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► **To cite this version:**

Nicola Bartolo, Sabino Matarrese, Ornella Pantano, Antonio Riotto. Second-order matter perturbations in a CDM cosmology and non-Gaussianity. *Classical and Quantum Gravity*, 2010, 27 (12), pp.124009. 10.1088/0264-9381/27/12/124009 . hal-00601894

**HAL Id: hal-00601894**

**<https://hal.science/hal-00601894>**

Submitted on 21 Jun 2011

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# Second-order matter perturbations in a $\Lambda$ CDM cosmology and non-Gaussianity

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**Abstract.** We obtain exact expressions for the effect of primordial non-Gaussianity on the matter density perturbation up to second order in a  $\Lambda$ CDM cosmology, fully accounting for the general relativistic corrections arising on scales comparable with the Hubble radius. We present our results both in the Poisson gauge and in the comoving and synchronous gauge, which are relevant for comparison to different cosmological observables.

CERN-PH-TH/2010-044

PACS numbers: 98.80.Cq

## 1. Introduction

In this paper we discuss how primordial non-Gaussianity (NG) in the cosmological perturbations is left imprinted in the Large-Scale Structure (LSS) of the universe in a  $\Lambda$ CDM cosmology. We show how the information on the primordial non-Gaussianity, set on super-Hubble scales, flows into smaller scales through a complete General Relativistic (GR) computation. Primordial NG thus leaves an observable imprint in the LSS. Another interesting finding is that, on sufficiently large scales, there is another additional source of non-Gaussianity which arises from GR corrections, the leading contributions of which are Post-Newtonian terms as first pointed out in Ref. [1]. The importance of the signatures of primordial non-Gaussianity in the evolution of the matter density perturbations is due to the fact that future high-precision measurements of the statistics of the dark matter density will allow to pin down the primordial non-Gaussianity, thus representing a tool complementary to studies of the Cosmic Microwave Background (CMB) anisotropies. It is beyond the scope of this paper to go into the details of the various theoretical and observational methods related to this issue. This paper will then serve as a basic guideline to capture the starting point expressions that relates primordial NG to the dark matter density and gravitational potentials, outlining some specific non-Gaussian signatures in the LSS that can be potentially interesting. We perform our calculations in assuming a flat Universe with pressure-less matter, i.e. Cold Dark Matter (CDM) plus non-relativistic ordinary matter and a cosmological constant, hereafter  $\Lambda$ CDM cosmology. We present our results both in the Poisson gauge and in the comoving time-orthogonal gauge, which are relevant for comparison to observations.

The primordial NG considered in our analysis is set at primordial (inflationary) epochs on large (super-Hubble) scales. At later times cosmological perturbations reenter the Hubble radius during the radiation or during the matter- and dark energy-dominated epochs. For scales re-entering the Hubble radius during the radiation dominated era one should include the radiation in the evolution equations, thus using a complete *second-order* matter transfer function also for those scales. A detailed treatment of it has been given in Refs.[2, 3]. Refs. [4, 5] also investigate the evolution of the dark matter perturbations up to second-order accounting for a radiation-dominated epoch, both analytically and numerically. Here we will focus on large scales for which the effects arising during the radiation-dominated epoch can be neglected.

The plan of the paper is as follows. In Section 2 we give the general form of the perturbed line-element and we introduce the gauge-invariant curvature perturbation of uniform density hypersurface at first and second order, which is used to provide the inflationary initial conditions, including the effect of primordial non-Gaussianity. In Section 3 we derive the second-order expression for the density perturbation in a  $\Lambda$ CDM cosmology, in the Poisson gauge, taking into full account both NG initial conditions and Post-Newtonian corrections arising from the non-linear evolution of perturbations according to the fully General Relativistic treatment. Section 4 contains a similar calculation in the comoving-synchronous gauge. Section 5 contains our concluding

remarks.

## 2. Metric perturbations and primordial non-Gaussianity

We consider a spatially flat Universe filled with a cosmological constant  $\Lambda$  and a non-relativistic pressureless fluid of Cold Dark Matter (CDM), whose energy-momentum tensor reads  $T^\mu_\nu = \rho u^\mu u_\nu$ . Following the notations of Ref. [6], the perturbed line element around a spatially flat FRW background reads

$$ds^2 = a^2(\tau) \{ -(1 + 2\phi)d\tau^2 + 2\hat{\omega}_i d\tau dx^i + [(1 - 2\psi)\delta_{ij} + \hat{\chi}_{ij}] dx^i dx^j \}. \quad (1)$$

where  $a(\tau)$  is the scale factor as a function of conformal time  $\tau$ . Here each perturbation quantity can be expanded into a first-order (linear) part and a second-order contribution, as for example, the gravitational potential  $\phi = \phi^{(1)} + \phi^{(2)}/2$ . Up to now we have not chosen any particular gauge. We can employ the standard split of the perturbations into the so-called scalar, vector and tensor parts, according to their transformation properties with respect to the 3-dimensional space with metric  $\delta_{ij}$ , where scalar parts are related to a scalar potential, vector parts to transverse (divergence-free) vectors and tensor parts to transverse trace-free tensors. Thus  $\phi$  and  $\psi$ , the gravitational potentials, are scalar perturbations, and for instance,  $\hat{\omega}_i^{(r)} = \partial_i \omega^{(r)} + \omega_i^{(r)}$ , where  $\omega^{(r)}$  is the scalar part and  $\omega_i^{(r)}$  is a transverse vector, *i.e.*  $\partial^i \omega_i^{(r)} = 0$  ( $(r) = (1, 2)$  stand for the  $r$ th-order of the perturbations). The symmetric traceless tensor  $\hat{\chi}_{ij}$  generally contains a scalar, a vector and a tensor contribution, namely  $\hat{\chi}_{ij} = D_{ij}\chi + \partial_i \chi_j + \partial_j \chi_i + \chi_{ij}$ , where  $D_{ij} \equiv \partial_i \partial_j - (1/3)\nabla^2 \delta_{ij}$ ,  $\chi_i$  is a solenoidal vector ( $\partial^i \chi_i = 0$ ) and  $\chi_{ij}$  represents a traceless and transverse (*i.e.*  $\partial^i \chi_{ij} = 0$ ) tensor mode $\ddagger$ . As for the matter component we split the mass density into a homogeneous  $\rho(\tau)$  and a perturbed part as  $\rho(\mathbf{x}, \tau) = \rho(\tau)(1 + \delta^{(1)} + \delta^{(2)}/2)$  and we write the four velocity as  $u^\mu = (\delta_0^\mu + v^\mu)/a$  with  $u^\mu u_\mu = -1$  and  $v^\mu = v^{(1)\mu} + v^{(2)\mu}/2$ .

The Friedmann background equations are

$$3\mathcal{H}^2 = a^2(8\pi G\rho(\tau) + \Lambda), \quad (2)$$

and

$$\rho'(\tau) = -3\mathcal{H}\rho(\tau), \quad (3)$$

where a prime stands for differentiation with respect to conformal time, and  $\mathcal{H} = a'/a$ . The matter density parameter is  $\Omega_m(\tau) = 8\pi G a^2(\tau)\rho(\tau)/(3\mathcal{H}^2(\tau))$ .

Before recalling how one can parametrize the primordial non-Gaussianity, we need to provide a general definition for the amplitude of non-Gaussianity characterizing the matter density contrast beyond the usual second-order Newtonian contributions. It

$\ddagger$  In what follows, for our purposes we will neglect linear vector modes since they are not produced in standard mechanisms for the generation of cosmological perturbations (as inflation), and we also neglect tensor modes at linear order, since they give a negligible contribution to LSS formation.

proves convenient to introduce an effective gravitational potential obeying the Poisson equation

$$-\nabla^2\Phi = \frac{3}{2}\Omega_m\mathcal{H}^2\delta, \quad (4)$$

and at an initial epoch, deep in matter domination, we write

$$\Phi_{\text{in}} = \Phi_{\text{in}}^{(1)} + f_{\text{NL}}(\Phi_{\text{in}}^{(1)2} - \langle \Phi_{\text{in}}^{(1)2} \rangle), \quad (5)$$

with the dimensionless non-linearity parameter  $f_{\text{NL}}$  setting the level of quadratic non-Gaussianity, and  $\Phi \propto g(\tau)$ ,  $g(\tau)$  being the usual growth suppression factor (see Sec. 3). Therefore we will write the matter density contrast in Fourier space in terms of the linear density contrast by defining the kernel  $\mathcal{K}_\delta(\mathbf{k}_1, \mathbf{k}_2; \tau)$  depending on the wavevector of the perturbation modes as

$$\begin{aligned} \delta_{\mathbf{k}}(\tau) &= \delta_{\mathbf{k}}^{(1)}(\tau) + \frac{1}{2}\delta_{\mathbf{k}}^{(2)}(\tau) \\ &= \delta_{\mathbf{k}}^{(1)}(\tau) + \int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^3} \mathcal{K}_\delta(\mathbf{k}_1, \mathbf{k}_2; \tau) \delta_{\mathbf{k}_1}^{(1)}(\tau) \delta_{\mathbf{k}_2}^{(1)}(\tau) \delta_D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}). \end{aligned} \quad (6)$$

We can write the kernel as

$$\mathcal{K}_\delta(\mathbf{k}_1, \mathbf{k}_2; \tau) = \mathcal{K}_\delta^N(\mathbf{k}_1, \mathbf{k}_2; \tau) + \frac{3}{2}\Omega_m\mathcal{H}^2 f_{\text{NL}}(\mathbf{k}_1, \mathbf{k}_2, \tau) \frac{g_{\text{in}}}{g(\tau)} \frac{k^2}{k_1^2 k_2^2}, \quad (7)$$

where  $k^2 \equiv |\mathbf{k}_1 + \mathbf{k}_2|^2$  and  $\mathcal{K}_\delta^N(\mathbf{k}_1, \mathbf{k}_2; \tau)$  is the second-order Newtonian kernel.

### 2.1. Primordial non-Gaussianity and initial conditions

We conveniently fix the initial conditions at the time when the cosmological perturbations relevant for LSS are outside the horizon. A standard and convenient way to account for any initial primordial non-Gaussianity is to consider the curvature perturbation of uniform density hypersurfaces  $\zeta = \zeta^{(1)} + \zeta^{(2)}/2 + \dots$ , where  $\zeta^{(1)} = -\hat{\psi}^{(1)} - \mathcal{H}\delta\rho^{(1)}/\rho'$  and at second-order [12, 7]

$$\begin{aligned} -\zeta^{(2)} &= \hat{\psi}^{(2)} + \mathcal{H} \frac{\delta^{(2)}\rho}{\rho'} - 2\mathcal{H} \frac{\delta^{(1)}\rho'}{\rho'} \frac{\delta^{(1)}\rho}{\rho'} - 2 \frac{\delta^{(1)}\rho}{\rho'} (\hat{\psi}^{(1)'} + 2\mathcal{H}\hat{\psi}^{(1)}) \\ &\quad + \left( \frac{\delta^{(1)}\rho}{\rho'} \right)^2 \left( \mathcal{H} \frac{\rho''}{\rho'} - \mathcal{H}' - 2\mathcal{H}^2 \right), \end{aligned} \quad (8)$$

where  $\hat{\psi}^{(r)} = \psi^{(r)} + \nabla^2\chi^{(r)}/6$ . This is a gauge-invariant quantity which remains constant on super-horizon scales after it has been generated during a primordial epoch (and possible isocurvature perturbations are no longer present). Therefore,  $\zeta^{(2)}$  provides all the necessary information about the primordial level of non-Gaussianity. The conserved value of the curvature perturbation  $\zeta$  allows to set the initial conditions for the metric and matter perturbations accounting for the primordial contributions. Different scenarios for the generation are characterized by different values of  $\zeta^{(2)}$ , while the post-inflationary nonlinear evolution due to gravity is common to all of them [13, 14, 15, 7]. For example, in standard single-field inflation  $\zeta^{(2)}$  is generated during inflation and its value is  $\zeta^{(2)} = 2(\zeta^{(1)})^2 + \mathcal{O}(\epsilon, \eta)$  [16, 17, 13], where  $\epsilon$  and  $\eta$  are the usual slow-roll

parameters. Therefore it turns out to be convenient to parametrize the primordial non-Gaussianity level in terms of the conserved curvature perturbation as in Ref. [20]

$$\zeta^{(2)} = 2a_{\text{NL}} \left(\zeta^{(1)}\right)^2, \quad (9)$$

where the parameter  $a_{\text{NL}}$  depends on the physics of a given scenario, and in full generality it can depend on scale and configuration. For example in the standard scenario  $a_{\text{NL}} \simeq 1$ , while in the curvaton case (see e.g. Ref. [7])  $a_{\text{NL}} = (3/4r) - r/2$ , where  $r \approx (\rho_\sigma/\rho)_{\text{D}}$  is the relative curvaton contribution to the total energy density at curvaton decay. In the minimal picture for the inhomogeneous reheating scenario,  $a_{\text{NL}} = 1/4$ . For other scenarios we refer the reader to Ref. [7, 8, 9, 10, 11]. One of the best techniques to detect or constrain the primordial large-scale non-Gaussianity is through the analysis of the CMB anisotropies, for example by studying the CMB bispectrum [7, 18, 19]. The non-linearity parameter  $f_{\text{NL}}$  as defined in Eq. (5) is defined also to make contact with the primordial non-Gaussianity entering in the CMB anisotropies. For large primordial non-Gaussianity, when  $|a_{\text{NL}}| \gg 1$ ,  $f_{\text{NL}} \simeq 5a_{\text{NL}}/3$  (see Refs. [7, 20]).

### 3. Dark matter density perturbations at second-order: Poisson gauge

The goal of this section is to compute the matter density contrast in the Poisson gauge [21], *i.e.* the generalization beyond linear order of the longitudinal gauge, by which a more direct comparison with the standard Newtonian approximation adopted in the interpretation of LSS observations and in N-body simulations in Eulerian coordinates is possible (see, however, Ref. [22] for a critical discussion of the potential problems connected to the use of the Poisson gauge on scales comparable with the Hubble radius). In the Poisson gauge one scalar degree of freedom is eliminated from the  $g_{0i}$  component of the metric, and one scalar and two vector degrees of freedom are eliminated from  $g_{ij}$ .

Let us briefly recall the results for the linear perturbations in the case of a non-vanishing cosmological  $\Lambda$  term. At linear order the traceless part of the  $(i-j)$ -components of Einstein equations gives  $\phi^{(1)} = \psi^{(1)} \equiv \varphi$ . Its trace gives the evolution equation for the linear scalar potential  $\varphi$

$$\varphi'' + 3\mathcal{H}\varphi' + a^2\Lambda\varphi = 0. \quad (10)$$

Selecting only the growing mode solution one can write

$$\varphi(\mathbf{x}, \tau) = g(\tau) \varphi_0(\mathbf{x}), \quad (11)$$

where  $\varphi_0$  is the peculiar gravitational potential linearly extrapolated to the present time ( $\tau_0$ ) and  $g(\tau) = D_+(\tau)/a(\tau)$  is the so called growth-suppression factor, where  $D_+(\tau)$  is the usual linear growing-mode of density fluctuations in the Newtonian limit, *i.e.* the no-decaying solution of the differential equation

$$D'' + \mathcal{H}D' - \frac{3}{2}\mathcal{H}^2\Omega_m D = 0. \quad (12)$$

We normalize the growth factor so that  $D_+(\tau_0) = a_0 = 1$ . The exact form of  $g$  can be found in Refs. [23, 24, 25]. In the  $\Lambda = 0$  case  $g = 1$ . A very good approximation for  $g$  as a function of redshift  $z$  is given in Refs. [23, 24]

$$g \propto \Omega_m [\Omega_m^{4/7} - \Omega_\Lambda + (1 + \Omega_m/2)(1 + \Omega_\Lambda/70)]^{-1}, \quad (13)$$

with  $\Omega_m = \Omega_{0m}(1+z)^3/E^2(z)$ ,  $\Omega_\Lambda = \Omega_{0\Lambda}/E^2(z)$ ,  $E(z) \equiv (1+z)\mathcal{H}(z)/\mathcal{H}_0 = [\Omega_{0m}(1+z)^3 + \Omega_{0\Lambda}]^{1/2}$  and  $\Omega_{0m}$ ,  $\Omega_{0\Lambda} = 1 - \Omega_{0m}$ , the present-day density parameters of non-relativistic matter and cosmological constant, respectively. According to our normalization,  $g(z=0) = 1$ . The energy and momentum constraints provide the density and velocity fluctuations in terms of  $\varphi$  (see, for example, Ref. [13] and [26, 27] for the  $\Lambda$  case)

$$\delta^{(1)} = \frac{1}{4\pi G a^2 \bar{\rho}} [\nabla^2 \varphi - 3\mathcal{H}(\varphi' + \mathcal{H}\varphi)], \quad (14)$$

$$v_i^{(1)} = -\frac{1}{4\pi G a^2 \bar{\rho}} \partial_i (\varphi' + \mathcal{H}\varphi). \quad (15)$$

In a similar way the expression of the second-order matter density contrast can be computed starting from the energy constraint given by the  $(0-0)$  Einstein equation, once the evolution of the gravitational potentials is known. The latter has been already computed in detail in Ref. [28] and in the following we summarize the main results.

The evolution equation for the second-order gravitational potential  $\psi^{(2)}$  is obtained from the trace of the  $(i-j)$ -Einstein equations §

$$\psi^{(2)''} + 3\mathcal{H}\psi^{(2)'} + a^2\Lambda\psi^{(2)} = S(\tau), \quad (16)$$

where  $S(\tau)$  is the source term

$$S(\tau) = g^2 \Omega_m \mathcal{H}^2 \left[ \frac{(f-1)^2}{\Omega_m} \varphi_0^2 + 2 \left( 2 \frac{(f-1)^2}{\Omega_m} - \frac{3}{\Omega_m} + 3 \right) \alpha_0(\mathbf{x}) + g^2 \left[ \frac{4}{3} \left( \frac{f^2}{\Omega_m} + \frac{3}{2} \right) \nabla^{-2} \partial_i \partial^j (\partial^i \varphi_0 \partial_j \varphi_0) - (\partial^i \varphi_0 \partial_i \varphi_0) \right] \right], \quad (17)$$

where, for simplicity of notation, we have introduced

$$\alpha_0(\mathbf{x}) = \left[ \nabla^{-2} (\partial^i \varphi_0 \partial_i \varphi_0) - 3 \nabla^{-4} \partial_i \partial^j (\partial^i \varphi_0 \partial_j \varphi_0) \right], \quad (18)$$

and

$$f(\Omega_m) = \frac{d \ln D_+}{d \ln a} = 1 + \frac{g'(\tau)}{\mathcal{H}g(\tau)}, \quad (19)$$

which can be written as a function of  $\Omega_m$  as  $f(\Omega_m) \approx \Omega_m(z)^{4/7}$  [23, 24]. In Eq. (17)  $\nabla^{-2}$  stands for the inverse of the Laplacian operator.

§ The second-order perturbations of the Einstein tensor  $G^\mu_\nu$  can be found for any gauge in Appendix A of Refs. [16, 7] and directly in the Poisson gauge, e.g., in Refs. [2, 3]. The perturbations of the energy-momentum tensor up to second order in the Poisson gauge have been computed in Ref. [13] for a general perfect fluid (see Ref. [7] for expressions in any gauge).

The solution of Eq. (16) is then obtained using Green's method with growing and decaying solutions of the homogeneous equation  $\psi_+(\tau) = g(\tau)$  and  $\psi_-(\tau) = \mathcal{H}(\tau)/a^2(\tau)$ , respectively. The second-order gravitational potentials then read [28]

$$\begin{aligned} \psi^{(2)}(\tau) = & \left( B_1(\tau) - 2g(\tau)g_{\text{in}} - \frac{10}{3}(a_{\text{NL}} - 1)g(\tau)g_{\text{in}} \right) \varphi_0^2 + \left( B_2(\tau) - \frac{4}{3}g(\tau)g_{\text{in}} \right) \alpha_0(\mathbf{x}) \\ & + B_3(\tau)\nabla^{-2}\partial_i\partial^j(\partial^i\varphi_0\partial_j\varphi_0) + B_4(\tau)\partial^i\varphi_0\partial_i\varphi_0, \end{aligned} \quad (20)$$

$$\begin{aligned} \phi^{(2)}(\tau) = & \left( B_1(\tau) + 4g^2(\tau) - 2g(\tau)g_{\text{in}} - \frac{10}{3}(a_{\text{NL}} - 1)g(\tau)g_{\text{in}} \right) \varphi_0^2 + \left[ B_2(\tau) + \frac{4}{3}g^2(\tau) \right. \\ & \times \left. \left( e(\tau) + \frac{3}{2} \right) - \frac{4}{3}g(\tau)g_{\text{in}} \right] \alpha_0(\mathbf{x}) + B_3(\tau)\nabla^{-2}\partial_i\partial^j(\partial^i\varphi_0\partial_j\varphi_0) + B_4(\tau)\partial^i\varphi_0\partial_i\varphi_0, \end{aligned} \quad (21)$$

where we have introduced  $B_i(\tau) = \mathcal{H}_0^{-2}(f_0 + 3\Omega_{0m}/2)^{-1}\tilde{B}_i(\tau)$  with the following definitions

$$\tilde{B}_1(\tau) = \int_{\tau_{\text{in}}}^{\tau} d\tilde{\tau} \mathcal{H}^2(\tilde{\tau})(f(\tilde{\tau}) - 1)^2 C(\tau, \tilde{\tau}), \quad (22)$$

$$\tilde{B}_2(\tau) = 2 \int_{\tau_{\text{in}}}^{\tau} d\tilde{\tau} \mathcal{H}^2(\tilde{\tau}) \left[ 2(f(\tilde{\tau}) - 1)^2 - 3 + 3\Omega_m(\tilde{\tau}) \right] C(\tau, \tilde{\tau}), \quad (23)$$

$$\tilde{B}_3(\tau) = \frac{4}{3} \int_{\tau_{\text{in}}}^{\tau} d\tilde{\tau} \left( e(\tilde{\tau}) + \frac{3}{2} \right) C(\tau, \tilde{\tau}), \quad \tilde{B}_4(\tau) = - \int_{\tau_{\text{in}}}^{\tau} d\tilde{\tau} C(\tau, \tilde{\tau}), \quad (24)$$

and

$$C(\tau, \tilde{\tau}) = g^2(\tilde{\tau})a(\tilde{\tau}) \left[ g(\tau)\mathcal{H}(\tilde{\tau}) - g(\tilde{\tau})\frac{a^2(\tilde{\tau})}{a^2(\tau)}\mathcal{H}(\tau) \right], \quad (25)$$

with

$$e(\Omega_m) \equiv \frac{f^2(\Omega_m)}{\Omega_m}. \quad (26)$$

The expression for  $\phi^{(2)}$  is obtained from the relation between  $\psi^{(2)}$  and  $\phi^{(2)}$

$$\nabla^2\nabla^2\psi^{(2)} = \nabla^2\nabla^2\phi^{(2)} - 4g^2\nabla^2\nabla^2\varphi_0^2 - \frac{4}{3}g^2\left(e + \frac{3}{2}\right) \left[ \nabla^2(\partial_i\varphi_0\partial^i\varphi_0) - 3\partial_i\partial^j(\partial^i\varphi_0\partial_j\varphi_0) \right], \quad (27)$$

which follows from the traceless part of the  $(i-j)$ -component of Einstein equations [28]. Here  $\varphi_{\text{in}} = g_{\text{in}}\varphi_0$  represents the initial condition taken at some time  $\tau_{\text{in}}$  deep in the matter dominated era on super-horizon scales. The solutions (20) and (21) properly account for the non-Gaussian initial conditions parametrized by  $(a_{\text{NL}} - 1)$ . These are obtained using the expression for  $\zeta^{(2)}$  during the matter-dominated epoch together with Eq. (27) and the second-order (0-0)-component of Einstein equations (both evaluated for a matter-dominated epoch), so that one can express  $\phi_{\text{in}}^{(2)}$  and  $\psi_{\text{in}}^{(2)}$  in terms of  $\zeta^{(2)}$  of Eq. (9), where  $\zeta^{(1)} = -5\varphi_{\text{in}}/3$  (see Refs. [13, 15, 7, 20, 28]).

Before proceeding further notice that in the expression for the second-order gravitational potentials of Eq. (20) and (21) we recognize two contributions. The term which dominates on small scales,  $[B_3(\tau)\nabla^{-2}\partial_i\partial^j(\partial^i\varphi_0\partial_j\varphi_0) + B_4(\tau)\partial^i\varphi_0\partial_i\varphi_0]$ , which gives rise to the second-order Newtonian piece and is insensitive to any non-Gaussianity in the initial conditions. The remaining pieces in Eqs. (20) and (21) correspond to



contributions which tend to dominate on large scales with respect to those characterizing the Newtonian contribution, and whose origin is purely relativistic. In particular these are the pieces carrying the information on primordial non-Gaussianity. For a flat matter-dominated (Einstein-de Sitter) universe  $g(\tau) = 1$  and  $B_1(\tau) = B_2(\tau) = 0$ , while  $B_3(\tau) \rightarrow (5/21)\tau^2$  and  $B_4(\tau) \rightarrow -\tau^2/14$ , so that one recovers the expressions of [3] (see also [29]). In Ref. [27] second-order cosmological perturbations have been computed in the  $\Lambda \neq 0$  case from the synchronous to the Poisson gauge, thus extending the analysis of Ref. [6], and the CMB temperature anisotropies induced by metric perturbations have been also considered by applying the expressions of Ref. [30] (see also Ref. [28]). However, an important point to notice is that both Refs. [6, 30] and Ref. [27] disregard any primordial non-linear contribution from inflation.||

The matter density contrast at second-order can now be calculated from the (0-0) Einstein equation

$$3\mathcal{H}\psi^{(2)'} + 3\mathcal{H}^2\phi^{(2)} - \nabla^2\psi^{(2)} + 12\mathcal{H}\psi^{(1)}\psi^{(1)'} - 2\nabla^2(\psi^{(1)})^2 - 12\mathcal{H}^2(\phi^{(1)})^2 - 12\mathcal{H}\phi^{(1)}\psi^{(1)'} - 3(\psi^{(1)'})^2 + \partial_i\psi^{(1)}\partial^i\psi^{(1)} - 4\psi^{(1)}\nabla^2\psi^{(1)} = -3\mathcal{H}^2\Omega_m\left(\frac{1}{2}\delta^{(2)} + v^{(1)2}\right). \quad (28)$$

We thus arrive at the density contrast in the Poisson gauge

$$\begin{aligned} \delta^{(2)} = & \frac{1}{\Omega_m} \left[ (f-1)^2 - \frac{2}{g^2} \frac{A'(\tau)}{\mathcal{H}} - \frac{2}{g^2} A(\tau) - 1 \right] \varphi^2 - \frac{2}{\Omega_m g^2} \left[ \frac{B_2'(\tau)}{\mathcal{H}} - \frac{4}{3} \frac{g'}{\mathcal{H}} g_{\text{in}} + B_2(\tau) \right. \\ & \left. - \frac{4}{3} g g_{\text{in}} + \frac{4}{3} g^2 \left( e + \frac{3}{2} \right) \right] \alpha(\mathbf{x}, \tau) - \frac{2}{\Omega_m g^2} \frac{B_3'(\tau)}{\mathcal{H}} \nabla^{-2} \partial_i \partial^j (\partial^i \varphi \partial_j \varphi) - \frac{2}{\Omega_m g^2} \frac{B_4'(\tau)}{\mathcal{H}} \partial_i \varphi \partial^i \varphi \\ & + \frac{2}{3\Omega_m g^2} \left[ B_1(\tau) - 2g g_{\text{in}} - \frac{10}{3} (a_{\text{NL}} - 1) g g_{\text{in}} + 2g^2 \right] \frac{\nabla^2 \varphi^2}{\mathcal{H}^2} + \frac{2}{3\Omega_m g^2} \left[ B_2(\tau) - \frac{4}{3} g g_{\text{in}} \right. \\ & \left. - \frac{4}{3} \frac{f^2 g^2}{\Omega_m} - g^2 - 3\mathcal{H}^2 B_4(\tau) \right] \frac{\partial_i \varphi \partial^i \varphi}{\mathcal{H}^2} + \frac{2}{3\Omega_m g^2} \left[ -3B_2(\tau) + 4g g_{\text{in}} - 3\mathcal{H}^2 B_3(\tau) \right] \\ & \times \frac{1}{\mathcal{H}^2} \nabla^{-2} \partial_i \partial^j (\partial^i \varphi \partial_j \varphi) + \frac{8}{3\Omega_m} \frac{\varphi \nabla^2 \varphi}{\mathcal{H}^2} + \frac{2}{3\Omega_m g^2} \frac{B_3(\tau)}{\mathcal{H}^2} \partial_i \partial^j (\partial^i \varphi \partial_j \varphi) + \frac{2}{3\Omega_m g^2} \\ & \times \frac{B_4(\tau)}{\mathcal{H}^2} \nabla^2 (\partial^i \varphi \partial_i \varphi), \end{aligned} \quad (29)$$

where  $A(\tau) \equiv [B_1(\tau) - 2g g_{\text{in}} - (10/3)(a_{\text{NL}} - 1)g g_{\text{in}}]$  and  $\alpha(\mathbf{x}, \tau)$  has the same expressions as  $\alpha_0(\mathbf{x})$  introduced in Eq. (18) with  $\varphi$  in place of  $\varphi_0$ . Notice that we have explicitly verified that Eq. (29) in the limit of an Einstein-de Sitter universe recovers the expression for the matter density contrast obtained in Ref. [1].

Eq. (29) is the main result of this section. It shows how the primordial NG, which is initially generated on large scales, is transferred to the density contrast on subhorizon scales. The expression for the density contrast is made of three contributions: a second-order Newtonian piece (the last two terms in Eq. (29), proportional to  $\tau^4$  in an Einstein-de Sitter universe) which is insensitive to the non-linearities in the initial conditions; a Post-Newtonian (PN) piece (related to two gradient terms of the gravitational potential) which carries the most relevant information on primordial NG; the super-horizon terms

|| The results in Refs. [6, 30, 27] have initial conditions corresponding to our  $a_{\text{NL}} = 0$ .

(corresponding to the first two lines of Eq. (29)). Our findings show in a clear way that the information on the primordial NG set on super-Hubble scales flows into the PN terms, leaving an observable imprint in the LSS. Another interesting result which shows up in Eq.(29) is the presence in the Post-Newtonian term of contributions different from the primordial NG which are due to weakly non-linear corrections which switch on when the modes cross inside the Hubble radius and which can constitute an interesting additional source of non-Gaussianity since they can probe large-scale GR corrections.

### 3.1. The non-linearity parameter $f_{\text{NL}}$

We can rewrite the matter density contrast in Fourier space in terms of the linear density contrast as in Eq. (6). We find

$$\mathcal{K}_\delta^N(\mathbf{k}_1, \mathbf{k}_2; \tau) = \frac{3}{4} \frac{\Omega_m}{g^2} \mathcal{H}^2 \left[ B_3(\tau) \frac{(\mathbf{k} \cdot \mathbf{k}_1)(\mathbf{k} \cdot \mathbf{k}_2)}{k_1^2 k_2^2} + B_4(\tau) \frac{k^2 (\mathbf{k}_1 \cdot \mathbf{k}_2)}{k_1^2 k_2^2} \right], \quad (30)$$

and for the non-linearity parameter

$$\begin{aligned} f_{\text{NL}}^P(\mathbf{k}_1, \mathbf{k}_2; \tau) = & \left[ \frac{5}{3}(a_{\text{NL}} - 1) + 1 - \frac{g}{g_{\text{in}}} - \frac{1}{2} \frac{B_1(\tau)}{g g_{\text{in}}} \right] - \frac{(k_1^2 + k_2^2)}{k^2} \frac{g}{g_{\text{in}}} \\ & + \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)}{k^2} \left[ \frac{2}{3} e(\tau) \frac{g}{g_{\text{in}}} + \frac{1}{2} \frac{g}{g_{\text{in}}} + \frac{2}{3} - \frac{1}{2} \frac{B_2(\tau)}{g g_{\text{in}}} + \frac{3}{2} \mathcal{H}^2 \frac{B_4(\tau)}{g g_{\text{in}}} \right] \\ & + \frac{(\mathbf{k} \cdot \mathbf{k}_1)(\mathbf{k} \cdot \mathbf{k}_2)}{k^4} \left[ \frac{3}{2} \mathcal{H}^2 \frac{B_3(\tau)}{g g_{\text{in}}} - 2 + \frac{3}{2} \frac{B_2(\tau)}{g g_{\text{in}}} \right] \\ & - \frac{(\mathbf{k} \cdot \mathbf{k}_1)(\mathbf{k} \cdot \mathbf{k}_2)}{k^2} \frac{k_1^2 + k_2^2}{k_1^2 k_2^2} \frac{3}{2} \mathcal{H}^2 f(\tau) \frac{B_3(\tau)}{g g_{\text{in}}} - (\mathbf{k}_1 \cdot \mathbf{k}_2) \frac{k_1^2 + k_2^2}{k_1^2 k_2^2} \frac{3}{2} \mathcal{H}^2 f(\tau) \frac{B_4(\tau)}{g g_{\text{in}}}. \end{aligned} \quad (31)$$

The non-linearity parameter  $f_{\text{NL}}$  is defined via Eq. (4) and in this way it generalizes the standard definition of Ref. [31] inferred from the Newtonian gravitational potential. In order to obtain the expressions in Eqs. (30) and (31) we have performed an expansion in  $(\mathcal{H}/k_{1,2}) \ll 1$  up to terms  $(\mathcal{H}/k_i)^2$  starting from Eq. (29). In Eq. (31) the primordial NG is clearly evident in the piece proportional to  $(a_{\text{NL}} - 1)$ . The remaining terms are due to the horizon scale Post-Newtonian corrections we commented about in Eq. (29). The non-linearity (NG) induced by these terms show a specific and non-trivial shape dependence that can help in detecting them. Of course their relative importance increases with the scale, and in fact it has been shown in Ref. [32] that, through the large-scale halo bias techniques investigated in Refs. [33, 34], these GR corrections are potentially detectable.

It is also worth noticing that some of the terms entering in the non-linearity parameter vanish in the limit of a vanishing cosmological constant (e.g.  $B_1(\tau)$  and  $B_2(\tau)$  go to zero in this limit).

A final comment on Eq. (6). In this expression, when the various modes are well inside the horizon, one can take the usual configuration for the linear density perturbations with  $\delta_{\mathbf{k}}^{(1)}(\tau) \propto (2k^2 T(k)/3\Omega_{m0} H_0^2) D_+(a)$ , where  $T(k)$  is the usual linear matter transfer function. In this way one partially accounts for the effects of the transition from a radiation- to a matter-dominated epoch. In fact a full computation would require to study up to second-order the evolution of the density perturbations

also during the radiation dominated era. In this way one can recover a full matter transfer function up to second-order. Details about such a computation can be found in Refs. [3, 4, 5]. For example, Ref. [5] shows that the corrections from the full matter transfer function, when accounting for a matching at second-order to the radiation epoch, gives a relative correction with respect to the Newtonian kernel that is of order  $a_{\text{eq}} \sim 10^{-4}$  and that such a correction is equivalent to the effect of a primordial NG of  $f_{\text{NL}} \sim 4$ . That these small-scale corrections are tiny is easy to understand. By looking at Eq. (28) one realizes that accounting for the radiation epoch at second-order the matter density perturbations from, say approximately the matter-radiation equality epoch onwards, will get a correction which scales like  $\delta^{(2)}|_{\text{equiv}} \tau^2$ , which rescales the matching initial conditions to the radiation epoch. However we expect such term to be negligible w.r.t. the Newtonian part which scales like  $(\delta_{\mathbf{k}}^{(1)}(\tau))^2 \propto \tau^4$ , and also w.r.t. a sizable primordial NG.

#### 4. Dark matter density perturbations at second-order: comoving-synchronous gauge

Let us now see how the second-order matter perturbations and gravitational potentials are obtained in the comoving-synchronous gauge.

We make use of the formalism developed in Refs. [35, 36, 37, 6] which the reader is referred to for more details. The synchronous, time-orthogonal gauge is defined by setting  $g_{00} = -a^2(\tau)$  and  $g_{0i} = 0$ , so that the line-element takes the form

$$ds^2 = a^2(\tau)[-d\tau^2 + \gamma_{ij}(\mathbf{x}, \tau)dx^i dx^j]. \quad (32)$$

For our fluid containing irrotational, pressure-less matter plus  $\Lambda$ , this also implies that the fluid four-velocity field is given by  $u^\mu = (1/a, 0, 0, 0)$ , so that  $x$  represent comoving ‘‘Lagrangian’’ coordinates for the fluid element (indeed, the possibility of making the synchronous, time-orthogonal gauge choice and comoving gauge choice simultaneously is a peculiarity of fluids with vanishing spatial pressure gradients, i.e. vanishing acceleration, which holds at any time, i.e. also beyond the linear regime). A very efficient way to write down Einstein and continuity equations is to introduce the peculiar velocity-gradient tensor [37]

$$\vartheta_j^i \equiv u^i{}_{;j} - \frac{a'}{a}\delta_j^i = \frac{1}{2}\gamma^{ik}\gamma'_{kj}, \quad (33)$$

where we have subtracted the isotropic Hubble flow. Here semicolons denote covariant differentiation. From the continuity equation  $T^{\mu\nu}{}_{;\nu} = 0$ , we infer the exact solution for the density contrast  $\delta = \delta\rho/\rho$  [37, 6]

$$\delta(\mathbf{x}, \tau) = (1 + \delta_0(\mathbf{x}))[\gamma(\mathbf{x}, \tau)/\gamma_0(\mathbf{x})]^{-1/2} - 1, \quad (34)$$

where  $\gamma = \det\gamma_{ij}$ . The subscript ‘‘0’’ denotes the value of quantities evaluated at the present time.

From Eq. (34) it is evident that in the comoving-synchronous gauge the only independent degree of freedom is the spatial metric tensor  $\gamma_{ij}$ . The energy constraint reads

$$\vartheta^2 - \vartheta^i \vartheta^j_{|i} + 4\mathcal{H}\vartheta + \mathcal{R} = 6\mathcal{H}^2\Omega_m\delta, \quad (35)$$

where  $\mathcal{R}^i_j$  is the Ricci tensor associated with the spatial metric  $\gamma_{ij}$  with scalar curvature  $\mathcal{R} = \mathcal{R}^i_i$ . The momentum constraint reads  $\vartheta^i_{j|i} = \vartheta_{,j}$ , where bars stand for covariant differentiation in the three-space with metric  $\gamma_{ij}$ . Finally, one can use the Raychaudhuri equation

$$\vartheta' + \mathcal{H}\vartheta + \vartheta^i \vartheta^j_{|i} + \frac{3}{2}\mathcal{H}^2\Omega_m\delta = 0, \quad (36)$$

which is obtained from the energy constraint and the trace of the evolution equation

$$\vartheta^{i'}_j + 2\mathcal{H}\vartheta^i_j + \vartheta\vartheta^i_j + \frac{1}{4}(\vartheta^k_l\vartheta^l_k - \vartheta^2)\delta^i_j + \mathcal{R}^i_j - \frac{1}{4}\mathcal{R}\delta^i_j = 0. \quad (37)$$

Notice that these equations are exact and describe the fully non-linear evolution of cosmological perturbations (up to the time of caustic formation). In order to show how the primordial non-Gaussianities appear in the matter density contrast, we then perform a perturbative expansion up to second order in the fluctuations of the metric.

The spatial metric tensor can be expanded as

$$\gamma_{ij} = (1 - 2\psi^{(1)} - \psi^{(2)})\delta_{ij} + \chi_{ij}^{(1)} + \frac{1}{2}\chi_{ij}^{(2)}, \quad (38)$$

where  $\chi_{ij}^{(1)}$  and  $\chi_{ij}^{(2)}$  are traceless tensors and include scalar, vector and tensor (gravitational waves) perturbations. As usual we split the density contrast into a linear and a second-order part as  $\delta(\mathbf{x}, \tau) = \delta^{(1)}(\mathbf{x}, \tau) + \frac{1}{2}\delta^{(2)}(\mathbf{x}, \tau)$ . At linear order the growing-mode solutions in the comoving-synchronous gauge are given by

$$\begin{aligned} \psi^{(1)}(\mathbf{x}, \tau) &= \frac{5}{3}\varphi_{\text{in}}(\mathbf{x}) + \frac{2}{9\mathcal{H}^2(\tau)\Omega_m(\tau)}\nabla^2\varphi(\mathbf{x}, \tau), \\ \chi_{ij}^{(1)}(\mathbf{x}, \tau) &= D_{ij}\chi^{(1)}(\mathbf{x}, \tau), \quad \chi^{(1)}(\mathbf{x}, \tau) = -\frac{4}{3\mathcal{H}^2(\tau)\Omega_m(\tau)}\varphi(\mathbf{x}, \tau), \end{aligned} \quad (39)$$

where  $\varphi(\mathbf{x}, \tau)$  is the growing-mode scalar potential defined in Eq. (11). The linear density contrast  $\delta^{(1)}$  in this gauge is related to  $\varphi$  via the usual Poisson equation, namely

$$\nabla^2\varphi(\mathbf{x}, \tau) = \frac{3}{2}\mathcal{H}^2(\tau)\Omega_m(\tau)\delta^{(1)}(\mathbf{x}, \tau). \quad (40)$$

In writing  $\chi_{ij}^{(1)}$  we have eliminated the residual gauge ambiguity of the synchronous gauge as in Ref. [6] ¶. We have also assumed that linear vector modes are absent, since they are not produced in standard mechanisms for the generation of cosmological perturbations (as inflation). We have also neglected linear tensor modes, since they play a negligible role in LSS formation.

¶ More in general, at any order  $n$  in perturbation theory the scalar potentials  $\psi^{(n)}$  and  $\chi^{(n)}$  can be shifted by arbitrary constant amounts  $\delta\psi_0^{(n)}$  and  $\delta\chi_0^{(n)}$ , only provided  $\delta\psi_0^{(n)} + (1/6)\nabla^2\delta\chi_0^{(n)} = 0$ .

By perturbing Eq. (34) up to second order, we get

$$\begin{aligned} \delta^{(2)} = & \delta_0^{(2)} + 3 \left( \psi^{(2)} - \psi_0^{(2)} \right) + \frac{1}{2} (D_+^2 - 1) \left[ \frac{1}{2} \left( \nabla^2 \chi_0^{(1)} \right)^2 + \right. \\ & \left. + \partial^i \partial^j \chi_0^{(1)} \partial_i \partial_j \chi_0^{(1)} \right] - (D_+ - 1) \left( 2\psi_0^{(1)} + \frac{1}{3} \nabla^2 \chi_0^{(1)} \right) \nabla^2 \chi_0^{(1)}. \end{aligned} \quad (41)$$

To compute the metric perturbation  $\psi^{(2)}$ , we can use the evolution equation, Eq. (37). To this aim it proves convenient to write

$$\begin{aligned} \psi^{(2)} = & \psi_0^{(2)} - \frac{1}{3} \delta_0^{(2)} + \frac{1}{6} \partial^i \partial^j \chi_0^{(1)} \partial_i \partial_j \chi_0^{(1)} - \\ & - \frac{1}{3} \left( 2\psi_0^{(1)} + \frac{1}{3} \nabla^2 \chi_0^{(1)} \right) \nabla^2 \chi_0^{(1)} + \frac{1}{12} \left( \nabla^2 \chi_0^{(1)} \right)^2 + \xi, \end{aligned} \quad (42)$$

where we have introduced the variable  $\xi$  which is determined by the equation

$$\begin{aligned} \xi'' + \mathcal{H}\xi' - \frac{3}{2} \mathcal{H}^2 \Omega_m \xi = \\ = \frac{1}{4} \mathcal{H}^2 \Omega_m D_+^2 \left[ \frac{1}{2} \left( \nabla^2 \chi_0^{(1)} \right)^2 - \left( 1 + \frac{2}{3e(\Omega_m)} \right) \partial^i \partial^j \chi_0^{(1)} \partial_i \partial_j \chi_0^{(1)} \right]. \end{aligned} \quad (43)$$

An approximate solution of this equation can be obtained by making use of the usual approximation (see e.g. Ref. [38, 39])  $e(\Omega_m) \approx 1$ , which holds true for reasonable values of  $\Omega_m$ . Under these circumstances we can write

$$\xi(\tau) \approx A D_+(\tau) + \frac{D_+^2(\tau)}{14} \left[ \frac{1}{2} \left( \nabla^2 \chi_0^{(1)} \right)^2 - \frac{5}{3} \partial^i \partial^j \chi_0^{(1)} \partial_i \partial_j \chi_0^{(1)} \right], \quad (44)$$

where  $A$  is an integration constant which can be determined by the energy constraint at the initial time. Notice that it is precisely the sub-leading, Post-Newtonian term proportional to the linear growing mode  $D_+$  which brings all the relevant information about primordial and GR-induced non-Gaussianity. It is also important to stress that the time dependence of this term, which comes from the homogeneous solution of the above equation is *exact*, while the above approximation only affects the fastest growing Newtonian terms, i.e. those proportional to  $D_+^2$ . Using this procedure and providing the initial data (formally at  $\tau \rightarrow 0$ ) in terms of the gauge-invariant curvature perturbation, which in this gauge reads  $\zeta_{\text{in}}^{(1)} = -\psi_{\text{in}}^{(1)} - (1/6)\nabla^2 \chi_{\text{in}}^{(1)} = -5\varphi_{\text{in}}/3$  and  $\zeta_{\text{in}}^{(2)} = -\psi_{\text{in}}^{(2)} - (1/6)\nabla^2 \chi_{\text{in}}^{(2)} = 50a_{\text{nl}}\varphi_{\text{in}}^2/9$ , we finally obtain

$$\begin{aligned} \delta^{(2)}(\tau) = & \frac{100}{9\mathcal{H}_0^2} \left[ f(\Omega_{0m}) + \frac{3}{2} \Omega_{0m} \right]^{-1} \left\{ D_+(\tau) \left[ \left( \frac{3}{4} - a_{\text{NL}} \right) (\nabla \varphi_{\text{in}})^2 + (2 - a_{\text{NL}}) \varphi_{\text{in}} \nabla^2 \varphi_{\text{in}} \right] \right. \\ & \left. + \frac{D_+^2(\tau)}{14\mathcal{H}_0^2} \left[ f(\Omega_{0m}) + \frac{3}{2} \Omega_{0m} \right]^{-1} \left[ 5 (\nabla^2 \varphi_{\text{in}})^2 + 2\partial^i \partial^j \varphi_{\text{in}} \partial_i \partial_j \varphi_{\text{in}} \right] \right\}, \end{aligned} \quad (45)$$

where we made use of the following property

$$\mathcal{H}D'_+ + \frac{3}{2} \mathcal{H}^2 \Omega_m D_+ = \text{const.}, \quad (46)$$

whose validity can be easily proven on the basis of Eq. (12) and of the Friedmann equation  $\mathcal{H}' - \mathcal{H}^2 + (3/2)\mathcal{H}^2 \Omega_m = 0$ . One may notice that even the Newtonian part (i.e. the one proportional to  $D_+^2$ ) of the second-order matter perturbations differs from

the Poisson gauge expression. This is a well-known feature that is to be ascribed to the different meaning of the mass density when moving from Eulerian to Lagrangian coordinates, which only appears at second and higher orders (see, e.g. Ref. [40]).

As we did in the Poisson-gauge case, we can re-express this result in terms of a suitable potential  $\Phi$  in order to introduce a non-linearity parameter  $f_{\text{NL}}$  with the usual meaning. We obtain (see also Ref. [32])

$$f_{\text{NL}}^C(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{3} \left[ (a_{\text{NL}} - 1) - 1 + \frac{5}{2} \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k^2} \right] \quad (47)$$

Notice that the non-linearity parameter is different w.r.t. the one in Eq. (31), not only because they have been computed in two different gauges, but also because in the comoving synchronous gauge the Post-Newtonian term giving rise to  $f_{\text{NL}}^S$  can be easily written in an exact form where the growing mode  $D_+(\tau)$  is factored out, while this is not the case for the Poisson gauge. It is also important to stress here that the expression for  $f_{\text{NL}}$  obtained in this gauge is the one to be used to evaluate the effect of NG on the Lagrangian bias of dark matter halos, as recently stressed in Ref. [41]. In Ref. [32] it was shown that tiny effects in the above equation which come purely from the General Relativistic evolution, i.e. the term which survives in the limit  $a_{\text{NL}} = 1$ , are potentially detectable for some planned LSS surveys.

## 5. Concluding remarks

In this paper we have investigated the effect of primordial and GR-induced non-Gaussianities on the second-order matter density perturbation in a  $\Lambda$ CDM cosmology. The calculation has been performed in two popular gauges, the Poisson gauge and the comoving time-orthogonal one, which are useful for comparison with observations, depending on the particular quantity under study. For instance, in evaluating the effect of NG on the mass function and Lagrangian bias of dark matter halos, the comoving gauge expression is more appropriate, while the Poisson gauge formulae are more suitable for gravitational lensing studies. The strongest present limits on  $f_{\text{NL}}$  come from the analysis of the angular bispectrum of WMAP temperature anisotropy data. Indeed, Komatsu et al. [42], analyzing the 7-years WMAP data obtain the 95% limits  $-10 < f_{\text{NL}}^{\text{local}} < 74$ , and  $-214 < f_{\text{NL}}^{\text{equilateral}} < 266$ . The analysis of Planck data both in temperature and E-mode polarization is expected to improve the accuracy by almost an order of magnitude. A complementary and very powerful information on the amplitude and shape of primordial NG will come from the study the galaxy clustering (e.g. Ref. [43]) and other LSS datasets, such as weak gravitational lensing (e.g. Refs [44, 45]) and redshifted 21cm background anisotropy (e.g. Refs. [46, 47]). Primordial NG in LSS data can be searched for by various techniques: abundance of massive and/or high-redshift objects [48, 49, 50, 51, 52, 53, 54], abundance of voids [55], higher-order statistics such as bispectrum and trispectrum (see, e.g. Ref. [56]), large-scale clustering of halos, thought as rare high peaks of the dark matter distribution [57, 33, 34, 58, 59, 51], and

their cross-correlation with the CMB via the Integrated Sachs-Wolfe effect [60], small-scale NG corrections to the matter power-spectrum [59, 61]. For instance, Slosar et al. [62], exploiting the scale-dependence of the NG correction to the halo linear bias  $\Delta b_{\text{NG}} \propto f_{\text{NL}}^{\text{local}} k^{-2}$ , obtained the 95% confidence range  $-29 < f_{\text{NL}}^{\text{local}} < 70$ , using the two-point function of a combination of datasets. The prospects for sensibly narrowing these limits with the advent of galaxy surveys sampling regions comparable to the Hubble volume are analyzed in Ref. [63]. Indeed, as shown in Ref. [32], there are very promising prospects to observe NG signatures down to the limits of the order unity GR corrections discussed in this paper. This largely motivates theoretical efforts to obtain accurate predictions of these effects.

## Acknowledgments

A.R. acknowledges partial support by the EU Marie Curie Network UniverseNet (HPRNCT2006035863). This research has been partially supported by the ASI Contract No. I/016/07/0 COFIS, the ASI/INAF Agreement I/072/09/0 for the Planck LFI Activity of Phase E2.

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