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► **To cite this version:**

Lawrence Michael Brown. An Interpretation of the Haasen-Kelly Effect. Philosophical Magazine, 2010, pp.1. 10.1080/14786431003705355 . hal-00603439

**HAL Id: hal-00603439**

**<https://hal.science/hal-00603439>**

Submitted on 25 Jun 2011

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### An Interpretation of the Haasen-Kelly Effect

Journal:	<i>Philosophical Magazine &amp; Philosophical Magazine Letters</i>
Manuscript ID:	TPHM-09-Jul-0316.R2
Journal Selection:	Philosophical Magazine
Date Submitted by the Author:	18-Dec-2009
Complete List of Authors:	Brown, Lawrence; University of Cambridge, Physics
Keywords:	dislocations, aluminium, nickel
Keywords (user supplied):	flow stress



Only

## An Interpretation of the Haasen-Kelly Effect

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*Abstract:* In 1957, P. Haasen and A. Kelly presented an account of experiments which showed that in Al and Ni single crystals the mere act of unloading during a tensile test caused a small but reproducible transient increase in flow stress, a 'yield-point'. The increase is proportional to the current work-hardened stress level. It occurs at the lowest temperatures, where diffusion is not possible, but it is enhanced at higher temperatures. The effect was interpreted as arising from dislocations which 'anchor themselves during unloading and that this process of anchoring, which prevents reverse plastic flow, is also responsible for the yield-point effect on reloading'. Recent ideas emphasising the role of dislocation dipoles as sources for slip provide a natural explanation for these effects.

(Ms date 18/12/2009)

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1. *Introduction:* A simplified description of the work-hardened state envisages three idealised types of dislocation sources: (1) segments in small-angle boundaries which are mainly found in the ‘carpets’ or regions roughly parallel to glide planes, but also in the incidental dislocation boundary (IDB) structure within the geometrically necessary boundary structure (GDB) encasing slip bands (Kuhlmann-Wilsdorf and Hansen 1992); (2) primary screw dislocation dipoles which limit slip in a direction perpendicular to the Burgers vector; (3) primary edge dislocation dipoles which limit slip parallel to it, as sketched by Hirsch (1975). The dipole arrays act as staging posts for dislocations, exchanging with the mobile segments to produce ever finer and more numerous dipoles, but allowing the mobile segments to penetrate the array with a certain probability. The arrays are thus both sources and sinks for mobile dislocations. Edge dipoles are the most numerous sources.

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In thinking about the dipole arrays, it is important to recognise<sup>1</sup> that the local resistance they pose to primary dislocations is not proportional to the square root of their dislocation density, nor even to their dipole density, but varies from dipole to dipole depending only on the height<sup>2</sup> of each dipole. The dipoles are spaced about ten times further apart than their average height. Because the dipole arrays consist of a distribution of heights, the arrays are ‘statistically deformable’, meaning that they have a mixture of weak and strong points. For screw arrays, there is a minimum height of dipole which can survive without cross-slipping to extinction. For edge arrays, the smallest dipoles transform to faulted dipoles, which can act as sources of deformation twinning (Steeds 1967, Niewcas and Saada, 2002). As the applied stress increases, the maximum height of stable dipole diminishes, until at the onset of Stage III hardening, there are no screw dipoles remaining in the crystal, only edge dipoles

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<sup>1</sup> This seems first to have been stated by Friedel (*Dislocations*, 1964, Pergamon Press, p 248): ‘. . . in many cases the distance  $h$  between slip planes. . . is so small that the maximum elastic interaction. . . between dislocations in neighbouring planes is large compared with the applied stress. The whole Taylor model then breaks down.’

<sup>2</sup> The ‘height’ of a dipole is the distance  $h$  measured perpendicular to the slip plane between the two dislocations of opposite sign making it up.

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4 and small-angle boundaries are left. Full confinement of the slip  
5 bands is then lost.  
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9 Within the dipole arrays there is opportunity to exchange  
10 partners. In the case of the edge dipoles, screw dislocations  
11 connecting one dipole with another can move to transfer  
12 partners. In general, higher (coarser) dipoles have more energy  
13 per unit length than finer (narrower) ones, and because of non-  
14 linear effects, interstitial dipoles (half-planes pointing inwards)  
15 have greater energy than vacancy dipoles. As deformation  
16 proceeds, the dipole array thus not only becomes finer, it tends  
17 to contain more narrow vacancy dipoles than interstitial ones.  
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24 Niewcas (2002) presents beautiful detailed observations of the  
25 dipole debris produced by the monotonic deformation of Cu.  
26 Although the screw dislocations are not retained in the thin foils,  
27 the distribution of edge dislocation dipoles is broadly consistent  
28 with this simplified description.  
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34 *2. The Haasen-Kelly (1957) Effect:* When their classic paper on  
35 yield phenomena in face-centred cubic single crystals was  
36 published, what was at issue was whether or not the small yield  
37 points observed after ageing deformed crystals were due to the  
38 formation of Cottrell atmospheres at dislocations. Haasen and  
39 Kelly showed that the mere act of unloading the crystals  
40 produces a small but measurable yield point. Fig 1 shows a  
41 schematic diagram, redrawn from their paper. The crystals must  
42 be unloaded by more than 50% to produce the full effect. The  
43 effect is observed at temperatures too low for any deformation-  
44 induced defect to migrate, but it is enhanced at room  
45 temperature. The presence of impurities seems to reduce it.  
46 Upon reloading after unloading, the size of the yield point as  
47 defined in Fig. 1 is approximately proportional to the applied  
48 stress above the level at the end of stage I of the hardening  
49 curve. However, the effect is very small, the yield point at the  
50 lowest temperature amounting to about 1.5% of the applied  
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stress. It appears that similar effects are observed in zone-refined iron (Wray and Horne 1966).

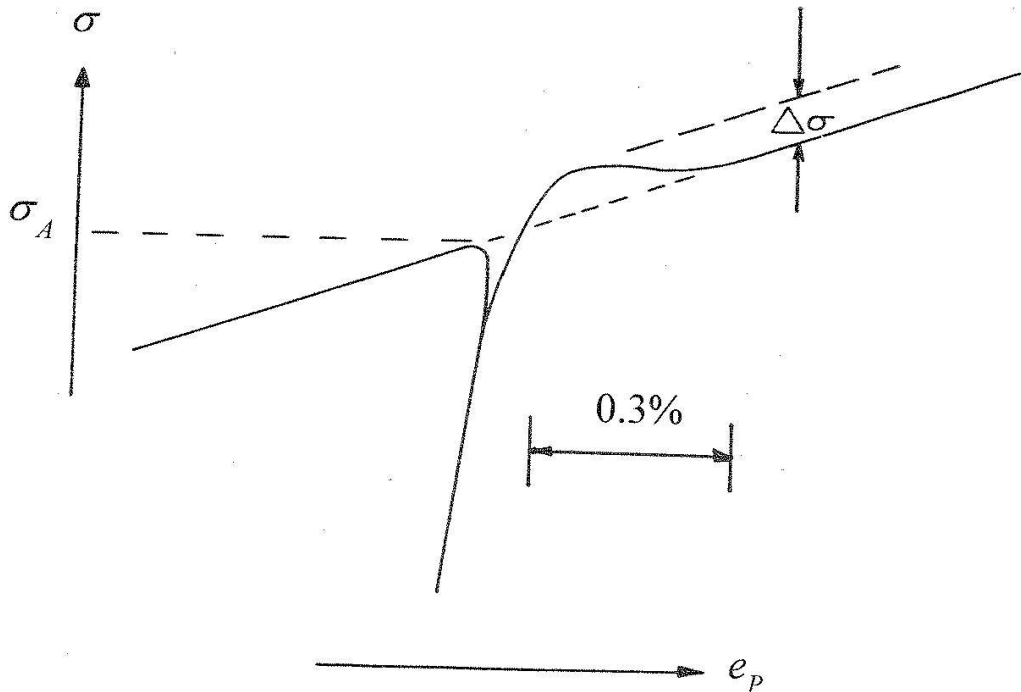


Fig. 1 (adapted from Haasen and Kelly (1957)). The definition of the observed yield point,  $\Delta\sigma$ , is shown. The plastic strain interval is estimated from another diagram in their paper.

3. *A simple interpretation:* The stress required to split an edge dislocation dipole is given by Brown and Nabarro (2004) as

$$\sigma_c \approx \frac{\mu b}{8\pi(1-\nu)h} \left( 1 \pm \frac{6\gamma_G(1+\nu)(1-2\nu)^3}{\pi^2(1-\nu)^2} \right) = \sigma_A \left( 1 \pm \frac{\Delta\sigma_A}{\sigma_A} \right). \quad (1)$$

Here,  $\mu$  is the shear modulus,  $b$  the magnitude of the Burgers vector,  $h$  is the dipole height,  $\nu$  is Poisson's ratio, and  $\gamma_G$  is the Grüneisen constant, about equal to two for most materials, and a measure of non-linear effects. The term outside the bracket is the usual splitting stress in linear elasticity, as derived for

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4 example by Kelly and Groves (1970). Under such a stress,  
5 linear elasticity shows that a dipole can be flipped from one  
6 stable configuration to another and split entirely, each  
7 dislocation then becoming mobile, free to travel separately to  
8 another obstacle. For the dipole to act as a source of  
9 dislocations, the splitting stress equals the applied stress,  $\sigma_A$ .  
10 The term inside the bracket is an estimate of the correction due  
11 to non-linear elasticity. It *increases* the flipping stress but  
12 *decreases* the splitting stress for interstitial dipoles, and does the  
13 opposite for vacancy dipoles. One can imagine that  
14 compressive regions are elastically harder than tensile ones, so it  
15 becomes more difficult to force dislocation half-planes over one  
16 another when they point toward each other, but easier when they  
17 point away from each other.  
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27 Brown and Nabarro also show that screw dislocations  
28 connecting dipoles of opposite sign are subject to an effective  
29 stress tending to convert interstitial dipoles into vacancy ones.  
30 The magnitude of the stress is about 10% of the stress required  
31 to split the dipoles, if the dipoles are spaced about ten times  
32 their height. But the stress required to split the dipoles is equal  
33 to the current level of the work-hardened flow stress, so the  
34 screws cannot respond to the small conversion forces unless the  
35 crystal is unloaded.  
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43 One has here a possible basis for an interpretation of the  
44 Haasen-Kelly effect: upon unloading, screw dislocations can  
45 move so as to convert interstitial dipoles into vacancy ones,  
46 thereby increasing slightly the stress required to split them so as  
47 to release the dislocations. The magnitude of the effect is just  
48 proportional to the flow stress. The process requires slip alone,  
49 no diffusion is necessary.  
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55 Equation (1) predicts a surprisingly large effect. The fractional  
56 change in splitting stress is predicted to be about 13% if  
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Poisson's ratio<sup>3</sup> is taken to be about 1/3. The experimental value is much less than this. In order to achieve agreement between theory and experiment, a slightly more complicated analysis is required.

4. *A more realistic interpretation:* According to Nabarro and Brown (2004), the action of the mobile screw dislocations is to convert dipole arrays with equal sizes of vacancy and interstitial dipoles into arrays with smaller vacancy dipoles and larger interstitial ones, whilst conserving both the number of dipoles and total vacancy/interstitial content of the array. Only if the array connects with external surfaces can material be extruded or intruded from the slip bands. This happens in persistent slip bands in cyclic plasticity, but in unidirectional deformation most slip bands do not connect with outer surfaces. The unloading impact on interstitial and vacancy dipoles of equal size is to create smaller vacancy dipoles, harder to split on reloading, as well as larger interstitial dipoles, easier to split on reloading. Not all dipoles can be modified in this way, only some of the edge dipoles – those connected to screw dislocations – whereas the screw dipoles (if any) and the links of dislocations in the small-angle boundaries are likely to be unmodified on unloading.

Fig. 2 shows schematically what can happen. On the left is sketched  $n_s(\sigma)$ , the number of dipoles per unit area which can act as a source of mobile dislocations under the stress  $\sigma$ . Under load, all the dipoles which can be split at stresses lower than  $\sigma_A$  are wiped out, whereas the stronger dipoles remain. On unloading, the step-function discontinuity in dipole density is modified: some narrower vacancy dipoles are created, which increases the density above the step, but also some wider interstitial dipoles, which increases the density below it. At the

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<sup>3</sup> The value of Poisson's ratio to be inserted here is *not* the value one uses to produce accurately the ratio between the line energies of edge and screw dislocations, but it is the value which gives the ratio between the shear modulus and the bulk modulus. Using the tables given by Hirth and Lothe (1992) one finds that for all the metals discussed here the value can be taken to be  $0.33 \pm .03$ .



step itself, the density is reduced by an appreciable fraction. The non-linear forces producing the rearrangement of the dipoles are small, so only dipoles very close to those at the step are affected. The dipole distribution is modified without external plastic strain, only the motion of the short segments of screw dislocations can produce some reverse strain

The relationship between the dipole density and the plastic strain increment  $de_p$  caused by a stress increment  $d\sigma$  is

$$de_p = 2\lambda b n_s(\sigma) d\sigma, \quad (2)$$

where  $\lambda$  is the mean free path of each of the two liberated dislocations split from the dipole. For strains very near to the unloading and reloading event, we can assume a constant source density and a constant mean free path, so

$$\frac{d\sigma}{de_p} \approx 2\lambda b \frac{1}{n_s(\sigma)}. \quad (3)$$

On the right-hand side of Fig. 2 is sketched the stress-strain curve upon reloading. We see that the 'premature yielding' of Fig. 1 results from the extra interstitial dipoles produced by the rearrangement, and the 'yield point' is produced by the extra vacancy dipoles. The shape of the observed stress-strain curve is roughly reproduced if about half the sources at the step undergo rearrangement. We see that the magnitude of the yield-point, as defined in Fig. 1, is lower than the fractional change in splitting stress estimated in Eq. (1) by a factor of about 6, bringing it much more in line with the experimental observations.

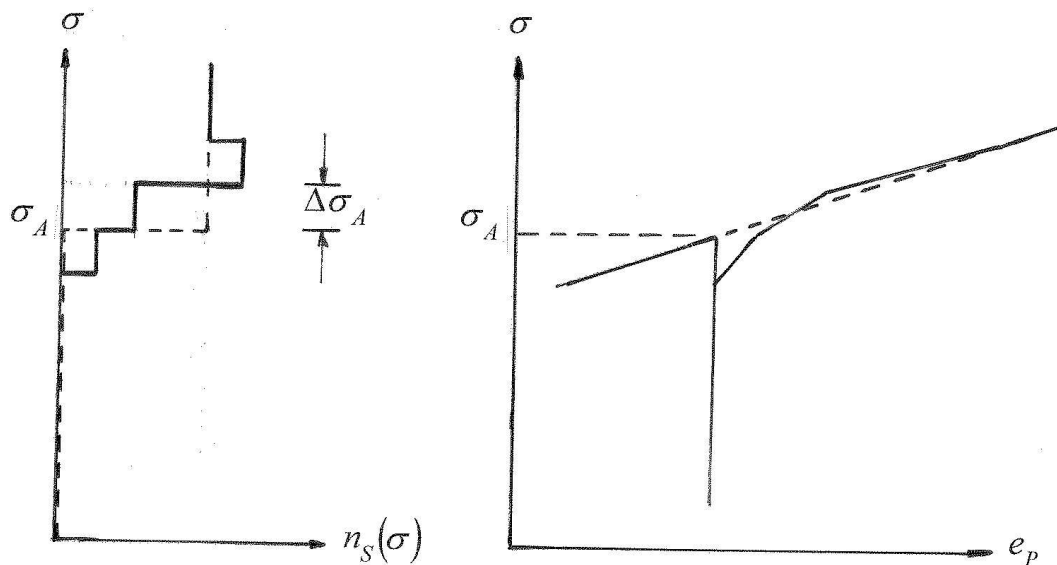


Fig. 2. On the left hand side is sketched the density of dislocation sources (abscissa) as a function of the applied stress under load (ordinate). There is a step discontinuity at the current stress level. On unloading, screw dislocations move to create larger interstitial dipoles and smaller vacancy dipoles, thereby modifying the distribution in the immediate neighbourhood of the discontinuity. The corresponding stress-strain curve is sketched in the right-hand diagram. The slope of the stress-strain curve is proportional to the reciprocal of the source density. If no rearrangement occurs, it follows the dashed line. Rearrangement produces a yield point at the onset of plastic flow. The slopes of the straight-line segments on the reloading curve have been estimated from equation (3), using the known slope of the basic work-hardening curve in Fig. 1.

The sketch of Fig. 2 is the outcome of a crude attempt to reproduce the experimental curve. It is based upon the assumption that no macroscopic instability occurs upon yielding. However, the curve of Fig. 1 shows a short range of strain in which the stress-strain curve shows zero or even negative slope. In this range, the Considère condition for plastic

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4 stability is clearly violated. Although Haasen and Kelly make  
5 no mention of it, some sort of Lüders band probably propagates  
6 through their specimens at yield. The reduced slope of the curve  
7 produced by modelling in Fig. 2 can produce plastic instability  
8 and result in an apparent zero or negative slope in the output of  
9 the tensile testing machine. A more sophisticated model must  
10 take this into account before one can reliably derive the source  
11 density distribution from the experimental data.  
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18 At temperatures where cross-slip of the screw dislocations can  
19 occur, the slip required for the rearrangement will be facilitated  
20 because the screws will be more flexible, not constrained to one  
21 glide plane. Rearrangement of more pairs of dipoles will occur.  
22 Cross-slip is accomplished by the thermally activated motion of  
23 jogs, with an activation energy around 0.1eV, that is, cross-slip  
24 becomes fast between 20K and 77K, the temperature range  
25 where an enhancement of the Haasen-Kelly effect is observed.  
26 The action of impurities is to anchor the jogs, greatly slowing  
27 the cross-slip, so impeding the rearrangement. Thus one finds a  
28 simple semi-quantitative explanation for the temperature and  
29 impurity dependence of the effect.  
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39 There is one further point worth mentioning: for edge dipoles in  
40 the shape of elongated isolated loops, there may be a bowing  
41 stress to overcome, as well as a passing stress. Bowing under  
42 stress forces the dislocations to be curved at the point of  
43 splitting, so it reduces the nominal splitting stress for an infinite  
44 straight dipole. The bowing stress is equal to the nominal  
45 passing stress if the dipole is about  $8\pi(1-\nu) \approx 17$  times longer than  
46 its height. Under these circumstances, it is easy to use the  
47 calculations of Brown (2006) to estimate the reduction in  
48 passing stress: it is reduced by about 50%. However, the  
49 diagram of Fig. 2 is unchanged. All that happens is that the  
50 contribution of the passing (splitting) stress to the overall flow  
51 stress is reduced by a factor that depends upon the shape of the  
52 dipole loops. Such a factor will further reduce the size of the  
53 yield point.  
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5 The interpretation of the Haasen-Kelly effect proposed here is  
6 unlikely to be unique. However in the voluminous literature on  
7 work-hardening I can find no other attempt at an explanation  
8 more detailed than that given by Haasen and Kelly in their  
9 original paper. Their explanation, quoted in the abstract, is  
10 succinct and correct!  
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16 5. *Conclusions:* The assumption that a major component of  
17 dislocation sources in unidirectional deformation comprises  
18 edge dislocation dipoles leads to a simple semi-quantitative  
19 explanation of the Haasen-Kelly effect.  
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26 6. *Acknowledgements:* It is a pleasure to acknowledge the  
27 many years of discussions and encouragement from Tony Kelly.  
28 I am grateful to a referee for helpful criticism, and to the  
29 Cavendish Laboratory and to Robinson College for support.  
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### Captions for Figures

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