ'^d on the destination side. Assume further that the minimum network costs $t_{\alpha\beta}^*$ have been obtained by shortest path search onto the network, and that the origin and destination terminal costs are independent.

Then the $T_{\alpha\beta}^{od}$ variables have mean values $\bar{T}_{\alpha\beta}^{od} = \mu_{\alpha}^o + t_{\alpha\beta}^* + \mu_{\beta}^{\prime d}$ and general covariance between $T_1 = (\alpha_1, \beta_1)$ and $T_2 = (\alpha_2, \beta_2)$ of $\chi_{12} \equiv \text{cov}(T_1, T_2) = v_{\alpha(1)\alpha(2)}^o + v_{\beta(1)\beta(2)}^d$.

These make up the inputs to a logit or probit model, as in Section 3. The application of a logit model requires to estimate a common, somewhat average variance for the χ_{rr} by OD pair of zones, and to drop the other covariance terms. The probit model is more suitable since it takes into account both heteroskedasticity and covariance. Although no closed-form formulas are available in the general case beyond three options, the formulas given by Clark (1961) indeed yield excellent approximations, as pointed out by Maher and Hughes (1998) and checked by Samadzad (on-going PhD Thesis).

4.4. Network and terminal assignment

Overall, the assignment of trip demand to the transport system, both network and terminal (subnetwork), can be performed efficiently by the following algorithm:

0. Characterize the terminal vectors of origin zones and of destination zones.
1. Initialize cumulated flow by network link (and any other relevant network element) at zero.
2. By destination zone $d \in D$:
 - (2a) By anchor $\beta \in d$, search for shortest paths $r_{\alpha\beta}$ on the main network from all anchors α of all origin zones $o \in O$. Initialize at zero a vector of node inflows associated to β .
 - (2b) By origin zone o , evaluate the choice probabilities $p_{\alpha\beta}^{od}$ of the anchor pairs of zonal OD pair (o, d) , and add the associated flow $x_{\alpha\beta}^{od} = q_{od} p_{\alpha\beta}^{od}$ to component α in the vector of node inflows associated to β .
 - (2c) By anchor $\beta \in d$, load the inflow vector, from all anchors α of every origin $o \in D$, onto the shortest network tree rooted at β , and add the resulting local flows to the cumulated local flows.

This yields the vector of link flows cumulated over all OD pairs. To evaluate the subnetwork flows, in Step 2c the $x_{\alpha\beta}^{od}$ flow could be added to subnetwork cumulated flows x_{α}^o and x_{β}^d .

5. Numerical experiment

The roadway network of the Paris metropolitan area is modelled by the French Department for Transport (Dreif, 2006) for planning purposes. The network model includes about 15,000 nodes, 40,000 unidirectional links and 1,300 TAZs. This makes up our main network.

We derived the terminal travel conditions from combination of three detailed databases as follows: (i) 'BD Topo' by the French Geographical Institute (IGN) provides a comprehensive description of roads with metric accuracy; (ii) 'MOS' by the regional Institute for Land Planning (IAU-IDF) provides the type and intensity of land use; (iii) The General Population Census by the French Institute for Statistical and Economic Studies (Insee) yields the number of people and jobs at the block level. Applying the TransCad GIS to the comprehensive roadway database, three or four anchor nodes were selected for each TAZ. Other assumptions include the OD matrix of trip flows at the morning peak hour and the vector of link travel times, on the basis of a classical user equilibrium assignment.

Fig. 3 depicts the topology of the roadway network. Fig. 4 is focused on a sub-area located near the centre; it depicts the link flows under disaggregate trip ends, whereas Fig. 5 indicates the difference between these volumes and those obtained from standard all-or-nothing assignment. It appears that the magnitude of change may reach up to 2,000 veh/h, which amounts to 25% of flow on the busiest motorways; on urban arterials the absolute magnitude is smaller (up to 500 veh/h) but the relative magnitude can be much higher, up to 80%. At the present stage however, such changes are indicative rather than firmly-grounded evidence, since they stem from all-or-nothing assignment. Indeed, under traffic equilibrium some important re-routing might take place in the model with disaggregate trip ends.

More significant are the aggregate indicators about travel time as minimized by every network user – according to each of the two models. Average OD travel time is decreased from 9.55 min in the classical model to 9.44 min in the disaggregate model – a relatively small variation. Considering each OD pair as a class, the interclass variance of average travel time is decreased from 25.2 min² to 23.4 min² in the disaggregate model, in which there is also intraclass variance which amounts to 4.97 min² on average, yielding total variance of 28.4 min² which is 10% higher than in the classical model. This reveals that the zoning system accounts for about 80% of the variance in trip travel times – the remaining part is far from negligible.

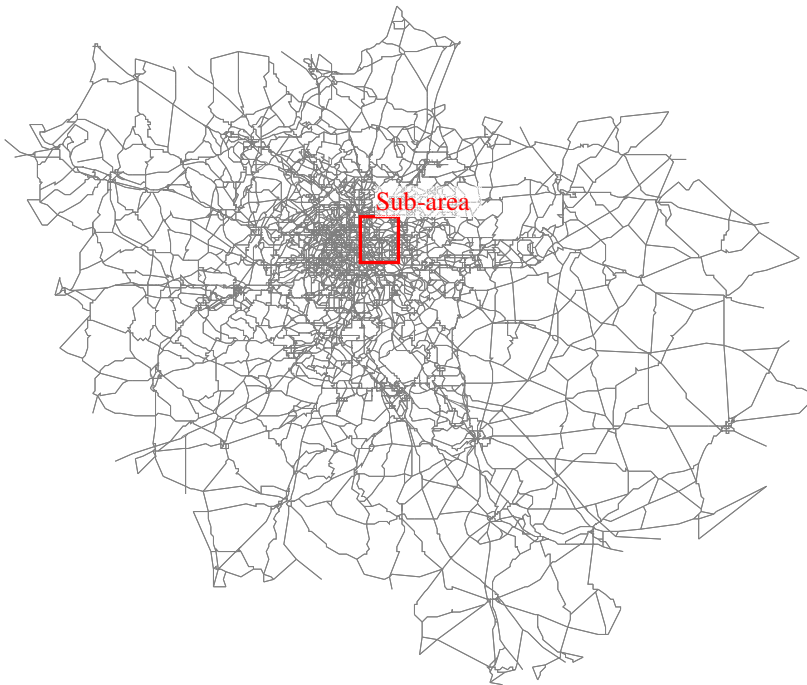


Fig. 3. Network map.

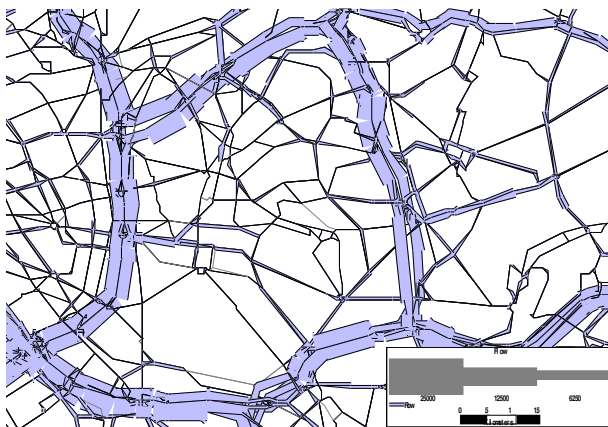


Fig. 4. Link flow within sub-area under disaggregate trip ends.



Fig. 5. Difference in link flow between disaggregate model and aggregate one.

6. Conclusion

Modeling principles have been provided for traffic assignment with disaggregate trip ends, along with an application methodology. On assuming that the terminal costs and the network costs are independent, the assignment problem is decomposed into two stages respectively network and terminal. As the disaggregate assignment model is focused on terminal travel conditions, further research may be targeted to not only network issues (e.g. transit, multimodal, equilibrium, dynamic) but also “supernetwork” issues of combining trip distribution and even trip generation with network assignment in an integrated, far-reaching assignment model (Cf. Sheffi, 1985; Oppenheim, 1995). Our probit treatment of terminal times makes up an attractive alternative to the previous, logit-based supernetwork models.

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