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Exploring the polycentric city with an agent-based model

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Abstract

The standard Urban Economics model of Alonso, Muth, Mills, describes analytically an equilibrium location of households in urban areas. We present an agent-based model, using simple interactions between agents, and able to reach this equilibrium in a dynamic way. The agent-based model allows us to simulate the development of a city by combination of heterogeneous agents and the introduction of several work centers. This tool allows the addition of a wide variety of features to the Urban Economics model to study their effects. We focus here on the study of the polycentric city.

Keywords: agent-based model, urban economics, location choice, polycentric city

Introduction

Agent-based models are widely used to simulate traffic at a microscopic level. The global aim of the work presented here is to use this tool in Urban Economics to deal with research questions regarding urban systems, for instance the location of households with respect to their income or the study of the polycentric city.

There are in the literature numerous analytical works on the Urban Economics model of Alonso, Muth, Mills (AMM model), studying the factors which explain the location choices of households within this model ([1], [2]), so that it is an interesting benchmark to start an exploration using agent-based systems. The first stage of this work is the reproduction of the classical results of the AMM model. To build an agent-based model, the analytical model has to be discretized: there is a finite number of agents who interact individually. This is a difference with the analytical model, which is continuous. This model provides an illustration of a discrete model converging to the continuous AMM model for large population sizes ([3]). A comparison between the results of the simulations and those of the AMM model is presented in section 2.1. The use of agent-based models allows us to handle easily agents' states, rules of behaviour and environments. Sets of agents such as neighborhoods can be used, so that it is easy to introduce neighborhood effects. Individual and collective behaviours can be monitored in a simple way. In addition, the agent-based simulations are dynamic. This time dependence allows us to see the equilibrium emerge from the interactions between agents, which is described in section 2.2. Out of equilibrium dynamics could also be studied. Once the results of the agent-based model are validated by the comparison with the analytical model, the model can be made more complex by adding different ingredients, and firstly income groups. This allows us to explore phenomena which are difficult or impossible to treat analytically. This work is presented in section 3 with the modelization of a city with several work centers.

1 Description of the model

1.1 Urban Economics model

The AMM model was developed to study the location choices of economic agents in a urban space, with agents competing for housing (which is identified with land in the simplest version of the model). Agents have a transport cost to commute for work. Their workplace is located in a central business district (CBD), which is represented by a point in the urban space. Amenities can be introduced in certain versions of the model to study their influence on the location of agents ([4], [5]). Agents usually represent single workers, but they can also be used to describe households, which can be made more complex in further versions of the model. Housing is rented by landowners who rent to the highest bidder, which introduces a competition for housing between agents. They also compete with an agricultural use of land, which is represented by an agricultural rent R_a .

Agents have a utility function describing their welfare, which is here a Cobb-Douglas function $U = z^\alpha s^\beta$, where z is a composite good representing all consumer goods except housing and transport, s is the surface of housing, α and β are parameters describing agents' preferences for composite good and housing surface, with $\alpha + \beta = 1$. Agents also have a budget constraint $Y = z + tx + ps$, where Y is their income, t the transport cost per unit distance, x their distance to the CBD and p the price of a unit surface of land at location x . See [6] for a more detailed description of this model.

The analytical model reproduced in this work with agent-based simulations is a closed city model: the number of agents N in the city is chosen exogenously and remains constant during a simulation. This model can be solved analytically in a two-dimensional space if $R_a = 0$. For $R_a > 0$ it can be solved numerically for one income group. With a population divided into several income groups, one needs to build a specific algorithm for the resolution of the model, whose general form is described in [6].

1.2 Agent-based implementation

Let us describe in this section the agent-based implementation of the standard monocentric AMM model. In the agent-based system, the simulation space is a two-dimensional grid where each cell can be inhabited by one or several agents, or used for agriculture. These cells have a fixed land surface s_{tot} . The unit of distance is taken as the side length of a cell. The CBD is represented by a point at the center of the space.

At the initialization, a population of N agents is created. These agents are placed at random locations. Land prices are equal to the agricultural rent $p_0 = R_a$. At a given location x , they occupy a quantity of land which is the optimal consumption of land conditional on price p : $s = \beta \frac{Y - tx}{p}$. This determines the quantity of composite good they consume and also their utility.

1.2.1 Dynamics of moves

The main feature of the model consists in agent-based dynamics of moves and bids in the urban space. The rules defining agents' moves are suggested by the competition for land in the analytical model.

Agents move with no cost. Let us describe an iteration n of a simulation, changing the variables from their value at step n to their value at step $n + 1$. An agent which will be candidate to a move and a cell are chosen randomly. The price of this cell, located at a distance x of the center, is p_n at step n . The optimal housing surface that the agent can choose in the candidate cell is $s = \beta \frac{Y - tx}{p_n}$, which allows us to compute his composite good

consumption and the utility that he would get thanks to the move and to compare it to his current utility.

If the agent candidate can have a utility gain $\Delta U > 0$ by moving into the candidate cell, then he moves. In this purpose he raises the price of the candidate cell by proposing a bid $p_{n+1} = p_n(1 + \epsilon \frac{s_{occ}}{s_{tot}} \frac{\Delta U}{U})$, where ϵ is a parameter that we introduce to control the magnitude of this bid. Prices evolve quicker if this parameter is high. s_{occ} is the surface of land occupied by other agents in the cell and s_{tot} the total land surface of the cell. The factor $\frac{s_{occ}}{s_{tot}}$ makes the bid higher if the cell is more occupied, that is to say, more attractive. The first agent to move in an empty cell does not raise the price.

The price is a price per unit surface, linked to a cell. When an agent bids higher, the price is changed for all agents in the cell. Their consumption of land is also changed according to $s = \beta \frac{Y-tx}{p_{n+1}}$ and their utility is computed again. This feature of the model defines a competition for land between agents, as in the standard analytical model¹.

1.2.2 Surface constraint, time decrease of price

We described how prices increase in the model. Due to the stochastic choice of agents and cells, prices can rise above their equilibrium level at some locations, making some cells unattractive. Indeed, the price of a cell where several agents move in successively can increase so much that it reaches a value which makes the cell unattractive. In this case, agents living there will progressively leave the cell for more attractive locations.

Therefore we choose to decrease exponentially the price of cells which are not completely full², according to $p_{n+1} = p_n - (p_n - R_a \times 0.9)/T_p \cdot s_{av}/s_{tot}$, where T_p is a parameter determining the speed of decrease of prices, $s_{av} = s_{tot} - s_{occ}$ and s_{tot} are the non occupied surface and the total surface of the cell. If no agent moves in, the price decreases according to this formula until it reaches the agricultural rent, where the decrease stops: the cell is then used for agriculture. The factor $\frac{s_{av}}{s_{tot}}$ makes this time decrease quicker as the cell is emptier and thus less attractive.

1.2.3 Parameters

The different parameters of the model are listed in table 1. Their value has been chosen according to several criteria. First for the parameters of the model itself: α , β , Y , t , N , R_a , s_{tot} . Their values have been chosen mainly for technical reasons regarding the comparison between the (continuous) analytical model and the (discrete) agent-based model, but naturally other values could have been chosen, without changing the qualitative behaviour of the model. For instance, a higher population N could have improved the agreement between the analytical and the agent-based model, but it would have slowed down the simulations.

Parameters ϵ and T_p are specific to the agent-based model. Their values have been chosen such that the competition between agents on the housing market is efficient and

¹Specific situations arise which do not appear in an equilibrium (static) model. For instance, an agent may want to move into a candidate cell that is already full, proposing a higher bid on the price of housing there. Then we make the following choice: the price of housing is raised for all agents living in the cell to the level of this new bid, but the agent candidate does not move. Then agents' surfaces of housing and utilities are computed again. As the price is raised, housing surfaces are decreased and there is a chance that enough space is freed for the candidate agent to move in, in which case he does. Else, he has to wait until he is proposed another move.

²With two income groups, it is difficult to determine whether a cell is full or not: we choose to let the price decrease if the smallest surface of housing s_{min} of agents there is smaller than the available surface of the cell s_{av} .

Parameters	Description	Default value
α, β	Preferences for composite good and housing	0.75; 0.25
Y_p, Y_r	Incomes of poor and rich agents	300, 480
t	Transport cost (unit distance)	10
N	Population	1000×2
R_a	Agricultural rent	5
s_{tot}	Surface of a cell	50
ϵ	Bidding parameter	0.5
T_p	Time decrease of the price of non-full cells	100

Table 1: Parameters of the model

the system reaches the equilibrium in the whole city, as described in section 2.3.2. The agent-based model has different behaviours and for instance does not reach an equilibrium (the utility of agents does not become completely homogeneous across the city) for certain values of these parameters, but the study of these different behaviours is beyond the scope of the present paper. Let us just note that these parameters are introduced here to provide a minimal framework allowing an agent-based simulation to reach the equilibrium of the AMM model. They do not seem to have an immediate correspondence with relevant or measurable variables explaining the dynamics of urban land markets.

1.3 Socio-economic outcome

The goal of this work is to study the urban social structure and the socio-economic outcomes of the models developed here. Thus we study especially some variables of the model, which characterize these outcomes. Our benchmark is a reference simulation of a monocentric city with two income groups. Then we compare the values of the socio-economic variables in the reference simulation and in more complex models to observe the effects of the modifications which are introduced in the standard model.

To this end, we study variables which we find most relevant to describe the outcomes of the models. The utility of individuals is associated to their welfare and gives an economic outcome of the models. The cumulated distances of agents' trips to work, associated with housing surfaces, give their environmental outcome, which could be conveyed for instance in terms of greenhouse gases emissions associated to transport and land use (heating and cooling). The evolution of social inequalities are given by the difference in the utility of individuals of different income groups.

The use of agent-based systems allows naturally an easy access to any individual or global variable of the model, so that effects of the models on land rents for instance can also be studied.

2 Comparison with the analytical model and time evolution

2.1 Results with two income groups: model 1

The simulations allow us to reach the equilibrium of the AMM model, as can be seen on figure 1. This equilibrium corresponds to a configuration where no agent can raise his utility by moving, and therefore no agent has an incentive to do so. In each income group, individuals have an identical utility across the city. With two income groups, the utility of "rich" agents is still higher than that of "poor" agents, because they do not have the same exogenous parameters (they have different incomes). The cells which are occupied are those closest to the city center (CBD), the prices at the border of the city are equal to the agricultural rent and prices increase with the proximity to the center. The surfaces of

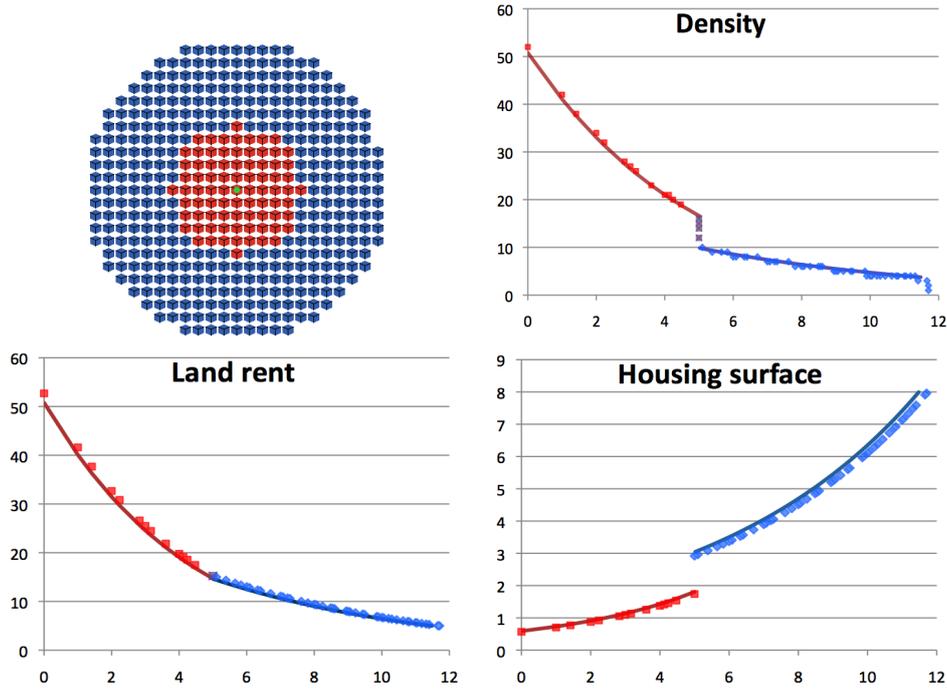


Figure 1: Top left panel, shape of the city. Other panels: comparison between the results of the agent-based model and the analytical results. Density, land rents and housing surfaces as functions of the distance to the center. The continuous lines represent the analytical results, whereas the symbols indicate the results of the agent-based model. Poor agents are in red and rich agents in blue, purple symbols on the density graph indicate cells where agents of both income groups live together.

housing increase when the distance to the center increases. As in the analytical equilibrium, rich agents are located at the periphery of the city, where they pay lower land prices and have higher housing surfaces, but also with higher transport costs. This reproduces the pattern observed in most North-American cities ([6]).

The results of the simulations do not match exactly the analytical results because of the effects of the discretization (which leads for instance to a border of the city which is not exactly at the same distance from the center all around the city). The discrete character of the simulation appears in particular on the density curve, which is like a step function in the simulations and a continuous function in the analytical results. The equilibrium of the agent-based model is described in the following sections.

2.2 Emergence of a city

We now describe how a city emerges from the interactions between individuals during a simulation. Initially, all agents are located randomly and all prices are equal to the agricultural rent. Density is also random and quite low as agents are dispersed over the simulation space. Then agents move mainly towards the CBD as shown on figure 2 and bid higher, so that the rent curve evolves from a flat rent to the equilibrium rent. At the beginning of the simulation (figures 2(a) and 2(b)), agents gather at the city center without competing much for land, because many cells close to the center are still not full. But when all agents are concentrated around the center (from figure 2(d) on), most bids do not result in an agent moving, for few cells have a sufficient available surface to allow an agent to move in with an interesting utility. This feature of the model arises because

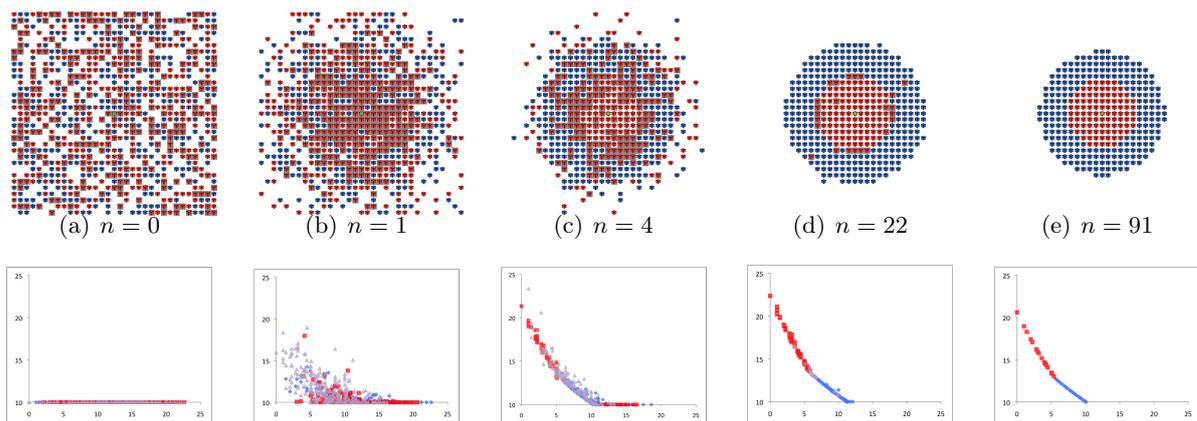


Figure 2: Evolution of the shape of the city (first line) and of the price of land as a function of the distance to the center (second line) during a simulation. On the first line, the CBD is indicated by a green point. Cells whose background is grey indicate that poor and rich agents live there; these cells are displayed as purple symbols on the second line. At the equilibrium, the city is completely segregated and there are no more such cells. n indicates the mean number of moves per agent since the beginning of the simulation.

the vacancy rate of the standard Urban Economics model we reproduce is zero.

The main variable which indicates the proximity to the equilibrium is the homogeneity of the utility of agents. To describe this homogeneity, we use the relative inhomogeneity of the utility defined as $\Delta U_{\max} = (U_{\max} - U_{\min})/U_{\max}$. With two income groups, we use the maximum of this variable within income groups. Initially, this variable has a quite high value as all agents are located randomly, and it decreases during the simulations. We choose to stop the simulations when the relative variations of utility within income groups are smaller than 10^{-6} , which means that ΔU_{\max} has decreased by approximately five orders of magnitude, as shown on figure 3.

The model allows to test explicitly if no agent has an incentive to move: when $\Delta U_{\max} < 10^{-6}$, each agent tests if he can raise his utility by moving into any other cell, regardless of a sufficient or not sufficient available surface in the cell. The relative possible variations of utility are found to be of the same order of magnitude as ΔU_{\max} .

The standard deviation of the utility can also be computed. It gives a more precise idea of the variations of utility in the model. On figure 3 are displayed the evolution of the relative inhomogeneity of utility in each income group, and the corresponding evolution of the standard deviation of utility in each income group (also divided by the maximal utility) during a simulation. The latter is always smaller than the former, as should be. This evolution is given as an illustration: because of the stochastic dynamics of the model, it varies across simulations. The equilibrium of the agent-based model is described in more detail in section 2.3.2.

2.3 Equilibrium

2.3.1 Existence and uniqueness of the analytical equilibrium

It is shown in [7] that there exists one unique equilibrium for the standard monocentric AMM model with one or more income groups. The proof of this result is based on boundary rent curves between income groups and between agricultural and residential land uses. Although this result seems difficult to extend to any polycentric city, we give intuitive

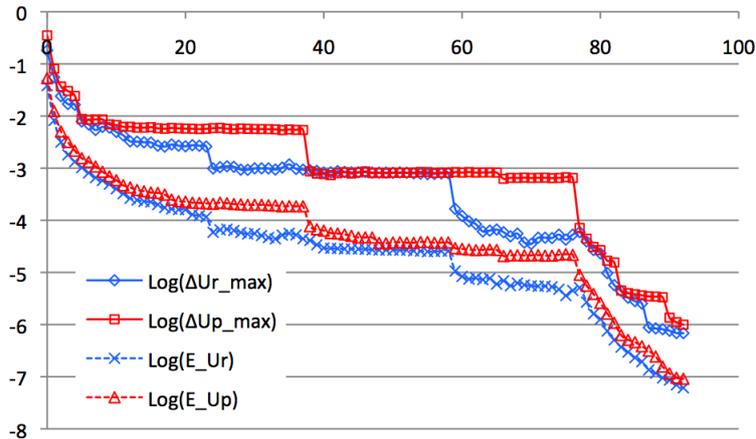


Figure 3: Evolution of the common logarithms of the relative inhomogeneity ΔU and of the relative standard deviation E_U of poor (in red) and rich (in blue) agents' utility during a simulation. One unit of the abscissa represents 50000 iterations.

arguments to support the fact that there is also one unique equilibrium in the models which are studied in this paper.

The existence of (at least) one equilibrium for all simple polycentric models studied here is proved in [8] using fixed-point methods. Hence, it remains to be argued that with the polycentric changes added here to the standard monocentric model, there is no apparition of multiple equilibria, contrarily to what can happen with more complicated models, for instance [1] or [9]. It can be observed that these works add important changes to the standard model by adding variables to the utility function, while our work only considers a more complex transport. Indeed, introducing several work centers breaks the circular symmetry of the transport cost which is found in the monocentric city.

Let us first consider the case of model 2 with two centers separated by a distance d , the simplest polycentric model we study, where agents work at the employment center which is closest to their housing location (see section 3). When centers are sufficiently far apart, two cities are present and do not interact, with an equal share of the whole population residing in each city. In this case, the result of [7] ensuring existence and uniqueness of the equilibrium is clearly still valid. When centers are brought closer and cities begin to interact, the situation is a bit more complicated.

Our approach then consists in mapping this simple polycentric model onto a fictitious monocentric model verifying the assumptions required in [7] to ensure existence and uniqueness of the equilibrium. This mapping allows us to prove the existence and uniqueness of the equilibrium for the polycentric model. In the urban models studied in [7], as well as in our model 2, a given location is completely characterized by the distance of commuting for an agent residing in this place. Equivalently, space is characterized by the amount of land available at each commuting distance x . Let us note $L(x)dx$ the amount of land available between commuting distances x and $x + dx$. A monocentric model with a distribution of land equivalent to that of our simple duocentric model would have $L(x) = 4\pi x$ for $0 \leq x \leq d/2$ and $L(x) = 4x(\pi - \arccos(d/x))$ for $x \geq d/2$. This fictitious monocentric model verifies all assumptions ensuring that it has one unique equilibrium, which is also true for our duocentric model as a consequence. This result could be extended to models with 3 or more centers, as this would only make the function $L(x)$ more complicated. With several income groups, the result still holds.

The case of model 4 with $m = 1$ is almost similar. In this model two work centers

(East and West) separated by a distance d are considered, and each agent represents a two-workers household, with each worker of the household working at a different center (see section 3). Thus the total commuting distance of the household is the sum $d_E + d_W$ of distances between the household's housing location and both centers. The function $L(x)$ of the corresponding monocentric model is now $L(x) = 0$ for $0 \leq x \leq d$ and $L(x) = 2xE(d/x)$ for $x \geq d$, with $E(e)$ the complete elliptic integral of the second kind. This last formula corresponds to the circumference of an ellipse of major axis x , distance between focal points d , and eccentricity d/x . A similar argument of correspondence proves the uniqueness of the equilibrium in this case³, and is still valid with several income groups. However, it seems impossible to extend this result to more than two work centers in this case.

In the cases of model 3, model 4 and model 5 with $0 < m < 1$ (see section 3), the previous arguments supporting the uniqueness of the analytical equilibrium seem difficult to reproduce. But it remains true that no important change is brought to the utility function when compared with the standard monocentric model. Only the transport cost (seen as a function defined on the two-dimensional space of the model) is changed.

In addition, an important argument in favor of this uniqueness is the fact that for all models presented here, the agent-based model converges to the same equilibrium situation for every run of a simulation, as shown in appendix A. This agent-based equilibrium is further described in the next section.

2.3.2 Analytical and agent-based equilibria

Assuming that the previous arguments are convincing of the fact that our simple polycentric models all have one unique analytical equilibrium, it should still be argued that the agent-based model is able to reach a discrete version of this equilibrium. Figure 1 shows that it is so for the standard monocentric AMM model with two income groups. Let us describe more precisely the hypotheses ensuring that the agent-based model reaches an equilibrium which is similar to the analytical one. Section 2.2 shows that the equilibrium of the agent-based model is defined as a situation in which utility is homogeneous within each group, ensuring that no agent has an incentive to move. But this condition alone does not guarantee that the equilibrium is reached, as shown in appendix B. Indeed, a supplementary condition is needed: that every cell is optimally used, either for agriculture or for housing.

From the comparison with the analytical equilibrium, it follows that only two situations should be observed at equilibrium for the cells of the agent-based model. The first is the case of an agricultural cell, whose price should be equal to the agricultural rent, and where no agent should reside. The second case is a residential cell, where no space should be left for another agent to move in. Indeed, if the cell can accommodate (at least) one more agent, it indicates that equilibrium is not reached as the city could be made more compact, providing a higher utility for agents.

We want to monitor the number or share of cells which are not optimally used. Indeed, it follows from the previous discussion that the measure of the distance to the equilibrium in terms of the homogeneity of utility should be completed by another one, linked to residential location. The variable we use is the quotient of a surface we call "empty" S_{empty} to the total housing surface of agents S_{tot} . Let us now describe how this "empty" surface is computed. Each cell of the simulation space is visited. If the cell has inhabitants, the smallest housing surface of the inhabitants s_{min} is stored. If the surface still available in the cell s_{av} is greater than s_{min} , then a part of the cell is considered "empty". To

³In [7], it is assumed that $L(x) > 0$ on $[0, x_1]$ with x_1 a positive number. We assume that the result of existence and uniqueness of the equilibrium is still valid under the condition $L(x) > 0$ on $[x_0, x_1]$ with $0 < x_0 < x_1$, though we do not provide a proof supporting this assertion.

determine how much exactly, it is computed how many agents with housing surface s_{\min} could still fit in the cell. The corresponding surface is considered "empty". The values of parameters ϵ and T_p are chosen so as to minimize the quotient $S_{\text{empty}}/S_{\text{tot}}$ (which is checked to be smaller than 0.5% at the equilibrium of the simulations presented here) with an acceptable simulation time. It is also verified that every cell without inhabitants has a price which is equal to the agricultural rent.

With these conditions, each model we study converges to a unique equilibrium, as described in appendix A. Appendix B presents a simulation using values of parameters ϵ and T_p which do not allow the system to reach a state where $S_{\text{empty}}/S_{\text{tot}} < 0.5\%$.

3 Additions to the standard model

3.1 Polycentric city: model 2

The agent-based mechanism introduced in this work to reproduce the results of the AMM model is robust enough for us to introduce effects which are difficult to treat analytically. For instance, several centers can be introduced, which to our knowledge has not been done analytically for a two-dimensional city. Agents work at the center which is the closest to their housing, and as a consequence, can change jobs as they move. This last feature seems unrealistic but allows to prevent market frictions and reach more rapidly the equilibrium. The results of a such model are given on figure 4.

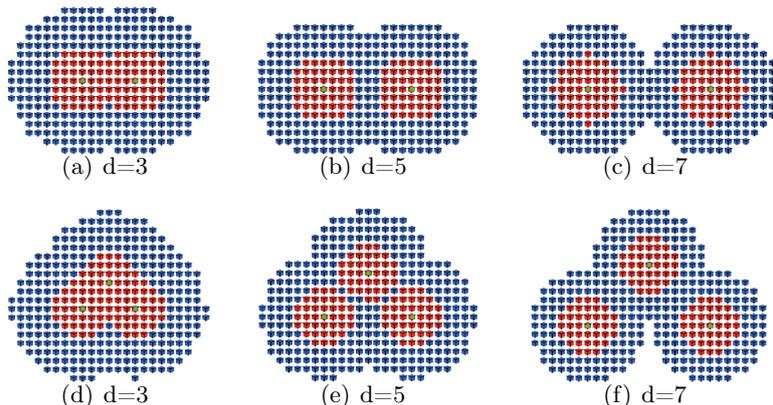


Figure 4: Cities with two centers separated by $2d$ cells (first line) and cities with three centers located at $(-d; 0)$, $(d; 0)$ and $(0; d)$, for different values of d . Centers are indicated by green dots, and agents work in the center closest to their housing location.

Rents, housing surface and density represented as functions of the distance to the nearest center produce curves which are qualitatively similar to those of figure 1. Table 2 allows to see the evolution of different variables for this polycentric model, such as agents' utility, the mean commuting distance for each income group, the total commuting distance, the total rent and the total surface of the city, compared with the reference configuration with two income groups from paragraph 2.1. The mean density is given by the inverse of the total surface, as the population is fixed.

Raising the number of centers amounts to raising the surface available at a given commuting distance in the city, or equivalently to reducing transport costs. As a consequence, this reduces the competition for housing. Agents have greater housing surfaces, smaller commuting distances and a higher utility. The total rent increases, which can seem surprising but can be explained by the fact that housing surfaces are greater. The mean density

decreases while housing surfaces increase. These effects are more pronounced when the centers are further away from one another.

Model	U_r	U_p	$U_r - U_p$	D_{mean}^r	D_{mean}^p	D_{tot}	R_{tot}	p_{mean}	S_{tot}
2 income groups (§ 2.1)	100	100	100	100	100	100	100	100	100
2 centers $d = 3$ fig. 4(a)	101.6	102.6	99.0	83.1	78.1	81.7	101.3	94.9	106.7
2 centers $d = 5$ fig. 4(b)	102.2	103.1	99.1	78.2	80.3	78.8	101.5	92.9	109.7
2 centers $d = 7$ fig. 4(c)	102.6	103.4	99.3	78.8	81.9	79.7	101.4	88.5	114.5
3 centers $d = 3$ fig. 4(d)	102.1	103.4	98.7	77.4	67.0	74.4	101.8	94.3	108.0
3 centers $d = 5$ fig. 4(e)	103.1	104.4	98.7	70.6	69.2	70.2	102.1	88.8	114.9
3 centers $d = 7$ fig. 4(f)	103.9	105.3	98.7	66.7	71.3	68.0	102.3	83.8	122.1
100% $d = 2$ fig. 5(a)	100.0	100.49	99.5	87.2	120.5	96.8	100.2	100	99.5
100% $d = 6$ fig. 5(b)	100.8	102.0	99.8	72.0	122.8	86.6	100.9	99.0	102.0
100% $d = 10$ fig. 5(c)	101.4	103.1	98.4	70.1	124.4	85.8	101.0	92.5	109.3
80% $d = 2$ fig. 5(d)	100.4	100.9	99.6	90.8	97.2	92.7	100.5	100	101.1
80% $d = 6$ fig. 5(e)	101.3	102.4	98.9	79.9	91.0	83.2	101.2	96.1	105.2
80% $d = 10$ fig. 5(f)	101.9	103.2	98.8	75.5	95.3	81.3	101.3	91.8	110.3

Table 2: Comparison between the different polycentric models. Variables are rich and poor agents' utility U_r and U_p , their difference, rich and poor agents' mean commuting distances D_{mean}^r and D_{mean}^p , the total commuting distance D_{tot} , total rent R_{tot} , mean unit surface price p_{mean} and the total surface of the city S_{tot} .

Thus economic and environmental outcomes of the introduction of several centers in this model are positive: agents' utility increases and commuting distances decrease. Agents' utility increases when the distance between centers increases, but the effect on commuting distances is more complex (see table 2). Commuting distances are always smaller than in the reference simulation, but they can increase again when the centers are moved away from each other, as the decreasing competition for land results in increasing housing surfaces, and thus city size. It should also be noted that bigger housing surfaces result in greater heating (and cooling) needs, which are a major source of greenhouse gases emissions. This puts the environmental outcome of this model in another perspective. The social outcome is not intuitive. Poor agents' utility increases more than rich agents', but the utility gap depends on the number of centers. With two centers, this utility gap is smaller when the centers are closer. With three centers, it remains constant when centers are further away.

The third line of figure 8 and appendix C present respectively the shape of the city and the evolution of different variables for this same model 2, with only one income group, as a limiting case of model 5, presented in section 3.4.

In model 2, the CBD is splitted in two work centers of equal size. This is the simplest polycentric city which can be imagined. Let us now introduce a version of this simple polycentric model, with two work centers of different sizes.

3.2 Constrained polycentric city: model 3

It is also possible to assign agents to a given employment center at the beginning of the simulation and to keep it. The computation of the equilibrium in this configuration on a two-dimensional city has not been done to our knowledge, but the bid mechanism used here allows us to find this equilibrium. For instance, all rich agents work in a given center and all poor ones in another center at another location. The result of a such model is given on the first line of figure 5, where the center on the right can be seen as a center with low-skill jobs (or an industrial zone) in the east of the city, and the center of the left, a center with high-skilled workers on the west of the city.

It is also possible, as shown on the second line of figure 5, to have only a part of each income group working at each center, that is to say, to suppose that centers are not completely specialized. In this case, as agents in a certain income group have different

constraints, their utility is not homogeneous within an income group. Indeed, utility is homogeneous among agents of the same income group working at the same center. On the second line of figure 5, the urban system is composed of two work centers, which are not indifferent for agents (contrarily to what is done in model 2) and two income groups, that is, four (2×2) utility groups at equilibrium.

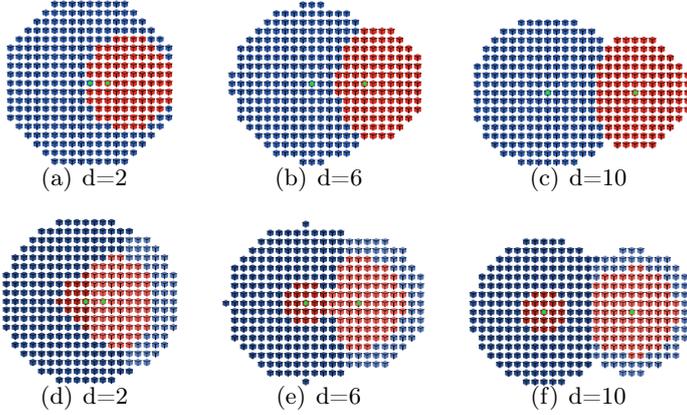


Figure 5: First line: cities with two centers where poor agents work in the east center and rich agents in the west center. Second line: 80% of poor agents work in the east center and 20% in the west center, and conversely for rich agents. The distance d between both centers is indicated. On the second line, agents working in the center on the right have paler colors.

As can be seen on table 2, the global effect of the introduction of centers with constraints for agents is quite similar to the effect of centers without constraints: the competition for housing decreases. The economic outcome is positive, as agents' utility increases when the distance between centers increases. The housing surfaces increase, and they increase when the distance between centers increases. However, the simulation presented on figure 5(a) is an exception: the city surface is reduced and the mean density is higher than in the reference configuration.

Partial or total segregation of rich and poor agents in job locations decreases in fact mainly the competition for housing between both income groups: poor agents are less pushed toward the center by their competition with rich agents, and rich agents are less pushed toward the outskirts of the city. Two effects appear on commuting distances. This decrease of the competition between income groups for housing raises poor agent's commuting distances and decreases those of rich agents. And the increase in the surface available at a given commuting distance decreases all commuting distances. So the environmental outcome is positive from the transport point of view, as commuting distances decrease globally when the distance between centers increases. But it has to be balanced with the negative environmental effect of increasing housing surfaces. The effects on commuting distances of each income group are more complex, as can be seen on table 2. The social outcome is globally positive. The utility gap between rich and poor agents decreases. When the segregation linked to employment is total, the effect of increasing the distance between centers is not monotonous (see table 2). When this segregation is partial, social inequalities decrease when the distance between centers increases. Though it must be remembered that in the last simulations, a new disparity has appeared within each income group.

3.3 Spreading the CBD: model 4

The polycentric models presented in the two last sections give interesting indications on the outcomes of polycentrism in the standard Urban Economics model. However, it is quite unrealistic to imagine a city planner who could for instance have the power to share the CBD of his city in half and put both halves apart, driving as a consequence both halves of the population apart. In this section, we wish to study a more realistic phenomenon: the decentralization of employment locations, which can also be seen as a spread of the CBD.

To this end, we study a model with only one income group, divided into 5 subgroups labeled by $i = 0..4$. Within each of these subgroups, agents work at a distance d_i of the (punctual) center of the city, given by $d_i = d \times i$. We study the influence of the decentralization of employment locations, as a function of this distance d . For $d = 0$, this model corresponds to model 1 with only one income group.

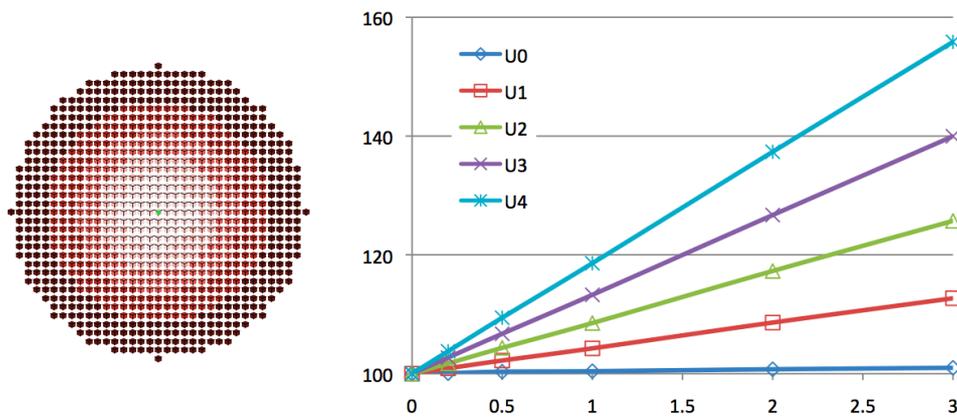


Figure 6: Left panel: shape of the city in model 4, where darker colors indicate higher values of i and of the utility. Right panel: evolution of the utility of agents of each group $i = 1..5$ as functions of the difference d of radius of the employment rings of two consecutive groups.

Figure 6 shows the location of different employment subgroups at the equilibrium. Agents working further from the center of the city are also located further from the center. This is logical, because they have a comparative advantage there as they benefit of a lower transport cost than agents working closer to the center. They also have a higher utility, as shown on the right panel of figure 6, where their utility is compared with their value in the reference simulation $d = 0$, corresponding to model 1 with only one income group. Their utility increases with the distance d , including the utility of agents of group 0, who benefit of a reduction of the competition for housing in the center, even if their transport cost does not change. So that the economic outcome of this model is obviously positive.

Figure 7 gives the outcomes of this model on the different variables studied previously. The total commuting distance decreases strongly when d increases, which gives a positive environmental outcome. But it seems unrealistic that agents living and working at the periphery of the city never commute to the center. A simple way to deal with this question is studied in the next section. The total rent increases when d increases, as a result of the decrease of the mean price in the city being outweighed by the increase of the total housing surface. This has a negative environmental effect which mitigates the positive one on commuting distances.

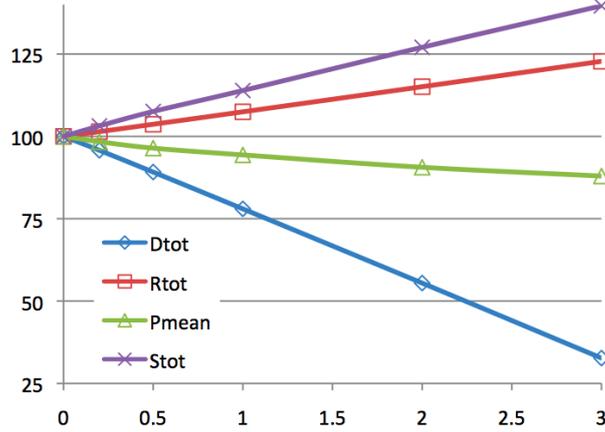


Figure 7: Outcomes of model 4 as functions of the difference d of radius of the employment rings of two consecutive groups.

3.4 Polycentric city and two-workers households: model 5

One important flaw of the previous models is the fact that as the only transport motive considered is the daily commuting for work, the location choice of each agent is only linked to one work center, so that with several centers, the interaction between centers is poor.

A simple way to add more coherence to the city as a whole while keeping the same framework consists in studying two-workers households. Each agent described previously represents now a household composed of two workers. For simplicity, we consider only a city with two centers, as the ones of the first line of figure 4. In addition, and also for the sake of simplicity, we study a model with only one income group. Households are divided in two groups. In the first group, which we denote by "common", both persons in the household work at the same employment center. In the second group, which we denote by "splitted", they work in different centers. This is imposed exogenously and does not change during a simulation.

We study the outcome of this model depending on two variables: the distance d between centers, and the share $m \in [0, 1]$ of households of the "splitted" group. Note that the case $m = 0$ has already been studied, as it corresponds to model 2 with two centers. Let us label employment centers by "East" and "West", and note d_E and d_W the distances between a given household's location and centers East and West. Then if both persons in this household work at the same employment center ("common" group), the East center for instance, the transport cost associated with the commuting of the household is $2 \times \tilde{t} \times d_E$. If they work at different centers ("splitted" group), their transport cost is $\tilde{t} \times (d_E + d_W)$. The transport cost for a unit distance \tilde{t} is chosen so that in the limit $d = 0$, this model 5 with households corresponds to model 1 (with only one income group): $\tilde{t} = t/2$.

One important consequence of the new ingredient added here is that a minimal commuting distance of d is imposed for all households of the "splitted" group. It is their commuting distance if they are located on the segment linking both employment centers. So that the minimal total commuting distance D_{\min}^{tot} of agents in the city is $D_{\min}^{\text{tot}} = d \times m \times N$. This minimal distance is exogenously imposed, and is a special feature of this model 5.

To begin with, let us study what happens in the case $m = 1$, where all households belong to the "splitted" group. To minimize their transport cost, agents choose their location by minimizing $d_E + d_W$. As a result, the shape of the city is elliptic with both employment centers as focal points, as can be seen on the first line of figure 8. Indeed,

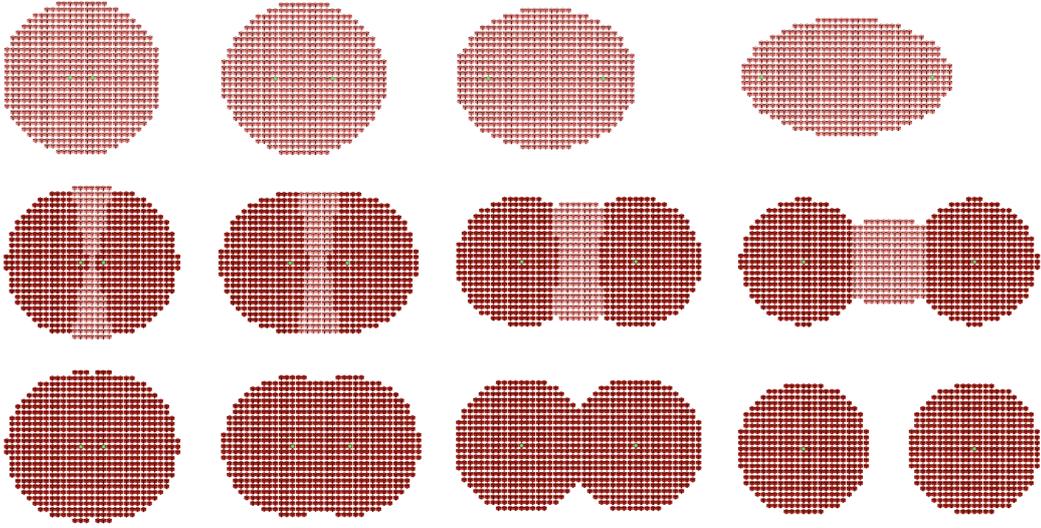


Figure 8: Shape of the city with households (model 5), with $m = 1$ (first line), $m = 0.2$ (second line) and $m = 0$ (third line). The different columns correspond to different values of the distance d between centers: $d = 4, 10, 20, 30$ from left to right. Agents of the "common" group have a darker color than agents of the "splitted" group.

the figure defined by the set of points verifying $d_E + d_W = k$, with k a given constant, is an ellipse. The effect of increasing d on the transport cost of agents can be described as follows: the transport cost is increased (everywhere, except at both employment centers themselves, where it does not change when compared when the monocentric case) because of the increasing minimal commuting distance described previously, and the center of the city (seen as the place where transport cost is minimal) is spread on a segment linking both employment centers.

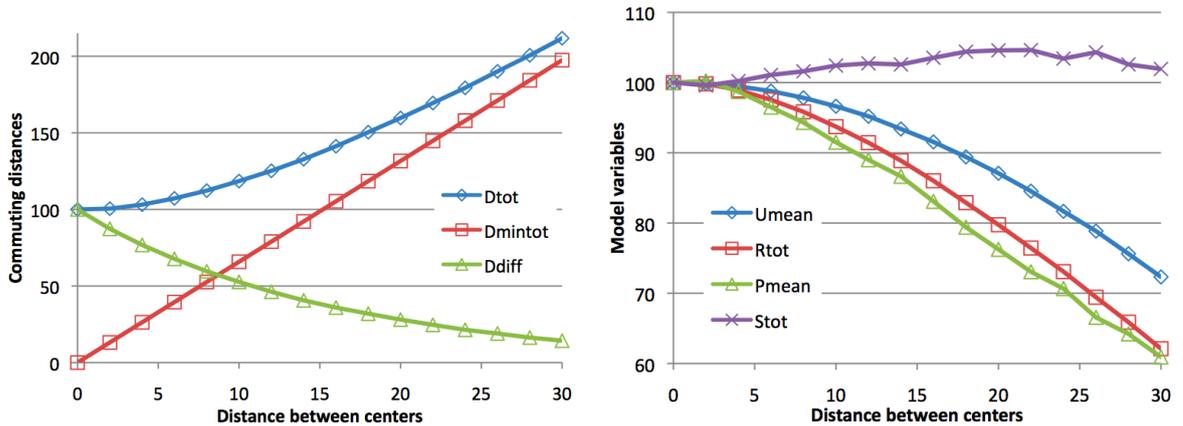


Figure 9: Outcomes of model 5 with $m = 1$ as functions of the distance d between centers. Left panel: evolution of the total commuting distance, the minimal distance and their difference. Right panel: evolution of agents' utility, of the total rent, of the mean price and of the total surface of the city.

As a consequence, the total commuting distance D_{tot} of agents increases when centers are moved apart, mainly because of the contribution of the minimal commuting distance $D_{\text{min}}^{\text{tot}}$, as can be seen on the left panel of figure 9. $D_{\text{diff}} = D_{\text{tot}} - D_{\text{min}}^{\text{tot}}$ is also indicated. The variables are given on the basis of their value in a reference simulation with $d = 0$

(corresponding to model 1 with only one income group), to allow an easy comparison. The utility of agents U_{mean} decreases when d increases, very slowly when centers are close to each other and then more rapidly. The total surface of the city is always bigger than in the reference (monocentric) simulation, but it decreases when d is high. The mean price of housing and the total rent decrease when d increases, as the share of income used for transport increases.

Within this model, polycentrism is undesirable. It has both a negative economic outcome with the decreasing utility of agents, and a negative environmental outcome, as housing surfaces increase and commuting distances increase. However, it has to be remembered that commuting distances increase mainly because of the minimal commuting distance shown on the left panel of figure 9. This effect could be seen as the worst case scenario of a monocentric city evolving towards a polycentric shape: all households increase their travel distances accordingly. A more realistic scenario is given by the case where only a part of the households increase their travel distances, which we study now.

When $0 < m < 1$, simulations show that the utility of agents of the "common" group is always higher than that of agents of the "splitted" group. This is logical, as agents of the "splitted" group have more constraints, as they want to stay close to two places. The outcomes of this model with $0 < m < 1$ are intermediate between the outcomes of this model with only agents of the "splitted" group, shown on the first line of figure 8 and on figure 9, and those of model 2 with two centers and only one income group, which are presented on the last line of figure 8 and in appendix C. The second line of figure 8 gives the shape of the city with $m = 0.2$ for different values of d , and figure 10 gives the corresponding outcomes of model 5. The city shape is an intermediary between $m = 0$ and $m = 1$, that is to say, between two disks and an ellipse. Agents of the "splitted" group (in a paler shade) are located between both centers, separating agents of the "common" group in two parts.

Figure 10 shows that the outcomes of the model in this case $m = 0.2$ are also intermediary between those obtained for $m = 0$ and $m = 1$. The total commuting distance decreases at first when d increases, and then increases again, mainly because of the contribution of the minimal commuting distance imposed on agents of the "splitted" group. So that it becomes higher than its value at $d = 0$ when centers are far away from each other. The utility U_0 of "common" agents increases with d , as their competition for land with "splitted" agents decreases. The utility U_1 of "splitted" agents increases at first when d increases, and then decreases again, below its value at $d = 0$. The total surface of the city increases with d , while the mean price of land decreases. The total rent increases at first when d increases, and then decreases, below its value at $d = 0$.

In this case, which seems more realistic than the same model with $m = 1$ presented before, polycentrism is desirable, as long as centers are not moved too far apart from each other. Indeed, the utility of agents of both groups increases when d increases for small values of d , which gives a positive economic outcome of this model. The environmental outcome is also positive, as the total commuting distance decreases when d increases, for small value of d . But this positive effect is mitigated by the fact that housing surfaces increase, which tends to increase emissions of greenhouse gases. Thus this more realistic model tends to confirm the conclusions of model 2, as long as centers are kept not too far away from each other.

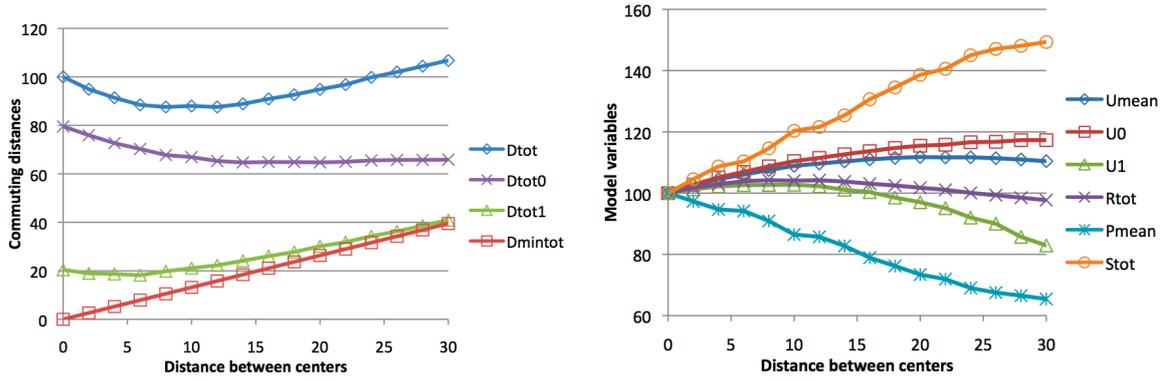


Figure 10: Outcomes of model 5 with $m = 0.2$ as functions of the distance d between centers. Same variables as on figure 9. On the left panel, the total commuting distances D_{tot}^0 and D_{tot}^1 of agents of both groups are presented. On the right panel, the mean utility of agents U_{mean} , the utility U_0 of "common" agents and U_1 of "splitted" agents are given.

4 Perspectives

Historical evolution with endogenous amenities

The aim is to explore the historical evolution of a "semi-open" city, where population grows in time with the arrival of new inhabitants. This is inspired by [1] and [10]. We introduce an endogenous amenity which is linked to the presence of rich households: the cells where rich households live become more and more attractive, whereas cells where poor agents live become unattractive. Using a log-linear utility, rich agents have to be more sensitive to this endogenous amenity in order to obtain the situation presented on figure 11, where rich agents are located in the center of the city. At the beginning of the

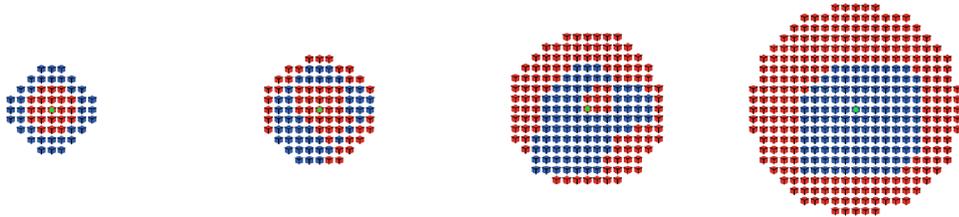


Figure 11: Historical evolution of a city with endogenous amenities linked to the presence of rich neighborhoods.

simulation, the population is ten times lower than at the end, and no endogenous amenity is present, so that with this utility function the city has a "North American" structure, with rich households at the periphery of the city. The "European" structure emerges over time, rich households outbidding progressively poorer ones in the center of the city.

Calibration

Another perspective of this work is the calibration of the agent-based model with values which are coherent with empirical values for the different parameters (income, transport cost, agricultural rent...), and which give coherent results (densities, housing surfaces, size of the city...). Indeed, using only Alonso's model, where land and housing are not distinguished, this calibration is impossible because of the absence of vertical housing: all

agents live on the ground, and the densities can not come close to real densities. The city size is also unrealistic as a result.

In order to make such a calibration possible, a mechanism of building construction (Muth model, see [6]) is added to the agent-based model, representing housing industry and determining the housing surface available in each cell.

Polycentric city with endogenous centers

This paper presents a study of different simple polycentric models, to explore the outcomes of the AMM model beyond the monocentric framework. But employment centers are still given exogenously, so that the location of jobs can not be studied within this model. It is an interesting perspective of this work to study models with endogenous location of employment centers. The present study can be seen as a first step in this direction. It is indeed important to know what happens with given employment centers, before studying models where these location mechanisms are endogenous.

To study an endogenous location of employment centers, a new type of agents must be created to represent firms, and these new agents should compete with other agents for land. They also make location choices, but trying to maximize a profit function whereas households maximize their utility function. Such analytical models (of one-dimensional cities) are described in [11].

Broader perspectives and discussion

This work is interesting as a complement of analytical works when analytical results are difficult to obtain. It is not meant to come in competition with a mathematical treatment of urban models. A complete analytical study of the different models presented in this work would surely bring other insights on these simple polycentric models.

Hence, two important perspectives can be considered: first, a research perspective is to study dynamic urban models, which are difficult to treat analytically. For instance, once the models presented here have reached an equilibrium, a parameter value is changed (e.g. a raise in transport cost) and the consecutive dynamic changes on the urban systems can be studied, until another equilibrium is reached. Second, a more applied perspective is to design simulation models which could be of an easy use for city planners to help decision-making. Using the robustness of the agent-based dynamics presented here, and applying it to real-world data, for instance a urban road network, simulation models could indeed be designed to study economic, environmental and social consequences of different urban planning policies, within the AMM model.

Conclusion

From a methodological point of view, this work shows the interest of agent-based systems in the study of collective phenomena carried out by social sciences. With the example of the standard Urban Economics model, we use this simulation tool to reproduce the results of an equilibrium model. To this end, we introduce an interaction between agents which allows to lead the system towards the equilibrium. These agent-based simulations allow us to model phenomena which are difficult to deal with analytically, like the introduction of polycentrism in a circular closed city with two income groups. This can also be seen as an improvement of the equilibrium model because simple or more refined interaction mechanisms could be studied in this way. And as this agent-based model is dynamic, it

can be an interesting tool to study the dynamics of urban location as a perspective of this work.

The introduction of several centers in the AMM model has a positive impact on agents' welfare, as transport expenses and competition on the housing market decrease. Commuting distances are reduced, which gives a positive environmental outcome of the polycentric city in this model. However, the increase of housing surfaces may counterbalance this decrease of greenhouse gases emissions. Although the global effect of a reduction of competition for land between agents is clear, its impact on the different variables of this simple urban model and on different income groups is not obvious, as the results show.

The use of agent-based systems on calibrated urban models could allow to test the effect of different urban policies, and to have a global view of their effect on the urban system. In this goal a calibration of a version of this model where housing construction is endogenous is an interesting perspective of research.

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A Reproducibility of the results

In order to confirm that the equilibrium reached by the agent-based model is unique, we perform the same simulation numerous times. In spite of the stochasticity of the dynamics of the model, each run converges to the same equilibrium, in a sense which is defined more precisely here.

The simulations are stopped only once the two conditions ensuring that the equilibrium is reached, described in section 2, are verified : the homogeneity of utility ΔU_{\max} is smaller than 10^{-6} (section 2.2) and the share of "empty" surface $S_{\text{empty}}/S_{\text{tot}}$ is smaller than 0.5% (section 2.3.2).

The results of these simulations are given in table 3 for two models presented in this work: the first part corresponds to model 1, the reference monocentric model with two income groups. The second corresponds to model 5 with $d = 9$ and $m = 0.2$. The equilibrium values of the variables characterizing the models have only very small variations across different simulations. The maximal variation observed is of approximately 0.1% under the two previous conditions.

Model 1	U_r	U_p	$U_r - U_p$	D_{mean}^r	D_{mean}^p	D_{tot}	R_{tot}	p_{mean}	S_{tot}
Variations (in %)	0.009	0.001	0.02	0.09	0.06	0.08	0.02	0.11	0.08
Model 5	U_p	U_0	U_1	D_{mean}^0	D_{mean}^1	D_{tot}	R_{tot}	p_{mean}	S_{tot}
Variations (in %)	0.008	0.01	0.02	0.09	0.05	0.06	0.02	0.09	0.08

Table 3: Reproducibility of the results: maximal relative variations of the variables characterizing the models across 15 runs of the same simulation.

These values can be seen as the approximate magnitude of error bars of the results presented in this work.

B Parameters of the agent-based model

In this section, we give an example of stationary configuration⁴ of the agent-based model when the parameters specific to the agent-based model, ϵ and T_p , are not chosen so as to minimize the inhomogeneity of utility ΔU_{\max} and the share of "empty" surface $S_{\text{empty}}/S_{\text{tot}}$. As a consequence, the system does not reach an equilibrium which corresponds to the analytical one.

We keep the values of parameters given in table 1, except T_p , which we take as $T_p = 3000$. The results of this simulation are shown on figure 12. They should be compared with the results of figure 1. A much higher value of T_p is used in the present case, so that

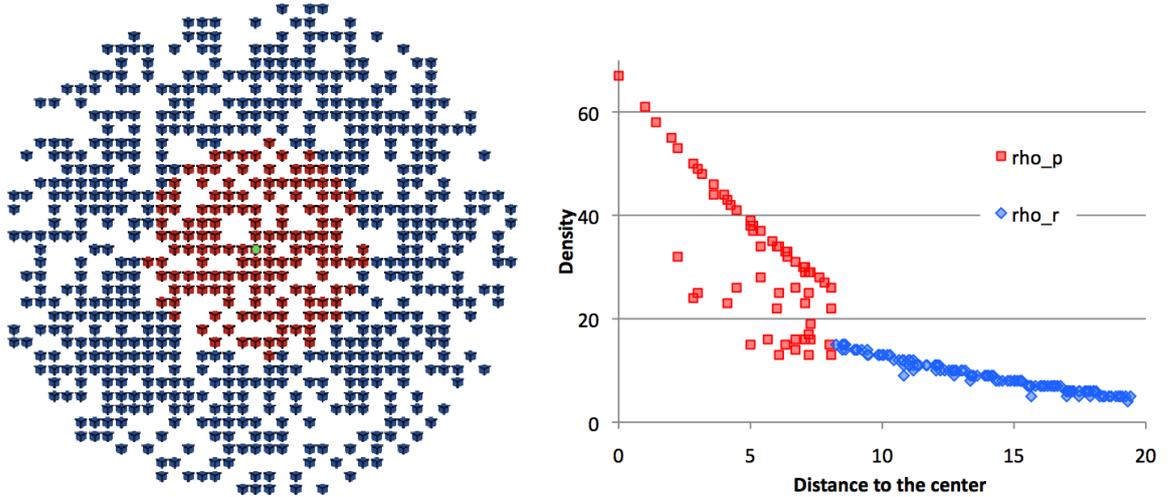


Figure 12: Monocentric city with two income groups, with $T_p = 3000$. Left panel: shape of the city. Right panel: density as a function of the distance to the center.

the price of vacant cells decreases very slowly. It even decreases too slowly to manage to compensate price increases due to agents' bids, which prevents the system from reaching an equilibrium corresponding to the analytical one. Indeed, as can be seen on the left panel of figure 12, as the price of cells decreases too slowly, some cells, even close to the CBD, are left vacant after their price has increased too much. The bid mechanism still manages to bring the system to a state with homogeneous utility, where $\Delta U_{\max} < 10^{-6}$. However, a lot of space is not optimally used, which is indicated by the value of the share of "empty" surface $S_{\text{empty}}/S_{\text{tot}} \simeq 130\%$ (in the other simulations presented in this work, this variable is smaller than 0.5% – see section 2.3.2). Numerous cells where no agents live have a price which is higher than the agricultural rent, a situation which can not be observed if space is optimally used.

⁴This configuration corresponds actually to a state of the system where the evolution is very slow, so that the configuration seems stationary. We do not study this configuration more precisely here and present it as an illustration of a simulation not converging to an equilibrium corresponding to the analytical one.

C Simple polycentric city with one income group

The model presented in this section corresponds to model 2 with only one income group, or equivalently, to model 5 with $m = 0$. The shape of the city in this framework is given on the last line of figure 8. The corresponding outcomes on the different variables characterizing the model are given on figure 13. One obvious result can already be seen on figure 8: when

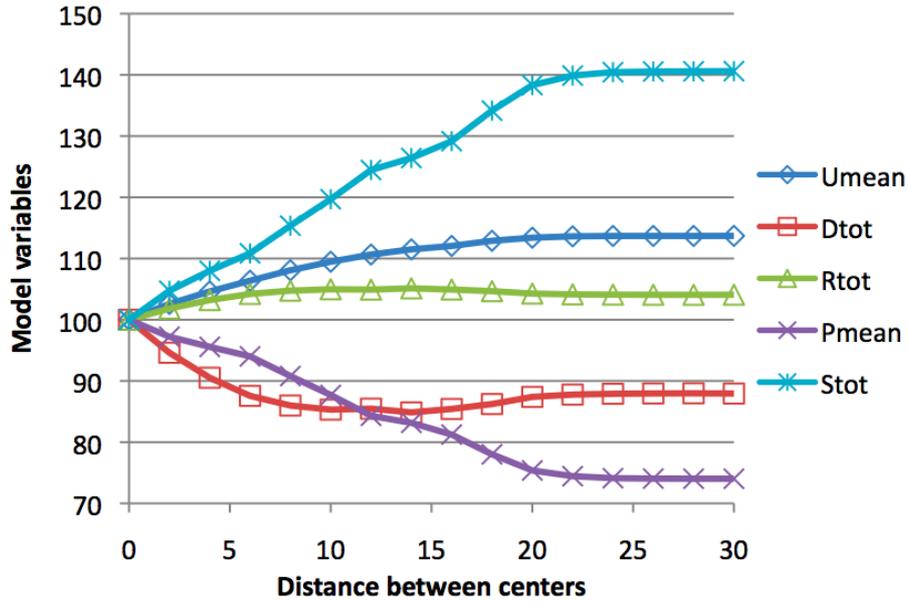


Figure 13: Evolution of the variables characterizing the model as a function of the distance between centers.

centers are sufficiently far apart so that both halves of the city do not interact anymore (when the distance between centers is approximately 25 cells), the variables do not evolve when centers are pushed further away from one another.