

Encoding Phases using Commutativity and Non-commutativity in a Logical Framework

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Outline

- 1** Minimalist Categorical Grammars
 - Partially Commutative Logic (PCL)
 - MCG rules
- 2** Phases
 - Definition
 - Phases in MCG
- 3** Example: simple sentence
- 4** Phase Impenetrability Condition

Overview

1 Minimalist Categorical Grammars - MCG

- string (words) level: non-commutativity
- hypotheses: commutativity

2 Phases

- proposal: a phase is a non-commutative point (in sequence of hypotheses)

⇒ Encoding Phases with non-commutativity.

Partially Commutative Logic (PCL)

Rules of PCL: (introduction and elimination)

- Lambek calculus connectives: \odot , \backslash and $/$ (non-commutatives)

$$\frac{\Gamma \vdash A \quad \Delta \vdash A \backslash C}{\langle \Gamma; \Delta \rangle \vdash C} [\backslash_e]$$

- commutative multiplicative linear connectives: \otimes and \multimap

$$\frac{\Delta \vdash A \otimes B \quad \Gamma, (A, B), \Gamma' \vdash C}{\Gamma, \Delta, \Gamma' \vdash C} [\otimes_e]$$

- entropy rule
- axiom

Labelled proofs

A label r will be considered as $r = (r_s, r_h, r_c)$.

String operations

- concatenation: \bullet

$$\frac{\Delta \vdash_G r_2 : B \quad \Gamma \vdash_G r_1 : B \setminus A}{\langle \Gamma; \Delta \rangle \vdash_G (\text{Concat}(r_2) \bullet r_{1s}, r_{1t}, r_{1c}) : A} [\setminus_e]$$

- substitution: $[t/x]$

$$\frac{\Gamma \vdash_G r_1 : A \otimes B \quad \Delta[x : A, y : B] \vdash_G r_2 : C}{\Delta[\Gamma] \vdash_G r_2[\text{Concat}(r_1)/x, \epsilon/y] : C} [\otimes_e]$$

Minimalist Program

Minimalist Logic, a restriction of PCL to:

- elimination rules
- \backslash and $/$ (and \odot for phases)
- \otimes
- axiom
- entropy

Rules of MCG are defined over two principles:

- combining two pieces of derivation: *merge*
- redefining internal relations in a derivation: *move*

Merge

- combines two pieces of proofs together (non-commutative)
- combine hypotheses of different proofs (commutative)
- elimination of \backslash or $/$
- entropy

For labels: concatenation of strings

$$\begin{array}{c}
 \Delta \vdash s : B \quad \Gamma \vdash (r_s, r_h, r_c) : B \backslash A \\
 \hline
 \Delta; \Gamma \vdash (\text{Concat}(s) \bullet r_s, r_h, r_c) : A \quad [\backslash_e] \\
 \hline
 \Delta, \Gamma \vdash (\text{Concat}(s) \bullet r_s, r_h, r_c) : A \quad [\text{entropy}]
 \end{array}
 \Rightarrow
 \frac{\Delta \vdash s : B \quad \Gamma \vdash (r_s, r_h, r_c) : B \backslash A}{\Delta, \Gamma \vdash (\text{Concat}(s) \bullet r_s, r_h, r_c) : A} [\text{mg}]$$

Head Movement

Rules which modifies the position of the string in the label:

- *head movement*
- *affix hopping*

Use of $<$ and $>$.

Head-Movement:

$$\frac{\Delta \vdash s : B \quad \Gamma \vdash (r_{spec}, r_{head}, r_{comp}) : B_{>} \setminus A}{\Delta, \Gamma \vdash (\text{Concat}(s_{-head}) \bullet r_{spec}, s_{head} \bullet r_{head}, r_{comp}) : A} [mg]$$

$$\frac{\Delta \vdash s : B \quad \Gamma \vdash (r_{spec}, r_{head}, r_{comp}) : B \setminus < A}{\Delta, \Gamma \vdash (\text{Concat}(s_{-head}) \bullet r_{spec}, r_{head} \bullet s_{head}, r_{comp}) : A} [mg]$$

Move

In MCG move is structurally different from in MG.

- It assumes that a phrase is included in the proof if and only if all its hypotheses are
- Discharge of commutative hypotheses: \otimes

For labels: the concatenation of the moved phrase is substituted in the position of the newest hypothesis.

$$\frac{\Gamma \vdash r_1 : A \otimes B \quad \Delta[u : A, v : B] \vdash r_2 : C}{\Delta[\Gamma] \vdash r_2[\text{Concat}(r_1)/u, \epsilon/v] : C} [mv]$$

Structure of items

Strong structure of lexical items

$$\begin{array}{lcl}
 L & ::= & (B) / P_1 \quad | \quad C \\
 B & ::= & P_1 \setminus (B) \quad | \quad P_2 \setminus (B) \quad | \quad C \quad | \quad D \\
 C & ::= & P_2 \otimes (C) \quad | \quad C_1 \\
 D & ::= & P_2 \odot (D) \quad | \quad C_1 \\
 C_1 & ::= & P_1
 \end{array}$$

$$(c_m \setminus \dots \setminus c_1 \setminus (b_1 \otimes \dots \otimes b_n \otimes a)) / d$$

Note that there are no lexical hypothesis

Definition

According to Chomsky, a phase is a syntactic domain.

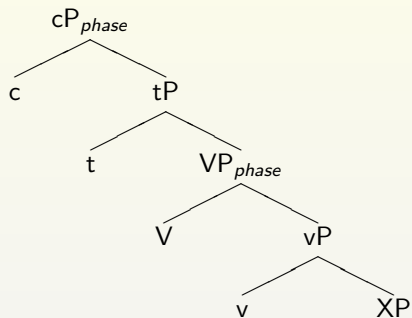
He assumes that the analysis of a sentence is driven by the verb which goes by at least two specific phases:

- VP
- cP

Neither tP nor the decomposition over simple verb form are *phases*

Phases define syntactic islands: the content of a *phase* must be moved to its left-hand side in order to let it accessible. (**transfer**).

Definition



Definition

According to Chomsky:

A *phase* is a node of the syntactic tree which triggers moves.

It is not possible to simulate this with a unique lexical item

Also **transfert** is the realization of

- possible *moves*
- direct linking of hypotheses in *cyclic moves*.

Definition

Semantic:

Different works claim that at particular step of the verb derivation some specific thematic roles could be assigned.

The usual semantic of MCG require specific simplification points which correspond to *phases*.

Phases in MCG

1 deep syntactic:

$$\vdash v /< d$$

2 Mode:

$$\vdash k \setminus d \setminus V /< v$$

3 Inflection:

$$\vdash k \setminus t /< V$$

4 Comp:

$$\vdash c / t$$

(Use of head movement)

Phases in MCG (Extension of the Minimalist Logic)

Use of non-commutativity in lexical items

$$\Delta_1, H_1; H_2 \Delta_2 \vdash A \quad (1)$$

Using the \odot_e rule of the PCL:

$$\frac{\Delta_s, \Delta_h, \Delta_c \vdash (s_s, s_h, s_c) : X \odot Y \quad \Gamma_s, X; Y, \Gamma_c \vdash (r_s, r_h, r_c) : Z}{\Gamma_s, \Delta_s, \Delta_h \vdash (r_s \bullet s_s, r_h, s_h \bullet s_c \bullet r_c) : Z} \text{ [phase]}$$

Phases in MCG

$$\frac{\Delta_s, \Delta_h, \Delta_c \vdash (s_s, s_h, s_c) : X \odot Y \quad \Gamma_s, X; Y, \Gamma_c \vdash (r_s, r_h, r_c) : Z}{\Gamma_s, \Delta_s, \Delta_h \vdash (r_s \bullet s_s, r_h, s_h \bullet s_c \bullet r_c) : Z} [phase]$$

- Discharge of hypotheses [$phase_1$]
- Realization of **all possible moves** [$phase_{trans}$]
i.e. transfert (on the left part of the phase)

Phases in MCG

Lexical consequences on phase entries

- 1** deep syntactic:

$$\vdash v /< d$$

- 2** Mode:

$$\vdash k \setminus d \setminus V /< v \Rightarrow V; v \vdash k \setminus d \setminus V$$

- 3** Inflection:

$$\vdash k \setminus t /< V$$

- 4** Comp:

$$\vdash c / t \Rightarrow c; t \vdash c$$

Phases in MCG

Lexical consequences on other verb entries

1 deep syntactic:

$$\vdash v /< d \Rightarrow \vdash (V \odot v) / d$$

2 Mode:

$$\vdash k \setminus d \setminus V /< v \Rightarrow V; v \vdash k \setminus d \setminus V$$

3 Inflection:

$$\vdash k \setminus t /< V \Rightarrow \vdash k \setminus (c \odot t) / V$$

4 Comp:

$$\vdash c / t \Rightarrow c; t \vdash c$$

Example

The child reads a book

Lexicon

<i>Determiners</i>	$\vdash (\epsilon, \textit{the}, \epsilon) : k \otimes d / n$ $\vdash (\epsilon, \textit{a}, \epsilon) : k \otimes d / n$
<i>noun</i>	$\vdash (\epsilon, \textit{child}, \epsilon) : n$ $\vdash (\epsilon, \textit{book}, \epsilon) : n$
<i>verb</i> (mode) (inflection) (comp)	$\vdash (\epsilon, \textit{read}, \epsilon) : (V \odot v)_{<} / d$ $V; v \vdash (\epsilon, \epsilon, \epsilon) : k \setminus d \setminus V$ $\vdash (\epsilon, -s, \epsilon) : k \setminus (c \odot t) /_{<} V$ $c; t \vdash (\epsilon, \epsilon, \epsilon) : c$

Step 1

- 1 Saturation of object position.

$$\frac{\vdash (\epsilon, \text{read}, \epsilon) : (V \odot v)_{<} / d \quad d \vdash (\epsilon, u, \epsilon) : d}{d \vdash (\epsilon, \text{read}, u) : (V \odot v)} [mg]$$

Step 2

1 Saturation of hypotheses

$$\frac{d \vdash (\epsilon, w, \epsilon) : d \quad \frac{k \vdash (\epsilon, v, \epsilon) : k \quad V; v \vdash (\epsilon, \epsilon, \epsilon) : k \setminus d \setminus V}{k, V; v \vdash (v, \epsilon, \epsilon) : d \setminus V} [mg]}{d, k, V; v \vdash (w \ v, \epsilon, \epsilon) : V} [mg]$$

Step 2

- 1 combine the result with the first verb step,

$$\frac{d \vdash (\epsilon, \text{read}, u) : (V \odot v)_{<} \quad d, k, V; v \vdash (w \ v, \epsilon, \epsilon) : V}{d, k, d \vdash (w \ v, \text{read}, u) : V} [\text{phase}_1]$$

- 2 build the object NP

$$\frac{\vdash (\epsilon, a, \epsilon) : (k \otimes d) / n \quad \vdash (\epsilon, \text{book}, \epsilon) : n}{\vdash (\epsilon, a, \text{book}) : k \otimes d} [\text{mg}]$$

- 3 introduce the object of the sentence with a *move* in the transfer part of the *phase*.

$$\frac{\vdash (\epsilon, a, \text{book}) : k \otimes d \quad d, k, d \vdash (w \ v, \text{read}, u) : V}{d \vdash (w \ a \ \text{book}, \text{read}, \epsilon) : V} [\text{phase}_{\text{trans}}]$$

Step 2

- 1 combine the result with the first verb step,

$$\frac{d \vdash (\epsilon, \text{read}, u) : (V \odot v)_{<} \quad d, k, V; v \vdash (w \ v, \epsilon, \epsilon) : V}{d, k, d \vdash (w \ v, \text{read}, u) : V} [\text{phase}_1]$$

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Step 3

- 1 combine the result with the first verb step,

$$\frac{\frac{\frac{\vdash (\epsilon, -s, \epsilon) : k \setminus (c \odot t) / < V \quad d \vdash (w \text{ a book}, \text{read}, \epsilon) : V}{k \vdash (\epsilon, z, \epsilon) : k \quad d \vdash (\epsilon, \text{reads}, w \text{ a book}) : k \setminus (c \odot t)}}{k, d \vdash (z, \text{reads}, w \text{ a book}) : (c \odot t)}}{[mg]}$$

- 2 build the subject NP

$$\frac{\frac{\vdash (\epsilon, \text{the}, \epsilon) : (k \otimes d) / n \quad \vdash (\epsilon, \text{child}, \epsilon) : n}{\vdash (\epsilon, \text{the}, \text{child}) : k \otimes d}}{[mg]}$$

- 3 unloaded it:

$$\frac{\frac{\vdash (\epsilon, \text{the}, \text{child}) : k \otimes d \quad k, d \vdash (z, \text{reads}, w \text{ a book}) : (c \odot t)}{\vdash (\text{the child}, \text{reads}, \text{a book}) : (c \odot t)}}{[mv]}$$

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- 3 unloaded it:

$$\frac{\vdash (\epsilon, \text{the}, \text{child}) : k \otimes d \quad k, d \vdash (z, \text{reads}, w \text{ a book}) : (c \odot t)}{\vdash (\text{the child}, \text{reads}, \text{a book}) : (c \odot t)} [mv]$$

Step 4

1 ending the derivation

$$\frac{\vdash (\textit{the child, reads, a book}) : (c \odot t) \quad c; t \vdash (\epsilon, \epsilon, \epsilon) : c}{\vdash (\epsilon, \epsilon, \textit{the child reads a book}) : c} [\textit{phase}]$$

Phase Impenetrability Condition

The encoding of PIC directly uses properties of the MCG.

$$\frac{\Delta_s, \Delta_h, \Delta_c \vdash (s_s, s_h, s_c) : X \odot Y \quad \Gamma_s, X; Y, \Gamma_c \vdash (r_s, r_h, r_c) : Z}{\Gamma_s, \Delta_s, \Delta_h \vdash (r_s \bullet s_s, r_h, s_h \bullet s_c \bullet r_c) : Z} [phase]$$

The derivation strictly controls the structure and check internal relations.

Phase Impenetrability Condition

- Derivations with *phases* immediately block the process unlike traditional MCG or MG which perform the full derivation before concluding that a specific feature stand at the end (and reject the derivation)
- PIC encoding do not need new rules (Unlike the other constraints)

Conclusion

- extension of the fragment of PCL used
- introduction of Phases in MCG just by the use of non-commutativity
- give a stricter drive for the analysis
- encode the Phase Impenetrability Condition (almost for free)

Perspectives

- semantic interpretation of phases
- equivalence between MCG with phase and MCG
- generative power of MCG with Phases